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BEAM LOADING COMPENSATION SCHEMES FOR THE MUON RECIRCULATING LINACS OF THE CERN NEUTRINO FACTORY

M. Ferrario, V. Fusco, M. Migliorati

INFN- Laboratori Nazionali di Frascati, Via E. Fermi 40, 00044 Frascati (Roma), Italy

Abstract

A train of 100 muon bunches, 2.2 μs long, with an average current of 0.1 A, has to be accelerated by the first recirculating linac (μRLA1) from 2 GeV up to 10 GeV (in 4 turns) with a rms relative energy spread $\sigma_e < 5\ 10^{-3}$. Despite the huge amount of energy stored in the 352 MHz cavities adopted for this linac (~100 Joule @ 10 MV), the energy spread induced by beam loading effects without compensation results to be σ_e=1.4 x10^{-2}. We discuss in this note the results of a preliminary study about possible schemes for beam loading compensation in μRLA1. Simple scaling laws are derived by means of the phasor description of beam loading effects according to P. Wilson treatment. The code HOMDYN is used for multi-bunch computations whose main features are recalled in the appendix..

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1. — INTRODUCTION

A train of 100 muon bunches, 2.27 μs long, with an average current of 0.1 A, has to be accelerated by the first recirculating linac (μRLA1) from 2 GeV up to 10 GeV (in 4 turns) with a rms relative energy spread $\sigma_e < 5\ 10^{-3}$. Since the μRLA1 circumference is designed to fit exactly with the 2.27 μs long train, each accelerating structure will be loaded by a 4 x 2.27 μs long train (400 bunches) per RF pulse [1], repeated at 75 Hz.

Despite the huge amount of energy stored in the 352 MHz cavities adopted for this linac (~100 Joule @ 10 MV), the energy spread induced by beam loading effects without compensation results to be σ_e=1.4 x10^{-2} as shown in Fig. 1

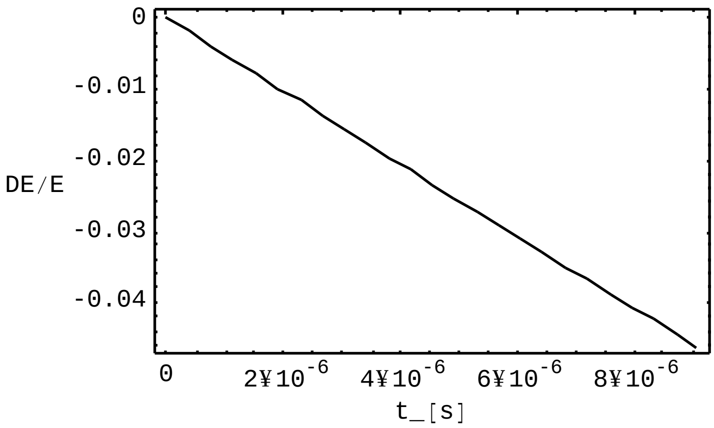

Fig. 1 – Normalized energy gain along the beam without beam loading compensation, $\frac{DE}{E} = \frac{E_n - E_1}{E_1}$; E_n being the energy gained by the n-th bunch, (σ_e=1.4 x10^{-2}).

We discuss in this report two possible beam loading compensation schemes able to reduce the induced energy spread to an acceptable level.

The first method is based on a well know technique also adopted by the TESLA project [2]. In this case the beam is injected during the cavity filling so that the beam induced voltage is compensated by the additional voltage provided by the generator, as schematically shown in Fig. 2.

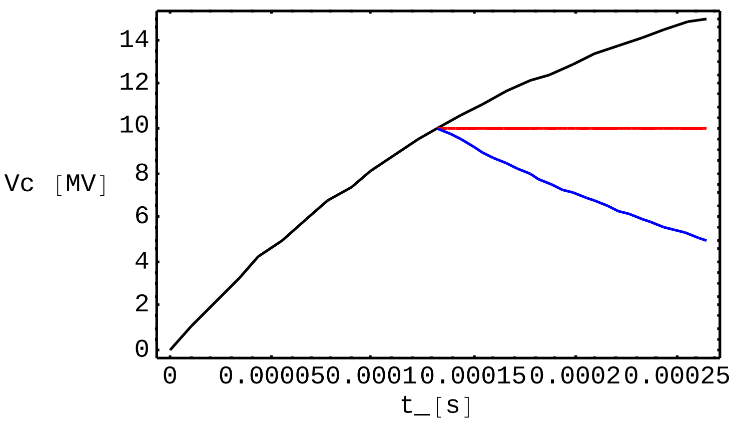

Fig. 2 – The beam induced voltage (blue line) is compensated by the generator voltage (black line) resulting in a constant cavity voltage (red line).

