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MODIFICATION OF RADIATION BY RELATIVISTIC PARTICLES IN THIN TARGETS DUE TO TRANSITION RADIATION

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Abstract

Influence of transition radiation by relativistic electrons crossing a thin target on other emission processes involved in electron-target interaction is considered. Strong modification of the characteristics of such emission mechanisms as bremsstrahlung and polarization bremsstrahlung as well as parametric X-ray radiation in crystalline targets is discussed in this work.

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1 Introduction

There are many different mechanisms responsible for an emission by fast charged particles moving in a medium. As a rule, yields of such emission processes are considered separately. Meanwhile simultaneous manifestation of two or more emission processes may change substantially characteristics of the total emission. In well known experiment [1] devoted to the study of Landau-Pomeranchuk-Migdal (LPM) effect in bremsstrahlung by relativistic elections in a thin layer, observed form of the emission spectrum was different from that given by the theory of LPM effect in unbounded medium [2]. Later it was shown that the mentioned difference was caused by the contribution of transition radiation (TR) [3].

Modification of several emission processes due to the influence of TR is analyzed in this work. Such modification can be manifested variously by: independent TR contribution to total emission yield; interference of TR amplitude with that of another emission mechanism; TR contribution to the formation of alternative emission process. TR influence tends often to new phenomena in emission processes. This paper reviews some of these phenomena.

Section 2 is devoted to the study of density effect in a new emission mechanism known as polarization bremsstrahlung (PB) [4,5]. Suppression of this effect due to TR contribution to the formation of PB yield from a thin amorphous target is shown in the paper.

Strong modification of the structure of coherent PB from relativistic electrons crossing a thin crystalline target (the coherent part of PB is widely known as parametric X-ray radiation, or PXR [6,7]), caused by diffracted TR, is considered in Section 3 in the case that observed photons propagate to Bragg scattering direction.

The theory of PXR predicts an existence of two PXR peaks propagating along both Bragg scattering direction and the velocity of an emitting particle. Second peak (forward PXR) is presently the subject of intensive experimental studies [8],[9],[10]. It turns out that TR contribution is competitive with PXR one. This circumstance may influence substantially on the interpretation of experimental data as is shown in Section 4.

Section 5 of the paper is devoted to the description of anomalous Ter-Mikaelian effect of dielectric suppression of bremsstrahlung from thin layer of amorphous medium. This effect different from the ordinary Ter-Mikaelian effect is caused by the interference between bremsstrahlung and TR emission amplitudes.

Analogous effect but due to the interference between TR and coherent bremsstrahlung from relativistic electrons crossing an aligned crystal is considered in Section 6. Peculiarities in the emission spectrum caused by the coherent azimuthal scattering of emitting electrons by atomic strings are studied in this Section.

Studies of this work is accomplished in an analysis of the LPM effect in the coherent bremsstrahlung from relativistic electrons. It is shown in Section 7 of this paper that the Migdal's function describes correctly the spectrum of coherent bremsstrahlung in high enough frequency range and for high enough emitting electron energy only.

Conclusions are collected in Section8.

2 Suppression of density effect for polarization bremsstrahlung in a thin target

Fast charged particle transversing medium produces polarization bremsstrahlung due to the excitation of medium's electrons by electromagnetic field of a projectile [4,5]. In the case of condensed medium, the projectile field is screened due to polarization of the medium. This effect (the density effect) causes moderation of relativistic particle ionization energy losses (Fermi effect [11]). An analogous effect has been predicted for PB of relativistic particles moving through an unbounded medium [12].

It is well known that the Fermi effect is suppressed in the case of a relativistic particle crossing a sufficiently thin target [13]. The physical reason for such suppression is very simple. The particle's field in vacuum in front of the target is transformed to the screened field inside the target over the emission formation length $l_{coh} \approx \gamma^2/\omega$ [14], γ is the Lorentz factor of the projectile. Therefore, the structure of the particle field does not change substantially in the frequency range where the inequality $l_{coh} \gg L$ is valid (L is the thickness of the target).

One can expect suppression of the density effect to occur as well in PB from relativistic particle crossing a thin layer of a medium. To elucidate this question let us consider the structure of electromagnetic field excited by a relativistic particle moving with a constant velocity V along the axes $\mathbf{e_x}$ which is normal to a plate of amorphous medium with thickness L. To find the Fourier transform of the electric field $\mathbf{E}_{\omega\mathbf{k}} = (\mathbf{2}\pi)^{-\mathbf{4}} \int \mathbf{dtd^3re^{i\omega t - ikr}E(r,t)}$ by means of Maxwell equations, it is necessary to determine the induced current density for medium electrons. We use the following simple expression in this paper

$$\mathbf{J} = -\frac{\mathbf{e^2}}{\mathbf{m}}\mathbf{A}(\mathbf{r},\mathbf{t})\hat{\mathbf{n}}, \quad \hat{\mathbf{n}} = \sum_{\alpha} \sum_{\beta=\mathbf{1}}^{\mathbf{Z}} \delta(\mathbf{r} - \mathbf{r}_{\alpha} - \mathbf{r}_{\alpha\beta}), \tag{1}$$

which is generally accepted within the framework of X-ray scattering theory [15]. Formula (1) is valid in the frequency range $I << \omega << m$ (I is the mean ionization potential of an atom, m is the electron mass). These relations allow us to consider atomic electrons as free during the emission process and to neglect the Compton shift of the frequency of emitted photon. It is very important that electron coordinates $\mathbf{r}_{\alpha\beta}$ are approximately constant during the process of relativistic particle collision with an atom. In formula (1) $\mathbf{A}$ is the vector-potential of electromagnetic field, α is the index for individual atoms, while β is the index for electrons in a given atom of atomic number Z.

Substituting (1) into the Maxwell equations permits us to obtain the following equation for the transverse component of the electrical field $\mathbf{E}^{\mathbf{tr}}_{\omega\mathbf{k}} = \sum_{\lambda=1}^{\mathbf{2}} \mathbf{e}_{\lambda\mathbf{k}}\mathbf{E}_{\lambda\mathbf{k}}$:

$$(k^2-\omega^2)\mathbf{E}_{\lambda\mathbf{k}} + \int \mathbf{d^3k'} \sum_{\lambda'=1}^{\mathbf{2}} \mathbf{e}_{\lambda'\mathbf{k'}}\mathbf{e}_{\lambda\mathbf{k}}\mathbf{G}(\mathbf{k'}-\mathbf{k})\mathbf{E}_{\lambda'\mathbf{k'}} = \frac{\mathbf{i}\omega\mathbf{e}}{\mathbf{2}\pi^{\mathbf{2}}}\mathbf{e_x}\mathbf{e}_{\lambda\mathbf{k}}\delta\left(\mathbf{k_x}-\frac{\omega}{\mathbf{V}}\right),$$

$$G = \frac{e^2}{2\pi^2 m}\sum_{\alpha}\sum_{\beta=1}^{Z}\exp\left[i(\mathbf{k}'-\mathbf{k})(\mathbf{r}_\alpha+\mathbf{r}_{\alpha\beta})\right], \tag{2}$$

where $\mathbf{e}_{\lambda\mathbf{k}}$ are the polarization vectors, $\mathbf{k}\mathbf{e}_{\lambda\mathbf{k}} = \mathbf{0}$.

The function $G(\mathbf{k}'-\mathbf{k})$ describes both reflecting and scattering properties of the target. Considering PB as being due to the scattering of total electromagnetic field associated with penetration of a fast particle consisting of the primary projectile field and the generated transition radiation field on the fluctuations of the target electron density, one may separate the average and random quantities in Eq.(2),

$$E_{\lambda\mathbf{k}} \equiv \bar{E}_{\lambda\mathbf{k}} + \tilde{E}_{\lambda\mathbf{k}},\ \ G(\mathbf{k}'-\mathbf{k}) \equiv \bar{\mathbf{G}}(\mathbf{k}'-\mathbf{k}) + \tilde{\mathbf{G}}(\mathbf{k}'-\mathbf{k}),$$

$$\bar{G} = \langle G\rangle = \frac{4e^2 n_0}{m}F(\mathbf{k}'-\mathbf{k})\delta(\mathbf{k}'_{||}-\mathbf{k}_{||})\frac{\sin(\mathbf{k'_x}-\mathbf{k_x})\mathbf{L}/\mathbf{2}}{\mathbf{k'_x}-\mathbf{k_x}}. \tag{3}$$

Here the brackets $\langle\rangle$ mean averaging over the coordinates $\mathbf{r}_\alpha$ and $\mathbf{r}_{\alpha\beta}$, n_0 is the atomic density of the medium, $F(\mathbf{k}'-\mathbf{k})$ is the formfactor of an atom, $\mathbf{k}_{||}$ is the $\mathbf{k}$ component parallel to the target surface.

