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TOWARDS THE COMPLETE N=2 SUPERFIELD BORN-INFELD ACTION WITH PARTIALLY BROKEN N=4 SUPERSYMMETRY

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Abstract

We propose a systematic way of constructing $N = 2$, $d = 4$ superfield Born-Infeld action with a second nonlinearly realized $N = 2$ supersymmetry. The latter, together with the manifest $N = 2$ supersymmetry, form a central-charge extended $N = 4$, $d = 4$ supersymmetry. We embed the Goldstone-Maxwell $N = 2$ multiplet into an infinite-dimensional off-shell supermultiplet of this $N = 4$ supersymmetry and impose an infinite set of covariant constraints which eliminate all extra $N = 2$ superfields through the Goldstone-Maxwell one. The Born-Infeld superfield Lagrangian density is one of these composite superfields. The constraints can be solved by iterations to any order in the fields. We present the sought $N = 2$ Born-Infeld action up to the 10th order. It encompasses the action found earlier by Kuzenko and Theisen to the 8th order from a self-duality requirement. This is a strong indication that the complete $N = 2$ Born-Infeld action with partially broken $N = 4$ supersymmetry is also self-dual.

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1 Introduction

For many reasons it is important to know off-shell superfield actions of supersymmetric extensions of the Born-Infeld (BI) theory [1]-[6] and to understand the geometry behind them. One of the basic sources of interest in such actions is that their notable subclass, the BI actions with a hidden extra nonlinearly realized supersymmetry, provides a manifestly worldvolume supersymmetric description of various Dp-branes in a static gauge [2]. As was demonstrated in [3] (see also [4]), this sort of BI actions supplies a nice example of systems with partial spontaneous breaking of global supersymmetry (PBGS). The covariant superfield gauge strengths in terms of which such actions are formulated, can be identified with the Goldstone superfields supporting a nonlinear realization of some underlying extended supersymmetry. The manifest supersymmetry of the given BI action is the linearly realized half of the underlying supersymmetry.

At present, the Goldstone superfield BI actions are known in a closed explicit form only for the 1/2 PBGS options $N = 2 \to N = 1$ in $d = 4$ [3, 4] and $d = 3$ [5]. They amount to the worldvolume actions of the spacetime-filling D3- and D2-branes in a fixed gauge and involve, respectively, the $N = 1$, $d = 4$ and $N = 1$, $d = 3$ vector multiplets as the Goldstone ones.

In [7, 8] it was suggested that, by analogy with the construction of ref. [3], $N = 2$, $d = 4$ vector multiplet could serve as the Goldstone multiplet for the 1/2 spontaneous breaking of $N = 4$, $d = 4$ supersymmetry. The associated Goldstone superfield action should be a particular representative of the $N = 2$ supersymmetric BI actions, such that it possesses a hidden $N = 2$ supersymmetry in parallel with the manifest one. By inspection of the component field content of the $N = 2$ vector multiplet, it is obvious that such action should describe a D3-brane in $D = 6$, with the scalar component fields parameterizing two transverse directions. The $N = 2$ BI action constructed in [6] reveals no hidden extra supersymmetry [9] and so it can be regarded merely as a part of the hypothetical genuine $N = 4 \to N = 2$ BI action.

In recent papers [10, 11] we showed how the full set of superfield equations describing the $N = 2 \to N = 1$ BI system in $d = 3$ and the $N = 2 \to N = 1$, $N = 4 \to N = 2$ and $N = 8 \to N = 4$ ones in $d = 4$ can be deduced from the customary nonlinear realizations approach applied to the relevant PBGS patterns. A characteristic common feature of these superfield systems is that the pure BI part of the corresponding bosonic equations always appears in a disguised form in which the Bianchi identity for the Maxwell field strength and the dynamical equation are mixed in a tricky way. On the other hand, the equations for the scalar fields (in the $N = 4 \to N = 2$ and $N = 8 \to N = 4$ cases in which the Goldstone vector multiplets include such fields) come out in a form explicitly derivable from the standard static-gauge Nambu-Goto actions. The disguised form of the BI equations can be split into the kinematical and dynamical parts by a nonlinear equivalence redefinition of the corresponding bosonic component field. As was demonstrated in [11] for the $N = 4 \to N = 2$ example, the superfield version of this redefinition is an equivalence transformation from the original basic $N = 2$ Goldstone superfield to the standard $N = 2$ Maxwell superfield strength. It enables one to divide the original system of superfield equations into the pure kinematical and dynamical parts which are separately invariant under the original hidden supersymmetry, and to construct the correct $N = 2$ superfield action yielding the dynamical part as the equation of motion. In this way we reconstructed the $N = 2$ BI action with the hidden extra $N = 2$ supersymmetry up to the sixth order in $N = 2$ Maxwell superfield strength.

Although this approach can in principle be applied to restore, step by step, the $N = 4 \to$

$N = 2$ BI action to any order, it would be desirable to develop a more direct way for constructing such an action, similar to the method which was used in [3] in the $N = 2 \to N = 1$, $d = 4$ case (see also [4] and [5]). It deals with the linear Maxwell superfield strength from the very beginning, and it is based upon completing the latter to a linear off-shell supermultiplet of the full supersymmetry by adding a few extra superfields of the unbroken supersymmetry. After imposing a nonlinear covariant constraint on the superfields of the linear supermultiplet one ends up with a nonlinear realization of the full supersymmetry in terms of the Maxwell superfield strength as the only independent Goldstone superfield. In all the cases studied so far, both the BI superfield Lagrangian density and the Goldstone-Maxwell superfield strength belong to the linear supermultiplet just mentioned.

