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N=2 AND N=4 SUPERSYMMETRIC BORN-INFELD THEORIES FROM NON LINEAR REALIZATIONS

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Abstract

Starting from nonlinear realizations of the partially broken central-charge extended $N = 4$ and $N = 8$ Poincaré supersymmetries in $D = 4$, we derive the superfield equations of $N = 2$ and $N = 4$ Born-Infeld theories. The basic objects are the bosonic Goldstone $N = 2$ and $N = 4$ superfields associated with the central charge generators. By construction, the equations are manifestly $N = 2$ and $N = 4$ supersymmetric and enjoy covariance under another nonlinearly realized half of the original supersymmetries. They provide a manifestly worldvolume supersymmetric static-gauge description of D3-branes in $D = 6$ and $D = 10$. For the $N = 2$ case we find, to lowest orders, the equivalence transformation to the standard $N = 2$ Maxwell superfield strength and restore, up to the sixth order, the off-shell $N = 2$ Born-Infeld action with the second hidden $N = 2$ supersymmetry.

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1 Introduction

Supersymmetric extensions of the Born-Infeld (BI) theory in diverse dimensions are currently under intensive study. This vast interest is mainly due to the fact that the corresponding actions naturally arise in string theory as the worldvolume actions of Dp-branes (see [1] and refs. therein). It is of importance to have superfield formulations of supersymmetric BI theories, with all linearly realized supersymmetries being manifest.

The $N = 1, D = 4$ super BI action constructed in [2] is not entirely determined by $N = 1$ supersymmetry. As was shown in [3], this freedom can be fixed by requiring the action to possess an extra hidden nonlinearly realized $N = 1$ supersymmetry completing the manifest one to $N = 2, D = 4$ supersymmetry. Such a form of the $N = 1$ BI action naturally comes out within the nonlinear realization of $N = 2$ supersymmetry partially broken to $N = 1$, with the vector gauge $N = 1$ multiplet as the Goldstone one. In components, the action is reduced to the static-gauge form of the worldvolume action of the “space-filling” D3-brane [3,4].

Until now, the action of refs. [3,4] remains the only example of a superfield BI action in $D \geq 4$ having both manifest and hidden supersymmetries, and so admitting an interpretation as the Goldstone superfield action (or a worldvolume-supersymmetric form of some D3-brane action). The $N = 2$ BI action constructed in [5] possesses no hidden supersymmetry [6]. So it can be viewed merely as a part of the as yet unknown action of the Goldstone $N = 2$ vector multiplet supporting the spontaneous partial breaking $N = 4 \rightarrow N = 2$ as was suggested in [7,8]. As for $N = 4$ BI action which is a candidate for the Goldstone multiplet action associated with the breaking $N = 8 \rightarrow N = 4$ in $D = 4$ (the static-gauge action of D3-brane in $D = 10$ [9]), only the first (quartic) nonlinear correction to the $N = 4$ Maxwell action is known [1].

Because of lacking the Goldstone superfield BI actions for these cases, it is natural to try to deduce the relevant superfield equations of motion as a non-polynomial generalization of the $N = 2$ and $N = 4$ super Maxwell equations. In [10,11] we derived covariant superfield equations for a few examples of superbranes in $D = 3$ and $D = 4$ by applying the formalism of nonlinear realizations to the appropriate partially broken supersymmetries. In particular, we recovered the $N = 2 \rightarrow N = 1$ BI system in this new setting [10].

Here we apply the same method to derive the superfield equations for the partial-breaking patterns $N = 4 \rightarrow N = 2$ and $N = 8 \rightarrow N = 4$ in $D = 4$, with the $N = 2$ and $N = 4$ vector multiplets as the Goldstone ones. In both cases, the covariant gauge field strength satisfies a disguised form of the BI equation which takes the familiar form after a field redefinition. We elaborate in more detail on the first case. In particular, we show that the equation for the complex scalar field corresponds to the static-gauge Nambu-Goto action of 3-brane in $D = 6$. To first orders in the Goldstone superfield, we find the equivalence transformation to the ordinary $N = 2$ vector multiplet superfield strength and restore the invariant off-shell action to sixth order. The first correction to the action of [5] arises in sixth order and it coincides with the one found in [6] from a different reasoning. We speculate on a possibility to generate non-abelian versions of super BI theories from the nonlinear-realizations approach.

2 Vector Goldstone multiplet for $N = 4 \rightarrow N = 2$

We wish to derive the $N = 2$ supersymmetric BI theory as a theory of the partial breaking of $N = 4, D = 4$ supersymmetry down to $N = 2$ supersymmetry, with the vector $N = 2$ multiplet as the Goldstone one. To apply the nonlinear realizations techniques [10], we firstly need to specify $N = 4, D = 4$ supersymmetry to start with. In the case of $N = 2 \rightarrow N = 1$ breaking the basic object was the Goldstone $N = 1$ spinor superfield associated with the spontaneously broken half of the $N = 2$ fermionic generators. This superfield is related by a field redefinition to the standard chiral spinor $N = 1$ Maxwell superfield strength. In the $N = 2$ Maxwell theory, the basic object is a complex scalar $N = 2$ off-shell superfield strength $\mathcal{W}$ which is chiral and satisfies one additional Bianchi identity:

$$(a)\ \bar{D}^i_{\dot\alpha}\mathcal{W} = 0\ ,\ D^\alpha_i\bar{\mathcal{W}} = 0\ , \quad (b)\ D^{ik}\mathcal{W} = \bar{D}^{ik}\bar{\mathcal{W}}\ . \tag{1}$$

Here,

$$D^i_\alpha = \frac{\partial}{\partial\theta^\alpha_i} + i\bar\theta^{\dot\alpha i}\partial_{\alpha\dot\alpha}\ , \quad \bar{D}_{\dot\alpha i} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha i}} - i\theta^\alpha_i\partial_{\alpha\dot\alpha}\ , \quad \left\{D^i_\alpha, \bar{D}_{\dot\alpha j}\right\} = -2i\delta^i_j\partial_{\alpha\dot\alpha}\ , \tag{2}$$

$$D^{ij} \equiv D^{\alpha\, i}D^j_\alpha\ , \quad \bar{D}^{ij} \equiv \bar{D}^i_{\dot\alpha}\bar{D}^{\dot\alpha\, j}\ . \tag{3}$$

The superfield equation of motion for $\mathcal{W}$ reads

$$D^{ik}\mathcal{W} + \bar{D}^{ik}\bar{\mathcal{W}} = 0\ , \tag{4}$$

and, together with ($1b$), amounts to

$$D^{ik}\mathcal{W} = \bar{D}^{ik}\bar{\mathcal{W}} = 0\ . \tag{5}$$

In order to incorporate an appropriate generalization of $\mathcal{W}$ into the nonlinear realization scheme as the Goldstone superfield, we need to have the proper bosonic generator in the algebra. The following central extension of $N = 4, D = 4$ Poincaré superalgebra suits this purpose

$$
\begin{gathered}
\left\{Q^i_\alpha, \bar{Q}_{\dot{\alpha}j}\right\} = 2\delta^i_j P_{\alpha\dot{\alpha}}\ , \quad \left\{S^i_\alpha, \bar{S}_{\dot{\alpha}j}\right\} = 2\delta^i_j P_{\alpha\dot{\alpha}}\ , \\
\left\{Q^i_\alpha, S^j_\beta\right\} = 2\varepsilon^{ij}\varepsilon_{\alpha\beta} Z\ , \quad \left\{\bar{Q}_{\dot{\alpha}i}, \bar{S}_{\dot{\beta}j}\right\} = -2\varepsilon_{ij}\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{Z}\ , \ \ (i,j=1,2)\ , \qquad (6)
\end{gathered}
$$

with all other (anti)commutators vanishing. Note an important feature that the complex central charge Z appears in the crossing anticommutator, while the generators $(Q, \bar{Q})$ and $(S, \bar{S})$ on their own form two $N = 2$ superalgebras without central charges. The full internal symmetry automorphism group of (6) (commuting with $P_{\alpha\dot{\alpha}}$ and Z) is $SO(5) \sim Sp(2)$. Besides the manifest R-symmetry group $U_R(2) = SU_R(2) \times U_R(1)$ acting as uniform rotations of the doublet indices of all spinor generators and the opposite phase transformations of the S- and Q-generators, it also includes the 6-parameter quotient $SO(5)/U_R(2)$ transformations which properly rotate the generators Q and S through each other. The superalgebra (6) is a $D = 4$ form of $N = (2, 0)$ (or $N = (0, 2)$) Poincaré superalgebra in $D = 6$.