To find PB field $\tilde{E}_{\lambda\mathbf{k}}$ in a vacuum behind the target one should solve the system (2), (3) and corresponding equations outside the target. This task has been solved earlier [16], where the general expression for PB spectral-angular distributions has been obtained in the form

$$\omega\frac{dN}{d\omega d\Omega} = \omega\frac{dN^{PB}}{d\omega d\Omega} + \omega\frac{dN^{STR}}{d\omega d\Omega} + \omega\frac{dN^{INT}}{d\omega d\Omega}, \tag{4}$$

where the first item describes the ordinary PB caused by the scattering of the fast particle Coulomb field, the second item corresponds to scattered transition radiation field and last one is interference term. Very simple formula may be obtained for $\omega\frac{dN^{PB}}{d\omega d\Omega}$ in the range of observation angles Θ

$$\gamma^{-2} + \frac{\omega_0^2}{\omega^2},\ \Theta^2 \ll \frac{1}{\gamma^2\omega_0^2 R^2} \ll 1, \tag{5}$$

where ω_0 is the plasma frequency, R is the screening radius in the statistical model of the atom. On condition (5) the contribution of transition radiation and bremsstrahlung is

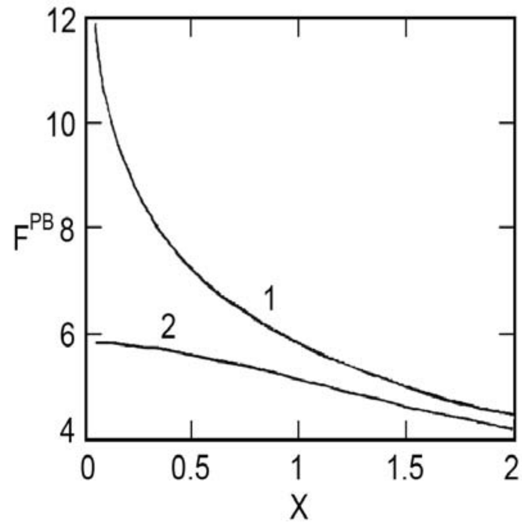

Figure 1: The influence of density effect on ordinary polarization bremsstrahlung in an unbounded medium ($x = \omega/\gamma\omega_0$). $\omega dN^{PB}/d\omega d^2\Theta = N_0 F^{PB}$, $N_0 = Z^2 e^6 n_0 L/\pi m^2$, $\omega_0 R = 5\omega^{-3}$. **1**: without taking into account the density effect; **2**: taking into account the density effect.

small relative do that of PB and coherent part of PB from a single atom, proportional to Z^2 dominates. The formula

$$\omega \frac{dN^{PB}}{d\omega d^2\Theta} = \frac{Z^2 e^6 n_0 L}{\pi m^2} \left[\ln \left(\frac{1}{\omega_0^2 R^2 (1 + \omega^2/\gamma^2 \omega_0^2)} \right) - 2 \right], \tag{6}$$

takes into account the density effect, which is illustrated by the curves presented in Fig.1.

Formula for $\omega \frac{dN^{STR}}{d\omega d\Omega}$, obtained in the same conditions has the form

$$\omega \frac{dN^{STR}}{d\omega d^2\Theta} = \frac{Z^2 e^6 n_0 L}{\pi m^2} \left[\left(1 + 2\frac{\omega^2}{\gamma^2 \omega_0^2} \right) \ln \left(1 + \frac{\gamma^2 \omega_0^2}{\omega^2} \right) - 2 \right]. \tag{7}$$

In contrast to the broad spectrum (6) due to scattering of the spectrally wide projectile Coulomb field, the spectrum (7) is concentrated within the narrow frequency range $\omega \leq \gamma\omega_0$ corresponding to the TR spectrum.

It should be noted that in the frequency range $\omega \ll \gamma\omega_0$, where the ordinary PB saturates due to the density effect, the sum of (6) and (7) is given by the expression

$$\omega \frac{dN}{d\omega d^2\Theta} \simeq \frac{Z^2 e^6 n_0 L}{\pi m^2} \left(\frac{\gamma^2}{\omega^2 R^2} \right), \tag{8}$$

which coincides with the distribution (6) for ordinary PB without taking into account the density effect, that is to say, the density effect is suppressed in PB from relativistic particle crossing a thin layer of a medium. To explain this result it should be noted that in the frequency range $\omega \ll \gamma\omega_0$ the screened field of penetrating particle is suppressed, but the TR field becomes similar to the vacuum field of a particle. Therefore, the main contribution to total emission yield determined by the scattered TR field becomes comparable to that due to the scattering of fast particle Coulomb field in vacuum. Obviously, nature of the effect considered is the same as that in atomic K-shell ionization by relativistic electrons crossing a thin layer of a dense medium [17].

Very important is the presence of interference term in a general formula (4)

$$\omega\frac{dN^{INT}}{d\omega d^2\Theta} = \frac{Z^2e^6n_0L}{\pi m^2}\left[\left(1+2\frac{\omega^2}{\gamma^2\omega_0^2}\right)\frac{\sin(\eta)}{\eta} + \cos(\eta) + \left(\eta - \frac{2}{\sigma}\frac{\omega^3}{\gamma^3\omega_0^3}\right)\mathrm{si}(\eta)\right.$$

$$\left. -\, 2\frac{\omega^2}{\gamma^2\omega_0^2}\mathrm{ci}(\eta) + \frac{2}{\sigma}\frac{\omega^3}{\gamma^3\omega_0^3}\left(\cos\left(\frac{\omega_0^2L}{2\omega}\right)\mathrm{si}\left(\frac{\omega L}{2\gamma^2}\right) + \sin\left(\frac{\omega_0^2L}{2\omega}\right)\mathrm{ci}\left(\frac{\omega L}{2\gamma^2}\right)\right)\right], \quad (9)$$

where $\eta = \sigma\left(\frac{\omega}{\gamma\omega_0} + \frac{\gamma\omega_0}{\omega}\right)$, $\sigma = \omega_0L/2\gamma$. This term is responsible for the dependence of discussed effect on the target thickness. Such dependence has been illustrated by the curves presented in Fig.2, where the thickness dependence of the PB spectrum is shown. In accordance with Fig.2 suppression of the density effect in PB process takes place for rather thin targets only, when the thickness L is less than the maximum value of the emission formation length $(l_{coh})_{max} = 2\gamma/\omega_0$.

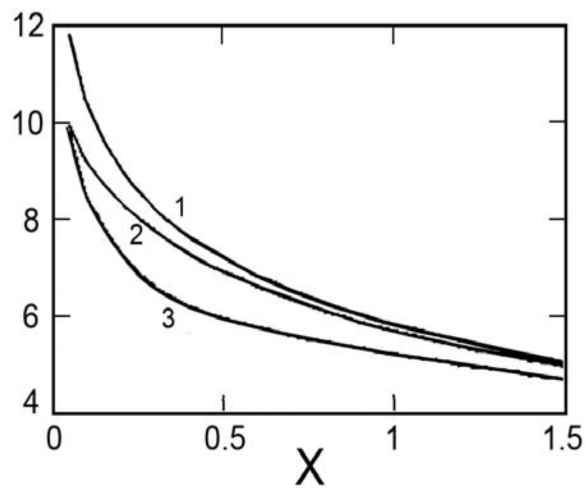

Figure 2: The spectrum of total polarization bremsstrahlung versus the target thickness L. **1**: F^{PB} without the density effect; **2**: $F^{PB} + F^{STR} + F^{INT}$ for $y = \omega_0L/2\gamma = 0.1$; **3**: $F^{PB} + F^{STR} + F^{INT}$ for $y = 1$.

3 Modification of parametric X-ray angular distribution due to the contribution of diffracted transition radiation

As above mentioned, PB from crystalline medium has a coherent part known as parametric X-rays or PXR [6,7]. In the case of relativistic emitting particles, PXR is observed as an intense X-ray peak with very small spectral and angular width. It is important to keep in mind that a zeroth-order electromagnetic field associated with relativistic particle, penetrating a target with a finite thickness, consists not only of the Coulomb field, but of the TR field as well. Obviously, the contribution of TR diffracted by the same system of crystalline atomic planes that are responsible for the PXR generation, may be substantial. Strong modification of the PXR angular distribution caused by the contribution of diffracted transition radiation (DTR) has been predicted in [18] and confirmed in further theoretical [19,20] and experimental [21] works.

Traditionally, the total emission amplitude describing the coherent emission from relativistic particle moving in a crystal is not separated into PXR and DTR parts. Meanwhile, such separation allows one to elucidate the relative contributions of above emission mechanisms to total emission yield [19,20]. This circumstance is of particular importance for the case when the DTR contribution dominates [21].

To demonstrate strong modification of the PXR angular distribution at the condition of substantial DTR contribution let us consider the emission from relativistic electrons crossing a boundary of a semi-infinite absorbing crystal with reflecting crystallographic plane parallel to the surface of a crystal (see Fig.2).

The equations for electromagnetic field excited in a crystal $\mathbf{E}^{\mathbf{tr}}_{\omega\mathbf{k}}$ may be obtained from (2) with account of regularity of the crystalline lattice. Within the frame of typical for PXR description two-wave approximation of the dynamical diffraction theory these equations have the form

$$(k^2-\omega^2(1+\chi_0))E_{\lambda\mathbf{k}}-\omega^2\chi_{-\mathbf{g}}\alpha_\lambda E_{\lambda\mathbf{k}+\mathbf{g}}=\frac{i\omega e}{2\pi^2}\mathbf{e}_{\lambda\mathbf{k}}\mathbf{V}\delta(\omega-\mathbf{kV})$$

$$((\mathbf{k}+\mathbf{g})^{\mathbf{2}}-\omega^{\mathbf{2}}(\mathbf{1}+\chi_{\mathbf{0}}))\mathbf{E}_{\lambda\mathbf{k}+\mathbf{g}}-\omega^{\mathbf{2}}\chi_{\mathbf{g}}\alpha_\lambda\mathbf{E}_{\lambda\mathbf{k}}=\mathbf{0}, \tag{10}$$

where $\mathbf{g}$ is the reciprocal lattice vector, $\mathbf{e}_{\mathbf{1k}}\sim[\mathbf{k},\mathbf{g}]$, $\mathbf{e}_{\mathbf{2k}}\sim[\mathbf{k},\mathbf{e}_{\mathbf{1k}}]$, dielectric susceptibilities χ_0 and $\chi_{\mathbf{g}}$ are determined as

$$\chi_0=-\frac{\omega_0^2}{\omega^2}+i\chi_0'',\ \ \chi_{\mathbf{g}}\equiv-\frac{\omega_0^2}{\omega^2}+i\chi_g'', \tag{11}$$

where $\omega_g^2=\omega_0^2(F(g)/Z(S(\mathbf{g})/\mathbf{N_0}))\mathbf{e}^{-\frac{1}{2}\mathbf{g^2u_T^2}}$, $F(g)$ is the atom form-factor, Z is the number of electrons in an atom, $S(\mathbf{g})$ is the structure factor of elementary cell containing

N_0 atoms, u_T is the mean square amplitude of thermal vibrations of atoms, $\alpha_1 = 1$, $\alpha_2 = (\mathbf{k}, \mathbf{k}+\mathbf{g})/\mathbf{k}|\mathbf{k}+\mathbf{g}|$.