In this paper we propose a generalization of the method of [3, 4] to the $N = 4 \to N = 2$ BI case. Two essentially novel closely related points of our construction, as compared to the previously elaborated cases, are as follows: (i) We start from a proper extension of $N = 4$, $d = 4$ Poincaré superalgebra by *a complex central charge*, in order to gain a geometric place for the complex bosonic $N = 2$ Maxwell superfield strength $\mathcal{W}$ as the Goldstone superfield [11]; (ii) The minimal linear $N = 4$ supermultiplet into which one can embed $\mathcal{W}$ necessarily involves an *infinite* tower of chiral $N = 2$ superfields of growing dimension interrelated by the central charge generators. As in the previous cases, the chiral $N = 2$ Lagrangian density of the $N = 4 \to N = 2$ BI theory is one of these extra superfields, but in order to express it in terms of $\mathcal{W}, \bar{\mathcal{W}}$ one is led to impose an *infinite* set of the covariant constraints which eliminate all the extra superfields as well. We give these constraints in the explicit form and solve them by iterations, in order to restore the correct $N = 4 \to N = 2$ BI action up to the 10th order in $\mathcal{W}, \bar{\mathcal{W}}$. Surprisingly, up to the 8th order it reproduces the $N = 2$ action found earlier to this order in [9] from the requirements of self-duality and invariance under a shift symmetry of $\mathcal{W}, \bar{\mathcal{W}}$ (in our approach it comes out as the symmetry generated by central charges). This is an indication that the requirements of [9] are equivalent to the single demand of hidden $N = 2$ supersymmetry. As a consequence, the full $N = 4 \to N = 2$, $d = 4$ BI action is expected to be self-dual.

2 Getting started

The basic object we shall deal with is a complex scalar $N = 2$ off-shell superfield strength $\mathcal{W}$. It is chiral and satisfies one additional Bianchi identity

$$(a)\ \bar{D}_{\dot\alpha i}\mathcal{W} = 0\ ,\ D^i_\alpha\bar{\mathcal{W}} = 0\ ,\quad (b)\ D^{ik}\mathcal{W} = \bar{D}^{ik}\bar{\mathcal{W}}\ . \tag{2.1}$$

Here

$$D^i_\alpha = \frac{\partial}{\partial\theta^\alpha_i} + i\bar\theta^{\dot\alpha i}\partial_{\alpha\dot\alpha}\ ,\quad \bar{D}_{\dot\alpha i} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha i}} - i\theta^\alpha_i\partial_{\alpha\dot\alpha}\ ,\quad \left\{D^i_\alpha, \bar{D}_{\dot\alpha j}\right\} = -2i\delta^i_j\partial_{\alpha\dot\alpha}\ , \tag{2.2}$$

$$D^{ij} \equiv D^{\alpha\, i}D^j_\alpha\ ,\quad \bar{D}^{ij} \equiv \bar{D}^i_{\dot\alpha}\bar{D}^{\dot\alpha\, j}\ . \tag{2.3}$$

Due to the basic constraints (2.1) the superfields $\mathcal{W}, \bar{\mathcal{W}}$ obey the following useful relations:

$$D^4\mathcal{W} = -\frac{1}{2}\Box\bar{\mathcal{W}}\ ,\quad \bar{D}^4\bar{\mathcal{W}} = -\frac{1}{2}\Box\mathcal{W}\ , \tag{2.4}$$

where

$$D^4 \equiv \frac{1}{48} D^{\alpha i} D^j_\alpha D^\beta_i D_{\beta j} \ , \quad \bar{D}^4 = \overline{(D^4)} \equiv \frac{1}{48} \bar{D}^i_{\dot\alpha} \bar{D}^{\dot\alpha j} \bar{D}_{\dot\beta i} \bar{D}^{\dot\beta}_j \ , \quad \Box \equiv \partial_{\alpha\dot\alpha} \partial^{\alpha\dot\alpha} \ . \tag{2.5}$$

The irreducible field content of $\mathcal{W}$, $\bar{\mathcal{W}}$ defined by (2.1) is the off-shell $N = 2$, $d = 4$ abelian vector multiplet. It consists of a $SU(2)$-singlet complex scalar field, a Maxwell field strength, a real $SU(2)$-triplet of the scalar bosonic auxiliary fields and a $SU(2)$-doublet of Weyl spinors, i.e. a total of $(8 + 8)$ independent components.

In order to be able to treat $\mathcal{W}, \bar{\mathcal{W}}$ as the Goldstone superfields associated with the PBGS pattern $N = 4 \to N = 2$ in $d = 4$ we should define, first of all, an appropriate modification of the standard $N = 4$, $d = 4$ Poincaré superalgebra. It should involve a complex bosonic generator to which one could put in correspondence $\mathcal{W}, \bar{\mathcal{W}}$ as the Goldstone superfields. As was shown in [11], the proper extension is given by the following superalgebra:

$$\begin{aligned} &\left\{ Q^i_\alpha, \bar{Q}_{\dot\alpha j} \right\} = 2\delta^i_j P_{\alpha\dot\alpha} \ , \quad \left\{ S^i_\alpha, \bar{S}_{\dot\alpha j} \right\} = 2\delta^i_j P_{\alpha\dot\alpha} \ , \\ &\left\{ Q^i_\alpha, S^j_\beta \right\} = 2\varepsilon^{ij} \varepsilon_{\alpha\beta} Z \ , \quad \left\{ \bar{Q}_{\dot\alpha i}, \bar{S}_{\dot\beta j} \right\} = -2\varepsilon_{ij} \varepsilon_{\dot\alpha\dot\beta} \bar{Z} \ , \quad (i, j = 1, 2) \ , \end{aligned} \tag{2.6}$$

with all other (anti)commutators vanishing. It was chosen in [11] as the starting point for constructing a nonlinear realization of the PBGS pattern $N = 4 \to N = 2$, with the generators $Q^i_\alpha, \bar{Q}_{\dot\alpha j}$, $P_{\alpha\dot\alpha}$ corresponding to the unbroken $N = 2$ supersymmetry and the remaining ones to the spontaneously broken symmetries. Obviously, a linear realization of the same version of PBGS should proceed from the same superalgebra. The necessary presence of the complex central charge Z in the anticommutators of the broken and unbroken $N = 2$ spinor generators is the crucial difference of the case under consideration from the PBGS case $N = 2 \to N = 1$ in $d = 4$ [3] and its $d = 3$ counterpart [5]. In the latter two cases one proceeds from the $N = 2$ Poincaré supersymmetries in $d = 4$ and $d = 3$ with no central charge generators; the elementary Goldstone superfields are the fermionic ones which are eventually identified with the corresponding $N = 1$ Maxwell superfield strengths. Note that the superalgebra (2.6) is a $d = 4$ notation for the $N = (2, 0)$ (or $N = (0, 2)$) Poincaré superalgebra in $D = 6$. In what follows, we shall not actually need to resort to the $D = 6$ interpretation. We shall entirely deal with the $N = 2$, $d = 4$ superfields, viewing (2.6) as a central-charge extension of the standard $N = 4$ Poincaré supersymmetry in $d = 4$.