As a first step towards the corresponding nonlinear realization, let us split the set of generators of the $N = 4$ superalgebra (6) into the unbroken $\left\{Q^i_\alpha, \bar{Q}_{\dot{\alpha}j}, P_{\alpha\dot{\alpha}}\right\}$ and broken $\left\{S^i_\alpha, \bar{S}_{\dot{\alpha}j}, Z, \bar{Z}\right\}$ parts. A coset element g is then defined by:

$$
g = \exp i\left(-x^{\alpha\dot{\alpha}}P_{\alpha\dot{\alpha}} + \theta^\alpha_i Q^i_\alpha + \bar{\theta}^i_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}_i\right) \exp i\left(\psi^\alpha_i S^i_\alpha + \bar{\psi}^i_{\dot{\alpha}}\bar{S}^{\dot{\alpha}}_i\right) \exp i\left(WZ + \bar{W}\bar{Z}\right)\ . \qquad (7)
$$

Acting on (7) from the left by various elements of the supergroup corresponding to (6), one can find the transformation properties of the coset coordinates.

For the unbroken supersymmetry $\left(g_0 = \exp i\left(-a^{\alpha\dot{\alpha}}P_{\alpha\dot{\alpha}} + \epsilon^\alpha_i Q^i_\alpha + \bar{\epsilon}^i_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}_i\right)\right)$ one has:

$$
\delta x^{\alpha\dot{\alpha}} = a^{\alpha\dot{\alpha}} - i\left(\epsilon^\alpha_i\bar{\theta}^{\dot{\alpha}i} + \bar{\epsilon}^{\dot{\alpha}i}\theta^\alpha_i\right)\ , \quad \delta\theta^\alpha_i = \epsilon^\alpha_i\ , \quad \delta\bar{\theta}^i_{\dot{\alpha}} = \bar{\epsilon}^i_{\dot{\alpha}}\ . \qquad (8)
$$

Broken supersymmetry transformations $\left(g_0 = \exp i\left(\eta^\alpha_i S^i_\alpha + \bar{\eta}^i_{\dot{\alpha}}\bar{S}^{\dot{\alpha}}_i\right)\right)$ are as follows:

$$
\begin{aligned}
&\delta x^{\alpha\dot{\alpha}} = -i\left(\eta^\alpha_i\bar{\psi}^{\dot{\alpha}i} + \bar{\eta}^{\dot{\alpha}i}\psi^\alpha_i\right)\ , \quad \delta\psi^\alpha_i = \eta^\alpha_i\ , \quad \delta\bar{\psi}^i_{\dot{\alpha}} = \bar{\eta}^i_{\dot{\alpha}}\ , \\
&\delta W = -2i\eta^\alpha_i\theta^i_\alpha\ , \quad \delta\bar{W} = -2i\bar{\eta}^i_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}_i\ . \qquad (9)
\end{aligned}
$$

Finally, the broken $Z, \bar{Z}$-translations $\left(g_0 = \exp i\left(cZ + \bar{c}\bar{Z}\right)\right)$ read

$$
\delta W = c\ , \quad \delta\bar{W} = \bar{c}\ . \qquad (10)
$$

The next standard step is to define the left-invariant Cartan 1-forms:

$$
\begin{aligned}
\omega^{\alpha\dot{\alpha}}_P &= dx^{\alpha\dot{\alpha}} - i\left(d\bar{\theta}^{\dot{\alpha}i}\theta^\alpha_i + d\theta^\alpha_i\bar{\theta}^{\dot{\alpha}i}\right) - i\left(d\bar{\psi}^{\dot{\alpha}i}\psi^\alpha_i + d\psi^\alpha_i\bar{\psi}^{\dot{\alpha}i}\right)\ , \\
\omega^\alpha_{Q\ i} &= d\theta^\alpha_i\ , \quad \bar{\omega}^i_{Q\ \dot{\alpha}} = d\bar{\theta}^i_{\dot{\alpha}}\ , \quad \omega^\alpha_{S\ i} = d\psi^\alpha_i\ , \quad \bar{\omega}^i_{S\ \dot{\alpha}} = d\bar{\psi}^i_{\dot{\alpha}}\ , \\
\omega_Z &= dW - 2id\theta^\alpha_i\psi^i_\alpha\ , \quad \bar{\omega}_Z = d\bar{W} + 2id\bar{\theta}^{\dot{\alpha}i}\bar{\psi}_{\dot{\alpha}i}\ . \qquad (11)
\end{aligned}
$$

The covariant derivatives of some scalar $N=2$ superfield Φ are defined by expanding the differential $d\Phi$ over the covariant differentials of the $N=2$ superspace coordinates

$$
\begin{aligned}
& d\Phi \equiv \omega_P^{\alpha\dot\alpha}\nabla_{\alpha\dot\alpha}\Phi + d\theta_i^\alpha \mathcal{D}^i_\alpha\Phi + d\bar\theta^i_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}_i\Phi \quad \Rightarrow \\
& \nabla_{\alpha\dot\alpha} = \left(E^{-1}\right)^{\beta\dot\beta}_{\alpha\dot\alpha}\partial_{\beta\dot\beta}\ , \quad E^{\beta\dot\beta}_{\alpha\dot\alpha} \equiv \delta^\beta_\alpha\delta^{\dot\beta}_{\dot\alpha} + i\psi^\beta_i\partial_{\alpha\dot\alpha}\bar\psi^{\dot\beta i} + i\bar\psi^{\dot\beta i}\partial_{\alpha\dot\alpha}\psi^\beta_i\ , \\
& \mathcal{D}^i_\alpha = D^i_\alpha + i\left(\psi^\beta_j D^i_\alpha\bar\psi^{\dot\beta j} + \bar\psi^{\dot\beta j}D^i_\alpha\psi^\beta_j\right)\nabla_{\beta\dot\beta}\ , \\
& \bar{\mathcal{D}}_{\dot\alpha i} = \bar D_{\dot\alpha i} + i\left(\psi^\beta_j\bar D_{\dot\alpha i}\bar\psi^{\dot\beta j} + \bar\psi^{\dot\beta j}\bar D_{\dot\alpha i}\psi^\beta_j\right)\nabla_{\beta\dot\beta}\ ,
\end{aligned} \tag{12}
$$

where $D^i_\alpha, \bar D_{i\dot\alpha}$ are defined in (2). The derivatives (12) obey the following algebra:

$$
\begin{aligned}
& \left\{\mathcal{D}^i_\alpha, \bar{\mathcal{D}}_{\dot\alpha j}\right\} = -2i\delta^i_j\nabla_{\alpha\dot\alpha} + 2i\left(\mathcal{D}^i_\alpha\psi^\gamma_k\bar{\mathcal{D}}_{\dot\alpha j}\bar\psi^{\dot\gamma k} + \mathcal{D}^i_\alpha\bar\psi^{\dot\gamma k}\bar{\mathcal{D}}_{\dot\alpha j}\psi^\gamma_k\right)\nabla_{\gamma\dot\gamma}\ , \\
& \left\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\right\} = 2i\left(\mathcal{D}^i_\alpha\psi^\gamma_k\mathcal{D}^j_\beta\bar\psi^{\dot\gamma k} + \mathcal{D}^i_\alpha\bar\psi^{\dot\gamma k}\mathcal{D}^j_\beta\psi^\gamma_k\right)\nabla_{\gamma\dot\gamma}\ , \\
& \left[\mathcal{D}^i_\alpha, \nabla_{\beta\dot\beta}\right] = -2i\left(\mathcal{D}^i_\alpha\psi^\gamma_k\mathcal{D}_{\beta\dot\beta}\bar\psi^{\dot\gamma k} + \mathcal{D}^i_\alpha\bar\psi^{\dot\gamma k}\mathcal{D}_{\beta\dot\beta}\psi^\gamma_k\right)\nabla_{\gamma\dot\gamma}\ .
\end{aligned} \tag{13}
$$

As in several previously studied examples [14,7,8,15], the Goldstone fermionic superfields ψ^i_α, $\bar\psi_{\dot\alpha i}$ can be covariantly expressed in terms of the central-charge Goldstone superfields $\mathcal{W}$, $\bar{\mathcal{W}}$ by imposing the inverse Higgs constraints [16] on the central-charge Cartan 1-forms. In the present case these constraints are

$$
\omega_Z|_{d\theta,d\bar\theta} = \bar\omega_Z|_{d\theta,d\bar\theta} = 0\ , \tag{14}
$$

where $|$ means the covariant projections on the differentials of the spinor coordinates. These constraints amount to the sought expressions for the fermionic Goldstone superfields

$$
\psi^i_\alpha = -\frac{i}{2}\mathcal{D}^i_\alpha W\ , \quad \bar\psi_{\dot\alpha i} = -\frac{i}{2}\bar{\mathcal{D}}_{\dot\alpha i}\bar W\ , \tag{15}
$$

and, simultaneously, to the covariantization of the chirality conditions $(1a)$

$$
\bar{\mathcal{D}}_{\dot\alpha i}W = 0\ , \quad \mathcal{D}^i_\alpha\bar W = 0\ . \tag{16}
$$

Actually, eqs. (15) are highly nonlinear equations serving to express ψ^i_α, $\bar\psi_{\dot\alpha i}$ in terms of $W, \bar W$ with making use of the definitions (12).

It is also straightforward to write the covariant generalization of the dynamical equation of the $N=2$ abelian vector multiplet (1), (5)

$$
\mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha W = 0\ , \quad \bar{\mathcal{D}}^{(i}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha j)}\bar W = 0\ . \tag{17}
$$

The equations (16), (17) with the superfield Goldstone fermions eliminated by (15) constitute a manifestly covariant form of the superfield equations of motion of $N=2$

Dirac-Born-Infeld theory with the second hidden nonlinearly realized $N=2$ supersymmetry. It closes, together with the manifest $N=2$ supersymmetry, on the $N=4$ supersymmetry (6).

As a first step in proving this statement, let us show that the above system of equations reduces the component content of W just to that of the on-shell $N=2$ vector multiplet. It is convenient to count the number of independent covariant superfield projections of W, $\bar{W}$.

At the dimensions (-1) and $(-1/2)$ we find W, $\bar{W}$ and $\psi_{i\alpha} = -\frac{i}{2}\mathcal{D}_{i\alpha}W$, $\bar{\psi}^i_{\dot\alpha} = \overline{(\psi_{i\alpha})} = -\frac{i}{2}\bar{\mathcal{D}}^i_{\dot\alpha}\bar{W}$, with a complex bosonic field and a doublet of gaugini as the lowest components.

At the dimension (0) we have, before employing (16), (17),

$$
\begin{aligned}
&\mathcal{D}^i_\alpha\psi^j_\beta = \varepsilon^{ij}f_{\alpha\beta} + i\varepsilon_{\alpha\beta}F^{(ij)} + F^{(ij)}_{(\alpha\beta)}\,, \quad \bar{\mathcal{D}}_{\dot\alpha i}\psi_{j\alpha} = \varepsilon_{ij}X_{\alpha\dot\alpha} + X_{(ij)\alpha\dot\alpha}\,,\\
&\bar{\mathcal{D}}_{\dot\alpha i}\bar{\psi}_{\dot\beta j} = -\varepsilon_{ij}\bar{f}_{\dot\alpha\dot\beta} + i\varepsilon_{\dot\alpha\dot\beta}\bar{F}_{(ij)} + \bar{F}_{(ij)(\dot\alpha\dot\beta)} \quad \mathcal{D}^i_\alpha\bar{\psi}^j_{\dot\alpha} = \varepsilon^{ij}\bar{X}_{\dot\alpha\alpha} - \bar{X}^{(ij)}_{\dot\alpha\alpha}\,, \qquad (18)
\end{aligned}
$$

$$
f_{\alpha\beta} \equiv \epsilon_{\alpha\beta}A + iF_{\alpha\beta},\ \bar{f}_{\dot\alpha\dot\beta} = \overline{(f_{\alpha\beta})} = \epsilon_{\dot\alpha\dot\beta}\bar{A} - i\bar{F}_{\dot\alpha\dot\beta}, \qquad (19)
$$

$$
f^\alpha_\beta f_{\alpha\gamma} = \epsilon_{\beta\gamma}\left(A^2 - \frac{1}{2}F^2\right)\,, \quad \bar{f}^{\dot\alpha}_{\dot\beta}\bar{f}_{\dot\alpha\dot\gamma} = \epsilon_{\dot\beta\dot\gamma}\left(\bar{A}^2 - \frac{1}{2}\bar{F}^2\right)\,. \qquad (20)
$$

The dynamical equations (17) imply

$$
F^{(ij)} = \bar{F}^{(ij)} = 0\,. \qquad (21)
$$

The lowest component of these superfields is a nonlinear analog of the auxiliary field of the $N=2$ Maxwell theory.

