



15 Febbraio 2001
LNF-01/009 (P)

Total Cross-Sections

R.M.Godbole¹

Center for Theoretical Studies, Indian Institute of Science, Bangalore 560 012, India

A. Grau²

Universitat de Granada, Granada, Spain

Giulia Pancheri³

INFN Frascati National Laboratories, I00044 Frascati, Italy

Y.N. Srivastava⁴

INFN, Physics Department, U. of Perugia, Perugia, Italy
Northeastern university, Boston, Mass.02115, USA

A unified approach to total cross-sections, based on the QCD contribution to the rise with energy, is presented for the processes pp , $p\bar{p}$, γp , $\gamma\gamma$, $e^+e^- \rightarrow hadrons$. For proton processes, a discussion of the role played by soft gluon summation in taming the fast rise due to mini-jets is presented. For photon-photon processes, a comparison with other models indicates the need for precision measurements in both the low and high energy region, likely only with measurements at future Linear Colliders.

1 Introduction

This paper describes our approach towards a QCD description of total cross-sections, examining both proton and photon processes. In recent years, data have become available for a complete set of processes. In addition to proton-proton and proton-antiproton, γp [1, 2] and $\gamma\gamma$ [3, 4, 5] total cross-sections are now known over a sufficiently extended energy range to allow for the observation of the rise with energy of all these cross-sections and one can aim to obtain a unified description. In what follows, we shall describe a QCD based approach for the energy dependence of the total cross-section, based on the Mini-jet model[6, 7] concerning the rise, and the use of soft gluon summation techniques, not only, to tame this rise[8] but also to obtain the early decrease. We start with purely hadronic cross-sections and then look in detail to photon-photon processes. For these, we discuss various models[9] and the precision needed for discrimination between them. Some of these models are based on the above mentioned QCD approach, others rely strictly on factorization[10] while some others are a mixture of both[11, 12, 13, 14, 15].

¹E-mail address: rohini@cts.iisc.ernet.in

²E-mail address: igräu@ugr.es

³E-mail address: pancheri@lnf.infn.it

⁴E-mail address: srivastava@pg.infn.it

2 A QCD Approach

The task of describing the energy behaviour of total cross-sections can be broken down into three parts:

- the rise
- the initial decrease
- the normalization

It has been known for quite some time, now, that the rise[16] can be obtained using the QCD calculable contribution from the parton-parton cross-section, whose total yield increases with energy[17]. It is not easy however to properly model this rise so as to reproduce correctly the slope, and we believe that soft gluon radiation, is the key to tame the rise due to hard gluon radiation. The distinction between soft and hard is so far arbitrary, but in this context we call soft gluons as those which do not undergo scattering against another parton in the colliding hadron, hard gluons as those which participate in the perturbative parton-parton scattering. This distinction corresponds to saying that the soft gluons have wavelength too long to see the content of the scattering area. It should be noted that the definition of the scattering area is energy dependent, since at very high energy, our physical picture of the scattering region is that of an expanding disk. We shall return to this point in later sections.

To see how the jet cross-section can contribute to the rise, consider the quantity

$$\sigma_{jet}(s; p_{tmin}) = \int_{p_{tmin}} dp_t \frac{d\sigma^{jet}}{dp_t} = \sum_{i,j,k,l} \int f_{i/a}(x_1) dx_1 \int f_{j/b}(x_2) dx_2 \int \frac{d\hat{\sigma}(ij \rightarrow kl)}{dp_t} dp_t \quad (1)$$

where the sum goes over all parton types. This quantity is a function of the minimum transverse momentum p_{tmin} of the produced jets and can be calculated using the, phenomenologically determined, parton densities for protons and photons. We show in Fig.1 the QCD jet cross-sections with $p_{tmin} = 2 \text{ GeV}$ obtained using GRV[18] densities for the three processes *proton – proton*, γ *proton* and $\gamma\gamma$, normalized so as to be compared with each other. The rise with energy for any fixed value of p_{tmin} is clearly observed. Notice that the rise is stronger for smaller values of p_{tmin} , whose value is a measure of the smallness of the x -values probed in the collision. Thus different densities, which correspond to different small- x behaviour, give different results for the same p_{tmin} . In all cases however, what one observes is that σ_{jet} rises too fast to describe σ_{tot} . For a unitary description, the jet cross-sections are embedded into the eikonal formalism[19], in which

$$\sigma_{pp(\bar{p})}^{tot} = 2 \int d^2\vec{b} [1 - e^{-\chi_I(b,s)} \cos(\chi_R)] \quad (2)$$

with $\chi = \chi_R + i\chi_I$, the eikonal function which contains both the energy and the transverse momentum dependence of matter distribution in the colliding particles, through the impact parameter distribution in b-space. The physical picture described by the