The task considered is well known [20,21], and we present here the final results only. The spectral-angular distribution of total emission has the form

$$\omega \frac{dN_\lambda}{d\omega d^2\theta} = \left\langle \left| A_\lambda^{PXR} + A_\lambda^{DTR} \right|^2 \right\rangle ,$$

$$A_\lambda^{PXR} = \frac{e}{\pi}\delta_\lambda \frac{\Omega_\lambda}{\gamma^{-2}+\gamma_*^{-2}+\Omega^2} \frac{1}{\xi + sign(\xi - \delta_\lambda \kappa_\lambda) f'_\lambda - \sigma - i f''_\lambda},$$

$$A_\lambda^{DTR} = \frac{e}{\pi}\delta_\lambda \Omega_\lambda \left(\frac{1}{\gamma^{-2}+\Omega^2} - \frac{1}{\gamma^{-2}+\gamma_*^{-2}+\Omega^2} \right) \frac{1}{\xi + sign(\xi - \delta_\lambda \kappa_\lambda) f'_\lambda - i(\eta + f''_\lambda)}, \tag{12}$$

where $\delta_\lambda = \omega_g^2 \alpha_\lambda / \omega_0^2 \sim 1,\ \Omega_1 = \Theta_\perp - \Psi_\perp,\ \Omega_2 = 2\Theta' + \Theta_\| + \Psi_\|,\ \Omega^2 = \Omega_1^2 + \Omega_2^2$, the angle $\mathbf{\Theta} = \mathbf{\Theta}_\perp + \mathbf{\Theta}_\|$ describes the emission angular distribution, the angle $\mathbf{\Psi} = \mathbf{\Psi}_\perp + \mathbf{\Psi}_\|$ describes the angular speed in emitting electron beam, θ' is the orientation angle describing a possible turning of the crystal by goniometer (see Fig.3), $\gamma_* = \omega/\omega_0 \approx \omega_B/\omega_0$, $\omega_B = g/2\sin(\varphi/2)$ is the Bragg frequency in the vicinity of which the emission spectrum is concentrated, $\eta = \omega^2 \chi''_0/\omega_0^2,\ \kappa_\lambda = \chi''_g \alpha_\lambda/\chi''_0,\ \sigma = \gamma_*^2(\gamma^{-2} + \gamma_*^{-2} + \Omega^2)$, the brackets $\langle\rangle$ mean averaging over the angles $\Psi_\perp$ and $\Psi_\|$, the very important quantities $\xi,\ f'_\lambda$ and f''_λ are defined by the formulae

$$\xi = \frac{g^2}{2\omega_0^2}\left(1 - \frac{\omega}{\omega'_B}\right),\ \omega'_B = \omega_B \left[1 + (\theta' + \theta_\|)\cot\frac{\varphi}{2}\right],$$

$$f'_\lambda{}^{,\prime\prime} = \frac{1}{\sqrt{2}}\sqrt{\sqrt{(\xi^2 - \delta_\lambda^2)^2 + 4\eta^2(\xi - \delta_\lambda\kappa_\lambda)^2} \pm (\xi^2 - \delta_\lambda^2)}. \tag{13}$$

In accordance with the expression for PXR amplitude this emission appears due to the scattering of screened Coulomb field of emitting particle. As a consequence, the maximum in PXR angular distribution is located at the point $\Theta_{max}^{PXR} \approx \sqrt{\gamma^{-2} + \gamma_*^{-2}}$ (without account of multiple scattering), corresponding to the maximum in the angular distribution of screened transverse component of fast electron's Coulomb field. On the other hand, DTR angular distribution coincides practically with that of TR, therefore the maximum in DTR angular distribution is located at the point $\Theta_{max}^{DTR} \approx \gamma^{-1} < \Theta_{max}^{PXR}$. Thus, DTR contribution leads to effective narrowing of the angular distribution of total emission.

This effect depends strongly on two parameters: γ/γ_* and η/δ_λ. Obviously, DTR contribution is negligible as well as TR one if $\gamma < \gamma_*$ (see the expression for A_λ^{DTR} in (12)). Within the range $\gamma \gg \gamma_*$ relative contribution of PXR and DTR depends strongly on the parameter η/δ_λ which is approximately equal to the ratio l_{ext}/l_{ab} ($l_{ext} \sim \omega/\omega_g^2$ is

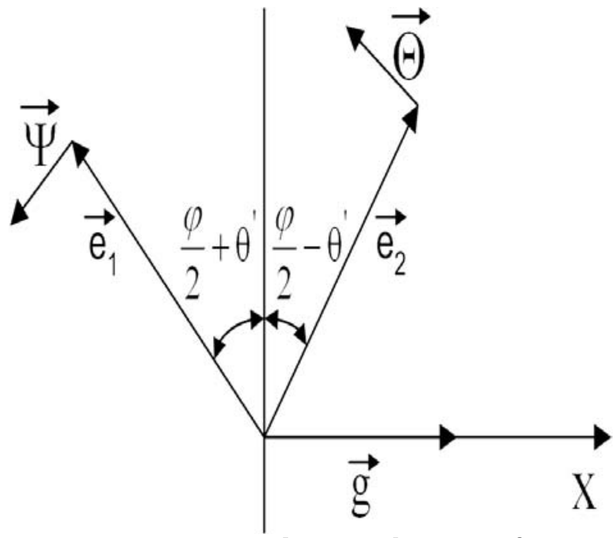

Figure 3: PXR for Bragg geometry. $\mathbf{e_1}$ and $\mathbf{e_2}$ are the axes of emitting electron beam and X-ray detector respectively, Θ' is the orientation angle, describing the possible turning of the crystal by goniometer.

the extinction length, on which DTR is formed, PXR is formed at the distance of the order of absorption length $l_{ab} \sim 1/\omega\chi''_?$).

Let us consider the orientation dependence of a strongly collimated photon flux (collimation angle $\Delta\Theta < \gamma^{-1}$) emitted from an electron beam with a small divergence ($\Delta\Psi < \gamma^{-1}$). Assuming X-ray detector to be placed in the plane $\Theta_\perp = 0$ (π-polarization contributes mainly at specified conditions) and neglecting the Bormann effect ($\kappa_\lambda = 0$) from (12) after integration over photon energies ω one can obtain the following expression

$$\frac{dN}{d^2\Theta} = \frac{2e^2\omega_g^2\gamma^2\cos(\varphi)}{\pi^2 g^2}\Phi(2\gamma\theta', \gamma/\gamma_*, \eta/\delta_2), \Phi = \Phi^{PXR} + \Phi^{DTR} + \Phi^{INT}. \qquad (14)$$

The functions (14) calculated for great values of the parameter γ/γ_*, when maxima in PXR and DTR orientational dependencies differ substantially, are presented in Figs.4 and 5. In accordance with presented figures the form of rocking curves can be changed dramatically at various photoabsorption coefficient values because of corresponding change of relative PXR and DTR contributions. It should be underlined that PXR angular distributions with one and two maxima have been observed experimentally [22].

4 Dynamical diffraction effect in TR from a thin crystal

PXR along the Bragg scattering direction has been considered in section3. The theory predicts an existence of additional PXR flux propagating along the velocity of emitting

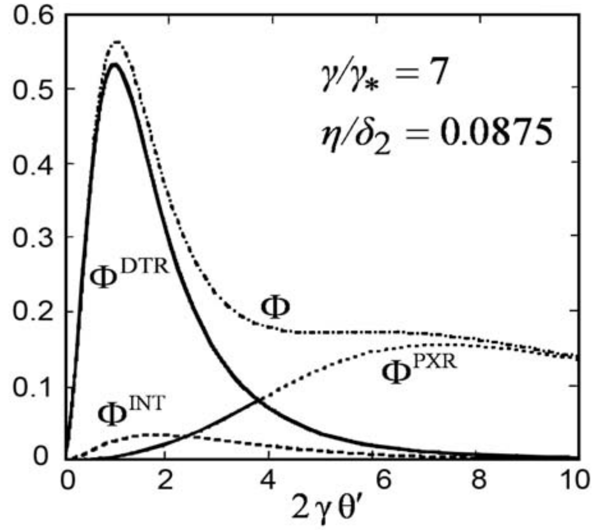

Figure 4: Dependence of the total emission on orientation. The functions Φ, Φ^{DTR}, Φ^{PXR} and Φ^{INT} are determined by Eq.14

electron [23]. This PXR flux known as the forward PXR was intensively studied theoretically and experimentally during more than 30 years, but the problem of the forward PXR observation has been solved recently [9,10].