The second method is similar to the one proposed by the CTF2 experiment [3]. By a suitable choice of the ratio between linac frequency and beam repetition rate (not anymore sub-harmonic of the linac frequency), the accelerating voltage drop induced by beam loading is compensated by the bunch to bunch injection phase shift towards the RF crest, as schematically shown in Fig. 3.

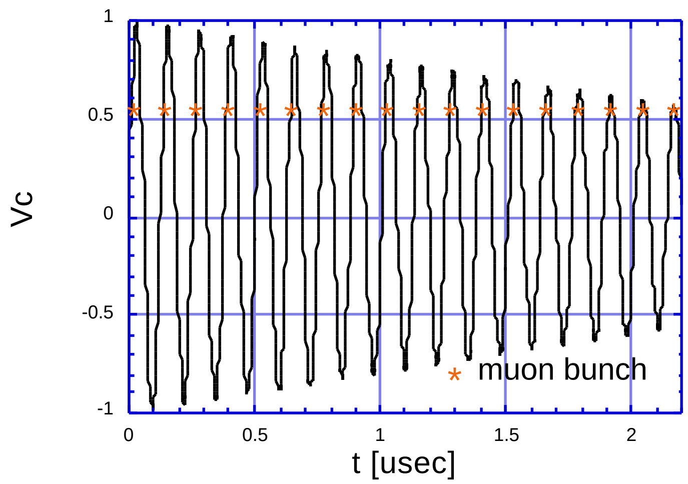

Fig. 3 – Voltage seen by different muon bunches due to the phase shift.

In the next sections we will discuss in more details the two proposed schemes including a sensitivity study with respect to the main parameter fluctuations. The code HOMDYN [4] is used for multi-bunch computations in μRLA1 and a brief description of the code is reported in the appendix.

We assume that the beam repetition rate is 44 MHz, and each bunch is 2 cm long carrying a 2.3 nC charge uniformly distributed in a cylinder. A detailed analysis of the longitudinal beam dynamics of the muon cooling channel could provide information about a more realistic bunch form factor and charge distribution, that will be taken in to account in a future work. We assume also that the accelerating cavities adopted for this linac are identical to the LEP [5] four cells 352 MHz superconducting cavities, operating in the $TM_{0,1,0\_\pi}$ standing wave mode at an accelerating field of 6 MV/m. The fundamental pass-band parameters are reported in Tab. 1. The LEP Klystrons are able to develop 1.3 MW in order to feed simultaneously many cavities equipped with power couplers tested up to 200 kW.

Table 1 – Fundamental pass-band modes.

Mode	π	3/4 π	π/2	π/4
Frequency [MHz]	352.018	351.086	348.897	346.748
R/Q [Ω]	470	0.005	0.099	0

According to the Wilson treatment [6], we can represent the accelerating voltage V_a, the real part of the total cavity voltage V_c, as a superposition of the generator voltage and the beam induced voltage, see also Fig. 4. Including transients the following relation holds for $t \geq t_o$:

$$V_a(t) = V_c(t)\cos\varphi$$

$$= e\left\{ V_{gr}\cos\psi_g e^{i(\psi_g+\vartheta)}\left[1-e^{-\frac{t}{\tau}(1-iTg(\psi_g))}\right] - V_{br}\cos\psi_b e^{i\psi_b}\left[1-e^{-\frac{(t-t_o)}{\tau}(1-iTg(\psi_b))}\right]\right\} \quad (1)$$

where V_{gr} and V_{br} are the steady state generator and beam induced voltages at resonance. The terms between rectangular brackets account for the transient state, t_o being the beam injection time. The angle between the beam current and the cavity voltage is ϕ , while ϑ is the angle between the beam current and the generator current. The tuning angle is defined as: $\psi = -Arctg(\Delta\omega\tau)$, where $\Delta\omega = \omega - \omega_\pi$ accounts for an off resonance excitation and $\tau = \frac{2Q_{ext}}{\omega_\pi}$ = 0.19 msec is the filling time of the fundamental mode resonating at ω_π with a matched external quality factor Q_{ext}.= 2.125 10^5.