3 Embedding N=2 vector multiplet into a linear N=4 multiplet

In [11], starting from a nonlinear realization of $N = 4$ supersymmetry defined by the superalgebra (2.6), we found, up to the 4th order in fields, an equivalence transformation from the $N = 2$ Goldstone superfield associated with the generator Z to the standard $N = 2$ Maxwell superfield strength $\mathcal{W}, \bar{\mathcal{W}}$ defined by the constraints (2.1). We found that the nonlinear hidden $N = 2$ supersymmetry and $Z, \bar{Z}$ symmetry are realized on $\mathcal{W}, \bar{\mathcal{W}}$ by the following transformations:

$$\delta\mathcal{W} = f \left(1 - \frac{1}{2} \bar{D}^4 \bar{\mathcal{A}}_0 \right) + \frac{1}{4} \bar{f} \Box \mathcal{A}_0 + \frac{1}{4i} \bar{D}^{i\dot\alpha} \bar{f} D^\alpha_i \, \partial_{\alpha\dot\alpha} \mathcal{A}_0, \quad \delta\bar{\mathcal{W}} = (\delta\mathcal{W})^* \ , \tag{3.1}$$

where the functions $f, \bar{f}$,

$$f = c + 2i\,\eta^{i\alpha}\theta_{i\alpha}\ , \quad \bar{f} = \bar{c} - 2i\,\bar{\eta}^i_{\dot\alpha}\bar{\theta}^{\dot\alpha}_i\ , \tag{3.2}$$

collect the parameters of broken supersymmetry $(\eta^{i\alpha}, \bar{\eta}^i_{\dot\alpha})$ and those of the central charge transformations $(c, \bar{c})$. The complex chiral function $\mathcal{A}_0$ was specified up to the fourth order [1]

$$\mathcal{A}_0 = \mathcal{W}^2\left(1 + \frac{1}{2}\bar{D}^4\bar{\mathcal{W}}^2\right) + O(\mathcal{W}^6)\ , \tag{3.3}$$

$$\bar{D}_{\dot\alpha i}\mathcal{A}_0 = 0\ . \tag{3.4}$$

Actually, the transformation law (3.1), (3.2) is the most general hidden supersymmetry transformation law of $\mathcal{W}, \bar{\mathcal{W}}$ compatible with the defining constraints (2.1), provided that the $N = 2$ superfield function $\mathcal{A}_0$ obeys the chirality condition (3.4). By analogy with the $N = 1$ construction of [3], in order to promote (3.1) to a *linear* (though still inhomogeneous) realization of the considered $N = 4$ supersymmetry, it is natural to treat $\mathcal{A}_0$ as a new *independent* $N = 2$ superfield constrained only by the chirality condition (3.4) and to try to define the transformation law of $\mathcal{A}_0$ under the $\eta, \bar{\eta}, c, \bar{c}$-transformations in such a way that the $N = 2$ superfields $\mathcal{A}_0$, $\mathcal{W}, \bar{\mathcal{W}}$ form a closed set. Then, imposing a proper covariant constraint on these superfields one could hope to recover the structure (3.3) as a solution to this constraint. In view of the covariance of this hypothetical constraint, the correct transformation law for $\mathcal{A}_0$ to the appropriate order can be reproduced by varying (3.3) according to the transformation law (3.1). Since we know $\mathcal{A}_0$ up to the 4th order, we can uniquely restore its transformation law up to the 3d order. We explicitly find

$$\delta\mathcal{A}_0 = 2f\mathcal{W} + \frac{1}{4}\bar{f}\Box\mathcal{A}_1 + \frac{1}{4i}\,\bar{D}^{i\dot\alpha}\bar{f}D^\alpha_i\,\partial_{\alpha\dot\alpha}\mathcal{A}_1, \tag{3.5}$$

where

$$\mathcal{A}_1 = \frac{2}{3}\,\mathcal{W}^3 + O(\mathcal{W}^5)\ , \quad \bar{D}_{\dot\alpha i}\mathcal{A}_1 = 0\ . \tag{3.6}$$

We observe the appearance of a new composite chiral superfield $\mathcal{A}_1$, and there is no way to avoid it in the transformation law (3.5). This is the crucial difference from the $N = 1$ case of ref. [3, 4] where a similar reasoning led to a closed supermultiplet with only one extra $N = 1$ superfield besides the $N = 1$ Goldstone-Maxwell one (the resulting linear multiplet of $N = 2$ supersymmetry is a $N = 1$ superfield form of the $N = 2$ vector multiplet with a modified transformation law [12, 13]).

