LABORATORI NAZIONALI DI FRASCATI
SIS-Pubblicazioni

LNF-00/020 (P)
4 Settembre 2000
hep-ph/0009073

$1/N_c$ AND ε'/ε *

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Abstract

We present a recent analysis of ε'/ε in the $1/N_c$ expansion. We show that the $1/N_c$ corrections to the matrix element of Q_6 are large and positive, indicating a $\Delta I = 1/2$ enhancement similar to the one of Q_1 and Q_2 which dominate the CP conserving amplitude. This enhances the CP ratio and can bring the standard model prediction close to the measured value for central values of the parameters. Several comments on the theoretical status of ε'/ε and the errors in its calculation are given.

PACS: 11.30.Er; 12.39.Fe; 13.25.Es

To appear in Nucl. Phys. B. Proc. Suppl.

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*Invited talk presented by T.H. at QCD 00, July 6-13th 2000, Montpellier, France

1 INTRODUCTION

Direct CP violation in $K \to \pi\pi$ decays was recently observed by the KTeV and NA48 collaborations [1,2]. The present world average [2] for the parameter ε'/ε is Re $\varepsilon'/\varepsilon = (19.3 \pm 2.4) \cdot 10^{-4}$. In the standard model CP violation originates in the CKM phase, and direct CP violation is governed by loop diagrams of the penguin type. The main source of uncertainty in the calculation of ε'/ε is the QCD non-perturbative contribution related to the hadronic nature of the $K \to \pi\pi$ decay. Using the $\Delta S = 1$ effective hamiltonian,

$$\mathcal{H}_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}\, \lambda_u \sum_{i=1}^{8} c_i(\mu)\, Q_i(\mu) \quad (\mu < m_c)\,, \tag{1}$$

the non-perturbative contribution, contained in the hadronic matrix elements of the four-quark operators Q_i, can be separated from the perturbative Wilson coefficients $c_i(\mu) = z_i(\mu) + \tau y_i(\mu)$ (with $\tau = -\lambda_t/\lambda_u$ and $\lambda_q = V^*_{qs} V_{qd}$). Introducing $\langle Q_i \rangle_I \equiv \langle (\pi\pi)_I | Q_i | K \rangle$, the CP ratio can be written as

$$\begin{aligned} \frac{\varepsilon'}{\varepsilon} \;\; &= \;\; \frac{G_F}{2} \frac{\omega\, \mathrm{Im}\lambda_t}{|\varepsilon|\, \mathrm{Re}A_0} \Big[\Big| \sum_i \, y_i \, \langle Q_i \rangle_0 \Big| \Big(1 - \Omega_{\rm IB} \Big) \\ &\quad - \frac{1}{\omega} \Big| \sum_i \, y_i \, \langle Q_i \rangle_2 \Big| \Big] \, . \end{aligned} \tag{2}$$

$\omega =$ReA_0/Re$A_2 = 22.2$ is the ratio of the CP conserving $K \to \pi\pi$ isospin amplitudes; $\Omega_{\rm IB}$ parameterizes isospin breaking corrections [3]. ε'/ε is dominated by $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$ which cannot be fixed from the CP conserving data [4,5]. Beside the theoretical uncertainties coming from the calculation of the $\langle Q_i \rangle_I$ and of $\Omega_{\rm IB}$, the analysis of the CP ratio suffers from the uncertainties on the values of various input parameters, in particular of the CKM phase in Imλ_t, of $\Lambda_{\rm QCD} \equiv \Lambda^{(4)}_{\overline{\rm MS}}$, and of the strange quark mass.

2 ON THE COUNTING IN $1/N_c$ AND THE USE OF THE LARGE-N_c VALUES FOR THE MATRIX ELEMENTS

To calculate the hadronic matrix elements we start from the effective chiral lagrangian for pseudoscalar mesons which involves an expansion in momenta where terms up to $\mathcal{O}(p^4)$ are included [6]. Keeping only terms of $\mathcal{O}(p^4)$ which contribute, at the order we calculate, to the $K \to \pi\pi$ amplitudes, for the lagrangian we obtain:

$$\begin{aligned} \mathcal{L}_{eff} \; &= \\ & \tfrac{f^2}{4} (\langle D_\mu U^\dagger D^\mu U \rangle + \tfrac{\alpha}{4N_c} \langle \ln U^\dagger - \ln U \rangle^2 \end{aligned}$$