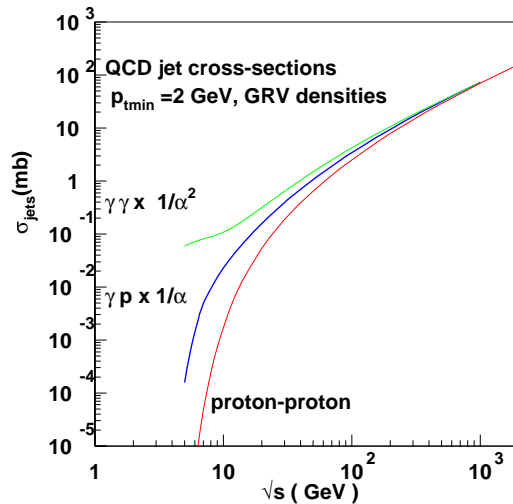


Fig. 1: Integrated jet cross-sections for $p_{tmin} = 2 \text{ GeV}$.

eikonal representation is that of two colliding hadronic disks, with partons providing the basic scattering constituents, distributed in the two hadrons according to an, *a priori*, unknown matter distribution. Thus, schematically, at any given c.m. energy \sqrt{s} and transverse distance, $\chi_I(b, s)$ should be obtained by integrating the differential cross-section over all subenergies and initial momenta of the colliding partons. In the Eikonal Minijet Model (EMM) one approximates $\chi_R \approx 0$ and calculates χ_I through the average number of collisions, from the definition of the inelastic cross-section. The simplest formulation, which incorporates the mini-jet assumption that it is the jet cross-section which drives the rise, is given by

$$2\chi_I(b, s) \equiv n(b, s) = A(b)[\sigma_{soft} + \sigma_{jet}] \quad (3)$$

so as to separate the calculable part, σ_{jet} , from the rest, to be parametrized. The normalization depends both on σ_{soft} and on the b -distribution. For the latter, the simplest hypothesis, is that it is given by the Fourier transform of the e.m. form factors of the colliding particles, i.e.

$$A_{ab}(b) \equiv A(b; k_a, k_b) = \frac{1}{(2\pi)^2} \int d^2\vec{q} e^{i\vec{q}\cdot\vec{b}} \mathcal{F}_a(q, k_a) \mathcal{F}_b(q, k_b) \quad (4)$$

where k_i are the scale factors entering into the form factors. The problem with such straightforward formulation for protons is that presently used gluon densities like GRV have such an energy dependence that it is not possible, with the above scheme, and

without further approximations, to simultaneously describe both the early rise and the high energy end. For instance, it is possible to describe the early rise, which takes place around $10 \div 30 \text{ GeV}$ for proton-proton and proton-antiproton scattering, using GRV densities and a $p_{tmin} \simeq 1 \text{ GeV}$, but then the cross-sections start rising too rapidly, whereas a $p_{tmin} \approx 2 \text{ GeV}$ can reproduce the Tevatron points[20, 21], but it misses the early rise. This can be seen in Fig.(2). To cure this difficulty, a QCD model for

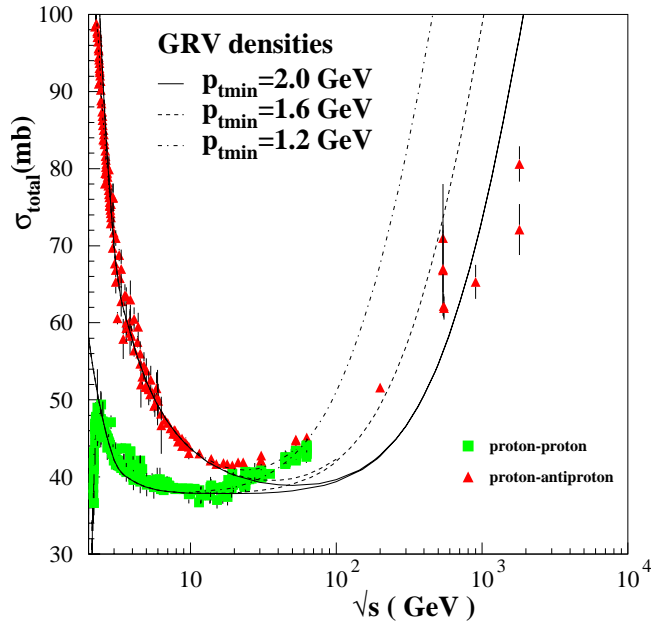


Fig. 2: Comparison between data[22] and predictions from the EMM (see text), for different minimum jet transverse momentum.