The experimental observation of the forward PXR is complicated mainly due to a small width of PXR spectrum compared to a typical energy resolution of X-ray detectors ($\Delta\omega \geq 150eV$) that results in efficient averaging of the forward PXR against the background of broadband bremsstrahlung and TR. TR contribution can be suppressed by using a negative interference between TR waves emitted from in and out surfaces of the target. But it is in this case TR can make the contribution as a very intense and narrow peak located at the same with the forward PXR angular and spectral region [24]. To show this let us consider an emission from relativistic electrons crossing a single crystal with a reflecting crystallographic plane arranged perpendicular to in and out surfaces of the target, so that the normal to the crystal surface lies in this plane. Assuming the velocity of emitting particle to be oriented at angle $\varphi/2$ relative to the reflecting plane and using equations analogous to (10) one can obtain the solution of discussed task in terms close to that obtained in previous section of this paper. Particularly, the general formula describing the total emission spectral-angular distribution has the form

$$\omega\frac{dN_\lambda}{d\omega d^2\theta} = \left\langle \left| A_\lambda^{PXR} + A_\lambda^{TR} \right|^2 \right\rangle, \tag{15}$$

close to (12). TR emission amplitude A_λ^{TR} we are interesting in is determined by the

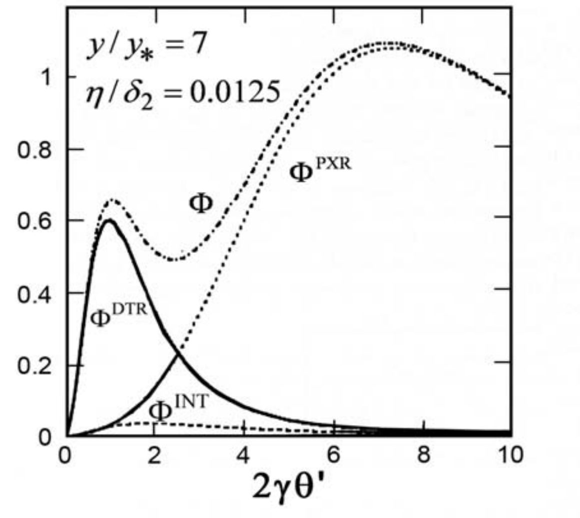

Figure 5: The same but for different values of the parameter $\eta/\delta_\perp$.

formula

$$A_\lambda^{TR} = \frac{e}{2\pi}\Theta_\lambda\left(\frac{1}{\gamma^{-2}+\Theta^2} - \frac{1}{\gamma^{-2}+\Theta^2+\omega_0^2/\omega^2}\right)\left[\left(1+\frac{\xi}{\sqrt{\xi^2+\delta_\lambda^2}}\right)\left(1-e^{-i\sigma_-}\right)\right.$$

$$\left. + \left(1-\frac{\xi}{\sqrt{\xi^2+\delta_\lambda^2}}\right)\left(1-e^{-i\sigma_+}\right)\right],$$

$$\sigma_\pm = \frac{\omega L}{2\cos(\varphi/2)}\left[\gamma^{-2}+\Theta^2+\frac{\omega_0^2}{\omega^2}\left(1+\xi\pm\sqrt{\xi^2+\delta_\lambda^2}\right)\right], \tag{16}$$

where $\Theta_1 = \Theta_\perp$, $\Theta_2 = \Theta_\parallel$, $\Theta^2 = \Theta_\perp^2 + \Theta_\parallel^2$, the quantity ξ coincides with that in Eq.(13), but it is well to bear in mind that $\omega_B' = \omega_B(1-\Theta_\parallel cot(\varphi/2))$. An influence of multiple scattering and a photoabsorption has been neglected in (16) for simplicity.

It should be noted that there are two different dependencies of TR amplitude on the emitted photon energy ω and the observation angle $\mathbf{\Theta}$. The "fast dependence" is concentrated in the function $\xi(\omega, \Theta_\parallel)$ and has the characteristic scales $\Delta\Theta \sim \Delta\omega/\omega \sim \omega_0^2/g^2 \ll 1$. This dependence manifests in the narrow vicinity of the Bragg frequency ω_B, or $|\xi| \leq \delta_\lambda \approx 1$ due to dynamical scattering of TR wave emitted from in surface of the target by the same crystallographic plane which is responsible for PXR generation. Obviously, the "slow dependence" is realized in the range $|\xi| \gg \delta_\lambda$, or far from the Bragg frequency where TR amplitude (16) takes the simple form

$$A_\lambda^{TR} \to \frac{e}{\pi}\Theta_\lambda\left(\frac{1}{\gamma^{-2}+\Theta^2} - \frac{1}{\gamma^{-2}+\Theta^2+\omega_0^2/\omega^2}\right)$$

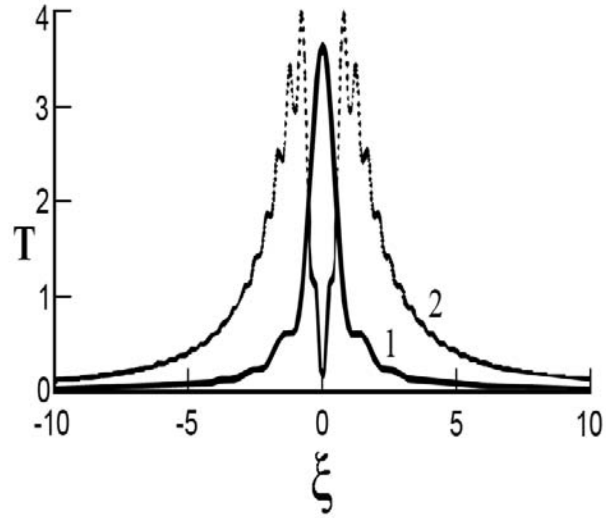

Figure 6: TR peak caused by dynamical diffraction effects. The presented curves have been calculated for fixed values of the parameters $\sigma = 2\pi$ and $\delta_\lambda = 0.9$. **1**: $p = 3$; **2**: $p = 8$.

$$\times \left[1 - \exp\left(- \frac{i\omega L}{2\cos(\varphi/2)} (\gamma^{-2} + \Theta^2 + \omega_0^2/\omega^2) \right) \right], \tag{17}$$

coinciding with the well-known emission amplitude for the ordinary TR emitted by a relativistic particle crossing a layer of amorphous medium [25]. Dynamical diffraction effects manifest in the close vicinity to the Bragg frequency only, where the TR contribution can be presented by the formula

$$\omega \frac{dN^{TR}}{d\omega d^2\Theta} = \frac{e^2}{\pi^2} \Theta_\lambda^2 \left(\frac{1}{\gamma^{-2} + \Theta^2} - \frac{1}{\gamma^{-2} + \Theta^2 + \omega_0^2/\omega^2} \right)^2$$

$$\times \left[\left(\cos(\sigma + p\xi) - \cos(p\sqrt{\xi^2 + \delta_\lambda^2}) \right)^2 + \left(\sin(\sigma + p\xi) - \frac{\xi}{\sqrt{\xi^2 + \delta_\lambda^2}} \sin(p\sqrt{\xi^2 + \delta_\lambda^2}) \right)^2 \right], \tag{18}$$

following from (15) and (16). Here $\sigma = \frac{\omega L}{2\cos(\varphi/2)} (\gamma^{-2} + \Theta^2 + \omega_0^2/\omega^2)$, $p = \frac{\omega_0^2 L}{2\omega \cos(\varphi/2)}$.

Obviously, in the range $|\xi(\omega, \Theta_{||})| \leq 1$ the energy of emitted photon ω and the observation angle $\mathbf{\Theta}$ may be assumed to be constant in the formula (18) with the exception of the "fast variable" $\xi(\omega, \Theta_{||})$.

Let us consider the expression in square brackets (18) $T(\xi, \sigma, p, \delta_\lambda)$ as a function of ξ. It is easy to see in the limit $|\xi| \gg \delta_\lambda \approx 1$ (far from the vicinity of Bragg frequency) the function T is described by the formula

$$T \to 2(1 - cos(\sigma)). \tag{19}$$

In accordance with (19) the function $T(\xi)$ appears as an isolated peak, if $\sigma = 2\pi k$. But this condition corresponds to negative interference between TR waves emitted from in and out surfaces of the target (see formula (17)). The form of this peak depends strongly on the value of coefficient p as may be seen from Fig.6. Its natural spectral width $\Delta\omega \approx \omega(2\omega_0^2/g^2) \ll \omega$ corresponds to the scale of Darwin table [15].

Since the peak considered appears in the frequency range where the forward PXR peak manifests, its existence must be taken into account in experimental studies of the forward PXR.

5 Anomalous Ter-Mikaelian effect in bremsstrahlung from relativistic electrons

Above considered emission processes are based on emission mechanisms corresponding to the scattering of a fast particle equilibrium electromagnetic field (the particle velocity change is not of principle). The modification of bremsstrahlung properties due to the contribution of TR is considered in this section.

Let us consider emission from relativistic electrons crossing a thin amorphous target near parallel to the normal $\mathbf{e}$ to its surface. Defining the unit vector $\mathbf{n}$ to the direction of emitted photon observation and the time-dependent velocity of emitting electron $\mathbf{V_t}$ by angular variables $\mathbf{\Theta}$ and $\mathbf{\Psi}$

$$\mathbf{n} = \mathbf{e}\left(\mathbf{1} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{\Theta^2}\right) + \mathbf{\Theta},\ (\mathbf{e}, \mathbf{\Theta}) = \mathbf{0}, \mathbf{V_t} = \mathbf{e}\left(\mathbf{1} - \frac{\mathbf{1}}{\mathbf{2}}\gamma^{-\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{\Psi_t^2}\right) + \mathbf{\Psi},\ (\mathbf{e}, \mathbf{\Psi}) = \mathbf{0}, \tag{20}$$

one can obtain within the frame of well-known approach [25] the following expression [20] for the total emission amplitude

$$\mathbf{A^{tot}} = \mathbf{A^{BS}} + \mathbf{A^{TR}},$$

$$\mathbf{A^{BS}} = \frac{\mathbf{e}}{\pi}\int\limits_{\mathbf{0}}^{\mathbf{L}} \mathbf{dt} \exp\left(\frac{\mathbf{i}\omega}{\mathbf{2}}\int\limits_{\mathbf{0}}^{\mathbf{t}} \mathbf{d}\tau(\gamma_{\mathbf{s}}^{-\mathbf{2}} + \mathbf{u}_{\tau}^{\mathbf{2}})\right)\frac{\mathbf{d}}{\mathbf{dt}}\frac{\mathbf{u_t}}{\gamma_{\mathbf{s}}^{-\mathbf{2}} + \mathbf{u_t^2}},$$