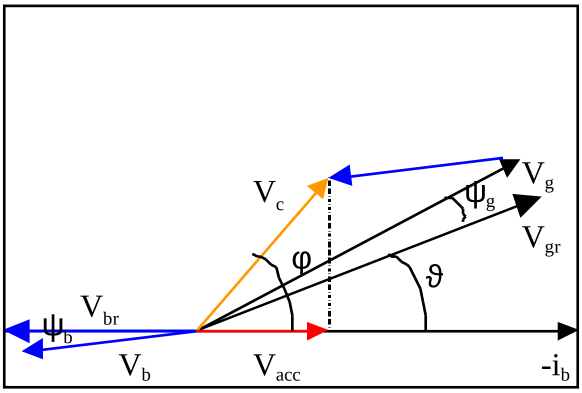

Fig. 4 – Wilson phasor diagram representing generator and beam loading voltage superposition.

2. — BEAM INJECTION DURING VOLTAGE TRANSIENT

In this case the beam is injected on crest ($\vartheta=\phi=0$), the cavity is excited on resonance ($\psi = 0, \Delta\omega = 0$) and the beam repetition rate (44 MHz) is a sub-harmonic of the cavity frequency, so that expression (1) reduces to:

$$V_a(t) = V_{gr}\left[1-e^{-\frac{t}{\tau}}\right] - V_{br}\left[1-e^{-\frac{(t-t_o)}{\tau}}\right] \quad (2)$$

The optimum beam injection time is the one that results in a constant accelerating voltage, as shown in Fig. 2, and is determined by the following relation:

$$t_o = \tau \ln\left(\frac{V_{gr}}{V_{br}}\right) \quad (3)$$

By injecting the beam at the optimum injection time t_o one could expect a perfect compensation of the beam induced voltage, as suggested by Fig. 2. This is not completely true, as shown if Fig. 5, in which the result of a multi-bunch simulation is reported for the 0.1 A beam
injected in a LEP superconducting cavity with Vacc= 10 MV. The beam induced voltage is

$V_{br} = \frac{R}{Q} Q_{ext} I_b$ = 10 MV, assuming a generator voltage $V_{gr}=2V_{br}$ the optimum injection time results to be t_o=0.1329 msec.

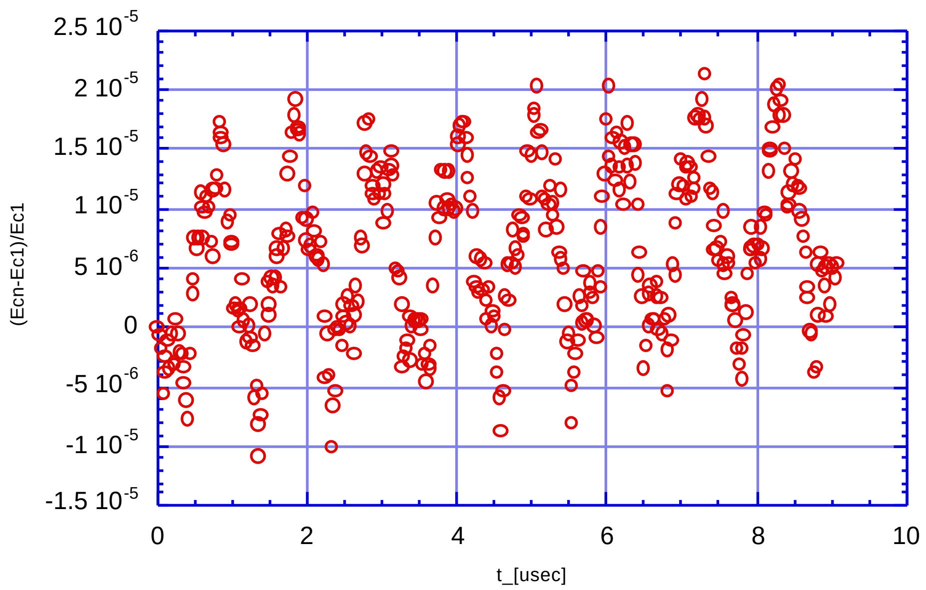

Fig. 5 – Normalized energy gain along a beam injected at t_o=0.1329 msec, as computed by HOMDYN, (σ_e=6.9 x10^{-6}).