the impact parameter distribution of partons has been proposed[8]. In this model the function $A(b, s)$ tames the rise at high energy induced by σ_{jet} . Technically this happens because the proposed function, as the energy increases, is more and more suppressed at large b . Physically this reflects the fact that as the energy increases, the partons become more and more acollinear, due to soft gluon emission, and the probability of collisions is reduced. To obtain such effect, it was proposed that the normalized function $A(b, s)$ is obtained as the Fourier transform of the transverse momentum distribution of colliding parton pair. In the leading order, the parton pair has zero transverse momentum as each parton is emitted along the direction of its parent proton. But, at high energy, this cannot be true, and the colliding valence quarks will have a degree of acollinearity, which can be calculated using soft gluon resummation techniques. The distribution

thus obtained, labelled Bloch-Nordsieck distribution[23], is the following

$$A_{BN}(b, s) = \mathcal{F}[\mathcal{P}_{BN}] = \frac{e^{-h(b,s)}}{\int d^2\vec{b} e^{-h(b,s)}} \quad (5)$$

with

$$h(b, s) = \frac{8}{3\pi} \int_0^{q_{max}} \frac{dk}{k} \alpha_s(k^2) \ln\left(\frac{q_{max} + \sqrt{q_{max}^2 - k^2}}{q_{max} - \sqrt{q_{max}^2 - k^2}}\right) [1 - J_0(kb)] \quad (6)$$

and q_{max} is a slowly increasing function of energy, which depends on the kinematics of the process[24].

Such a total cross-section formulation exhibits a scale dependence through the QCD coupling constant α_s , basically

- through the well known and clearly defined p_t -dependence in parton-parton collisions, which we take to be $\alpha_s(p_{tmin}^2)$ with $p_{tmin} \geq 1 \div 2 \text{ GeV}$,
- through the k_t dependence of the initial colliding partons, obtained from soft gluon emission.

This latter dependence needs to be clarified further. The integral in eq.(6) extends down to $k_t = 0$ and one needs to model the infrared behaviour of α_s in order to carry through the quantitative application of this Bloch-Nordsieck ansatz. It clearly follows from eqs.(5,6) that the more singular α_s is as $k_t \rightarrow 0$, the larger the function $h(b, s)$ is at large b -values and the faster is the fall of the function $e^{-h(b,s)}$ with increasing b . On the other hand, as the energy of the colliding particles increases, q_{max} is larger causing the function $h(b, s)$ to be larger, producing a suppression at high energy, for large b . The overall result is that the behaviour of $A(b, s)$ at large b is determined by

- higher \sqrt{s} producing larger q_{max} and more emission
- singular behaviour of α_s in the infrared region producing also many more soft gluons

The above considerations can be made quantitative, by introducing an average over the parton densities and assuming an approximate factorization between the transverse and the longitudinal degrees of freedom, i.e.

$$n(b, s) \approx n_{soft} + n_{hard} \approx n_{soft} + A_{BN}(b, < q_{max} >) \sigma_{jet} \quad (7)$$

The following average expression for $< q_{max} >$ was proposed in [8],

$$M \equiv < q_{max}(s) > = \frac{\sqrt{s} \sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \sqrt{x_1 x_2} \int_{z_{min}}^1 dz (1-z)}{2 \sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \int_{z_{min}}^1 dz} \quad (8)$$

with $z_{min} = 4p_{tmin}^2/(sx_1x_2)$ and $f_{i/a}$ the valence quark densities used in the jet cross-section calculation.

M establishes the scale which, on the average, regulates soft gluon emission in the collisions, whereas p_{tmin} provides the scale which characterizes the onset of hard parton-parton scattering. For any parton parton subprocess characterized by a p_{tmin} of $1 \div 2$ GeV, M has a logarithmic increase at reasonably low energy values and an almost constant behaviour at high energy[8]. In Fig.(3) we plot the quantity M as a function of \sqrt{s} .

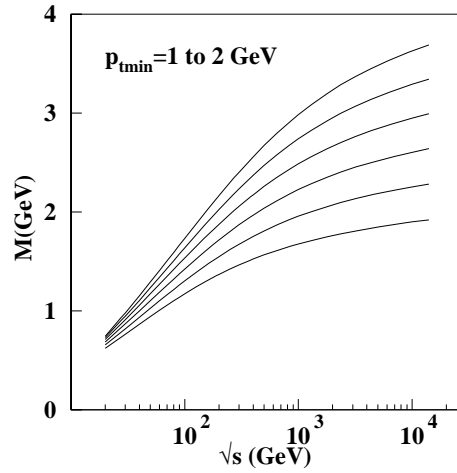


Fig. 3: The average maximum energy allowed to soft gluon emission in jet production as a function of \sqrt{s} for various p_{tmin} values and with GRV94 densities

With the quantity q_{max} thus determined, one can now calculate the Bloch-Nordsieck distribution function $A(b,s)$, using different ansätze for the functional expression for α_s in the k_t going to zero limit. It must be noticed that what enters in all calculations is not so much α_s as such, but rather its integral over the infrared region. Thus in principle from a phenomenological point of view, even a singular α_s can be used, provided it is integrable. Two models have been looked in detail so far, the frozen[25, 26] α_s model, in which

$$\alpha_s(k_t^2) = \frac{b}{\log(a^2 + k_t^2/\Lambda^2)} \quad (9)$$

and the singular[27] α_s model, in which

$$\alpha_s(k_t^2) = \frac{b'}{\log(1 + (k_t^2/\Lambda^2)^{2p})} \xrightarrow{k_t \rightarrow 0} \frac{1}{k_t^{2p}} \quad (10)$$

is singular, but integrable for $p < 1$. One can now calculate the overlap function $A_{BN}(b)$ for frozen and singular α_s cases and compare it with the Form Factor model, as shown in Fig. 4.

