



## **MODELING SKIN EFFECT IN LARGE MAGNETIZED IRON DETECTORS**

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### **Abstract**

The experimental problem of the calibration of magnetic field in large iron detectors is discussed. Emphasis is laid on techniques based on ballistic measurements as the ones employed by MINOS or OPERA. In particular, we provide analytical formulas to model the behavior of the apparatus in the transient regime, keeping into account eddy current effects and the finite penetration velocity of the driving fields. These formulas ease substantially the design of the calibration apparatus. Results are compared with experimental data coming from a prototype of the OPERA spectrometer.

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Massive magnetized iron detectors will play a leading role in neutrino physics in the forthcoming years. These detectors are used as instrumented target (MINOS [1]) or complementing high precision trackers (OPERA [2]) in the next generation long baseline neutrino experiments. Moreover, similar devices have been proposed for high precision atmospheric neutrino experiments (MONOLITH [3]) or in connection with the Neutrino Factories [4]. In most of their physical applications a calibration of the magnetic field in the bulk of the iron is mandatory. In particular, for MINOS or OPERA a relative precision on the knowledge in the local field of a few percent is enough to reach a negligible impact on the charge and momentum reconstruction of muons. Absolute calibration by stopping muons requires long data taking in the underground areas where these detectors will be located and will not provide locally the requested precision. On the other hand, Hall probes in the surrounding air test the field in iron only indirectly and need a detailed simulation to be rescaled. In fact, an absolute calibration of the average field in a given area can be obtained performing “ballistic measurements” [5], i.e. integrating the induced voltage in a set of pickup coils during the ramp-up of the driving current. This technique is currently employed by MINOS and OPERA [6,7]. In particular, the OPERA dipolar magnet consists of two vertical walls of rectangular cross section and of top and bottom flux return path (see Fig. 1). The walls consist of 12 iron layers (5 cm thick) interleaved with 2 cm of air allocated for the housing of the Resistive Plate Chambers. Each iron layer is made up of seven plates  $50 \times 1250 \times 8200 \text{ mm}^3$ . The driving coils will be installed in the return yokes. Several pickup coils (not shown in the figure) will be positioned along the walls, measuring the field in iron at a given height averaged over the 12 slabs. During the ramp-up, the variation of the current flowing in the drive coils induces a change in the magnetic flux cut by the pickup coil; the induced voltage is

$$V(t) = -\frac{d\Phi(t)}{dt} = -\frac{d}{dt} \int_S \vec{B}(t) \cdot \vec{n} ds = -SN \frac{d}{dt} \langle B \rangle(t) \quad (1)$$

where  $S$  is the cross sectional area (in iron) of one turn of the pickup coil,  $N$  is the number of turns and  $\langle B \rangle$  is the field averaged on the surface  $S$ . For a ramp-like waveform of the current flowing in the driving coils

$$i(t) = \begin{cases} 0 & : t < 0 \\ kt & : 0 < t < T \\ kT & : t > T \end{cases} \quad (2)$$

after integration of (1), we get

$$\frac{-1}{SN} \int_0^\infty dt' V_i(t') = \frac{1}{SN} (\Phi_{\text{fin}} - \Phi_0) = \langle B \rangle_{\text{fin}} - \langle B \rangle_0 \quad (3)$$

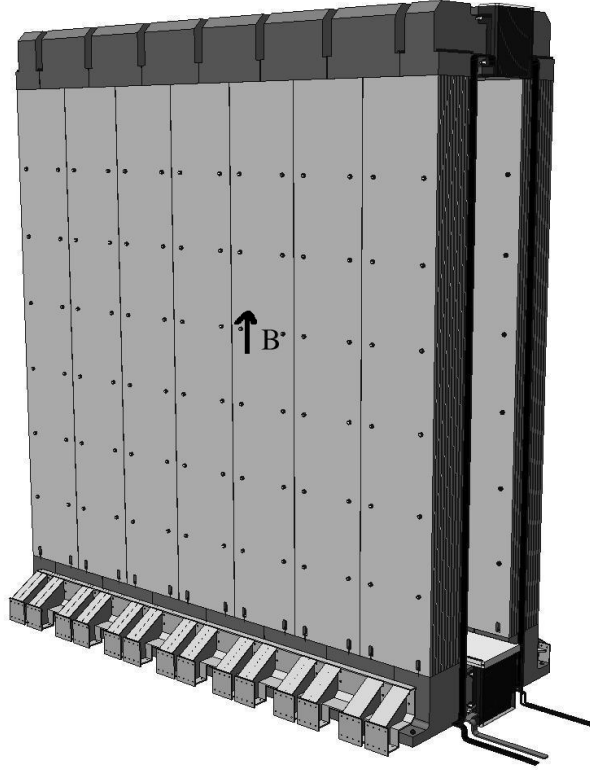


Figure 1: Schematic view of the OPERA spectrometer.

where the subscript “fin” indicates the value reached for  $t \rightarrow +\infty$ , i.e. when the power supply provides a current  $i = i_{\max} \equiv kT$  and the transient due to eddy currents is faded out. Clearly, this measurement provides a difference of fields. To get  $B$  at the nominal value of  $i_{\max}$  it is necessary to compute the residual field  $B_r$ ; it can be done running through a whole hysteresis loop, i.e. ramping up and down twice, and reconstructing  $B_r$  and  $B(i_{\max})$  from the differences of fields.