$$
\begin{aligned}
&+\langle \chi U^\dagger + U\chi^\dagger\rangle) + L_8\langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger\rangle \\
&+L_5\langle D_\mu U^\dagger D^\mu U(\chi^\dagger U + U^\dagger \chi)\rangle \,,
\end{aligned}
\tag{3}
$$

with $\langle A\rangle$ denoting the trace of A, $\alpha = m_\eta^2 + m_{\eta'}^2 - 2m_K^2$, $\chi = r\mathcal{M}$, and $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$. f and r are parameters related to the pion decay constant F_π and to the quark condensate, with $r = -2\langle\bar{q}q\rangle/f^2$. The complex matrix U is a non-linear representation of the pseudoscalar meson nonet. The conventions and definitions we use are the same as those in [7–9]. The method we use is the $1/N_c$ expansion [10]. In this approach, we expand the matrix elements in powers of the momenta and of $1/N_c$. For the $1/N_c$ corrections we calculated chiral loops as described in refs. [8,9]. Especially important to this analysis are the non-factorizable corrections, which are UV divergent and must be matched to the short-distance part. They are regularized by a finite cutoff Λ_c which is identified with the short-distance renormalization scale. The definition of the momenta in the loop diagrams, which are not momentum translation invariant, is discussed in detail in ref. [8]. Other recent work on matrix elements in the $1/N_c$ approach can be found in refs. [11–13].

For the Wilson coefficients we use the leading logarithmic and the next-to-leading logarithmic values [4]. The absence of any reference to the renormalization scheme in the low-energy calculation, at this stage, prevents a complete matching at the next-to-leading order [14]. Nevertheless, a comparison of the numerical results obtained from the LO and NLO coefficients is useful as regards estimating the uncertainties and testing the validity of perturbation theory.

As it is well known the large-N_c approximation fails completely in explaining the $\Delta I = 1/2$ rule. By taking the large-N_c values for the two dominant operators Q_1 and Q_2 one obtains a $\Delta I = 1/2$ CP-conserving amplitude which underestimates the data by roughly a factor of three. However, as pointed out in ref. [10], $Q_{1,2}$ show an important specificity; they are expected to be largely affected by non-chirally suppressed corrections beyond the large-N_c limit. In the counting in p^2 and $1/N_c$, this property is attributed to the fact that the tree level mesonic representation of $Q_{1,2}$ from the leading $\mathcal{O}(p^2)$ chiral lagrangian introduces $\mathcal{O}(p^2)$ terms with two derivatives. As a result the one-loop contributions over these terms, which are $\mathcal{O}(p^2/N_c)$, are quadratically divergent, i.e. possibly very large because these corrections are not protected by any symmetry like the chiral one. Calculating the terms of $\mathcal{O}(p^2/N_c)$ one obtains a large enhancement of the $\Delta I = 1/2$ amplitude [10] in the range required to reproduce the experiment [9]. This illustrates how important are the non-chirally suppressed $1/N_c$ corrections for an understanding of the $K \to \pi\pi$ amplitudes.

In this context, for ε'/ε, it is very important to investigate the counting in p^2 and

$1/N_c$ for the dominant operators $Q_{6,8}$ and to compare it with the one for $Q_{1,2}$. For Q_8 the mesonic representation from the $\mathcal{O}(p^2)$ lagrangian is $\mathcal{O}(p^0)$ and does not have any derivative (because Q_8 is a density-density operator whereas $Q_{1,2}$ are of the current-current type). As a result we do not expect any quadratic term but only chirally suppressed logarithmic or finite terms. The large-N_c limit, $B_8^{(3/2)} = 1$, is therefore expected to be a much better approximation than for $Q_{1,2}$. Different is the case of Q_6 due to the fact that there is no tree level contribution from the leading $\mathcal{O}(p^2)$ lagrangian, and the large-N_c value $B_6^{(1/2)} = 1$ refers to the tree level contribution from the $\mathcal{O}(p^4)$ lagrangian which, as for $Q_{1,2}$, carries two derivatives [i.e. is $\mathcal{O}(p^2)$]. Therefore, as for $Q_{1,2}$, we expect the large-N_c value of $B_6^{(1/2)}$ to be affected by large $\mathcal{O}(p^2/N_c)$ corrections [$\sim \mathcal{O}(100\,\%)$] resulting from the loops over the $\mathcal{O}(p^2)$ tree operator. This shows clearly that, in the same way as for the $\Delta I = 1/2$ rule, no clear statement can be done on the expected size of ε'/ε without calculating these $1/N_c$ non-factorizable corrections.