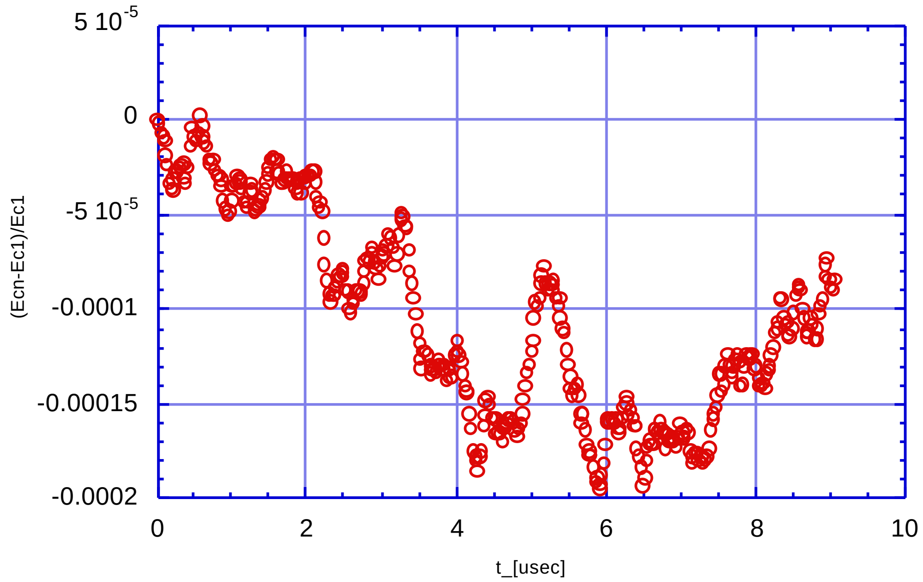

Fig.6- Normalized energy gain along a beam with ±10% random charge fluctuations, as computed by HOMDYN, (σ_e=5.37 x10^{-5}).

The resulting energy spread is very small σ_e=6.9 x10^{-6} but greater than zero with an energy oscillation along the bunch. This effect [7,4] is due to the transient excitation of the lower order modes of the fundamental pass-band in a multicell structure. Lower order modes contribute differently to the net energy gain from bunch to bunch, due to the fact that each bunch interacts with them with a different relative phase. Moreover in the transient state their voltage could reach a significant fraction of the accelerating mode voltage before decaying, with their damping times, to a very low steady state value. The energy oscillation shown in Fig. 5 has infact a beating frequency given by the frequency difference between the generator frequency and the mode closer on the dispersion curve to the accelerating mode. Since the bunch train duration, 9.08 μs, is very short compared to cavity time constant, τ = 0.19 msec, no damping effect is visible in Fig. 5.

This compensation scheme is very stable under charge, current and injection time fluctuations. One of the major concern in a muon accelerator is in fact charge fluctuations along the beam, that may change the loading condition from bunch to bunch and from train to train. We have investigated numerically random charge fluctuations along the beam, as shown for example in Fig. 6, verifying a very small influence on the final energy spread that remains below 10^{-4} also for a ± 10 % charge fluctuation. Also the time injection error doesn't affect the final energy spread significantly up to ±10 μs. While current fluctuations have to be kept below 1 % not to exceed $\sigma_e = 10^{-4}$.

The price to pay for such a reliable beam loading compensation scheme is the RF power demands. Since the Klystron has to replace the power absorbed by the beam: $P_b = V_{acc}\, I$ = 1 MW, each cavity should be fed by one klystron and power couplers should be overloaded at an unaxceptable level. Nevertheless if a reduction of beam average current could be envisaged, for example 0.02 A, this scheme could be a very suitable candidate providing an extremely good beam quality and stability.

3. — BEAM LOADING COMPENSATION BY PHASE SLIPPAGE

An alternative beam loading compensation scheme is described in this paragraph. If the compensation during voltage transient results unaffordable for power requirements, one has to properly rely on the stored energy in the cavity. An on crest injection at steady state would result in a huge energy spread, as shown in Fig. 1. But by injecting the beam off crest, with a proper ratio between cavity frequency and beam repetition rate, the beam slips in phase from bunch to bunch towards the RF wave crest while the voltage amplitude decreases because of the beam loading effect. We derive in this paragraph a simple way to optimize the process so that to reach the desired energy spread.