Next, substituting the expressions (15) for the spinor Goldstone fermions in the l.h.s. of eqs. (18) and making use of both (16) and (17), we represent these l.h.s. as

$$
\begin{aligned}
&\mathcal{D}^i_\alpha\psi^j_\beta = -\frac{i}{4}\left\{\mathcal{D}^i_\alpha,\mathcal{D}^j_\beta\right\}W + \frac{i}{4}\varepsilon^{ij}\mathcal{D}^k_{(\alpha}\mathcal{D}_{\beta)k}W\ ,\ \bar{\mathcal{D}}_{\dot\alpha i}\psi^j_\alpha = -\frac{i}{2}\left\{\bar{\mathcal{D}}_{\dot\alpha i},\mathcal{D}^j_\alpha\right\}W\ ,\\
&\bar{\mathcal{D}}_{\dot\alpha i}\bar{\psi}_{\dot\beta j} = -\frac{i}{4}\left\{\bar{\mathcal{D}}_{\dot\alpha i},\bar{\mathcal{D}}_{\dot\beta j}\right\}\bar{W} - \frac{i}{4}\varepsilon_{ij}\bar{\mathcal{D}}^k_{(\dot\alpha}\bar{\mathcal{D}}_{\dot\beta)k}\bar{W}\ ,\ \mathcal{D}^i_\alpha\bar{\psi}_{\dot\alpha j} = -\frac{i}{2}\left\{\mathcal{D}^i_\alpha,\bar{\mathcal{D}}_{\dot\alpha j}\right\}\bar{W} \qquad (22)
\end{aligned}
$$

Using the algebra (13) and comparing (22) with the definition (18) (taking into account (21)), it is straightforward to show that the objects $F^{(ij)}_{(\alpha\beta)}$, $\bar{F}_{(ij)(\dot\alpha\dot\beta)}$, $X_{(ij)\alpha\dot\alpha}$ and $\bar{X}^{(ij)}_{\dot\alpha\alpha}$ satisfy a system of homogeneous equations, such that the matrix of the coefficients in them is nonsingular at the origin $W=\bar{W}=0$. Thus these objects vanish as a consequence of the basic equations:

$$
F^{(ij)}_{(\alpha\beta)} = \bar{F}_{(ij)(\dot\alpha\dot\beta)} = X_{(ij)\alpha\dot\alpha} = \bar{X}^{(ij)}_{\dot\alpha\alpha} = 0. \qquad (23)
$$

As a result, on shell we are left with the following superfield content:

$$\mathcal{D}^i_\alpha \psi^j_\beta = \varepsilon^{ij} f_{\alpha\beta} \;, \quad \bar{\mathcal{D}}_{\dot\alpha i}\psi_{j\alpha} = \varepsilon_{ij} X_{\alpha\dot\alpha} \;,$$
$$\bar{\mathcal{D}}_{\dot\alpha i}\bar\psi_{\dot\beta j} = -\varepsilon_{ij}\bar f_{\dot\alpha\dot\beta} \;, \quad \mathcal{D}^i_\alpha \bar\psi^j_{\dot\alpha} = \varepsilon^{ij}\bar X_{\dot\alpha\alpha} \;. \tag{24}$$

The only new independent superfield at the dimension (0) is the complex one $F_{(\alpha\beta)}, \bar F_{(\dot\alpha\dot\beta)}$, while $A, \bar A$ and $X_{\alpha\dot\beta}, \bar X_{\dot\alpha\beta}$ are algebraically expressed through it and other independent superfields as will be shown below. In Sec. 3 we show that this superfield is related, by an equivalence field redefinition, to the Maxwell field strength obeying the BI equation of motion.

Substituting the explicit expressions for the anticommutators (13) into (22) and again using (24) in both sides of (22), we finally obtain:

$$A = -\frac{1}{2}\bar X_{\dot\gamma}{}^{\beta} f_{\beta\gamma}\nabla^{\gamma\dot\gamma} W \;, \quad \bar A = -\frac{1}{2} X_{\gamma}{}^{\dot\beta}\bar f_{\dot\beta\dot\gamma}\nabla^{\gamma\dot\gamma}\bar W \;,$$
$$X_{\alpha\dot\alpha} = \nabla_{\alpha\dot\alpha} W + \left(X_{\gamma\dot\alpha}\bar X_{\dot\gamma\alpha} + \bar f_{\dot\alpha\dot\gamma} f_{\alpha\gamma}\right)\nabla^{\gamma\dot\gamma} W \;,$$
$$\bar X_{\dot\alpha\alpha} = \nabla_{\alpha\dot\alpha}\bar W + \left(X_{\gamma\dot\alpha}\bar X_{\dot\gamma\alpha} + \bar f_{\dot\alpha\dot\gamma} f_{\alpha\gamma}\right)\nabla^{\gamma\dot\gamma}\bar W \;. \tag{25}$$

It is easy to see that these algebraic equations indeed allow one to express $A, \bar A$ and $X_{\alpha\dot\beta}, \bar X_{\alpha\dot\beta}$ in terms of $F_{(\alpha\beta)}$, $\bar F_{(\dot\alpha\dot\beta)}$ and $\nabla_{\alpha\dot\alpha} W$, $\nabla_{\alpha\dot\alpha}\bar W$:

$$A = \frac{i}{2}\, F^{(\beta}{}_{\gamma)}\nabla_{\beta\dot\gamma}\bar W \nabla^{\gamma\dot\gamma} W + \ldots \;, \quad X_{\alpha\dot\alpha} = \nabla_{\alpha\dot\alpha} W + \frac{1}{2}(\nabla W\cdot\nabla W)\nabla_{\alpha\dot\alpha}\bar W + \ldots \;. \tag{26}$$

As one more corollary of eqs (25), let us check the validity of additional integrability conditions which come out from our covariant chirality constraints (16)

$$\mathcal{D}^i_\alpha \bar W = \bar{\mathcal{D}}_{\dot\alpha i} W = 0 \quad \Rightarrow \quad \left\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\right\}\bar W = \left\{\bar{\mathcal{D}}_{\dot\alpha i}, \bar{\mathcal{D}}_{\dot\beta j}\right\} W = 0 \;. \tag{27}$$

In terms of the components (24) they read as follows:

$$Y \equiv \bar X_{\dot\beta}{}^{\beta} f_{\beta\gamma}\nabla^{\gamma\dot\beta}\bar W = 0 \;, \quad \bar Y \equiv X_{\beta}{}^{\dot\beta}\bar f_{\dot\beta\dot\gamma}\nabla^{\dot\gamma\beta} W = 0 \;. \tag{28}$$

Substituting the expression for $\bar X_{\dot\beta\beta}$ from (25) into Y, we find after some algebra

$$\left[1 - \left(X\cdot\nabla\bar W\right)\right] Y = \left(\nabla\bar W\right)^2 \left[\bar A\left(\frac{1}{2}F^2 - A^2\right) - A + \frac{1}{2}\bar X^{\dot\gamma\beta} f_{\beta\lambda} X^{\lambda}{}_{\dot\gamma}\right] \equiv \left(\nabla\bar W\right)^2 \mathcal{B} \;. \tag{29}$$

So, in order to prove (28) and (27), it suffices to show that

$$\mathcal{B} = \bar{\mathcal{B}} = 0 \;. \tag{30}$$

After some algebraic manipulations one gets

$$\left[1-\left(\bar{X}\cdot\nabla W\right)+\frac{1}{4}\bar{X}^{2}\left(\nabla W\right)^{2}\right]\mathcal{B}=\left(A^{2}-\frac{1}{2}F^{2}\right)\left[\left(\nabla W\cdot\nabla\bar{W}\right)\bar{\mathcal{B}}+\frac{1}{2}\left(\bar{X}\cdot\nabla W\right)\bar{Y}\right]. \tag{31}$$

Recalling that $Y,\bar{Y}$ are expressed through $\mathcal{B},\bar{\mathcal{B}}$ by division by non-singular factors (see eq. (29)), and substituting these expressions into (31) and its conjugate, we get for $\mathcal{B},\bar{\mathcal{B}}$ a system of homogeneous linear equations with a non-singular matrix of the coefficients. This proves (30) and (28).