$$\mathbf{A^{TR}} = -\frac{\mathbf{e}}{\pi}\left[\mathbf{u_i}\left(\frac{\mathbf{1}}{\gamma^{-\mathbf{2}} + \mathbf{u_i^2}} - \frac{\mathbf{1}}{\gamma_{\mathbf{s}}^{-\mathbf{2}} + \mathbf{u_i^2}}\right) - \mathbf{u_f}\left(\frac{\mathbf{1}}{\gamma^{-\mathbf{2}} + \mathbf{u_f^2}} - \frac{\mathbf{1}}{\gamma_{\mathbf{s}}^{-\mathbf{2}} + \mathbf{u_f^2}}\right)\right.$$
$$\left.\times \exp\left(\frac{i\omega}{2}\int\limits_0^L dt(\gamma_s^{-2} + u_t^2)\right)\right], \tag{21}$$

where $\gamma_s^{-2} = \gamma^{-2} + \omega_0^2/\omega^2$, $\mathbf{u_t} = \mathbf{\Psi_t} - \mathbf{\Theta}$, $\mathbf{u_i}$ and $\mathbf{u_f}$ are initial and final values of $\mathbf{u_t}$ respectively. $\mathbf{A^{BS}}$ in (21) is the bremsstrahlung amplitude. Obviously, $\mathbf{A^{BS}} = \mathbf{0}$ for the particle moving with a constant velocity. The quantity $\mathbf{A^{TR}}$ describes TR emission amplitude. It is easy to see that $\mathbf{A^{TR}} \to \mathbf{0}$ in the limit $\omega_0 \to 0$.

Formulae (21) allow to elucidate the problem of Ter-Mikaelian effect manifestation in bremsstrahlung from relativistic electrons crossing a thin layer of medium. Ter-Mikaelian effect in the bremsstrahlung generated in an unbounded medium appears as suppression of the bremsstrahlung yield in the frequency range $\omega < \gamma\omega_0$ due to the change of emitted photon phase velocity because of the polarization of medium electrons [14]. On the other hand, bremsstrahlung emitted from very thin layer of a medium is not suppressed by Ter-Mikaelian effect in accordance with the analysis performed in [25]. It is clear that there is bound to be the intermediate region of thickness where one should expect an existence of peculiarities in the manifestation of Ter-Mikaelian effect.

Using the general expressions (21) let us consider the spectral-angular distribution of total emission in the frequency range $\omega \sim \gamma\omega_0$, where Ter-Mikaelian effect manifests. Since TR can change very substantially characteristics of the total emission, the contribution of TR must be suppressed in the experiment devoted to observation of Ter-Mikaelian effect in bremsstrahlung. This problem can be solved by the collimation of emitted photon flux, because TR angular distribution has a dip gap in the range of observation angles $\Theta < \gamma^{-1}$ (it is clear that this property can be used under condition of weak multiple scattering only, when $\gamma^2\Psi_L^2 = L/L_{Sc} \ll 1$, $L_{Sc} = (e^2/4\pi)L_R$, L_R is the radiation length). Assuming the thickness of the target L to be small enough, so that the shift of the photon phase $(\omega/2)\int\limits_0^L dt\Psi_t^2$ caused by multiple scattering is much less than unity ($(\omega/2)\langle\int\limits_0^L dt\Psi_t^2\rangle = \frac{1}{2}(L/L_{Sc})(L/l_{coh})$, $l_{coh} = 2\gamma^2/\omega$), one can obtain from (21) the following expression for the spectrum of strongly collimated radiation:

$$\omega\frac{dN}{d\omega d\Omega} = \frac{e^2\gamma^2}{\pi^2}\left\{2\gamma^2\Psi_0^2\frac{1-\cos(y(x+x^{-1}))}{(1+x^2)^2}\right.$$
$$\left.+\frac{L}{L_{Sc}}\left[1-\frac{2x^2}{(1+x^2)^2}\left(1-\frac{\sin(y(x+x^{-1}))}{y(x+x^{-1})}\right)\right]\right\}, \qquad (22)$$

where Ψ_0 is the initial angular spread of emitting electron beam, $y = \omega_0 L/2\gamma$ is the ratio of the thickness of the target to the maximum possible value of the emission formation length, $x = \omega/\gamma\omega_0$.

The first term in (22) describes TR contribution appearing due to electron beam divergence, the second one, proportional to L/L_{Sc} corresponds to the contribution of bremsstrahlung, TR (this TR contribution is due to multiple scattering only) and an interference term. It is this contribution describes the modification of Ter-Mikaelian effect being considered. The function in square brackets in (22) is presented in Fig.7. Presented spectral curves (22) calculated for different values of the parameter y differ substantially from that following from both Ter-Mikaelian theory [14] for unbounded medium (curve **1**) and Garibian, Yang theory [25] for thin layer of a medium (curve **2**).

In order to explain the unexpected shape of presented spectral curves it should be noted that in the case of very thin target with the thickness L less than the effective formation length $l_{eff} \approx 2\gamma/\omega_0$ what is the same as $y \ll 1$, the equilibrium electromagnetic field of emitting electron is not changed substantially in the target. As a consequence, the emitted field in must be close to the ordinary bremsstrahlung field generated in a vacuum. Such "vacuum bremsstrahlung" appears in (21) due to the strong interference between bremsstrahlung $\mathbf{A^{BS}} \approx \frac{\mathbf{e}}{\pi}\gamma_{\mathbf{s}}^{\mathbf{2}}\mathbf{\Psi_L}$ and transition radiation $\mathbf{A^{TR}} \approx \frac{\mathbf{e}}{\pi}(\gamma^{\mathbf{2}} - \gamma_{\mathbf{s}}^{\mathbf{2}})\mathbf{\Psi_L}$ taking place with the proviso that $y \ll 1$. Thus, a Garibian-Yang theory is valid, if $y \ll 1$.

In the case $y > 1$ the electron's electromagnetic field is screened in the target and, therefore, the bremsstrahlung contribution is suppressed in the frequency range $\omega \leq \gamma\omega_0$ due to Ter-Mikaelian effect. Since TR contribution decreases quickly in the range $\omega \geq \gamma\omega_0$ and the interference between TR and bremsstrahlung is negligible due to phase oscillations in (21) [20,26], the total emission yield is suppressed in the vicinity of $\omega = \gamma\omega_0$ (see Fig.7).

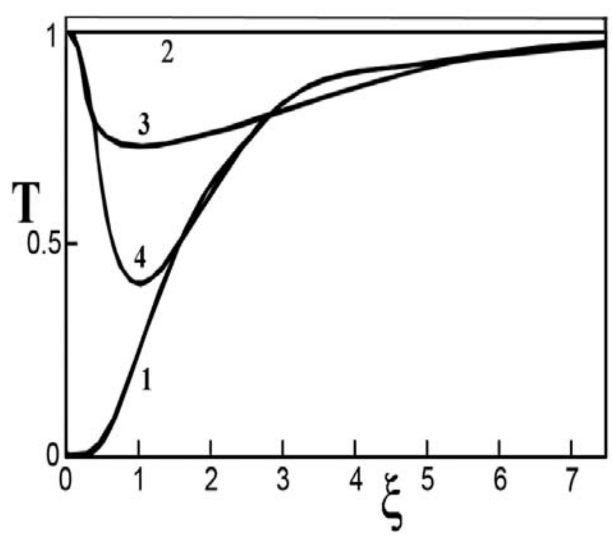

Figure 7: The emission spectrum under condition of anomalous Ter-Mikaelian effect. $\omega dN/d\omega d\Omega = (e^2\gamma^2/\pi^2)(L/L_{Sc})T(x,y)$. The curve **1** describes Ter-Mikaelian effect for unbounded medium ($T_1 = x^4/(1+x^2)^2$). The curve **2** corresponds to Garibian-Yang theory ($T_2 = 1$). **3**: $y = 1$; **4**: $y = 2$.

On the other hand, $\mathbf{A^{BS}} \sim \omega^{\mathbf{2}}/\omega_{\mathbf{0}}^{\mathbf{2}} \ll \gamma^{\mathbf{2}} \sim \mathbf{A^{TR}}$ as is clear from (21) ($\mathbf{A^{TR}} \approx \frac{\mathbf{e}}{\pi}\gamma^{\mathbf{2}}\mathbf{\Psi_L}\exp\left(-\frac{\mathbf{i}\omega_{\mathbf{0}}^{\mathbf{2}}\mathbf{L}}{\mathbf{2}\omega}\right)$ in the frequency range $\omega \ll \gamma\omega_0$) and the spectrum of strongly collimated emission increases in the range $\omega \ll \gamma\omega_0$ due to the scattering of TR photons to the range of small observation angles.

It is significant that the discussed anomalous Ter-Mikaelian effect has been observed experimentally [27].

6 Interference between TR and coherent bremsstrahlung from relativistic electrons in aligned crystals

It should be noted that the target's atomic structure is not fixed in the general formula (21), therefore we can use this formula for estimations of both incoherent bremsstrahlung in amorphous media and coherent bremsstrahlung in crystals.

Starting from (20) and (21), let us consider the total coherent emission from relativistic electrons crossing a thin aligned crystal, assuming the axis of a set of parallel atomic strings to be parallel to the normal $\mathbf{e}$ to the crystal surface, so the scattering angle $\mathbf{\Psi_t}$ is simultaneously the orientational angle of the emitting particle velocity $\mathbf{V(t)}$ to the string axis.