The different behaviour of $A(b, s)$ in the large b region, changes the energy behaviour of the average number of collisions in this region, with the result that the cross-section

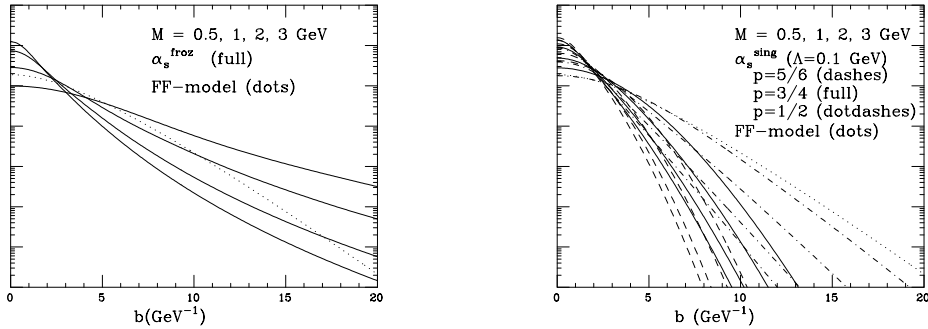


Fig. 4: Comparison of the $A(b,s)$ for the Form factor model with that in BN model, for frozen α_s (left panel) and for singular α_s , for various values of the parameter p (right panel).

indeed rises much less rapidly when the singular α_s model is used. We compare the three different models for the case of proton-proton and proton-antiproton scattering in Fig.(5), where the early decrease and normalization have been described, in all three cases, using the Form factor model and a parametrization of low energy data with 5 parameters. The rise on the other hand is described using σ_{jet} and the three different b -distribution functions just described. Notice that, for the three models, we have used different values of p_{tmin} in the jet cross-section, since we wanted to have curves passing through the high energy points. We see that the EMM model for protons using current parton densities like GRV does not reproduce well the initial rise with energy, and the same is also true for the frozen α_s model. For a comparison, we also show the QCD inspired description, labelled BGHP, used in the Aspen model [10] to predict photon cross-sections through factorization. What this exercise shows is that the energy behaviour of the total cross-section is determined both by soft gluon emission, and by hard gluons. Work is in progress to determine whether soft gluon emission, which produces a decrease of the cross-section with energy, plays a role also in the initial low energy region, the so called Regge region.

3 Photon processes

We now turn to discuss processes with photons, like photo-production or $e^+e^- \rightarrow hadrons$, a process which, at high energy, is dominated by $\gamma\gamma \rightarrow hadrons$. Again the main characteristics of the photonic cross-sections, are the overall normalization, the rise past $\sqrt{s} \approx 10 \text{ GeV}$, and an initial decrease. For $\gamma\gamma$, the errors on the overall normalization at low energy are so large that the initial decrease is hard to parametrized,

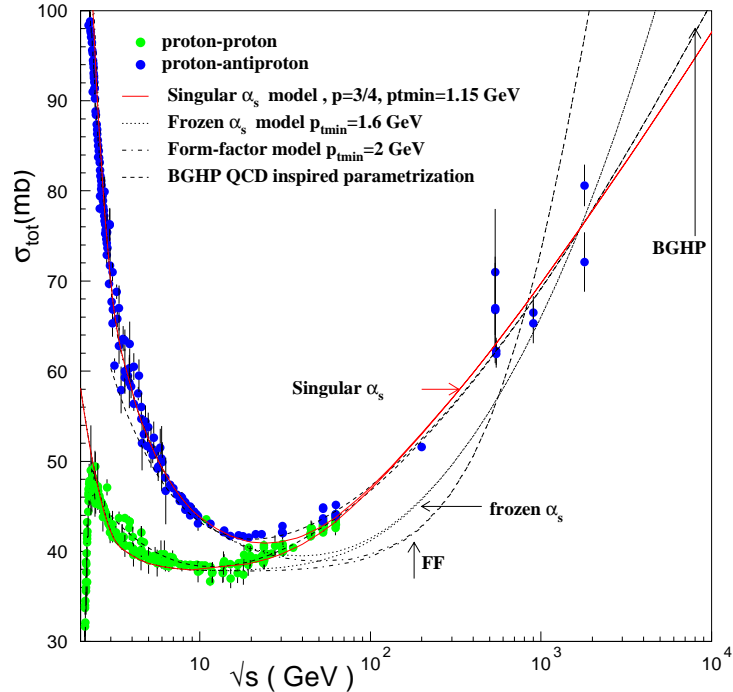


Fig. 5:

not so for γp . In this paper, we do not address the question of the low energy behaviour, which is simply obtained from the hadronic processes, through scaling hypotheses. For the normalization, one uses Vector Meson Dominance and Quark Parton Model ideas, by defining a quantity $P_{had} = \sum_{V=\rho,\omega,\phi} \frac{4\pi\alpha}{f_V^2}$ which represents the probability that a photon exhibits a hadronic content. The eikonal formulation then has to be modified to take this into account [28] and is written as [29]

$$\sigma_{tot} = 2P_{had} \int d^2\vec{b} [1 - e^{-\chi_I(b,s)}] \quad (11)$$

with $2\chi_I(b,s) = A(b)[\sigma_{soft} + \sigma_{jet}/P_{had}]$ with the notation of the previous section. In this approach, $P_{had}^{\gamma\gamma} = (P_{had}^{\gamma p})^2 \approx (1/240)^2$, at $\sqrt{s_{\gamma\gamma}} \approx 100 \text{ GeV}$. This factorization ansatz seems to work reasonably well. The soft part of the cross-section, which defines the normalization, is obtained from the number of quarks in the colliding particles, through a simple scaling factor, i.e. $\sigma_{soft}^{\gamma p} = \frac{2}{3}\sigma_{soft}^{pp}$ and $\sigma_{soft}^{\gamma\gamma} = \frac{2}{3}\sigma_{soft}^{\gamma p}$. As for the rise, the eikonal minijet model of course uses jet cross-sections with the appropriate photon densities. We should mention that there are at present at least two other models, which obtain not only the rise, but the entire cross-section from the proton cross-section, using factorization [10, 11] or a Regge-Pomeron type behaviour [12, 13]. We show in Fig.(6) old [22] and recent [30] data for γp total cross-section, together with BPC data

extrapolated [31] from DIS[32], compared with a band which represents the predictions from the EMM model, using two different formulations[33] with GRS[34] and GRV[18] type densities for the photonic partons. This band is compatible with the predictions for

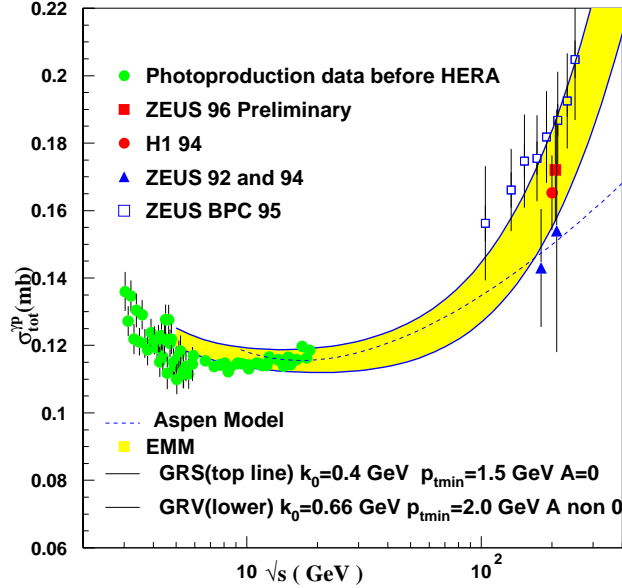


Fig. 6:

$\gamma\gamma$, for which we show in Fig. (7), our favourite curve for the recent data from LEP[4]. The b-dependence in these figures is obtained from an intrinsic transverse momentum description [6], using the experimentally measured transverse momentum distribution of the photonic partons [35]. Work is in progress to apply to photonic processes the Bloch-Nordsieck approach described in the previous section.

4 Precision needed to discriminate between models

It is instructive to compare different models among each other, also in view of the proposed electron-positron Linear Collider which should reach very high c.m. energies, with $\sqrt{s_{\gamma\gamma}}$ potentially as high as 500-700 GeV (if operated in the photon collider mode). This comparison between various model predictions is shown in Fig.(8). The stars for large energy values correspond to pseudo-data points, and illustrate a possible extrapolation of the measured cross-section with realistic errors[36].