Returning to the issue of extracting an irreducible set of covariant superfield projections of $W,\bar{W}$, it is easy to show that the further successive action by covariant spinor derivatives on (24) produces no new independent superfields. One obtains either the equations of motion (and Bianchi identities) for the independent basic superfields $W,\bar{W},\psi_{i\alpha},\bar{\psi}^{i}_{\dot{\alpha}}$ and $F_{(\alpha\beta)},\bar{F}_{(\dot{\alpha}\dot{\beta})}$, or some composite superfields which are expressed through x-derivatives of the basic ones (or as some appropriate nonlinear functions of the basic superfields). The useful relations which essentially simplify the analysis are the following ones:

$$\left\{\mathcal{D}^{(i}_{\alpha},\bar{\mathcal{D}}^{j)}_{\dot{\alpha}}\right\}=\left\{\mathcal{D}^{(i}_{\alpha},\mathcal{D}^{j)}_{\beta}\right\}=\left\{\bar{\mathcal{D}}^{(i}_{\dot{\alpha}},\bar{\mathcal{D}}^{j)}_{\dot{\alpha}}\right\}=0\,. \tag{32}$$

They follow by substituting (24) into the algebra (13). These relations are the covariant version of the integrability conditions for the Grassmann harmonic $N=2$ analyticity [17]. Thus the nonlinear $W,\bar{W}$ background specified by the equations (15)-(17) respects the Grassmann harmonic analyticity which plays a fundamental role in $N=2,D=4$ theories.

Before going further, let us make a few comments.

First, the nonlinear realization setting we used, in order to deduce our equations (15)-(17), drastically differs from the standard superspace differential-geometry setup of supersymmetric gauge theories (see, e.g., [12,13]). The starting point of the standard approach is the covariantization of the flat derivatives (spinor and vector) by the gauge-algebra valued connections with appropriate constraints on the relevant covariant superfield strengths. In our case (quite analogously to the previously considered $N=1,D=4$ [3] and $N=1,D=3$ [15] cases) the covariant derivatives include no connection-type terms. Instead, they contain, in a highly non-linear manner, the Goldstone bosonic $N=2$ superfields $W,\bar{W}$. These quantities, after submitting them to the covariant constraints (15)-(17), turn out to be the nonlinear-realization counterparts of the $N=2$ Maxwell superfield strength. As we show below, the Bianchi identities needed to pass to the gauge potentials are encoded in the set (15)-(17).

In the differential-geometry approach the constraints like (23) emerge before going on shell, they are a consequence of the Bianchi identities. In our nonlinear system we cannot separate in a simple way the kinematical off-shell constraints from the dynamical on-shell ones. We could try to relax our system by lifting the basic dynamical equations (17) and retaining only the chirality condition (16) together with (15) and an appropriate covariantization of the constraint ($1b$). But in this case we immediately face the same difficulty as in the $N=2 \to N=1$ case [3]: a naive covariantization of ($1b$) by replacing the flat spinor derivatives by the covariant ones proves to be not self-consistent. For self-consistency, it should be properly modified order by order, without any clear guiding principle. No such a problem arises when the dynamical equations (17) are enforced. The terms modifying the naive covariantization of ($1b$) can be shown to vanish, as in the $N=2 \to N=1$ case [10].

Nevertheless, as we argue below, there exists a highly nonlinear field redefinition which relates the nonlinear superfield Goldstone strength $W, \bar{W}$ to its flat counterpart $\mathcal{W}, \bar{\mathcal{W}}$ satisfying the off-shell irreducibility conditions (1). In this frame it becomes possible to divide the kinematical and dynamical aspects of our system and to write the appropriate off-shell action giving rise to the dynamical equations, in a deep analogy with the $N=2 \to N=1$ case [3,4].

As the last comment, we note that all the fields of the multiplet comprised by $W, \bar{W}$, except for $F_{(\alpha\beta)}, \bar{F}_{(\dot{\alpha}\dot{\beta})}$, can be given a clear interpretation as Goldstone fields: $W|, \bar{W}|$ for the spontaneously broken central-charge shifts, $\psi^i_\alpha|, \bar{\psi}_{i\dot{\alpha}}|$ for the spontaneously broken S-supersymmetry transformations and $F^{(ij)}|, \bar{F}^{(ij)}|$ for the spontaneously broken $SO(5)/U_R(2)$ transformations. This immediately follows from considering the transformation properties of the coset element (7). Eq. (21) following from the dynamical equations (17) explicitly breaks the $SO(5)/U_R(2)$ symmetry, leaving us with $U_R(2) \times U(1)$ as the only surviving internal symmetry (an extra $U(1)$ factor realized as a phase on $W, \bar{W}$ and spinor $N = 2$ superspace coordinates comes from the $D = 6$ Lorentz group). This is quite similar to the $N = 2 \to N = 1$ cases [14,3], where the $U(2)$ automorphism symmetry of the original $N = 2$ Poincaré superalgebra proves to be finally broken down to some its subgroup.

3 Bosonic equations of motion

As the next important step in examining the superfield system (15)-(17), we inspect its bosonic sector. The set of bosonic equations can be obtained by acting on both sides of (24) by two covariantized spinor derivatives, using the algebra (13) together with the relations (25) and omitting the fermions in the final expressions (which should contain only independent superfield projections and their x-derivatives). Instead of analyzing the bosonic sector in full generality, we specialize here to its two suggestive limits.

1. Vector fields limit. This limit amounts to

$$W\Big|_{\theta=\bar\theta=0} = \bar W\Big|_{\theta=\bar\theta=0} = 0\;. \tag{1}$$

From eqs. (25) with all fermions omitted, one can see that (1) imply

$$A = \bar A = X_{\alpha\dot\alpha} = \bar X_{\dot\alpha\alpha} = 0. \tag{2}$$

Thus, in this limit our superfields $W, \bar W$ contain only $F_{\alpha\beta}, \bar F_{\dot\alpha\dot\beta}$ as the bosonic components, which, owing to (17), obey the following simple equations

$$\partial_{\alpha\dot\alpha}F^{\alpha\beta} - F^{\gamma}_{\alpha}\bar F^{\dot\gamma}_{\dot\alpha}\partial_{\gamma\dot\gamma}F^{\alpha\beta} = 0\;, \quad \partial_{\alpha\dot\alpha}\bar F^{\dot\alpha\dot\beta} - F^{\gamma}_{\alpha}\bar F^{\dot\gamma}_{\dot\alpha}\partial_{\gamma\dot\gamma}\bar F^{\dot\alpha\dot\beta} = 0\;. \tag{3}$$

It was already shown in [10] that one can split eqs. (3) into the "true" Bianchi identities and "true" equations of motion

$$\partial_{\beta\dot\alpha}\left(fF^{\beta}_{\alpha}\right) - \partial_{\alpha\dot\beta}\left(\bar f\bar F^{\dot\beta}_{\dot\alpha}\right) = 0\;, \quad \partial_{\beta\dot\alpha}\left(gF^{\beta}_{\alpha}\right) + \partial_{\alpha\dot\beta}\left(\bar g\bar F^{\dot\beta}_{\dot\alpha}\right) = 0\;, \tag{4}$$

where

$$f = \frac{\bar F^2 - 2}{1 - \frac14 F^2\bar F^2}\;, \quad g = \frac{\bar F^2 + 2}{1 - \frac14 F^2\bar F^2}\;. \tag{5}$$

Now, in terms of the "genuine" field strengths

$$V^{\beta}_{\alpha} \equiv \frac{1}{2\sqrt2} fF^{\beta}_{\alpha}\;, \quad \bar V^{\dot\beta}_{\dot\alpha} \equiv \frac{1}{2\sqrt2}\bar f\bar F^{\dot\beta}_{\dot\alpha}\;, \tag{6}$$

the first equation (4) coincides with the standard Bianchi identity

$$\partial_{\beta\dot\alpha}V^{\beta}_{\alpha} - \partial_{\alpha\dot\beta}\bar V^{\dot\beta}_{\dot\alpha} = 0\;, \tag{7}$$

while the second one coincides with the equation of motion following from the $D=4$ BI action:

$$S = \int d^4x\sqrt{\left(V^2 - \bar V^2\right)^2 - 2\left(V^2 + \bar V^2\right) + 1}\;. \tag{8}$$

Thus the superfield system (15)-(17) encodes the BI equation, in accord with the statement that this system provides a supersymmetric extension of the latter.