Since the velocity $\mathbf{V(t)}$ might be changed substantially during small time intervals only, corresponding to collisions of this electron with atomic strings, one can represent the total emission amplitude as a sum of elementary amplitudes describing the emission on different strings. Within the small frequency range

$$\frac{\omega R}{2\gamma_s^2 \Psi} \ll 1, \tag{23}$$

when the emission formation length l_{coh} exceeds essentially the effective electron path in a string's potential R/Ψ (R is the screening radius in Fermi-Thomas atom model, R determines the effective transverse size of an atomic string) and, as a consequence, the emission appearing in k-th collision of the emitting electron with a single string, is determined by the scattering angle $\mathbf{\Psi_{k+1}} - \mathbf{\Psi_k}$ only ($\mathbf{\Psi_{k+1}}$ and $\mathbf{\Psi_k}$ are the values of the angle $\mathbf{\Psi_t}$ after and before k-th collision, respectively) the total amplitude (21) reduced to

$$\mathbf{A^{tot}} = \frac{\mathbf{e}}{\pi}\Bigg[\mathbf{u_i}\left(\frac{\mathbf{1}}{\gamma^{\mathbf{-2}} + \mathbf{u_i^2}} - \frac{\mathbf{1}}{\gamma_{\mathbf{s}}^{\mathbf{-2}} + \mathbf{u_i^2}}\right) - \mathbf{u_f}\left(\frac{\mathbf{1}}{\gamma^{\mathbf{-2}} + \mathbf{u_f^2}} - \frac{\mathbf{1}}{\gamma_{\mathbf{s}}^{\mathbf{-2}} + \mathbf{u_f^2}}\right)$$
$$\times \exp\left(\frac{i\omega}{2}\sum_k (\gamma_s^{-2} + u_k^2)\tau_k\right) - \sum_k \left(\frac{\mathbf{u_k}}{\gamma_s^{-2} + u_k^2} - \frac{\mathbf{u_{k-1}}}{\gamma_s^{-2} + u_{k-1}^2}\right) \times$$
$$\times \exp\left(\frac{i\omega}{2}\sum_{j\le k}(\gamma_s^{-2} + u_j^2)\tau_j\right)\Bigg], \tag{24}$$

where τ_k is the interval between $k-1$-th and k-th collisions.

The averaging of the general formula for the emission spectral-angular distribution $\omega dN/d\omega d^2\Theta = \langle|\mathbf{A^{tot}}|^2\rangle$ must be performed over accidental quantities τ_k and $\mathbf{\Psi_k}$. Taking into account an independence of different quantities τ_k and using the distribution function $f(\tau) = (1/\bar{\tau})\exp(-\tau/\bar{\tau})$ ($\bar{\tau} = \bar{l}_\perp/\Psi$, $\bar{l}_\perp = 1/\sqrt{n_0 d}$, n_0 is the density of atoms, d is the distance between neighboring atoms in the string), one can obtain the formula

$$\langle\exp\left(\frac{i\omega}{2}(\gamma_s^{-2}+u_k^2)\tau_k\right)\rangle = \frac{1}{1-\frac{i\omega}{2}\bar{\tau}(\gamma_s^{-2}+u_k^2)}, \tag{25}$$

which will be used in further calculations.

When averaging over $\mathbf{\Psi_k}$ one should take into account the unique property of electron coherent scattering by the average potential of atomic string consisting in the conservation law: $\Psi_k^2 = \Psi_{k+1}^2 = \Psi^2$. It is well known that only azimuthal angle χ_t of the vector $\mathbf{\Psi_t}$ is changed due to electron collision with atomic string. Therefore,

$$\mathbf{\Psi_k} = \mathbf{\Psi}\left(\mathbf{e_x}\cos\chi_\mathbf{k} + \mathbf{e_y}\sin\chi_\mathbf{k}\right), \quad \chi_\mathbf{k} = \chi_\mathbf{i} + \sum_{\mathbf{j}\le\mathbf{k}} \mathbf{\Delta}\chi_\mathbf{j}, \tag{26}$$

where $\Delta\chi_j$ is the change of azimuthal angle χ_t in j-th i collision. It is clear that all $\Delta\chi_j$ are independent accidental quantities. To average the expression for the emission spectral-angular distribution over $\Delta\chi_j$ one should use the following formula

$$\langle\cos\chi_k\rangle = \langle\cos\Delta\chi\rangle^k, \quad \langle\sin\chi_k\rangle = 0,$$

$$\langle\cos\Delta\chi\rangle = \frac{2}{l_\perp}\int_0^\infty db\cos\left(\Delta\chi(b)\right), \quad \Delta\chi = \pi - 2b\int_{\rho_0}^\infty \frac{d\rho}{\rho^2\sqrt{1-\frac{b^2}{\rho^2}+\frac{\Psi_{ch}^2}{\Psi^2}f(\rho)}}, \tag{27}$$

where $1-\frac{b^2}{\rho_0^2}+\frac{\Psi_c^2}{\Psi^2}f(\rho_0) = 0$, b is the impact parameter of electron collision with a string, the string potential is defined as $\varphi(\rho) = \varphi_0 f(\rho)$, $f(0) = 1$, $\Psi_{ch}^2 = \frac{2e\varphi_0}{m\gamma}$.

It should be noted that the procedure of averaging is very complicated in general case. Such procedure can be performed completely within the frame of dipole approximation of the emission theory (the scattering angle achievable at the distance of the order of l_{coh} must be small relative to characteristic emission angle γ^{-1}). In the case being considered the corresponding condition can be presented as

$$\gamma^2\langle(\mathbf{\Delta\Psi_{coh}})^2\rangle = 2\gamma^2\Psi^2[1-\langle\cos\Delta\chi\rangle^{\frac{l_{coh}}{\bar{\tau}}}]$$

$$\approx 2\gamma^2\Psi^2\left[1-\exp\left(-\frac{1-\langle\cos\Delta\chi\rangle}{\bar{\tau}}l_{coh}\right)\right] \ll 1. \tag{28}$$

Assuming that the condition (28) is fulfilled let us consider an interference between TR and coherent bremsstrahlung. Performing the necessary expansion in (24) one can obtain the following expression for the spectral-angular distribution of coherent bremsstrahlung:

$$\omega\frac{dN^{BS}}{d\omega\tilde{\Theta}d\tilde{\Theta}}=\frac{4e^2\Psi^2}{\pi}\frac{\gamma_s^{-4}+\tilde{\Theta}^4}{(\gamma_S s^{-2}+\tilde{\Theta}^2)^4}(1-\langle\cos\Delta\chi\rangle)\frac{\omega^2}{\omega^2+\omega_s^2(\tilde{\Theta})}\frac{L}{\bar{\tau}}, \tag{29}$$

where $\tilde{\boldsymbol{\Theta}}=\boldsymbol{\Theta}-\boldsymbol{\Psi}_{\mathbf{i}}$, the very important quantity $\omega_S(\tilde{\Theta})$ is defined by the formula

$$\omega_s(\tilde{\Theta})=\frac{2}{\bar{\tau}}\frac{1-\langle\cos\Delta\chi\rangle}{\gamma_s^{-2}+\tilde{\Theta}^2}. \tag{30}$$

Two effects are responsible for the emission yield suppression in small frequency range, as it is evident from (29) and (30). The well known Ter-Mikaelian effect of dielectric suppression is manifested within the frequency range $\omega<\gamma\omega_0$. More unexpected suppression effect appears in the frequency range $\omega<\omega_s(\tilde{\Theta})$ (obviously, the condition $\omega_s(\tilde{\Theta})\gg\gamma\omega_0$ must be fulfilled for real observation of this suppression effect).

The nature of this effect consists in a limitation by the value of emitting electron scattering angle achievable in the process coherent azimuthal scattering by atomic strings [28,29].

Spectral-angular distribution of TR is described by the formula

$$\omega\frac{dN^{TR}}{d\omega\tilde{\Theta}d\tilde{\Theta}}=\frac{4e^2}{\pi}\left(\frac{1}{\gamma^{-2}+\tilde{\Theta}^2}-\frac{1}{\gamma_s^{-2}+\tilde{\Theta}^2}\right)^2$$
$$\times\left[\tilde{\Theta}^2\left(1-\cos\frac{\omega L}{2}(\gamma_s^{-2}+\tilde{\Theta}^2)\right)+\Psi^2(1-\langle\cos\Delta\chi\rangle^{\frac{L}{\bar{\tau}}})\right]. \tag{31}$$

Where the first form in the brackets proportional to $\tilde{\Theta}^2$ corresponds to the ordinary TR from relativistic electron moving with a constant velocity. The second one describes an influence of coherent azimuthal multiple scattering of emitting electron.

To estimate the influence of azimuthal scattering on the total emission properties it is necessary to take into account an interference between TR and coherent bremsstrahlung. The corresponding interference term following from the general formula (24) is very complicated and therefore is not presented here (see [30]). But in conditions analogous to that considered in the previous Section (strong collimation of emitted photon flux and small value of the averaged azimuthal scattering angle, achievable in a single collision with atomic string, or $\langle\cos\Delta\chi\rangle\simeq1-\frac{1}{2}\langle\Delta^2\chi\rangle$, the condition $\Delta\chi\ll1$ is fulfilled if $\Psi^2\gg\Psi_{ch}^2$) the formula describing the total emission yield including coherent bremsstrahlung, TR and interference term has the simple form

$$\omega\frac{dN^{tot}}{d\omega\tilde{\Theta}d\tilde{\Theta}}=\frac{4e^2}{\pi}\gamma^2\Psi^2G(x,y,z),$$

$$G = yz\frac{x^4}{(1+x^2)^2+z^2x^2} + \left(1-e^{-yz}\right)\frac{1}{(1+x^2)^2} +$$
$$\left(1-e^{-yz}\right)\frac{z^2x^2(1+\cos y(x+x^{-1})) - zx(1+x^2)\sin y(x+x^{-1})}{(1+x^2)^2+z^2x^2}, \tag{32}$$

where the parameters x and y were determined in (**??**) and the new parameter z is determined as $z = \gamma\langle\Delta^2\chi\rangle/\omega_p\bar{\tau}$. The physical meaning of this parameter follows from the formula

$$\frac{\omega_s(0)}{\gamma\omega_0} = z\frac{x^2}{1+x^2} \leq z. \tag{33}$$

As outlined above, the field of existence of the effect coherent bremsstrahlung suppression due to azimuthal multiple scattering of emitting electrons is determined by inequalities $\gamma\omega_0 < \omega < \omega_s(\tilde{\Theta}) \leq \omega_*$, so that the discussed effect manifests with the constant $z \gg 1$ only in accordance with (33).