Since the cavity is excited on resonance by the generator the detuning angle is $\psi_g = 0$. Actually another detuning angle ψ_b should be consider, since the beam repetition rate is not anymore a subharmonic of the cavity resonant mode, but we are looking for minor perturbation of cavity frequency and/or beam repetition rate and we will assume also ψ_b=0. In addition the beam sees the generator voltage with a time dependent angle given by $\vartheta(t)=\vartheta_o+\Delta\omega_o(t-t_o)=\vartheta_o+\Delta\vartheta(t)$, t_o being the beam injection time. Given the previous assumptions equation (1) reads for $t \geq t_o$:

$$
\begin{aligned}
V_a(t) &= V_c(t)\cos\varphi \\
&= \operatorname{Re}\left\{ V_{gr} e^{i(\vartheta_o+\Delta\vartheta)} - V_{br}\cos\psi_b e^{i\psi_b}\left[1 - e^{-\frac{(t-t_o)}{\tau}(1-itg\psi_b)}\right]\right\} \\
&= V_{gr}\cos(\vartheta_o+\Delta\vartheta) - V_{br}\left[1 - e^{-\frac{(t-t_o)}{\tau}}\right]
\end{aligned}
\qquad (4)
$$

We request that the first and the last bunch of the train see the same cavity voltage $V_a(t_o) = V_a(t_b)$ where t_b is beam time duration. From the latter condition results the following relation:

$$
\Delta\vartheta = ar\cos\left[\cos\vartheta_o + \frac{V_{br}}{V_{gr}}\left(1 - e^{-\frac{t_b}{\tau}}\right)\right] - \vartheta_o \qquad (5)
$$

and by means of (5) one can compute the required cavity frequency shift as a function of the injection phase ϑ_o, as shown in Fig. 7 where $\Delta\nu = \dfrac{\Delta\vartheta}{2\pi t_b}$.

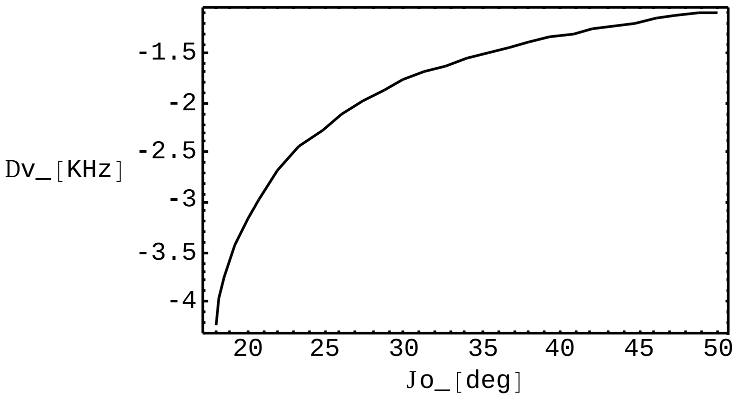

Fig. 7 - Cavity frequency shift as a function of the injection phase ϑ_o; $V_{gr}=V_b=10$ MV

For a given injection phase ϑ_o the energy gain variation along the beam is computed by means of (4) and shown in Fig. 8 as an example:

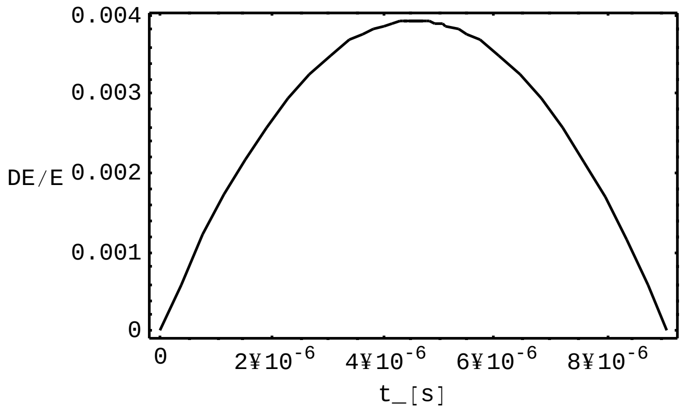

Fig. 8 – Normalized energy gain along the beam for $\vartheta_o = 20^o$ off crest , $\Delta\nu$= -3.3 KHz, $V_{gr}=V_b=10$ MV.