The measurement based on Eq. (3) is robust since it does not depend on the actual waveform of the induced voltage (if the integration time is sufficiently long) and even the requirement of a constant  $di/dt$  can be safely dropped. Hence, this technique can be extremely effective if  $k$  ( $\equiv di/dt$ ) and  $N$  are properly tuned, i.e. the induced voltage is enough and has a favorable signal-to-noise ratio against environmental noise. In particular, the tuning of  $k$  is critical and requires a careful design of the power supplies connected with the driving coils since the load inductance of the spectrometer can be huge<sup>1</sup>. Although the field measurement is independent of the waveform of the induced

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<sup>1</sup>The load inductance depends on the magnetic permeability and, hence, varies during the ramp-up. In

voltage, an approximate estimate of this voltage is needed to design the apparatus and provides useful information to assess systematic errors during the data taking. Neglecting eddy currents,  $V(t)$  has a simple form:

$$\begin{aligned} V(t) &= -SN \frac{d}{dt}(\mu(H) \cdot H) = -SN \left[ \frac{d\mu}{dH} \frac{dH}{dt} H(t) + \mu(H) \frac{dH}{dt} \right] \\ &= -SNnk \left[ \frac{d\mu}{dH} H(t) + \mu(H) \right] \end{aligned} \quad (4)$$

being  $H$  the magnetic intensity,  $\mu(H)$  the magnetic permeability of iron and  $n$  the effective number of driving turns per unit length ( $B = \mu H = \mu ni$ ). For very simple geometries  $n$  can be computed analytically. E.g. approximating the OPERA spectrometer as an Epstein frame [8],  $n$  would be the number of driving coils divided by the average path along the magnetic circuit. However, since the driving coils are not uniformly distributed along the path and the field losses in air are not negligible, the effective value of  $n$  at the height where the pickup coils are installed has to be computed numerically by finite-element calculation in the steady regime<sup>2</sup>. Once  $n$  is obtained, Eq. (4) can be computed analytically since  $\mu(H)$  and its derivatives are known from the B-H curve of the iron and  $dH/dt = nk$  during the ramp-up. Eq. (4) provides also a useful scaling law for different geometries. For instance, both the walls and the return yoke of the prototype of the OPERA spectrometer [7] are made up of 4 slabs instead of 12 but the magnetic properties of the steel are the same. The ratio between the maximum voltage obtained during the ramp-up will be just

$$\frac{V_{\max}}{V'_{\max}} = \frac{SNnk}{S'N'n'k'} \quad (5)$$

where the primed quantities refer to the prototype, the others to the final magnet. To prove Eq. (5) is enough noticing that  $V$  is maximized when  $f(H) \equiv (d\mu/dH) \cdot H + \mu$  reaches its maximum and  $\text{Max}\{f(H)\}$  is an universal property of the steel so it cancels out in the ratio (5).

Unfortunately Eqs.(4) and (5) are completely spoiled by the magnetic skin effect. Eddy currents induced in the bulk of the iron slow down the penetration of the field through the slabs; hence the assumption that  $H(t)$  is equal to  $nk t$  at any depth becomes the case of OPERA it has a value of about 0.14 H at nominal fields ( $i = i_{\max}$ ) but during the ramp-up reaches a maximum of 0.9 H.

<sup>2</sup>In the present case TOSCA [9] has been used to solve numerically the magnetostatic Maxwell equation for the actual geometry of the magnet.

unrealistic. To appreciate the relevance of this effect, Fig. 2-a shows the value of the field at one pickup coil of the OPERA prototype during a ramp-up from the residual field  $B_r$  ( $i = 0$ ) up to  $B^{\max}$  ( $i = 368$  A). The dashed line represents the expected field without eddy currents (i.e. assuming infinite resistivity for the steel), the continuous line shows the experimental data ( $\rho = 1 \cdot 10^{-7} \Omega\text{m}^{-1}$ ). Modeling the build up of eddy currents during the transient regime through finite-element calculation for these kind of devices is extremely cumbersome and analytic or semi-analytic formulas to reproduce the magnetic behavior of the spectrometer for time-varying currents would be of great practical importance. Moreover, in the case of OPERA or MINOS, the problems can be reduced to the calculation of the penetration rate of a time varying field  $H_x(t, y = 0) = H_x(t, y = D) = nkt$  through a semi-infinite plane of depth  $D$ <sup>3</sup>. Here  $D$  is the sum of the thicknesses of the slabs in the wall or in the return yoke:  $D = 4 \cdot 5 \text{ cm} = 20 \text{ cm}$  for the OPERA prototype.