In Table 1 we show total $\gamma\gamma$ cross-sections for three models of the "proton-is-like-the-photon" type. The last column shows the 1σ level precision needed to discriminate between Aspen[10] and BSW[11] models. The model labelled DL is obtained from

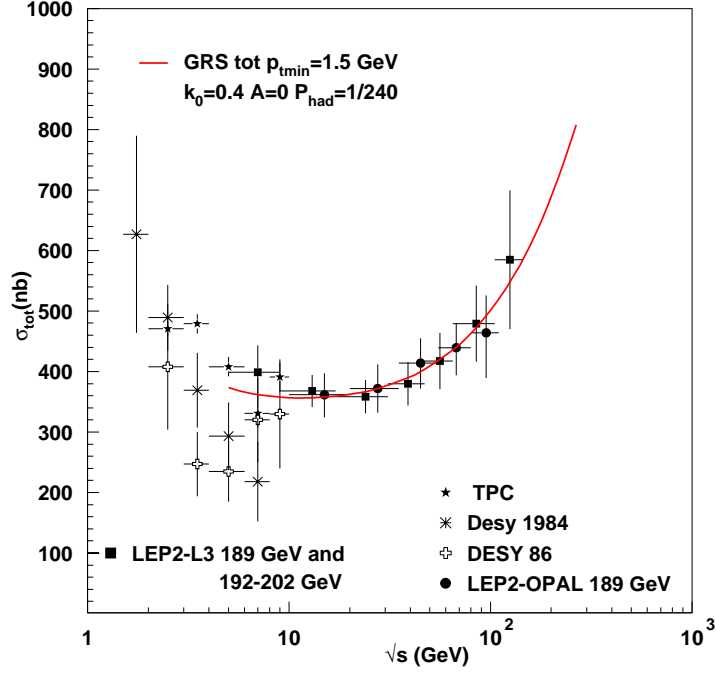


Fig. 7:

Table 1 Total $\gamma\gamma$ cross-sections and required precision for models based on factorization

$\sqrt{s_{\gamma\gamma}}(GeV)$	Aspen	BSW	DL	1σ
20	309 nb	330 nb	379 nb	7%
50	330 nb	368 nb	430 nb	11%
100	362 nb	401 nb	477 nb	10%
200	404 nb	441 nb	531 nb	9%
500	474 nb	515 nb	612 nb	8%
700	503 nb	543 nb	645 nb	8%

Regge/Pomeron exchange with parameters from Donnachie and Landshoff[12] and factorization at the residues. The difference between DL and either Aspen or BSW is bigger than between Aspen and BSW at each energy value.

Similar tables can be drawn for distinguishing among different formulations of the

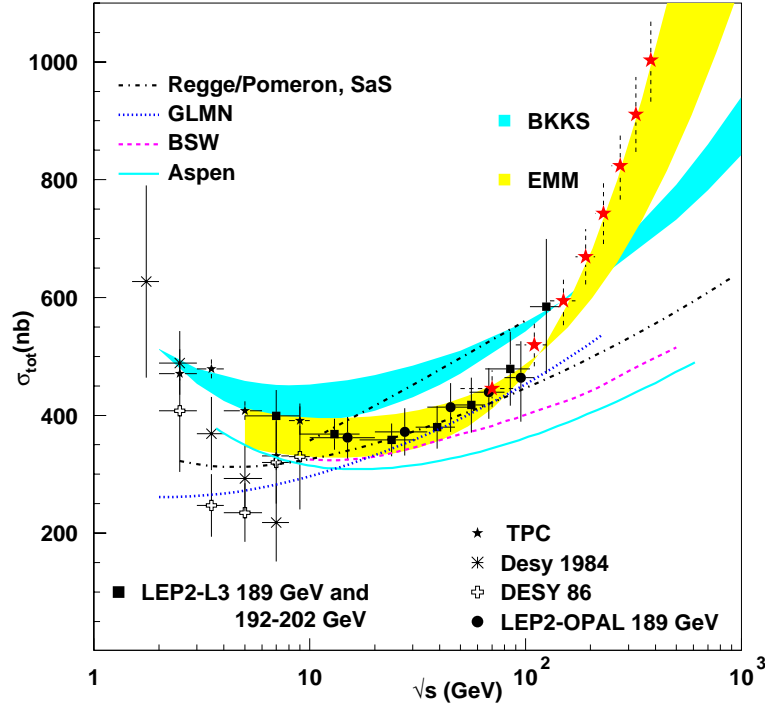


Fig. 8: Data and predictions from SaS[13], GLMN[14], BSW[11],[10], BKKS[15] and the EMM model, described in the text

Eikonal Minijet Model or between the EMM and other QCD based models, like BKKS [15]. Part of the problem in comparing data with theoretical expectations and hence make predictions for future machines, lies in the fact that the total cross-section for photon processes is difficult to measure, and theoretically difficult to define. A less uncertain quantity is actually the e^+e^- cross-section into hadrons, which is the quantity from which the $\gamma\gamma$ cross-sections are extracted. Thus it may be more appropriate to fold different model predictions for $\gamma\gamma$ cross-sections, with the photon distribution in the electrons taking into account the (anti)tagging of the electrons [37] and compare the resultant uncertainties. We show one such folding, using the Weizsäcker Williams approximation, with two different models for $\sigma_{\gamma\gamma}^{tot}$, EMM with GRV densities and the Aspen model, in Fig.(9). We see that as a result of the folding, the difference between different model predictions of a factor 2 or so get reduced to about 30%. Given the expected experimental errors at the future linear colliders it would be possible to discriminate between different theoretical models for $\gamma\gamma$ cross-sections even at a e^+e^- collider [36].