Note that our original variables $F_{\alpha\beta}, \bar F_{\dot\alpha\dot\beta}$ allow to avoid a square root in (8)

$$\sqrt{\left(V^2 - \bar V^2\right)^2 - 2\left(V^2 + \bar V^2\right) + 1} = \frac{\left(1 - \frac12 F^2\right)\left(1 - \frac12\bar F^2\right)}{1 - \frac14 F^2\bar F^2}\;. \tag{9}$$

However, the Bianchi identity (7) becomes very complicated in such a parametrization and it is unclear whether one can solve it in terms of an appropriate vector potential.

2. Scalar fields limit. This limit corresponds to the reduction

$$\mathcal{D}^i_{(\alpha}\mathcal{D}_{\beta)i}W\Big|_{\theta=\bar\theta=0} = \bar{\mathcal{D}}^i_{(\dot\alpha}\bar{\mathcal{D}}_{\dot\beta)i}\bar W\Big|_{\theta=\bar\theta=0} = 0\ . \tag{10}$$

From eqs. (25) and (28) one finds that the reduction conditions (10) imply

$$\begin{aligned} & A = 0\ , \quad \bar A = 0\ , \\ & X_{\alpha\dot\alpha} = \partial_{\alpha\dot\alpha}W + X_{\gamma\dot\alpha}\bar X_{\dot\gamma\alpha}\partial^{\gamma\dot\gamma}W, \quad \bar X_{\dot\alpha\alpha} = \partial_{\alpha\dot\alpha}\bar W + X_{\gamma\dot\alpha}\bar X_{\dot\gamma\alpha}\partial^{\gamma\dot\gamma}\bar W, \end{aligned} \tag{11}$$

while the equations of motion following from (17) read

$$\partial^\alpha_{\dot\alpha}X_{\alpha\dot\beta} + \bar X^{\dot\gamma\alpha}X^\gamma{}_{\dot\alpha}\partial_{\gamma\dot\gamma}X_{\alpha\dot\beta} = 0\ , \quad \partial_{\alpha\dot\alpha}\bar X^{\dot\alpha}{}_\beta + \bar X^{\dot\gamma}{}_\alpha X^\gamma{}_{\dot\alpha}\partial_{\gamma\dot\gamma}\bar X^{\dot\alpha}{}_\beta = 0\ . \tag{12}$$

The system (11) can be easily solved

$$X_{\alpha\dot\alpha} = \partial_{\alpha\dot\alpha}W + \frac{(\partial W)^2}{h}\partial_{\alpha\dot\alpha}\bar W\ , \quad \bar X_{\dot\alpha\alpha} = \partial_{\alpha\dot\alpha}\bar W + \frac{\left(\partial\bar W\right)^2}{h}\partial_{\alpha\dot\alpha}W\ , \tag{13}$$

where

$$h = \left[1 - \left(\partial W\cdot\partial\bar W\right)\right] + \sqrt{\left[1 - \left(\partial W\cdot\partial\bar W\right)\right]^2 - (\partial W)^2\left(\partial\bar W\right)^2}\ . \tag{14}$$

One can check that those parts of eqs. (12) which are symmetric in the free indices are identically satisfied with (13) and (14). The trace part of (12) can be cast into the form:

$$\partial_{\alpha\dot\alpha}\left(\frac{X^{\alpha\dot\alpha} + \frac12 X^2\bar X^{\dot\alpha\alpha}}{1 - \frac14 X^2\bar X^2}\right) = 0\ , \quad \partial_{\alpha\dot\alpha}\left(\frac{\bar X^{\dot\alpha\alpha} + \frac12\bar X^2 X^{\alpha\dot\alpha}}{1 - \frac14 X^2\bar X^2}\right) = 0\ . \tag{15}$$

Now, substituting (13), (14) in (15), one finds that the resulting form of these equations can be reproduced from the action

$$S = \int d^4x\left(\sqrt{1 + 2\left(\partial W\cdot\partial\bar W\right) + \left(\partial W\cdot\partial\bar W\right)^2 - (\partial W)^2\left(\partial\bar W\right)^2} - 1\right)\ . \tag{16}$$

This action is the static-gauge form of the Dirac-Nambu-Goto action of a 3-brane in D=6.

In terms of the variables $X_{\alpha\dot\alpha}, \bar X_{\dot\alpha\alpha}$ the action (16) becomes a rational function:

$$S = \int d^4x\left(\frac{1 + \left(X\cdot\bar X\right) + \frac14 X^2\bar X^2}{1 - \frac14 X^2\bar X^2} - 1\right)\ . \tag{17}$$

Note that one cannot vary $X_{\alpha\dot\alpha}, \bar X_{\dot\alpha\alpha}$ as independent fields, since, in view of (13), they satisfy some nonlinear integrability conditions.

Summarizing the discussion of Secs. 2 and 3, we have shown that the system of our superfield equations (15)-(17) is self-consistent and gives a $N=2$ superextension of both the equations of $D = 4$ BI theory and those of the static-gauge 3-brane in $D = 6$, with the nonlinearly realized second $N = 2$ supersymmetry. This justifies our claim that (15)-(17) are indeed a manifestly worldvolume supersymmetric form of the equations of D3-brane in $D = 6$ and, simultaneously, of $N = 2$ Born-Infeld theory. Similarly to the previous examples [10,11], the nonlinear realization approach yields the BI equations in a disguised form, with the Bianchi identities and dynamical equations mixed in a tricky way. At the same time, for the scalars we get the familiar static-gauge Nambu-Goto-type equations. This is in agreement with the fact that $W, \bar{W}$ undergo pure shifts under the action of the central charge generators $Z, \bar{Z}$, suggesting the interpretation of these superfields as the transverse brane coordinates conjugated to $Z, \bar{Z}$. These generators, in turn, can be interpreted as two extra components of the 6-momentum.

4 Towards a formulation in terms of $\mathcal{W}, \bar{\mathcal{W}}$

As was already mentioned, we expect that, like in the $N = 1$ case [3,4], there should exist an equivalence transformation to a formulation in terms of the conventional $N = 2$ Maxwell superfield strength $\mathcal{W}, \bar{\mathcal{W}}$ defined by the off-shell constraints (1).