This problem can be solved analytically for materials of constant permeability [10]. For ferromagnetic materials several approximate solutions have been proposed in literature. In particular it has been shown in [11] that if the B-H curve of the steel can be represented by the function

$$B = aH^b \quad 0 < b < 1 \quad (6)$$

and the driving field is sinusoidal with frequency  $\omega$  ( $H_{x0} = \hat{H}_{x0} \cos \omega t$ ), the complete solution for  $H_x$  in the slab is

$$H_x(y, t) = \begin{cases} \hat{H}_{x0}(1 - \alpha y)^\beta \cos[\omega t + \gamma \ln(1 - \alpha y)] & : \alpha y \leq 1 \\ 0 & : \text{elsewhere} \end{cases} \quad (7)$$

where

$$\alpha \equiv \left[ \frac{(1 - b)^2 |\omega| \mu}{(3 + b) \sqrt{2(1 + b)} \rho} \right]^{1/2} ; \quad \mu = a \hat{H}_{x0}^{b-1} \quad (8)$$

$$\beta \equiv \frac{2}{1 - b} ; \quad \gamma \equiv \frac{\sqrt{2(1 + b)}}{(1 - b)} \quad (9)$$

The condition  $\alpha y < 1$  has a simple physical interpretation. When  $y \rightarrow 1/\alpha$ ,  $H$  goes to zero, so that no electromagnetic fields exist beyond  $1/\alpha$ . Hence, every frequency penetrates up to a given depth and it is confined within a “skin” of thickness  $\alpha^{-1}$ . In many

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<sup>3</sup>In the following we use rectangular coordinates where the  $x$ - and  $z$ -directions are parallel to the surface of the slab; the  $y$ -direction is into the slab, normal to the surface. The interfaces between iron and air are at  $y = 0$  and  $y = D$ . The air between the slabs is neglected. Note also that in the present case the field  $H$  is always parallel to  $x$ :  $\vec{H}(t) = H_x(t) \vec{i}$ .

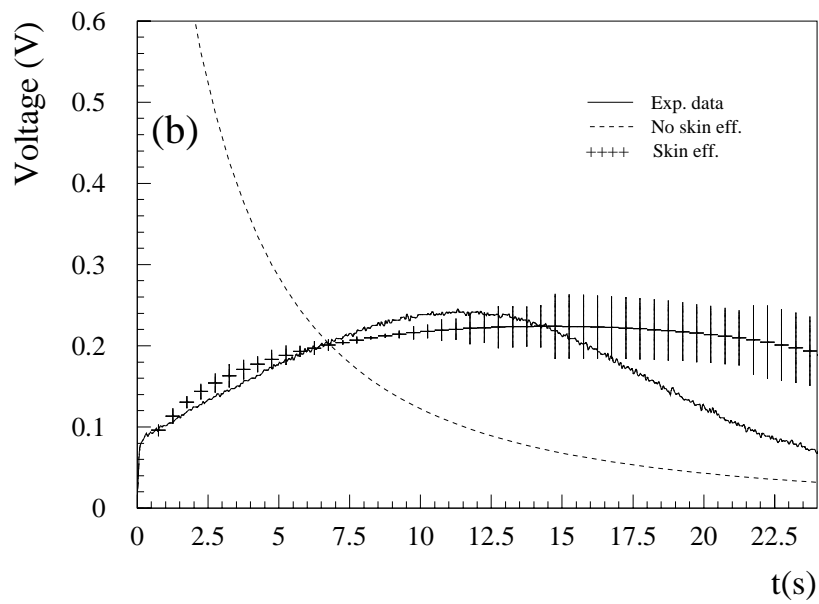
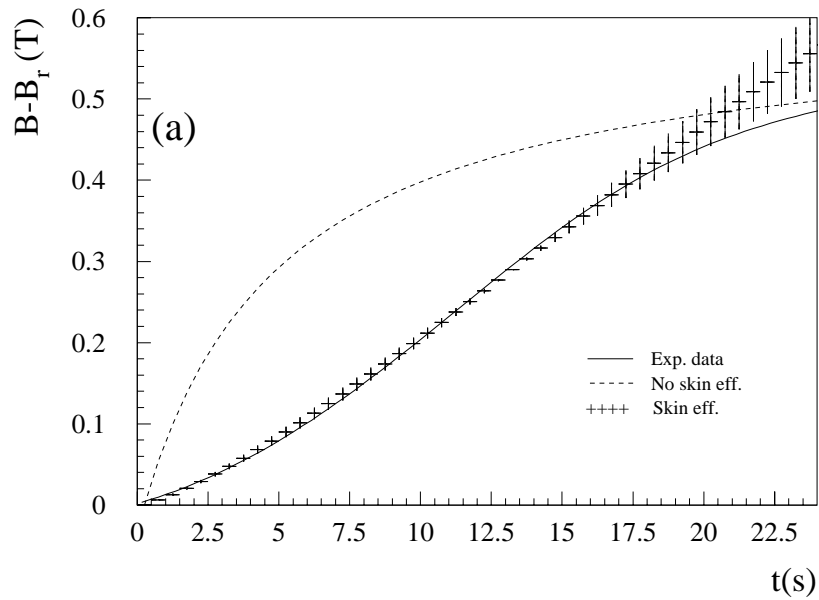