Thus, we are forced to incorporate a chiral superfield $\mathcal{A}_1$ as a new independent $N = 2$ superfield component of the linear $N = 4$ supermultiplet we are seeking. Inspecting the brackets of all these transformations suggests that the only possibility to achieve their closure in accord with the superalgebra (2.6) is to introduce an *infinite* sequence of chiral $N = 2$ superfields and their antichiral conjugates $\mathcal{A}_n$, $\bar{\mathcal{A}}_n$, $n = 0, 1, \ldots$,

$$\bar{D}_{\dot\alpha i}\mathcal{A}_n = 0\ ,\ D^i_\alpha\bar{\mathcal{A}}_n = 0\ , \tag{3.7}$$

with the following transformation laws:

$$\delta\mathcal{A}_0 = 2f\mathcal{W} + \frac{1}{4}\bar{f}\Box\mathcal{A}_1 + \frac{1}{4i}\,\bar{D}^{i\dot\alpha}\bar{f}D^\alpha_i\,\partial_{\alpha\dot\alpha}\mathcal{A}_1, \tag{3.8}$$

[1] For further convenience, here we use a slightly different notation for this function as compared to ref. [11].

$$
\begin{aligned}
&\delta\mathcal{A}_1 = 2f\mathcal{A}_0 + \frac{1}{4}\bar{f}\Box\mathcal{A}_2 + \frac{1}{4i}\,\bar{D}^{i\dot{\alpha}}\bar{f}D_i^{\alpha}\,\partial_{\alpha\dot{\alpha}}\mathcal{A}_2 \\
&\cdots\cdots\cdots \\
&\delta\mathcal{A}_n = 2f\mathcal{A}_{n-1} + \frac{1}{4}\bar{f}\Box\mathcal{A}_{n+1} + \frac{1}{4i}\,\bar{D}^{i\dot{\alpha}}\bar{f}D_i^{\alpha}\,\partial_{\alpha\dot{\alpha}}\mathcal{A}_{n+1}, \quad (n \geq 1) \\
&\delta\bar{\mathcal{A}}_n = (\delta\mathcal{A}_n)^* \ .
\end{aligned}
\tag{3.9}
$$

It is a simple exercise to check that these transformations close off shell both among themselves and with those of the manifest $N = 2$ supersymmetry just according to the $N = 4$ superalgebra (2.6).

Realizing (formally) the central charge generators as derivatives in some extra complex "central-charge coordinate" z

$$
Z = \frac{i}{2}\frac{\partial}{\partial z}\ , \quad \bar{Z} = \frac{i}{2}\frac{\partial}{\partial \bar{z}}\ ,
\tag{3.10}
$$

and assuming all the involved $N = 2$ superfields to be defined on a $z, \bar{z}$ extension of the standard $N = 2$ superspace, it is instructive to rewrite the transformation laws under the $c, \bar{c}$ transformations as follows:

$$
\frac{\partial\mathcal{W}}{\partial z} = \left(1 - \frac{1}{2}\bar{D}^4\bar{\mathcal{A}}_0\right)\ , \quad \frac{\partial\mathcal{W}}{\partial\bar{z}} = \frac{1}{4}\Box\mathcal{A}_0\ ,
\tag{3.11}
$$

$$
\frac{\partial\mathcal{A}_0}{\partial z} = 2\mathcal{W}\ , \quad \frac{\partial\mathcal{A}_0}{\partial\bar{z}} = \frac{1}{4}\Box\mathcal{A}_1\ ,
\tag{3.12}
$$

$$
\frac{\partial\mathcal{A}_n}{\partial z} = 2\mathcal{A}_{n-1}\ , \quad \frac{\partial\mathcal{A}_n}{\partial\bar{z}} = \frac{1}{4}\Box\mathcal{A}_{n+1}\ .
\tag{3.13}
$$

These relations imply, in particular,

$$
\left(\frac{\partial^2}{\partial z\partial\bar{z}} - \frac{1}{2}\Box\right)\mathcal{W} = 0\ , \quad \left(\frac{\partial^2}{\partial z\partial\bar{z}} - \frac{1}{2}\Box\right)\mathcal{A}_n = 0\ .
\tag{3.14}
$$

If we regard $z, \bar{z}$ as the actual coordinates, which extend the $d = 4$ Minkowski space to the $D = 6$ one, the relations (3.14) mean that the constructed linear supermultiplet is on shell from the $D = 6$ perspective. On the other hand, from the $d = 4$ point of view this multiplet is off-shell, and the relations (3.11) - (3.13), (3.14) simply give a specific realization of the central charge generators $Z, \bar{Z}$ on its $N = 2$ superfield components. In this sense this multiplet is similar to the previously known special $N = 2$, $d = 4$ and $N = 4$, $d = 4$ supermultiplets, which are obtained from the on-shell multiplets in higher dimensions via non-trivial dimension reductions and inherit the higher-dimensional translation generators as non-trivially realized central charges in $d = 4$ [14, 15] (a renowned example of this sort is the $(8 + 8)$ Fayet-Sohnius hypermultiplet [14]). Since the superalgebra (2.6) is just a $d = 4$ form of the $N = (2, 0)$ (or $N = (0, 2)$) $D = 6$ Poincaré superalgebra, it is natural to think that the above supermultiplet has a $D = 6$ origin and to try to reveal it.[2] We hope to come back to this interesting problem in the future. For the time being we prefer to treat the above infinite-dimensional representation in the pure

[2]By analogy with the previously known examples [3, 5], one could expect, at first glance, that this multiplet is a $d = 4$ form of the vector $N = 2$, $D = 6$ multiplet which is known to exist only on shell (in $D = 6$) [16]. However, this cannot be true because such a multiplet can be defined only for the $N = (1, 1)$ supersymmetry in $D = 6$ [16] while we are facing $N = (2, 0)$ or $N = (0, 2)$ supersymmetry in our case. Note that the PBGS option $N = 4 \to N = 2$, $d = 4$, with the $N = 4$, $d = 4$ supersymmetry being isomorphic just to the $N = (1, 1)$, $D = 6$ one, was discussed in [7]. It requires the $N = 2$ hypermultiplet, as the Goldstone multiplet.

$d = 4$ framework as a linear realization of the partial spontaneous breaking of the central-charge extended $N = 4$, $d = 4$ supersymmetry (2.6) to the standard $N = 2$ supersymmetry. The Goldstone character of this realization is manifested in the transformation law (3.1) which contains pure shifts by the parameters of the spontaneously broken symmetries. Therefore, the appropriate components of the superfield strength $\mathcal{W}$ are the Goldstone fields, and this superfield itself can be interpreted as the Goldstone $N = 2$ superfield of the linear realization of the considered PBGS pattern $N = 4 \to N = 2$.