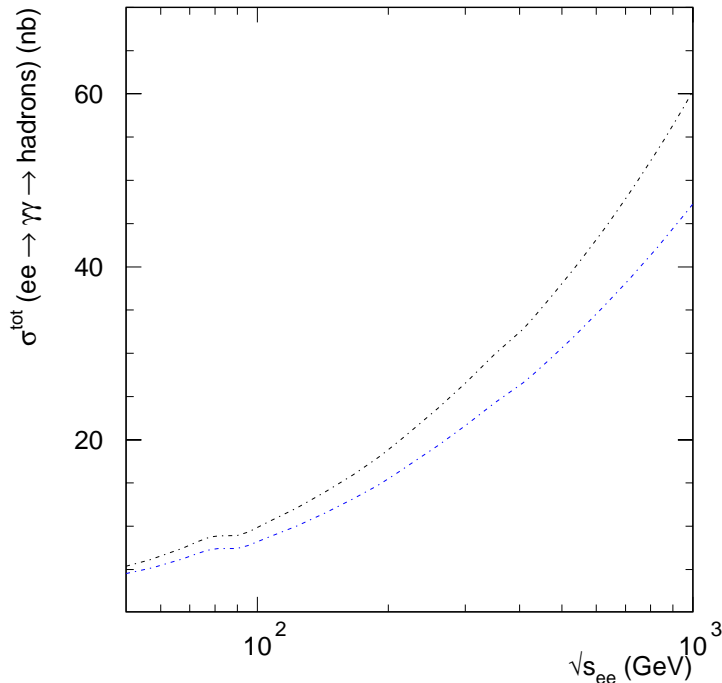


Fig. 9: Eikonal Minijet Model and Aspen Model, in $e^+e^- \rightarrow \text{hadrons}$

5 Conclusion

A QCD approach to the calculation of total cross-section based on the eikonal formulation, shows that QCD can describe the rise for both, photon and proton, processes. For protons there exist extensive and very accurate data, and it is seen that a treatment of both hard and soft gluon emission needs to be used in order to reproduce the rise at intermediate and high energies. Photon processes are still characterized by large experimental errors and the Eikonal Minijet Model, with only hard gluon scattering and fixed intrinsic transverse momentum for the photon, appears adequate. However, there exist models which are consistent with the current data within the experimental errors and which predict a much slower rise at higher energies. A more complete treatment would be required for data with much smaller experimental errors, likely only with measurements at the future Linear Colliders.

6 Acknowledgments

We acknowledge support from the EEC, TMR contract 98-0169.