3 ANALYSIS OF ε'/ε

Analytical formulas for all matrix elements, at next-to-leading order in the twofold expansion in powers of momenta and of $1/N_c$, are given in refs. [8,9]. In the pseudoscalar approximation, the matching has to be done below 1 GeV. Varying Λ_c between 600 and 900 MeV, the bag factors $B_1^{(1/2)}$ and $B_2^{(1/2)}$ take the values $8.2 - 14.2$ and $2.9 - 4.6$; quadratic terms in $\langle Q_1\rangle_0$ and $\langle Q_2\rangle_0$ produce a large enhancement which brings the $\Delta I = 1/2$ amplitude in agreement with the data [9]. Corrections beyond the chiral limit were found to be small.

For $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ the leading non-factorizable loop corrections, which are of $\mathcal{O}(p^0/N_c)$, are only logarithmically divergent [8]. Including terms of $\mathcal{O}(p^0)$, $\mathcal{O}(p^2)$, and $\mathcal{O}(p^0/N_c)$, $B_6^{(1/2)}$ and $B_8^{(3/2)}$ take the values $1.10 - 0.72$ and $0.64 - 0.42$. As a result, as with large-N_c values, ε'/ε is obtained generally much smaller than the data, except for input parameters taken close to the extreme of the ranges we considered. However, as stated above, since the leading $\mathcal{O}(p^0)$ contribution vanishes for Q_6, corrections from higher order terms beyond the $\mathcal{O}(p^2)$ and $\mathcal{O}(p^0/N_c)$ are expected to be large. In ref. [7] we investigated the $\mathcal{O}(p^2/N_c)$ contribution, i.e., the $1/N_c$ correction at the next order in the chiral expansion, because it brings about, for the first time, quadratic corrections on the cutoff. From counting arguments and more generally from the fact that the chiral limit is assumed to be reliable, the quadratic terms (which are not chirally suppressed) are expected to be dominant. It is still desirable to check that explicitly by calculating the corrections beyond the chiral limit, from logarithms and finite terms, as done for Q_1 and Q_2. Numerically, we observe a large positive correction from the quadratic term in $\langle Q_6\rangle_0$.

Table 1: Numerical values for ε'/ε (in units of 10^{-4}) as explained in the text.

LO	$14.8 \leq \varepsilon'/\varepsilon \leq 19.4$	
NDR	$12.5 \leq \varepsilon'/\varepsilon \leq 18.3$	central
HV	$7.0 \leq \varepsilon'/\varepsilon \leq 14.9$	
LO	$6.1 \leq \varepsilon'/\varepsilon \leq 48.5$	
NDR	$5.2 \leq \varepsilon'/\varepsilon \leq 49.8$	scanning
HV	$2.2 \leq \varepsilon'/\varepsilon \leq 38.5$	

This point was already emphasized in ref. [15]. The slope of the correction is qualitatively consistent and welcome since it compensates for the logarithmic decrease at $\mathcal{O}(p^0/N_c)$. Varying Λ_c between 600 and 900 MeV, the $B_6^{(1/2)}$ factor takes the values $1.50 - 1.62$. Q_6 is a $\Delta I = 1/2$ operator, and the enhancement of $\langle Q_6 \rangle_0$ indicates that at the level of the $1/N_c$ corrections the dynamics of the $\Delta I = 1/2$ rule applies to Q_6 as to Q_1 and Q_2. The size of the enhancement for $B_6^{(1/2)}$ appears however to be smaller for Q_6 than for $Q_{1,2}$ due to a smaller coefficient of the quadratic term. This coefficient is nevertheless large enough to increase ε'/ε by almost a factor of two.