4 Superfield action of the $N = 4 \to N = 2$ BI theory

As was already mentioned, in the approach proceeding from a linear realization of PBGS, the Goldstone superfield Lagrange density is, as a rule, a component of the same linear supermultiplet to which the relevant Goldstone superfield belongs. This is also true for the case under consideration. A good candidate for the chiral $N = 2$ Lagrangian density is the superfield $\mathcal{A}_0$. Indeed, the "action"

$$S = \int d^4x d^4\theta \mathcal{A}_0 + \int d^4x d^4\bar{\theta} \bar{\mathcal{A}}_0 \tag{4.1}$$

is invariant with respect to the transformation (3.8) up to surface terms, because, taking into account the basic constraints (2.1) and the precise form of these transformations, the integrand is shifted by x-derivatives. With the interpretation of the central charge transformations as shifts with respect to the coordinates $z, \bar{z}$, the action (4.1) does not depend on these coordinates in virtue of eqs. (3.12), though the Lagrangian density can bear such a dependence.

It remains for us to define covariant constraints which would express $\mathcal{A}_0$, $\bar{\mathcal{A}}_0$ in terms of $\mathcal{W}$, $\bar{\mathcal{W}}$, with preserving the linear representation structure (3.1), (3.8), (3.9). Because an infinite number of $N = 2$ superfields $\mathcal{A}_n$ is present in our case, there should exist an infinite set of constraints trading all these superfields for the basic Goldstone ones $\mathcal{W}$, $\bar{\mathcal{W}}$.

As a first step in finding these constraints let us note that the following expression:

$$\phi_0 = \mathcal{A}_0 \left(1 - \frac{1}{2}\bar{D}^4\bar{\mathcal{A}}_0\right) - \mathcal{W}^2 - \sum_{k=1} \frac{(-1)^k}{2 \cdot 8^k} \mathcal{A}_k \Box^k \bar{D}^4 \bar{\mathcal{A}}_k \tag{4.2}$$

is invariant, with respect to the f part of the transformations (3.1), (3.8) - (3.9). This leads us to choose

$$\phi_0 = 0 \tag{4.3}$$

as our first constraint. For consistency with $N = 4$ supersymmetry, the constraint (4.3) should be invariant with respect to the full transformations (3.1), (3.8), (3.9), with the $\bar{f}$ part taken into account as well. We shall firstly specialize to the $\bar{c}$ part of the $\bar{f}$ transformations. The requirement of the $\bar{c}$ covariance produces the new constraint

$$\phi_1 = \Box\mathcal{A}_1 + 2\left(\mathcal{A}_0\Box\mathcal{W} - \mathcal{W}\Box\mathcal{A}_0\right) - \sum_{k=0} \frac{(-1)^k}{2 \cdot 8^k} \left(\Box\mathcal{A}_{k+1}\Box^k\bar{D}^4\bar{\mathcal{A}}_k - \mathcal{A}_{k+1}\Box^{k+1}\bar{D}^4\bar{\mathcal{A}}_k\right) = 0\,. \tag{4.4}$$

It is invariant under the f transformations, but requiring it to be invariant also under the $\bar{c}$ part gives rise to the new constraint

$$\begin{aligned} \phi_2 \;=\; & \Box^2\mathcal{A}_2 + 2\left(\mathcal{A}_0\Box^2\mathcal{A}_0 - \Box\mathcal{A}_0\Box\mathcal{A}_0 + 2\Box\mathcal{A}_1\Box\mathcal{W} - \mathcal{A}_1\Box^2\mathcal{W} - \mathcal{W}\Box^2\mathcal{A}_1\right) \\ & - \sum_{k=0} \frac{(-1)^k}{2 \cdot 8^k} \left(\Box^2\mathcal{A}_{k+2}\Box^k\bar{D}^4\bar{\mathcal{A}}_k - 2\Box\mathcal{A}_{k+2}\Box^{k+1}\bar{D}^4\bar{\mathcal{A}}_k + \mathcal{A}_{k+2}\Box^{k+2}\bar{D}^4\bar{\mathcal{A}}_k\right) = 0\,. \end{aligned} \tag{4.5}$$

Applying the same procedure to (4.5), we find the next constraint

$$\begin{aligned}\phi_3 &= \Box^3\mathcal{A}_3 + 2\Big(3\Box^2\mathcal{A}_2\Box\mathcal{W} + 3\Box\mathcal{A}_1\Box^2\mathcal{A}_0 + \mathcal{A}_0\Box^3\mathcal{A}_1 - \mathcal{A}_1\Box^3\mathcal{A}_0 + \mathcal{A}_2\Box^3\mathcal{W} \\ &\quad - \mathcal{W}\Box^3\mathcal{A}_2 - 3\Box^2\mathcal{A}_1\Box\mathcal{A}_0 - 3\Box\mathcal{A}_2\Box^2\mathcal{W}\Big) + \ldots = 0, \end{aligned} \tag{4.6}$$

and so on. The full infinite set of constraints is by construction invariant under the f and $\bar{c}$ transformations. Indeed, using the relations (3.11)-(3.13) one may explicitly check that

$$\frac{\partial\phi_n}{\partial z} = 0\ , \quad \frac{\partial\phi_n}{\partial\bar{z}} = \frac{1}{4}\phi_{n+1}\ , \tag{4.7}$$

so the full set of constraints is indeed closed.