The non linear variation of the energy gain along the beam is due to the sinusoidal time dependency of the RF wave, with a maximum variation in the middle of the beam whose heigh depends from the injection phase ϑ_o. An additional condition useful to eliminate the arbitrary choice of ϑ_o is the request that DE/E=dV/V be lower than the desired energy spread in the middle of the beam i. e. at $t=t_b/2$. By solving the following relation:

$$\frac{DE}{E}(\vartheta_o) = \frac{V_a(\vartheta_o, t_b/2) - V_a(\vartheta_o, t_o)}{V_a(\vartheta_o, t_o)} \tag{6}$$

together with eq. (5), one can determine the optimal injection phase ϑ_o for a given energy spread as shown in Fig.9.

By increasing ϑ_o, infact , the beam experiences a more linear part of the RF wave but unfortunately at the same time the average energy gain of the beam decreases as shown in Fig.10. In addition the energy spread inside each bunch (intra-bunch energy spread) increases as shown in Fig.11 for two extreme cases, (depending also from the single bunch length , 2 cm in this example). There is anyway a possibility to partially compensate for the intra-bunch energy spread as proposed in [3], by running half of the linac with the opposite frequency shift $\Delta\omega$=-$\Delta\omega$ so that in the second half of the linac the beam phase slippage is backward with respect to the first half and the beam experience an opposite intra-bunch energy spread correlation that partially reduces this limitation. In our case anyway the intra-bunch energy spread is negligible compared to the total energy spread, as shown in Fig. 11 left plot.

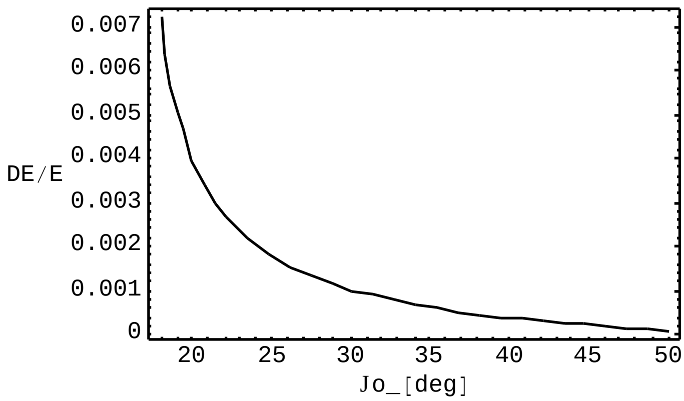

Fig. 9 - Normalized energy gain as a function of ϑ_o.

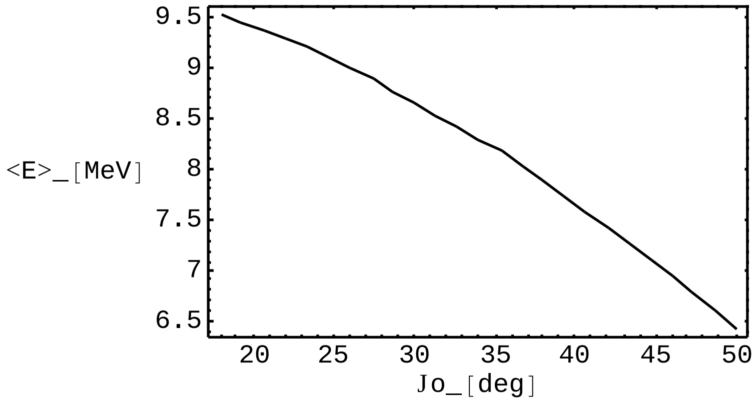

Fig. 10 –Average energy gain as a function of ϑ_o.