References

- [1] H1 Collaboration, Aid, S., et al.,: *Zeit. Phys. C* **69** (1995) 27; , **hep-ex/9405006**.
- [2] ZEUS Collaboration, Derrick, M., et al., *Phys. Lett. B* **293**, 465 (1992); Derrick, M., et al., *Zeit. Phys. C* **63**, 391 (1994).
- [3] L3 Collaboration, Paper 519 submitted to *ICHEP'98*, Vancouver, July 1998; Acciarri, M., et al., *Phys. Lett. B* **408**, 450 (1997); L3 Collaboration, Csilling, A., *Nucl.Phys.Proc.Suppl. B* **82**, 239 (2000); L3 collaboration, L3 Note 2548, Submitted to the *International High Energy Physics Conference*, Osaka, August 2000.
- [4] L3 Collaboration, Acciarri, M., et al., **CERN-EP 2001-012**, submitted for publication.
- [5] OPAL Collaboration. Waeckerle, F., *Multiparticle Dynamics 1997*, *Nucl. Phys. Proc. Suppl.B* **71**, 381 (1999) edited by G. Capon, V. Khoze, G. Pancheri and A. Sansoni; Söldner-Rembold, S., **hep-ex/9810011**, Proceedings of the *ICHEP'98*, Vancouver, July 1998; Abbiendi, G., et al., *Eur.Phys.J.C* **14**, 199 (2000), **hep-ex/9906039**.
- [6] Corsetti, A., Godbole, R.M., and Pancheri, G., *Phys.Lett. B* **435**, 441 (1998); **hep-ph/9807236**.
- [7] Godbole, R.M., and Pancheri, G., **hep-ph/0010104**, To appear in EPJC.
- [8] A. Grau, G. Pancheri and Y.N. Srivastava, *Phys. Rev.* **D60**, 114020 (1999), **hep-ph/9905228**.
- [9] Godbole, R.M., Grau, A., and Pancheri, G., *Nucl.Phys.Proc.Suppl.B* **82**, 246 (2000), **hep-ph/9908220**. In the proceedings of International Conference on the Structure and Interactions of the Photon (Photon 99), Freiburg, Germany, 23-27 May 1999.
- [10] Block, M.M., Gregores, E.M., Halzen, F., and Pancheri, G., *Phys.Rev. D* **58**, 17503 (1998); *Phys.Rev. D* **60**, 54024 (1999), **hep-ph/9809403**.
- [11] Bourrely, C., Soffer, J., and Wu, T.T., *Mod.Phys.Lett. A* **15**, 9 (2000).
- [12] Donnachie, A., and Landshoff, P.V., *Phys. Lett. B* **296**, 227 (1992), **hep-ph/9209205**.
- [13] Schuler, G.A., and Sjöstrand, T., *Z.Phys.C* **73**, 677 (1997), **hep-ph/9605240**.
- [14] Gotsman, E., Levin, E., Maor, U., and Naftali, E., *Eur. Phys. J. C* **14**, 511 (2000), **hep-ph/0001080**.
- [15] Badelek, B., Krawczyk, M., Kwiecinski, J., and Stasto, A.M., **hep-ph/0001161**.
- [16] Cline, D., Halzen, F., and Luthe, J., *Phys. Rev. Lett.* **31**, 491 (1973).
- [17] G. Pancheri and C. Rubbia, *Nucl. Phys.* **A 418**, 117c (1984). T.Gaisser and F.Halzen, *Phys. Rev. Lett.* **54**, 1754 (1985). G.Pancheri and Y.N.Srivastava, *Phys. Lett.* **B 158**, 402 (1986).
- [18] Glück, M., Reya, E., and Vogt, A., *Zeit. Physik C* **67**, 433 (1994); Glück, M., Reya, E., and Vogt, A., *Phys. Rev. D* **46**, 1973 (1992) 1973.
- [19] L. Durand and H. Pi, *Phys. Rev. Lett.* **58**, 58 (1987). A. Capella, J. Kwiecinsky, J. Tran Thanh, *Phys. Rev. Lett.* **58**, 2015 (1987). M.M. Block, F. Halzen, B. Margolis, *Phys. Rev.* **D 45**, 839 (1992). A. Capella and J. Tran Thanh Van, *Z. Phys. C* **23**, 168 (1984). P. l'Heureux, B. Margolis and P. Valin, *Phys. Rev. D* **32**, 1681 (1985).
- [20] CDF Collaboration, Abe, F., et al *Phys. Rev. D* **50**, 5550 (1994).

- [21] E710 Collaboration (N.A. Amos et al.): *Phys.Rev.Lett.* **63** (1989) 2784;
- [22] Review of Particle Physics: *EPJ* **C15** (2000) 1;
- [23] G.Pancheri-Srivastava and Y.N. Srivastava: *Pys. Rev* **D15** (77) 2915;
- [24] P. Chiappetta and M. Greco: *Nucl. Phys.* **B199** (82) 77;
- [25] F. Halzen, A.D. Martin and D.M. Scott: *Phys. Rev.* **D25** (1982) 754; ; *Phys.Lett.* 112B 1982 160
- [26] G. Altarelli, R.K. Ellis, M. Greco, and G. Martinelli: *Nucl. Phys.* **B246** (1984) 12;
- [27] J.L. Richardson: *Phys.Lett.* **B82** (79) 272; .
- [28] J.C. Collins and G.A. Ladinsky, *Phys. Rev.* **D 43**, 2847 (1991).
- [29] R.S. Fletcher , T.K. Gaisser and F. Halzen, *Phys. Rev.* **D 45**, 377 (1992); erratum *Phys. Rev.* **D 45**, 3279 (1992).
- [30] ZEUS Collaboration (C. Ginsburg et al.), Proc. 8th International Workshop on Deep Inelastic Scattering, April 2000, Liverpool, to be published in World Scientific.
- [31] B. Surrow, DESY-THESIS-1998-004. A. Bornheim, In the Proceedings of the *LISHEP International School on High Energy Physics*, Brazil, 1998, hep-ex/9806021.
- [32] Breitweg, J., et al., ZEUS collaboration, **DESY-00-071**, **hep-ex/0005018**.
- [33] Godbole, R.M., Grau, A., and Pancheri, G., **hep-ph/0101321**, To appear in the proceedings of Photon 2000, Ambelside, U.K., Aug. 2000.
- [34] Glück, M., Reya, E., and Schienbein, I., *Phys.Rev. D* **60**, 054019 (1999); Erratum, *ibid.* **62**, 019902 (2000).
- [35] ZEUS collaboration, Derrick, M., et al., *Phys. Lett.B* **354**, 163 (1995).
- [36] Pancheri, G., Godbole, R.M., and De Roeck, A., **LC-TH-2001-030**, *In preparation*.
- [37] See for example, Drees, M. and Godbole, R.M., *Phys. Rev. D* **50**, 3124 (1994), **hep-ph/9403229**; **hep-ph 9506241**, In the *Proceedings of PHOTON-95, incorporating the Xth International workshop on $\gamma\gamma$ collisions and related processes*, Sheffield.