The variation of the basic constraints with respect to the $\bar{f}$ transformations has the following general form:

$$\delta\phi_n = \bar{\eta}^{i\dot{\alpha}}\bar{\theta}_{i\dot{\alpha}}\mathcal{B}_n + \bar{\eta}^{i\dot{\alpha}}(\mathcal{F}_n)_{i\dot{\alpha}}\ . \tag{4.8}$$

Demanding this variation to vanish gives rise to the two sets of constraints

$$\text{(a)}\ \ \mathcal{B}_n = 0\ , \quad \text{(b)}\ \ (\mathcal{F}_n)_{i\dot{\alpha}} = 0\ . \tag{4.9}$$

The constraints (4.9a) are easily recognized as those obtained above from the $\bar{c}$ covariance reasoning. One can show by explicit calculations that

$$\bar{D}^{i\dot{\alpha}}(\mathcal{F}_n)_{j\dot{\beta}} \sim \delta^i_j\delta^{\dot{\alpha}}_{\dot{\beta}}\mathcal{B}_n\ . \tag{4.10}$$

Thus the fermionic constraints (4.9b) seem to be more fundamental. For example, for the constraint ϕ_1 (4.4) the basic fermionic constraint reads

$$\begin{aligned}(\mathcal{F}_1)_{i\dot{\alpha}} &= D^\alpha_i\partial_{\alpha\dot{\alpha}}\mathcal{A}_1 + 2\left(\mathcal{A}_0 D^\alpha_i\partial_{\alpha\dot{\alpha}}\mathcal{W} - \mathcal{W}D^\alpha_i\partial_{\alpha\dot{\alpha}}\mathcal{A}_0\right) \\ &\quad - \sum_{k=0}\frac{(-1)^k}{2\cdot 8^k}\left(D^\alpha_i\partial_{\alpha\dot{\alpha}}\mathcal{A}_{k+1}\Box^k\bar{D}^4\bar{\mathcal{A}}_k - \mathcal{A}_{k+1}\Box^k D^\alpha_i\partial_{\alpha\dot{\alpha}}\bar{D}^4\bar{\mathcal{A}}_k\right) = 0\ . \end{aligned} \tag{4.11}$$

In order to prove that the basic fermionic constraints (4.9b) are actually equivalent to the bosonic ones (4.9a), one has to know the general solution to *all* constraints. For the time being we have explicitly checked this important property only for the iteration solution given below. Taking for granted that this is true in general, we can limit our attention to the type (a) constraints only. The constraints (4.2), (4.4) are just of this type.

At present we have no idea, how to explicitly solve the above infinite set of constraints and find a closed expression for the Lagrangian densities $\mathcal{A}_0$, $\bar{\mathcal{A}}_0$ similar to the one known in the $N = 2 \to N = 1$ case [3]. What we are actually able to do, so far, is to restore a general solution by iterations. E.g., in order to restore the action up to the 10th order, we have to know the following orders in $\mathcal{A}_k$:

$$\begin{aligned}&\mathcal{A}_0 = \mathcal{W}^2 + \mathcal{A}_0^{(4)} + \mathcal{A}_0^{(6)} + \mathcal{A}_0^{(8)} + \ldots\ , \\ &\mathcal{A}_1 = \mathcal{A}_1^{(3)} + \mathcal{A}_1^{(5)} + \mathcal{A}_1^{(7)} + \ldots\ , \\ &\mathcal{A}_2 = \mathcal{A}_2^{(4)} + \mathcal{A}_2^{(6)} + \ldots\ , \quad \mathcal{A}_3 = \mathcal{A}_3^{(5)} + \ldots\ . \end{aligned} \tag{4.12}$$

These terms were found to have the following explicit structure:

$$
\begin{aligned}
\mathcal{A}_0^{(4)} &= \frac{1}{2}\mathcal{W}^2\bar{D}^4\bar{\mathcal{W}}^2\,, \quad \mathcal{A}_0^{(6)} = \frac{1}{4}\bar{D}^4\left[\mathcal{W}^2\bar{\mathcal{W}}^2\left(D^4\mathcal{W}^2+\bar{D}^4\bar{\mathcal{W}}^2\right)-\frac{1}{9}\mathcal{W}^3\Box\bar{\mathcal{W}}^3\right]\,,\\
\mathcal{A}_0^{(8)} &= \frac{1}{8}\bar{D}^4\Big[4\mathcal{W}^2\bar{\mathcal{A}}_0^{(6)}+4\bar{\mathcal{W}}^2\mathcal{A}_0^{(6)}+\mathcal{W}^2\bar{\mathcal{W}}^2D^4\mathcal{W}^2\bar{D}^4\bar{\mathcal{W}}^2-\frac{2}{9}\mathcal{W}^3\Box\left(\bar{\mathcal{W}}^3D^4\mathcal{W}^2\right)\\
&\quad -\frac{2}{9}\mathcal{W}^3\bar{D}^4\bar{\mathcal{W}}^2\Box\bar{\mathcal{W}}^3+\frac{1}{144}\mathcal{W}^4\Box^2\bar{\mathcal{W}}^4\Big]\,,\\
\mathcal{A}_1^{(3)} &= \frac{2}{3}\mathcal{W}^3\,, \quad \mathcal{A}_1^{(5)} = \frac{2}{3}\mathcal{W}^3\bar{D}^4\bar{\mathcal{W}}^2\,,\\
\mathcal{A}_1^{(7)} &= \bar{D}^4\left[\frac{1}{2}\mathcal{W}^3\bar{\mathcal{W}}^2\bar{D}^4\bar{\mathcal{W}}^2+\frac{1}{3}\mathcal{W}^3\bar{\mathcal{W}}^2D^4\mathcal{W}^2-\frac{1}{24}\mathcal{W}^4\Box\bar{\mathcal{W}}^3\right]\,,\\
\mathcal{A}_2^{(4)} &= \frac{1}{3}\mathcal{W}^4\,, \quad \mathcal{A}_2^{(6)} = \frac{1}{2}\mathcal{W}^4\bar{D}^4\bar{\mathcal{W}}^2\,, \quad \mathcal{A}_3^{(5)} = \frac{2}{15}\mathcal{W}^5\,.
\end{aligned}
\tag{4.13}
$$