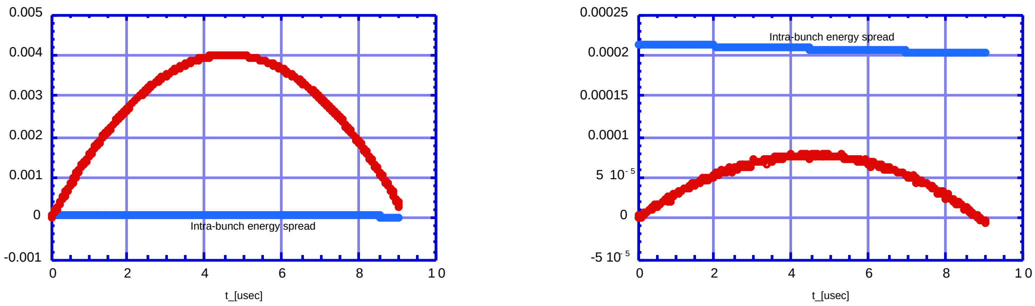

Fig. 11- Comparison between intra-bunch energy spread and normalized energy gain along the beam for two different cases: left plot ϑ_o =20˚ (possible working point), right plot ϑ_o =48˚, as computed by HOMDYN.

In conclusion, by solving simultaneously eq. (5) and (6) and taking into account the resulting average gain, one can determine the optimal injection phase and cavity frequency shift, provided that the intra-bunch energy spread doesn't overcome the required goal.

Considering a possible working point with $\vartheta_o = 20^o$, whose unperturbed result with DE/E= 0.004 are shown in Fig. 11 left plot, random charge fluctuations up to ± 10 % and nominal current fluctuation within ±5 % do not exceed the required energy spread. Since the power demand of this beam loading scheme is much lower than the previous case (<200 kW) and can be optimized by a proper choice of the Q_{ex}, we consider such a scheme very suitable for the μRLA.

4. — CONCLUSIONS

A preliminary analysis of possible beam loading compensation schemes for the muon recirculating linacs of the CERN neutrino factory has been discussed. A more systematic study of parameter sensitivity is in progress together with an analysis of the longitudinal beam dynamics in the Muon Cooling Channel, in order to optimize the longitudinal emittance matching with μRLA1

5. — ACKNOWLEDGEMENTS

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6. —REFERENCES

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7. —APPENDIX: THE HOMDYN MODEL

We studied the cavity-beam interaction by means of the code HOMDYN (see [4] and other references quoted there for a more detailed discussion). Originally the code was developed for single and multi-bunch dynamics computation in electron injectors devices, where transition from classical to relativistic dynamics takes place and space charge effects dominate the bunch transverse dynamics. Such a code relies on a simple self-consistent model that couples a current density description of beam evolution with the Maxwell equations in the normal modes expansion form. It takes into account single bunch space charge effects, beam loading of a long train of bunches, build-up effects of higher order modes and an on axis localized generator in order to describe the cavity re-filling from bunch to bunch passage. The code is of course suitable for a fully relativistic beam dynamics computation, especially when transient fields excitation plays an important role. An upgraded version of the code is under development including all the relevant physics of the Muon Cooling Channel.

We recall in this appendix the main equations of the model concerning the case under study, with some new features we added recently.

We represent the electric field in the cavity as a sum of normal orthogonal modes:

$$\mathbf{E}(t,\mathbf{r}) = \sum_n \Re e\left[A_n(t)\mathbf{e}_n(\mathbf{r})\right] \qquad \text{(A1)}$$

with complex amplitude

$$A_n(t) = \alpha_n(t)e^{i\omega_n t} = \frac{a_n(t)}{2}\, e^{i(\omega_n t+\phi_n(t))} \qquad \text{(A2)}$$

where $a_n(t)$ is a real amplitude. The field form factors $\mathbf{e}_n(\mathbf{r})$ are any normalized solution of the Helmholtz equation, satisfying the boundary condition $\hat{\mathbf{n}}\times\boldsymbol{e}_n = 0$ on the cavity surface and the solenoidal condition $\nabla\cdot\boldsymbol{e}_n = 0$ within the cavity volume. They can be computed by standard finite differences codes like SUPERFISH. In the following we will restrict our attention to the on axis longitudinal electric field components of TM modes.

The modes amplitude equations are:

$$\ddot{A}_n + \frac{\omega_n}{Q_n}\dot{A}_n + \omega_n^2 A_n = -\frac{1}{\varepsilon_o}\frac{d}{dt}\left[\int_V J(z,t)\cdot e_n(z)\,dv\right] \qquad \text{(A3)}$$

where as a driving current densities we consider the superposition of two terms $J = J_g + J_b$. The term J_g is a feeding sinusoidal current density, representing a point like power supply on the cavity axis located at z_g. The second term J_b represents the beam current density. The loaded quality factor Q accounts for the cavity losses.