A systematic, though as yet iterative procedure to find such a field redefinition starts by passing to the standard chirality conditions ($1a$) from the covariantly-chiral ones (16). After some algebra, (16) can be brought to the form

$$\bar{D}_{\dot{\alpha}i} R = 0 \ , \quad D^i_\alpha \bar{R} = 0 \ , \tag{1}$$

where

$$R = W + \frac{1}{2}\bar{W}\left(\partial W \cdot \partial W\right) + \frac{i}{4} D^\gamma_j W \bar{D}^{\dot{\gamma}j}\bar{W}\partial_{\gamma\dot{\gamma}}W + O(W^5) \ . \tag{2}$$

Now we pass to the new superfields $\mathcal{W}, \bar{\mathcal{W}}$ with preserving the flat chirality

$$\mathcal{W} \equiv R\left(1 - \frac{1}{2}\bar{D}^4\bar{R}^2\right) \ , \quad \bar{\mathcal{W}} \equiv \bar{R}\left(1 - \frac{1}{2}D^4 R^2\right) \ , \quad \bar{D}_{\dot{\alpha}i}\mathcal{W} = D^i_\alpha \bar{\mathcal{W}} = 0 \ , \tag{3}$$

where

$$D^4 \equiv \frac{1}{48} D^{\alpha i} D^j_\alpha D^\beta_i D_{\beta j} \ , \quad \bar{D}^4 = \overline{(D^4)} \equiv \frac{1}{48}\bar{D}^i_{\dot{\alpha}}\bar{D}^{\dot{\alpha}j}\bar{D}_{\dot{\beta}i}\bar{D}^{\dot{\beta}}_j \ . \tag{4}$$

Up to the considered third order, in terms of these superfields eqs. (17) can be rewritten as

$$D^{ij}\mathcal{W} = \bar{D}^{ij}\bar{\mathcal{W}} \ \Rightarrow \ D^4\mathcal{W} = -\frac{1}{2} \quad \bar{\mathcal{W}} \ , \ \bar{D}^4\bar{\mathcal{W}} = -\frac{1}{2} \quad \mathcal{W} \ , \quad \equiv \partial_{\alpha\dot{\alpha}}\partial^{\alpha\dot{\alpha}} \ , \tag{5}$$

$$D^{ij}\left(\mathcal{W} + \mathcal{W}\,\bar{D}^4\bar{\mathcal{W}}^2\right) + \bar{D}^{ij}\left(\bar{\mathcal{W}} + \bar{\mathcal{W}}\,D^4\mathcal{W}^2\right) = 0 \ . \tag{6}$$

Eq. (5) is recognized as the Bianchi identity ($1b$), so $\mathcal{W}, \bar{\mathcal{W}}$ can be identified with the conventional $N = 2$ vector multiplet superfield strength. Eq. (6) is then a nonlinear generalization of the standard free $N = 2$ vector multiplet equation of motion (4). The transformation properties of $\mathcal{W}, \bar{\mathcal{W}}$ can be easily restored from (8)-(10) and the definitions (2), (3).

The above procedure is an $N = 2$ superfield analog of separating Bianchi identities and dynamical equations for $F_{(\alpha\beta)}, \bar{F}_{(\dot{\alpha}\dot{\beta})}$ (see Sec. 3). In both cases we do not know the geometric principle behind the relevant field redefinitions. Though in the bosonic case we managed to find this redefinition in a closed form, we are not aware of it in the full superfield case. Nonetheless, we can move a step further and find the relation between $W, \bar{W}$ and $\mathcal{W}, \bar{\mathcal{W}}$, as well as the nonlinear dynamical equations for the latter, up to the fifth order. Then, using the transformation laws (8)-(10), we can restore the hidden S-supersymmetry and $Z, \bar{Z}$ transformations up to the fourth order. In this approximation, the transformation laws and equations of motion read

$$\delta\mathcal{W} = f - \frac{1}{2}\bar{D}^4(fA) + \frac{1}{4} \quad (\bar{f}\bar{A}) + \frac{1}{4i}\,\bar{D}^{i\dot{\alpha}}\bar{f}D^{\alpha}_{i}\,\partial_{\alpha\dot{\alpha}}\bar{A}\,, \quad \delta\bar{\mathcal{W}} = (\delta\mathcal{W})^{*}\,, \quad (7)$$

$$A = \bar{\mathcal{W}}^2\left(1 + \frac{1}{2}D^4\mathcal{W}^2\right)\,, \quad f = c + 2i\,\eta^{i\alpha}\theta_{i\alpha}\,, \quad \bar{f} = \bar{c} - 2i\,\bar{\eta}^{i}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}_{i}\,, \quad (8)$$

$$D^{ij}B + \bar{D}^{ij}\bar{B} = 0\,,$$

$$B = \mathcal{W} + \mathcal{W}\bar{D}^4\left(\bar{\mathcal{W}}^2 + \bar{\mathcal{W}}^2 D^4\mathcal{W}^2 + \frac{1}{2}\bar{\mathcal{W}}^2\bar{D}^4\bar{\mathcal{W}}^2 - \frac{1}{6}\mathcal{W} \quad \bar{\mathcal{W}}^3\right)\,. \quad (9)$$

The hidden supersymmetry transformations, up to the third order, close on the $c, \bar{c}$ ones in the η, ϵ and $\bar{\eta}, \bar{\epsilon}$ sectors, and on the standard $D = 4$ translations in the $\eta, \bar{\eta}$ sector. In the sectors η, η and $\bar{\eta}, \bar{\eta}$ the transformations commute, as it should be. Note that (7) is already of the most general form compatible with the chirality conditions and Bianchi identity (1). So this form will be retained to any order, only the functions A, $\bar{A}$ will get additional contributions.

It is straightforward to restore, to the sixth order, the off-shell $\mathcal{W}, \bar{\mathcal{W}}$ action which yields eq. (9) as the equation of motion

$$S^{(6)}_{bi} = \frac{1}{8}\left(\int d\zeta_L \mathcal{W}^2 + \text{c.c.}\right) + \frac{1}{16}\int dZ\left\{\mathcal{W}^2\bar{\mathcal{W}}^2\left[2 + \left(D^4\mathcal{W}^2 + \bar{D}^4\bar{\mathcal{W}}^2\right)\right] - \frac{1}{9}\mathcal{W}^3 \quad \bar{\mathcal{W}}^3\right\}. (10)$$

It is invariant, in the considered order, under the transformations (7). It differs from the sixth order of the $N = 2$ BI action of [5] just by the last term. The same correction term was found in [6] by requiring self-duality and invariance under the bosonic $c, \bar{c}$ symmetry. We uniquely recovered it from the hidden $N = 2$ supersymmetry. It would be interesting to inquire whether the equivalence of these two sets of requirements persists to higher orders. Using the above procedure, we in principle can restore the invariant action *to any order*.

5 N=4 Born-Infeld theory

Finally, we derive the superfield equations of $N = 4$ BI theory within the same approach.

The $N = 4, D = 4$ Maxwell theory [18] is described by the covariant strength superfield $\mathcal{W}_{ij} = -\mathcal{W}_{ji}, (i, j = 1, \ldots, 4)$, satisfying the following independent constraints [13]

$$\bar{\mathcal{W}}^{ij} \equiv (\mathcal{W}_{ij})^* = \frac{1}{2}\varepsilon^{ijkl}\mathcal{W}_{kl} \ , \tag{1}$$

$$D^k_\alpha \mathcal{W}_{ij} - \frac{1}{3}\left(\delta^k_i D^m_\alpha \mathcal{W}_{mj} - \delta^k_j D^m_\alpha \mathcal{W}_{mi}\right) = 0 \ . \tag{2}$$

In contrast to the $N = 2$ gauge theory, no off-shell superfield formulation exists in the $N = 4$ case: the constraints (1), (2) put the theory on shell.