Using the quoted values for $B_6^{(1/2)}$ together with the full leading plus next-to-leading order B factors for the remaining operators [7] the results we obtain for ε'/ε are given in tab. 1 for the three sets of Wilson coefficients LO, NDR, and HV and for Λ_c between 600 and 900 MeV. The numbers are close to the measured value for central values of the parameters (upper set). They are obtained by assuming zero phases from final state interactions. This approximation is very close to the results we would get if we used the small imaginary part obtained at the one-loop level [7].

Performing a scanning of the parameters [$125\,\text{MeV} \leq m_s(1\,\text{GeV}) \leq 175$ MeV, $0.15 \leq \Omega_{\text{IB}} \leq 0.35$, $1.04 \cdot 10^{-4} \leq \text{Im}\lambda_t \leq 1.63 \cdot 10^{-4}$, and $245\,\text{MeV} \leq \Lambda_{\text{QCD}} \leq 405\,\text{MeV}$] we obtain the numbers in lower set of tab. 1. They can be compared with the results of refs. [5,12,16–18]. The values of $B_8^{(3/2)}$ can also be compared with refs. [13,19]. Other recent calculations are reported in refs. [20–22]. The wide ranges reported in the table can be traced back, to a large extent, to the large ranges of the input parameters. This can be seen by comparing them with the relatively narrow ranges obtained for central values of the parameters. The parameters, to a large extent, act multiplicatively, and the large range for ε'/ε is due to the fact that the central value(s) for the ratio are enhanced roughly by a factor of two compared to the results obtained with B factors for Q_6 and Q_8 close to

Table 2: Same as in Tab. 1, but now with the phenomenological values for the phases.

LO	$19.5 \leq \varepsilon'/\varepsilon \leq 24.7$	
NDR	$16.1 \leq \varepsilon'/\varepsilon \leq 23.4$	central
HV	$9.3 \leq \varepsilon'/\varepsilon \leq 19.3$	
LO	$8.0 \leq \varepsilon'/\varepsilon \leq 62.1$	
NDR	$6.8 \leq \varepsilon'/\varepsilon \leq 63.9$	scanning
HV	$2.8 \leq \varepsilon'/\varepsilon \leq 49.8$	

the VSA. More accurate information on the parameters, from theory and experiment, will restrict the values for ε'/ε.

To estimate the uncertainties due to higher order final state interactions we also calculated ε'/ε using the real part of the matrix elements and the phenomenological values of the phases [23], $\delta_0 = (34.2 \pm 2.2)^\circ$ and $\delta_2 = (-6.9 \pm 0.2)^\circ$, i.e., we replaced $|\sum_i y_i \langle Q_i \rangle_I|$ in Eq. (2) by $\sum_i y_i \mathrm{Re}\langle Q_i \rangle_I / \cos\delta_I$. The corresponding results are given in tab. 2. They are enhanced by $\sim 25\,\%$ compared to the numbers in tab. 1. To reduce the FSI uncertainties in the $1/N_c$ approach it would be interesting to investigate the two-loop imaginary part. By doing so we expect to get phases very close to the ones of ref. [24] which have been obtained in Chiral Perturbation Theory at the same order and reproduce relatively well the data. In this sense we expect to get results close to the ones of tab. 2. A comparison of tabs. 1 and 2 will be however still useful to estimate higher order corrections (e.g. for the real part). In our analysis part of the uncertainty from higher order corrections is also included in the range due to the (moderate) residual dependence on the matching scale. In order to reduce the scheme dependence in the result, appropriate subtractions would be necessary [11,12,25]. Finally, it is reasonable to assume that the effect of the pseudoscalar mesons is the most important one. Nevertheless, the incorporation of vector mesons and higher resonances would be desirable in order to improve the treatment of the intermediate region around the rho mass and to show explicitly that the large enhancement we find at low energy at the level of the pseudoscalars remains up to the scale $\sim m_c$, where the matching with the short-distance part can be done more safely.

4 ON THE SIZE OF THE ERRORS IN THE ANALYSIS of ε'/ε

As shown in tabs. 1 and 2 the errors we obtain for ε'/ε are large. We believe however that the uncertainties are not largely overestimated and reflect well our present knowledge on

ε'/ε.