We have included the possibility to change the rf pulse rising time τ_g, representing the power supply term as follows:

$$J_g\left(t,z_g\right) = \frac{J_g^o}{2i}\,\delta\left(z - z_g\right)\left(1 - e^{-\frac{t}{\tau_g}}\right)e^{i\left(\omega_1 t+\psi_1\right)} \qquad \text{(A4)}$$

where J_g^o is the generator strength, ω_1 and ψ_1 are the generator frequency and phase respectively.

The basic assumption in the description of the beam term consists in representing each bunch as a uniform charged cylinder, whose length L and radius R can vary under a self-similar evolution, i.e. keeping anyway uniform the charge distribution inside the bunch. Further details are reported in [4], we recall here that the beam current density term J_b can be written for each bunch as follows:

$$J_b(t,z) = \frac{q\beta_{bar}c}{L}\left[\eta\left(z - z_t\right) - \eta\left(z - z_h\right)\right] \qquad \text{(A5)}$$

where q is the bunch charge, β=v(t)/c, η is a step function and the indexes h, t refer to bunch head and tail positions respectively. The equations for the longitudinal motion of the bunch barycenter are simply:

$$\dot{z}_{bar} = \beta_{bar}c \qquad\qquad \dot{\beta}_{bar} = \frac{e}{m_o c\gamma_{bar}^3}\,E_z\left(t,z_{bar}\right) \qquad \text{(A6)}$$

Substituting the definition (A2) in the modes amplitude equations (A3), under the slowly varying envelope (SVEA) approximation $\frac{d\alpha_n}{dt} << \omega_n\alpha_n$ we can neglect the second order derivatives $\frac{d^2\alpha_n}{dt^2} << \omega_n^2\alpha_n$, and we obtain a first order amplitude equation for each mode:

$$\dot{\alpha}_n + \frac{\omega_n}{2Q_n}\left(1 + \frac{i}{2Q_n}\right)\alpha_n = -\frac{1}{2\omega_n\varepsilon_o}\left(1 + \frac{i}{2Q_n}\right)\frac{d}{dt}\left[\int J(z,t)\;e_n(z)dz\right]e^{-i\omega_n t} \qquad \text{(A7)}$$

The SVEA approximation supposes small field perturbations produced by any single bunch, that add up to give an envelope of any field mode slowly varying on the time scale of its period T. Because the characteristic cavity reaction time is of the order of $\tau = \frac{2Q}{\omega} >> T$ we fulfill the SVEA hypothesis. This approximation allows to reduce the numerical and analytical computing time.

The evolution of the field amplitude during the bunch to bunch interval is given by an analytical solution of equation (A7) with J_b=0 , which connects successive numerical integration applied during any bunch transit. Taking in to account the generator feeding current (A4), with a general initial condition $\alpha_n\left(t_o\right) = \alpha_n^o$, the analytical solution of (A7) is:

$$\alpha_n(t) = K_n \left\{ \frac{i\omega_1}{i\Omega_n + \frac{\omega_n}{2Q_n}\left(1+\frac{i}{2Q_n}\right)} \left[e^{-\left(i\Omega_n + \frac{\omega_n}{2Q_n}\left(1+\frac{i}{2Q_n}\right)\right)(t-t_o)} - 1 \right] e^{i\Omega_n t} + \frac{g - i\omega_1}{i\Omega_n + \frac{\omega_n}{2Q_n}\left(1+\frac{i}{2Q_n}\right) - g} \left[e^{-\left(i\Omega_n + \frac{\omega_n}{2Q_n}\left(1+\frac{i}{2Q_n}\right) - g\right)(t-t_o)} - 1 \right] e^{(i\Omega_n - g)t} \right\} + \alpha_n^o e^{-\frac{\omega_n}{2Q_n}\left(1+\frac{i}{2Q_n}\right)(t-t_o)} \tag{A8}$$

where $\Omega_n = \omega_1 - \omega_n$, $g = \frac{1}{\tau_g}$, and $K_n = \frac{1}{4i\varepsilon_o\omega_n}\left(1+\frac{i}{2Q_n}\right) J_g^o e^{i\psi_1} e_n(z_g)$.