Let us analyze the function $G(x,y,z)$ in (32). As evident from this formula, the contribution of coherent bremsstrahlung (the first term in (32) is suppressed in the range $x < 1$ or $\omega < \gamma\omega_0$ due to the normal Ter-Mikaelian effect. The shape of the total spectrum is in the range $x \leq 1$ depends strongly on the values of parameters y and z. In the range $z \ll 1$ the spectrum is determined in the main by the relationship between TR (second term in (32)) and coherent bremsstrahlung contributions. This relationship depends on the coefficient yz (obviously, the coefficient $2yz = \langle\Delta^2\chi\rangle L/\bar{\tau}$ is equal to mean square of the total azimuthal angle of multiple scattering). In the special case that $yz \ll 1$ (the emission process being considered is close to that in amorphous target) contributions of coherent bremsstrahlung and TR are comparable in the range $x \leq 1$ and the shape of the total spectrum takes the form characteristic for the spectrum under condition of anomalous Ter-Mikaelian effect (see Fig.8).

In the case of thick enough target so that $yz \gg 1$ the contribution of coherent bremsstrahlung proportional to full number of electron's collisions with atomic strings $L/\bar{\tau}$ dominates in the frequency range $\omega > \gamma\omega_0$. As a consequence, the shape of the spectrum comes close to that for coherent bremsstrahlung suppressed by the normal Ter-Mikaelian effect (see the spectral curve presented in Fig.9).

The emission spectrum is changed substantially in the case $z \gg 1$. The spectral curve presented in Fig.10 has been calculated for $y > 1$ and $z \gg 1$. Since $L/l_{Coh} = y(x+x^{-1}) > 2y > 2$, the interference term in (32) oscillates in the case under consideration. Therefore the effect of strong suppression of coherent bremsstrahlung due to azimuthal multiple scattering in a wide frequency range is attended by oscillations in the spectrum.

It is interesting to note that the averaging of the general expression for emission spectral-angular distribution can be performed exactly beyond the frame of dipole ap-

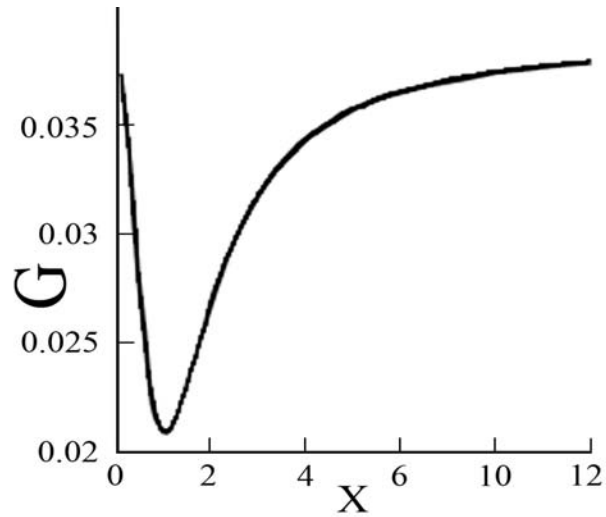

Figure 8: The spectrum of strongly collimated emission. The function $G(x, y, z)$, depended by (32) is presented here, $x = \omega/\gamma\omega_0$, $y = \omega_0 L/2\gamma$, $z = \gamma\langle\Delta^2\chi\rangle/\bar{\tau}\omega_0$. The curve has been calculated for fixed parameters $y = 0.1$ and $z = 0.4$.

proximation. Let us consider the characteristics of photons emitted along the atomic string axis and detected by X–ray detector with small angular size $\Delta\Theta \ll \gamma^{-1}$. Such conditions correspond to the experiment [31].

The expression (24) may be simplified very essentially on condition under consideration because of the important property of a fast charged particle coherent scattering by atomic string $\Psi_k^2 = \Psi_{k+1}^2 = \Psi^2$. The following from (24) expression

$$\mathbf{A^{tot}} = \frac{\mathbf{e}}{\pi}\left[\left(\frac{\mathbf{1}}{\gamma^{-\mathbf{2}} + \mathbf{\Psi^2}} - \frac{\mathbf{1}}{\gamma_{\mathbf{s}}^{-\mathbf{2}} + \mathbf{\Psi^2}}\right)\left(\mathbf{\Psi_i} - \mathbf{\Psi_f}\exp\left(\frac{\mathbf{i}\omega}{\mathbf{2}}(\gamma_{\mathbf{s}}^{-\mathbf{2}} + \mathbf{\Psi^2})\sum_{\mathbf{k}}\tau_{\mathbf{k}}\right)\right)\right.$$
$$\left. - \sum_k \frac{\mathbf{\Psi_k} - \mathbf{\Psi_{k-1}}}{\gamma_s^{-2} + \Psi^2}\exp\left(\frac{i\omega}{2}(\gamma_s^{-2} + \Psi^2)\sum_{j\le k}\tau_j\right)\right], \tag{34}$$

allows to calculate the emission spectral-angular distribution without complementary approximations. For example, the contribution of coherent bremsstrahlung is described by the formula

$$\omega\frac{dN^{Bs}}{d\omega d^2\Theta} = \frac{4e^2\Psi^2}{\pi}\frac{1 - \langle\cos(\Delta\chi)\rangle}{(\gamma_s^{-2} + \Psi^2)^2}\frac{\omega^2}{\omega^2 + \omega_s^2(\Psi)}\frac{L}{\bar{\tau}}, \tag{35}$$

where the quantity $\omega_s(\Psi)$ is defined by the formula (30) with precision of $\tilde{\Theta} \to \Psi$.

Comparison of the exact result (35) with dipole distribution (29) shows that the main characteristics of the collimated dipole coherent bremsstrahlung (the form of spectral distribution and the magnitude of this distribution in the frequency range where it is saturated) are presented for the emission beyond the frame of dipole approximation.

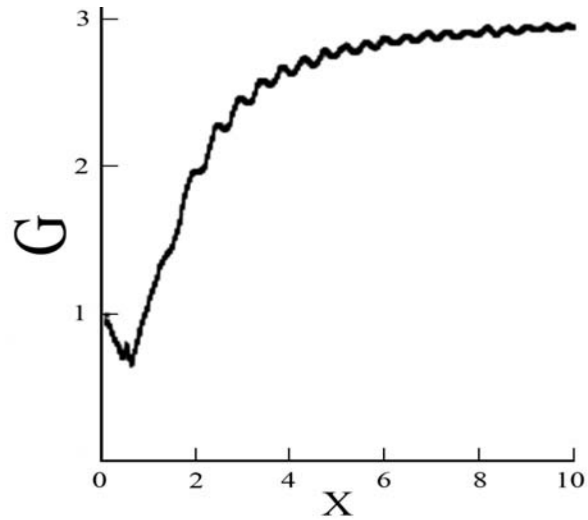

Figure 9: The same but for $y = 12$ and $z = 0.2$.

Let us use the result (35) to estimate the coherent bremsstrahlung yield from above-barrier fraction of an electron beam crossing the crystalline target along a string axis. The function $\omega dN^{CB}/d\omega d^2\Theta$, calculated by (35) with account of incoherent multiple scattering changing the orientation angle Ψ for $Si < 110 >$ crystal with fixed thickness and different electron energies, is presented in Fig.(11).

The curve **1** has been calculated for experimental conditions [31]. The theory and data correspond to each other with an accuracy of about $20 - 30\%$ within the range $30KeV < \omega < 150KeV$, but the discrepancy increases in the range $\omega > 150KeV$, where the calculated emission density is saturated in contrast with obtained data showing the continuing growth of the emission density. Such a discrepancy may be connected with the contribution of channeling particle emission in small photon energy range due to incoherent scattering processes.

7 Landau-Pomeranchuk-Migdal effect in coherent bremsstrahlung

As indicated above, the spectrum of strongly collimated coherent bremsstrahlung from relativistic electron, crossing an aligned crystal, is suppressed in a small frequency range due to the saturation of emitting particle scattering angle achievable in the process of coherent azimuthal scattering by atomic string's potential. On the other hand coherent bremsstrahlung can be suppressed due to Landau-Pomeranchuk-Migdal effect [32] appearing with the proviso that the scattering angle of emitting particle achievable at the distance of the order of emission formation length exceeds the characteristic angle γ^{-1}.

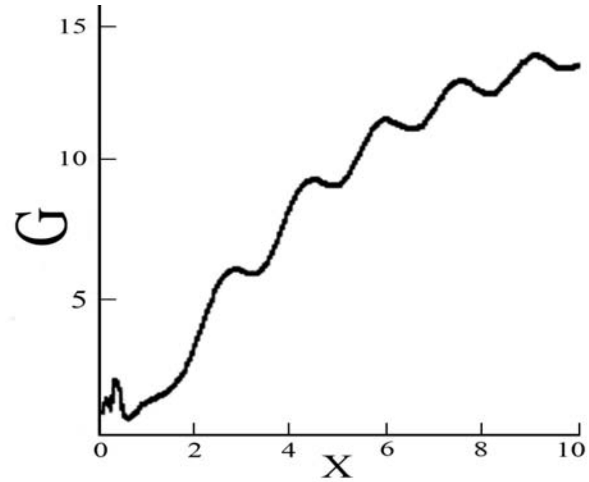

Figure 10: The same but for $y = 4$ and $z = 4$.