Note that, despite the presence of growing powers of the operator $\Box$ in our constraints, in each case the maximal power of $\Box$ can be finally taken off from all the terms in the given constraint, leaving us with this maximal power of $\Box$ acting on an expression which starts from the appropriate $\mathcal{A}_n$. Equating these final expressions to zero allows us to algebraically express all $\mathcal{A}_n$ in terms of $\mathcal{W},\bar{\mathcal{W}}$ and derivatives of the latter. For example, for $\mathcal{A}_3^{(5)}$ we finally get the following equation:

$$
\Box^3\mathcal{A}_3^{(5)} = \frac{2}{15}\Box^3\mathcal{W}^5 \;\Rightarrow\; \mathcal{A}_3^{(5)} = \frac{2}{15}\mathcal{W}^5\,. \tag{4.14}
$$

This procedure of taking off the degrees of $\Box$ with discarding possible "zero modes" can be justified as follows: we are interested in an off-shell solution that preserves the manifest standard $N=2$ supersymmetry including the Poincaré covariance. This rules out possible on-shell zero modes as well as the presence of explicit θ's or x's in the expressions which remain after taking off the appropriate powers of $\Box$. It can be checked to any desirable order that these "reduced" constraints yield correct local expressions for the composite superfields $\mathcal{A}_n$, which prove to transform just in accordance with the original transformation rules (3.1), (3.8), (3.9). We have explicitly verified this for our iteration solution (4.13). We do not know, for the time being, how to demonstrate the possibility to take off the powers of $\Box$ from the original constraints in general, without explicitly solving them. In Appendix we deduce a set of purely algebraic constraints which immediately give the above iteration solution and so are candidates for the general form of the "reduced" constraints.

The explicit expression for the action, up to the 8th order in $\mathcal{W},\bar{\mathcal{W}}$, reads

$$
\begin{aligned}
S^{(8)} &= \left(\int d^4xd^4\theta\mathcal{W}^2+\text{c.c.}\right)+\int d^4xd^4\theta d^4\bar{\theta}\left\{\mathcal{W}^2\bar{\mathcal{W}}^2\left[1+\frac{1}{2}\left(D^4\mathcal{W}^2+\bar{D}^4\bar{\mathcal{W}}^2\right)\right]\right.\\
&-\frac{1}{18}\mathcal{W}^3\Box\bar{\mathcal{W}}^3+\frac{1}{4}\mathcal{W}^2\bar{\mathcal{W}}^2\left[\left(D^4\mathcal{W}^2+\bar{D}^4\bar{\mathcal{W}}^2\right)^2+D^4\mathcal{W}^2\bar{D}^4\bar{\mathcal{W}}^2\right]\\
&\left.-\frac{1}{12}D^4\mathcal{W}^2\bar{\mathcal{W}}^3\Box\mathcal{W}^3-\frac{1}{12}\bar{D}^4\bar{\mathcal{W}}^2\mathcal{W}^3\Box\bar{\mathcal{W}}^3+\frac{1}{576}\mathcal{W}^4\Box^2\bar{\mathcal{W}}^4\right\}\,.
\end{aligned}
\tag{4.15}
$$

This action, up to a slight difference in the notation, coincides with the action found by Kuzenko and Theisen [9] from the requirements of self-duality and invariance under nonlinear shifts of $\mathcal{W},\bar{\mathcal{W}}$ (the $c,\bar{c}$ transformations in our notation). Let us point out that the structure of nonlinearities in the $c,\bar{c}$ transformations of $\mathcal{W},\bar{\mathcal{W}}$ in our approach is uniquely fixed by the

original $N = 4$ supersymmetry transformations and the constraints imposed. In [9] it was guessed order by order from the requirement that the action be invariant.

The next, 10th order part of the $N = 4$ invariant $N = 2$ BI action can be easily restored from eqs. (4.13). Its explicit form looks not too enlightening, so here we present only the relevant part of the chiral Lagrangian density in the condensed notation

$$\begin{aligned}\mathcal{A}_0^{(10)} &= \frac{1}{2}\bar{D}^4\Big[\mathcal{W}^2\bar{\mathcal{A}}_0^{(8)} + \bar{\mathcal{W}}^2\mathcal{A}_0^{(8)} + \mathcal{A}_0^{(4)}\bar{\mathcal{A}}_0^{(6)} + \bar{\mathcal{A}}_0^{(4)}\mathcal{A}_0^{(6)} - \frac{1}{8}\mathcal{A}_1^{(3)}\Box\bar{\mathcal{A}}_1^{(7)} - \frac{1}{8}\mathcal{A}_1^{(7)}\Box\bar{\mathcal{A}}_1^{(3)} \\ &\quad -\frac{1}{8}\mathcal{A}_1^{(5)}\Box\bar{\mathcal{A}}_1^{(5)} + \frac{1}{64}\mathcal{A}_2^{(4)}\Box^2\bar{\mathcal{A}}_2^{(6)} + \frac{1}{64}\mathcal{A}_2^{(6)}\Box^2\bar{\mathcal{A}}_2^{(4)} - \frac{1}{512}\mathcal{A}_3^{(5)}\Box^3\bar{\mathcal{A}}_3^{(5)}\Big] \ . \qquad (4.16)\end{aligned}$$

It would be interesting to compare it with the 10th order of the Kuzenko-Theisen action (which, unfortunately, was not explicitly given in [9]). Anyway, the coincidence of the action of [9] with the $N = 4 \to N = 2$ BI action, up to the 8th order, can be regarded as a strong indication that these actions coincide at any order and, hence, that the $N = 4 \to N = 2$ BI action is self-dual like its $N = 2 \to N = 1$ prototype [3, 4].

Finally, let us point out that after doing the θ integral, the pure Maxwell field strength part of the bosonic sector of the above action (and of the hypothetical complete action) comes entirely from the expansion of the standard Born-Infeld bosonic action. Just in this sense the above action is a particular $N = 2$ extension of the bosonic BI action. The difference from the action of ref. [6] is just in higher-derivative terms with the $\Box$ operators. These correction terms are crucial for the invariance under the hidden $N = 2$ supersymmetry, and they drastically change, as compared to ref. [6], the structure of the bosonic action, both in the pure scalar fields sector and the mixed sector involving couplings between the Maxwell field strength and the scalar fields. By a reasoning of [11], the additional terms are just those needed for the existence of an equivalence field redefinition bringing the scalar fields action into the standard static-gauge Nambu-Goto form.