As in the $N = 2$ case, in order to construct a nonlinear generalization of (1), (2) one should firstly define the appropriate algebraic framework. It is given by the following central charge-extended $N = 8, D = 4$ Poincaré superalgebra:

$$\begin{aligned}
&\left\{Q^i_\alpha, \bar{Q}_{\dot{\alpha}j}\right\} = 2\delta^i_j P_{\alpha\dot{\alpha}} \ , \quad \left\{S^i_\alpha, \bar{S}_{\dot{\alpha}j}\right\} = 2\delta^i_j P_{\alpha\dot{\alpha}} \ , \\
&\left\{Q^i_\alpha, S^j_\beta\right\} = \varepsilon_{\alpha\beta} Z^{ij} \ , \quad \left\{\bar{Q}_{\dot{\alpha}i}, \bar{S}_{\dot{\beta}j}\right\} = \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{Z}_{ij} \ ,
\end{aligned} \tag{3}$$

$$\bar{Z}_{ij} = \left(Z^{ij}\right)^* = \frac{1}{2}\varepsilon_{ijkl} Z^{kl} \ . \tag{4}$$

This is a $D = 4$ notation for the type IIB Poincaré superalgebra in $D = 10$.

We wish the $N = 4, D = 4$ supersymmetry $\left\{P_{\alpha\dot{\alpha}}, Q^i_\alpha, \bar{Q}_{\dot{\alpha}j}\right\}$ to remain unbroken, so we are led to introduce the Goldstone superfields

$$Z^{ij} \Rightarrow W_{ij}(x,\theta,\bar{\theta}) \ , \quad S^i_\alpha \Rightarrow \psi^\alpha_i(x,\theta,\bar{\theta}) \ , \quad \bar{S}_{\dot{\alpha}j} \Rightarrow \bar{\psi}^{\dot{\alpha}}_i(x,\theta,\bar{\theta}) \ . \tag{5}$$

The reality property (4) automatically implies the constraint (1) for W_{ij}:

$$\bar{W}^{ij} = \frac{1}{2}\varepsilon^{ijkl} W_{kl} \ . \tag{6}$$

On the coset element g

$$g = \exp i\left(-x^{\alpha\dot{\alpha}}P_{\alpha\dot{\alpha}} + \theta^\alpha_i Q^i_\alpha + \bar{\theta}^i_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}_i\right) \exp i\left(\psi^\alpha_i S^i_\alpha + \bar{\psi}^i_{\dot{\alpha}}\bar{S}^{\dot{\alpha}}_i + W_{ij}Z^{ij}\right) \tag{7}$$

one can realize the entire $N = 8, D = 4$ supersymmetry (3) by left shifts. The Cartan forms (except for the central charge one) and covariant derivatives formally coincide with (11)-(13), the indices $\{i, j\}$ now ranging from 1 to 4. The central charge Cartan form reads:

$$\omega^Z_{ij} = dW_{ij} + \frac{1}{2}\left(d\theta^\alpha_i \psi_{\alpha j} - d\theta^\alpha_j \psi_{\alpha i} + \varepsilon_{ijkl} d\bar{\theta}^k_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}l}\right) \ . \tag{8}$$

By construction, it is covariant under all transformations of the $N = 8, D = 4$ Poincaré supergroup. The Goldstone superfields $\psi_{\alpha i}$ and $\bar{\psi}^k_{\dot\alpha}$ can be covariantly eliminated by the inverse Higgs procedure, as in the previous case. The proper constraint reads as follows:

$$\omega^Z_{ij}|_{d\theta,d\bar\theta} = 0 \; . \tag{9}$$

It amounts to the following set of equations:

$$(a)\; \mathcal{D}^k_\alpha W_{ij} + \frac{i}{2}\left(\delta^k_i \psi_{\alpha j} - \delta^k_j \psi_{\alpha i}\right) = 0 \; , \quad (b)\; \bar{\mathcal{D}}^{\dot\alpha}_k W_{ij} + \frac{i}{2}\varepsilon_{ijkl}\bar{\psi}^{\dot\alpha l} = 0 \; , \tag{10}$$

which are actually conjugated to each other in virtue of (6). We observe that, besides expressing the fermionic Goldstone superfields through the basic bosonic one W_{ij}:

$$\psi_{\alpha i} = -\frac{2i}{3}\mathcal{D}^j_\alpha W_{ij} \; , \quad \bar{\psi}^{\dot\alpha i} = -\frac{2i}{3}\bar{\mathcal{D}}^{\dot\alpha}_j \bar{W}^{ij} \; , \tag{11}$$

eqs. (10) impose the nonlinear constraint

$$\mathcal{D}^k_\alpha W_{ij} - \frac{1}{3}\left(\delta^k_i \mathcal{D}^m_\alpha W_{mj} - \delta^k_j \mathcal{D}^m_\alpha W_{mi}\right) = 0 \tag{12}$$

(and its conjugate). This is the sought nonlinear generalization of (2).

It is straightforward to show that eq. (12) implies the disguised form of the BI equation (3) for the nonlinear analog of the abelian gauge field strength. For the six physical bosonic fields $W_{ij}|$ we expect the equations corresponding to the static-gauge of 3-brane in $D = 10$ to hold. Thus eqs. (6), (12) plausibly give a manifestly worldvolume supersymmetric description of D3-brane in a flat $D = 10$ Minkowski background. No simple off-shell action can be constructed in this case, since such an action is unknown even for the free $N = 4$ Maxwell theory. But even the construction of the physical fields component action for this $N = 4$ BI theory is of considerable interest. We hope to study this system in more detail elsewhere.

6 Conclusions

In this paper, generalizing the nonlinear-realizations approach of [10], we constructed superfield equations describing $N = 2$ and $N = 4$ supersymmetric BI theories with extra nonlinearly realized $N = 2$ and $N = 4$ supersymmetries. These systems, by construction, realize a $1/2$ partial breaking of the appropriate central-charge extended $N = 4$ and $N = 8$, $D = 4$ Poincaré supersymmetries which are in fact a $D = 4$ form of $N = 2$ supersymmetries in $D = 6$ and $D = 10$. Thus, the equations constructed admit a natural interpretation as providing a manifestly supersymmetric worldvolume description of D3-branes in $D = 6, 10$. In both cases the basic objects are the bosonic Goldstone superfields associated with the central-charge generators. They are nonlinear analogs of the $N = 2$ and $N = 4$ Maxwell superfield strengths.

Besides tasks for a future study, such as the construction of the full invariant actions for the considered systems (an off-shell action for $N = 2$ BI and an on-shell one for $N = 4$ BI) and deducing super BI theories in higher dimensions (e.g., $N = (1, 0)$, $D = 6$ BI theory associated with a nonlinear realization of $N = (2, 0)$, $D = 6$ supersymmetry [7,8]), let us mention a more ambitious problem. The above consideration raises the natural question as to how to make the nonlinear-realizations approach suitable also for deriving the equations of supersymmetric *non-abelian* BI theories with hidden supersymmetries (see [19] for a discussion of such systems in the Green-Schwarz formulation). Since the gauge superfield strengths always appear as Goldstone superfields in the nonlinear-realizations approach (associated with the spinor generators in the $N = 1, D = 4$ case [3] and the central-charge ones in the $N = 2$ and $N = 4$ cases), in order to be able to treat a non-abelian covariant superfield strength in a similar way, it seems necessary to pass to a new kind of superalgebras with the generators taking values in the gauge group algebra. Thus these generalized superalgebras should provide a nontrivial unification of supersymmetry with the gauge groups. The non-abelian analogs of the equations constructed here must inevitably involve the gauge connections for which there should also be a natural geometric place in nonlinear realizations of the hypothetical generalized supersymmetries. The treatment of gauge fields as the Goldstone fields [20] could be suggestive in this respect.

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