Figure 2: (a) Average magnetic field versus time measured experimentally (continuous line), computed without skin effect (dashed line) and computed including magnetic skin effect (crosses). (b) Induced voltage versus time measured experimentally (continuous line), computed without skin effect (dot-dashed line) and computed including magnetic skin effect (dots).

applications, the power law (6) is troublesome because it is impossible to parametrize the whole range of the B-H curve with a single pair of constants. In particular, the steepness  $b$  changes abruptly above the saturation knee when the rate of variation of  $B$  becomes much smaller. This means that during the ramp-up Eq. (6) remains valid up to a time  $\tilde{T}$  when the average field in the slabs starts saturating. Beyond this point Eq. (7) systematically overestimates the field, due to the failure of (6), and the model is no more effective. Fortunately this limitation is immaterial in the present case since we are interested in reproducing the induced voltage in the region contributing mostly to the integral (3), i.e. below the saturation knee. Moreover, since we need to describe  $V(t)$  just up to  $\tilde{T}$ , it is possible to choose a waveform different from (2), simplifying the treatment in the Fourier domain. For instance, we choose a triangular waveform

$$i(t) = \begin{cases} 0 & : t < -\tilde{T} \\ k\tilde{T} + kt & : -\tilde{T} < t < 0 \\ k\tilde{T} - kt & : 0 < t < \tilde{T} \\ 0 & : t > \tilde{T} \end{cases} \quad (10)$$

which is identical to (2) in the ramping-up phase. In this case, the time-varying field at  $y = 0$  ( $H(t, y = 0) = ni(t)$ ) has a non-singular real Fourier transform:

$$\frac{2\tilde{H}}{\tilde{T}\omega^2}(1 - \cos \omega\tilde{T}) \quad (11)$$

where  $\tilde{H} \equiv nk\tilde{T}$  is the maximum field at the surface. The field at a given depth  $d$  is

$$H_x^{\text{tot}}(t, y = d) = H_x(t, y = d) + H_x(t, y = D - d) \quad (12)$$

where

$$H_x(t, y) = \frac{1}{2\pi} \int_{\mathcal{D}} d\omega \frac{2\tilde{H}}{\tilde{T}\omega^2}(1 - \cos \omega\tilde{T})(1 - \alpha y)^\beta \cos[\omega t + \gamma \ln(1 - \alpha y)] \quad (13)$$

The integration domain  $\mathcal{D}$  is given by the subset of the real axis where the condition  $\alpha y < 1$  holds. This integral has been computed numerically in the case of the OPERA prototype. The crosses of Fig. 2-a represent the expected value of the field according to the model described above. The error band shows the systematic uncertainty coming from parametrization (6) and has been computed comparing (6) with the actual B-H curve of the steel. Other source of errors as the border effects coming from the finite length of the plane or the neglect of the air between the wall slabs have been computed by finite element calculation and are negligible compared with the uncertainty coming from the parametrization. Fig. 2-b shows the corresponding induced voltage versus time. Similar

results have been obtained at other hysteresis curves. Below the B-H knee the model is able to reproduce the experimental data within the instrumental precision (about 3% [7]). For high values of  $\tilde{H}$  the systematic bias is within the parametrization uncertainty discussed above.

In conclusion, in this letter we provided a semi-analytic treatment of the magnetic behavior of large magnetized spectrometers in the transient regime. This model can be fruitfully applied to design field calibration systems based on ballistic measurements, especially for underground detectors where absolute calibration from the range-curvature correlation of penetrating particles is not available.

## 1 Acknowledgments

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