The analysis of the auxiliary field sector shows that the equation for the auxiliary field $P^{(ik)}(x)$ has the following generic structure:

$$P^{(ik)}M^{(nl)}_{(ik)} = 0 \ ,$$

where M is a non-singular matrix, $M = I + \ldots$, and "dots" stand for terms involving fields and their derivatives. Hence, $P^{(ik)} = 0$ on shell, i.e. the auxiliary field is non-propagating, as in the standard $N = 2$ Maxwell theory.

5 Conclusion

In this paper we proposed a systematic way of constructing a $N = 2$ superfield BI action with a hidden second $N = 2$ supersymmetry. It is based on extending the $N = 2$ vector multiplet to an infinite-dimensional linear off-shell multiplet of the central-charge modified $N = 4$ supersymmetry and imposing an infinite set of covariant constraints which give rise to a nonlinear realization of the $N = 4$ supersymmetry in terms of the $N = 2$ Maxwell (Goldstone-Maxwell) superfield strengths $\mathcal{W}, \bar{\mathcal{W}}$. Solving these constraints by iterations, we have restored the $N = 4$ supersymmetric BI action to 10th order in $\mathcal{W}, \bar{\mathcal{W}}$. In order to construct the full action, we need to know the general solution of the constraints. For this purpose it seems necessary to work out another, technically more feasible way of tackling the infinite set of these

constraints, perhaps in a $z, \bar{z}$ extended $N = 4$ superspace, rather than in the $N = 2$ one. We hope to report soon on a progress in this direction. Another project for a future study is to apply our approach to construct a genuine non-abelian version of the $N = 4 \to N = 2$ BI action as a proper modification of the action proposed in [17].

In the course of writing this paper we learned that a construction conceptually closed to ours was independently worked out by A. Galperin [18].

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Appendix

Let us consider the following constraint

$$\varphi_1 = \mathcal{A}_1 \left(1 - \frac{1}{2}\bar{D}^4\bar{\mathcal{A}}_0\right) - \frac{2}{3}\mathcal{W}\mathcal{A}_0 - \sum_{k=1} \frac{(-1)^k}{8^k}\left(\frac{k}{3} + \frac{1}{2}\right)\mathcal{A}_{k+1}\Box^k\bar{D}^4\bar{\mathcal{A}}_k = 0\ . \tag{A.1}$$

Using (3.11),(3.12),(3.13) and the following useful relation:

$$\frac{\partial}{\partial z}\left(\sum_{k=1}\frac{(-1)^k}{8^k}a_k\mathcal{A}_{k+m}\Box^{k+p}\bar{D}^4\bar{\mathcal{A}}_{k+n}\right) = \frac{1}{4}\sum_{k=0}\frac{(-1)^k}{8^k}\left(a_k - a_{k+1}\right)\mathcal{A}_{k+m}\Box^{k+p+1}\bar{D}^4\bar{\mathcal{A}}_{k+n+1} \tag{A.2}$$

($a_0 \equiv 0$), one can easily check that

$$\frac{\partial}{\partial z}\varphi_1 = \frac{4}{3}\phi_0\ . \tag{A.3}$$

In other words, (A.1) is the result of "integrating" the basic constraint (4.2), with respect to z. The same "integration" procedure can be continued further to get the successive set of the algebraic constraints

$$\varphi_2 = \mathcal{A}_2\left(1 - \frac{1}{2}\bar{D}^4\bar{\mathcal{A}}_0\right) - \frac{1}{2}\mathcal{W}\mathcal{A}_1 - \sum_{k=1}\frac{(-1)^k}{8^k}\left(\frac{k^2}{8} + \frac{k}{2} + \frac{1}{2}\right)\mathcal{A}_{k+2}\Box^k\bar{D}^4\bar{\mathcal{A}}_k = 0\ , \tag{A.4}$$

$$\frac{\partial}{\partial z}\varphi_2 = \frac{3}{2}\varphi_1\ ,$$

$$\varphi_3 = \mathcal{A}_3\left(1 - \frac{1}{2}\bar{D}^4\bar{\mathcal{A}}_0\right) - \frac{2}{5}\mathcal{W}\mathcal{A}_2 - \sum_{k=1}\frac{(-1)^k}{8^k}\left(\frac{k^3}{30} + \frac{k^2}{4} + \frac{37k}{60} + \frac{1}{2}\right)\mathcal{A}_{k+3}\Box^k\bar{D}^4\bar{\mathcal{A}}_k = 0\ , \tag{A.5}$$

$$\frac{\partial}{\partial z}\varphi_3 = \frac{8}{5}\varphi_2\ ,$$

and so on.

We have checked that the iteration solution of the constraints (A.1), (A.4), (A.5), up to the 8th order, exactly coincides with (4.13), but we still have no general proof that this set of constraints is indeed fundamental. It is, by construction, covariant under the c- transformations and η-supersymmetry, but its covariance under the $\bar{f}$ transformations remains to be proved. Note the interesting relations betweeen the constraints (4.4) - (4.6) and (A.1), (A.4), (A.5)

$$\phi_1 = \frac{\partial}{\partial\bar{z}}\phi_0 = \frac{4}{3}\frac{\partial^2}{\partial z\partial\bar{z}}\varphi_1\ ,\quad \phi_2 = \frac{32}{9}\left(\frac{\partial^2}{\partial z\partial\bar{z}}\right)^2\varphi_2\ ,\quad \phi_3 = \frac{80}{9}\left(\frac{\partial^2}{\partial z\partial\bar{z}}\right)^3\varphi_3\ ,\dots\ . \tag{A.6}$$

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