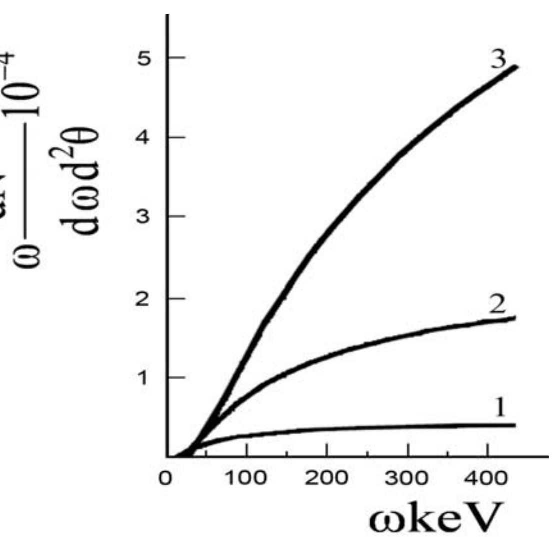

Figure 11: The spectrum of strongly collimated coherent bremsstrahlung. The presented curves have been calculated for $Si\langle 110\rangle$, $\Psi_0 = 0.25 \cdot 10^{-3} rad$, $L = 0.523 mm$. The curves **1–3** correspond to the electron energies 500, 1000 and 2000MeV respectively. Experimental data [31] are presented here.

It is clear than in general case the spectrum of coherent bremsstrahlung in small photon energy range is formed by two mentioned effects. To consider the relation between contributions of these effects let us start from the most general expression for spectral-angular distribution

$$\omega\frac{dN}{d\omega d\Omega}=\frac{e^2\omega^2}{4\pi^2}\left\langle\left|\int\limits_{-\infty}^{\infty}dt[\mathbf{n},\mathbf{v_t}]\mathbf{e}^{\mathbf{i}\omega(\mathbf{t}-\sqrt{\epsilon(\omega)}\mathbf{n},\mathbf{r_t})}\right|^2\right\rangle, \tag{36}$$

describing the bremsstrahlung properties within the frame of classical electrodynamics, here $\mathbf{r_t}=\mathbf{r}(\mathbf{t})$ is the emitting electron trajectory, $\mathbf{v_t}=\frac{\mathbf{d}}{\mathbf{dt}}\mathbf{r_t}$, $\epsilon=1-\omega_0^2/\omega^2$, $\mathbf{n}$ is the unit vector to the direction of emitted photon propagation. Integrating (36) over observation angles and using the approximations (**??**) one can obtain the following formula for the spectral distribution of the emission intensity

$$\omega\frac{dN}{dtd\omega}=-\frac{e^2\omega}{\pi}\left\langle\int\limits_0^{\infty}\frac{dt}{t}\left[\gamma_s^{-2}+\Psi^2-\Psi^2\cos(\Delta\chi_t)\right]\left[\sin(2\omega t)-\right.\right.$$

$$\left.\left.-\sin\left(\frac{\omega}{2}(\gamma_s^{-2}+\Psi^2)t-\frac{\omega\Psi^2}{t}\int\limits_0^t d\tau(t-\tau)\cos(\Delta\chi_\tau)\right)\right]\right\rangle. \tag{37}$$

Averaging in (37) can be performed analytically in the case of small azimuthal scattering angles $\Delta\chi\ll 1$ [28]. To perform a qualitative analysis in the general case we use the Landau-Pomeranchuk approach [32], replacing the function $\cos(\Delta\chi)$ in (37) by the averaged one $\overline{\cos(\Delta\chi_t)}=\int\limits_0^{2\pi}d\Delta\chi_t\cos(\Delta\chi_t)f(t,\Delta\chi_t)$, where the distribution function $f(t,\Delta\chi_t)=(2\pi)^{-1}\sum\limits_k\exp(ik\Delta\chi_t-\alpha_k t)$ is presented, for example, in [28]. Obviously,

$$\overline{\cos(\Delta\chi_t)}=\exp(-\alpha_1 t),\alpha_1=\frac{1-\langle\cos(\Delta\chi)\rangle}{\bar{\tau}}. \tag{38}$$

Substituting (38) into (37), one can reduce this formula to simple one

$$\omega\frac{dN}{dtd\omega}=\frac{e^2\Psi^2\omega}{\pi}\left\{\int_0^\infty\frac{dt}{t}\left(1-e^{-t}\right)\sin\frac{\omega}{\omega_s(\Psi)}\left(t-\frac{2\gamma_s^2\Psi^2}{1+\gamma_s^2\Psi^2}\frac{t-1+e^{-t}}{t}\right)-\right.$$

$$\left.-\frac{\pi}{2}\frac{1}{\gamma_s^2\Psi^2}[1-\frac{2}{\pi}\int_0^\infty\frac{dt}{t}\sin\frac{\omega}{\omega_s(\Psi)}\left(t-\frac{2\gamma_s^2\Psi^2}{1+\gamma_s^2\Psi^2}\frac{t-1+e^{-t}}{t}\right)]\right\} \tag{39}$$

The result (39) allows to obtain simple asymptotical formulae for the emission spectrum in different frequency ranges. For example, in the range of very small frequencies $\omega\ll\gamma\omega_0$ formula (39) is reduced to the expression

$$\omega\frac{dN}{dtd\omega}\simeq\frac{2e^2}{\pi}\gamma^2\Psi^2\frac{1-\langle\cos\Delta\chi\rangle}{\bar{\tau}}\frac{\omega^2}{\gamma^2\omega_0^2}, \tag{40}$$

describing the Ter-Mikaelian effect in coherent bremsstrahlung.

It is very important that the spectrum of coherent bremsstrahlung is described in the frequency range $\gamma\omega_0 \ll \omega \ll \omega_* = 2\gamma^2(1 - \langle\cos\Delta\chi\rangle)/(1+\gamma^2\Psi^2)\bar{\tau}$ by the formula

$$\omega\frac{dN}{dtd\omega} \simeq e^2\frac{\gamma^2\Psi^2}{1+\gamma^2\Psi^2}\frac{1-\langle\cos\Delta\chi\rangle}{\bar{\tau}}\frac{\omega}{\omega_*}, \tag{41}$$

which is correct independently of the energy of emitting electrons.

Formula (41) describes the above studied suppression effect, caused by the limitation on the value of azimuthal scattering angle.

LPM effect in the coherent bremsstrahlung can manifest in the range $\omega \gg \omega_*$, where the formula (39) can be reduced to more simple one

$$\omega\frac{dN}{dtd\omega} = \frac{e^2\Psi^2}{\pi}\omega\left[2\frac{\omega_*}{\omega}(1+\gamma^2\Psi^2)\int_0^\infty dt\sin\left(2t+\frac{\omega_L}{\omega}t^2\right) - \right.$$

$$\left. -\frac{\pi}{2}\frac{1}{\gamma^2\Psi^2}\left(1-\frac{2}{\pi}\int_0^\infty\frac{dt}{t}\sin\left(2t+\frac{\omega_L}{\omega}t^2\right)\right)\right] \tag{42}$$

where $\omega_L = \frac{8}{3}\gamma^4\Psi^2(1-\langle\cos\Delta\chi\rangle)/\bar{\tau}$ is the frequency of LPM effect. In the special case that $\omega_* \ll \omega_L$ the formula (42) is reduced within the frequency range $\omega_* \ll \omega \ll \omega_L$ to the expression [28]

$$\omega\frac{dN}{dtd\omega} \simeq e^2\Psi^2\gamma^2\frac{1-\langle\cos\Delta\chi\rangle}{\bar{\tau}}\sqrt{\frac{2}{\pi}\frac{\omega}{\omega_L}} \tag{43}$$

describing LPM effect in coherent bremsstrahlung.

In the opposite case $\omega \gg \omega_L$ the emission yield saturates

$$\omega\frac{dN}{dtd\omega} \simeq \frac{2e^2}{\pi}\Psi^2\gamma^2\frac{1-\langle\cos\Delta\chi\rangle}{\bar{\tau}} \tag{44}$$

Thus, the spectrum of non-collimated coherent bremsstrahlung consists of several segments, where spectral properties are very different; in this case the boundaries between these segments depend strongly on the emitting electron energy $m\gamma$, orientation angle Ψ and the crystal parameters. Among other things, the field of existence of LPM effect appears only with the proviso that

$$\frac{\omega_*}{\omega_L} \approx \frac{4}{3}\gamma^2\Psi^2(1+\gamma^2\Psi^2) \gg 1 \tag{45}$$

In accordance with (45) LPM effect is not realized if $\gamma\Psi < 1$. This conclusion is well founded because the inequality $\gamma\Psi < 1$ implies that the condition of dipole approximation (28) is fulfilled.

8 Conclusions

In accordance with performed analysis, the transition radiation from relativistic electron crossing a thin target can modified substantially the characteristics of other emission processes considered usually without account of contribution of TR.

Scattering of TR field by atomic electrons causes the suppression of the density effect manifestation in polarization bremsstrahlung from relativistic electrons crossing a thin layer of amorphous medium.

Similar process but in a crystalline target may change very substantially the angular distribution of parametric X-rays from relativistic electrons.

In the process of parametric X-rays along the emitting electron velocity (forward PXR) TR contribution may consist in the strong peak with small angular and spectral width. This peak caused by dynamical diffraction effects arranges near to the forward PXR peak and hampers therefore the observation of PXR peak.

Interference between TR and bremsstrahlung contributions is responsible for the manifestation of anomalous Ter-Mikaelian effect in the bremsstrahlung from relativistic electrons crossing a thin layer of amorphous medium.

Analogous and other effects take place in the process of coherent emission from relativistic electrons crossing a thin crystalline target near parallel to the axis of a set of atomic strings.

The manifestation of Landau-Pomeranchuk-Migdal effect in the coherent bremsstrahlung from relativistic electrons in aligned crystal differs substantially from that in amorphous medium due to the competition from the side of another suppression effect caused by the limitation by the angle of coherent azimuth scattering of emitting electron by string's potential.

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