Note that we performed a scanning of the parameters. Refs. [5,16–18] used in addition to the scanning method also a Monte Carlo procedure with gaussian distributions for the experimental input and flat distributions for the theoretical parameters. In this way a probability distribution is obtained for ε'/ε, and the authors gave the median and the $68\,\%$ confidence level interval. We would like to emphasize that the use of this C.L. interval removes part of the hadronic uncertainties and leads to a range for ε'/ε which is two times smaller than the one obtained from a full scanning over the ranges of the theoretical (mostly hadronic) parameters. In our opinion the Monte Carlo analysis is misleading because there is no justification for assuming *any* probability distribution for theoretical parameters, and it leads to an underestimate of the uncertainties in the calculation. To illustrate this point one might note that values for ε'/ε above the $68\,\%$ C.L. range given e.g. in ref. [16] can be obtained for central values of the experimental parameters and for quite reasonable values of the theoretical ones within the ranges considered in this reference. Therefore we think that a full scanning of the theoretical parameters, with gaussian distribution or full scanning for the experimental parameters[1], gives a better idea of the uncertainties in the CP ratio. A similar comment applies to the range for Imλ_t since the determination of the CKM phase involves many non-perturbative theoretical parameters (as e.g. $\hat{B}_K$).

Moreover, as we explained above, non-chirally suppressed corrections beyond the large N_c limit are essential for $B_6^{(1/2)}$. Therefore, in our opinion larger errors for $B_6^{(1/2)}$ should have been taken in ref. [16] since the authors did not calculate this parameter (in this case, from the counting in p^2 and $1/N_c$, the errors could be taken as large as $100\,\%$). The same comment applies to ref. [18] where a similar range for $\langle Q_6\rangle_0$ was adopted [$B_6^{(1/2)}$ was varied around its large-N_c value taking an error of $100\,\%$, but m_s was fixed adopting the value $(m_s+m_d)(2\,\mathrm{GeV}) = 130\,\mathrm{MeV}$ without considering any error on it]. A similar statement applies to the dispersive analysis of ref. [26] which does not give access to any of the scale dependent non-factorizable terms (the non-chirally suppressed quadratic corrections in particular). The calculation of the scale dependent terms is not easier for lower values of the squared momentum of the kaon, and dispersive techniques do not help in their calculation. To neglect these terms leads to a failure in reproducing the $\Delta I = 1/2$ rule and is also not justified for Q_6.[2] As pointed out in ref. [18], in the Chiral Quark Model [17] the correlation between the $\Delta I = 1/2$ amplitude and ε'/ε (used to fix

[1] Since the ranges in tabs. 1 and 2 are predominantly due to the theoretical uncertainties, the difference between these two procedures is moderate.

[2] Further comments on the dispersive analysis of the FSI effects in the calculation of ε'/ε can be found in ref. [27].

the parameters necessary to estimate the matrix element of Q_6) is subject to potentially large uncertainties. As for $\Omega_{\rm IB}$, a minimum error of $\sim\ 0.10$ seems to be required for a careful estimate of ε'/ε. The value $\Omega_{\rm IB}\ =\ 0.16 \pm 0.03$ in ref. [3] (which has been used in ref. [18]) was obtained by investigating the $\pi - \eta$ contribution to this parameter (including $\eta - \eta'$ mixing). However, corrections beyond this term could be non-negligible as suggested by the numerical results of refs. [28,29].

We would like to emphasize also that the $\sim\ 25\,\%$ error obtained by comparing tabs. 1 and 2 should be included by any analysis which either does not include final state interactions or does not reproduce well the numerical values of the phases. This estimate of part of the neglected higher order corrections is usually not taken into account.

We conclude that at present there is no method which can predict ε'/ε with an error much smaller than the one presented in tabs. 1 and 2 which give a good idea of the uncertainties involved in the calculation of the CP ratio. The statement, that the experimental data can be accommodated (only) if all the hadronic parameters are taking values at the extreme of their reasonable ranges (see e.g. ref. [30]), which is based on the use of the $68\,\%$ C.L. intervals of refs. [5,16,18], has only weak theoretical foundations.

5 COMMENTS ON ε'/ε

Following our analysis, described above, a series of comments can be made:

- The use of the large-N_c value or values close to it is not justified for Q_6 in ε'/ε in the same way as for $Q_{1,2}$ in the $\Delta I = 1/2$ rule.

- The related claim that we expect in general in the standard model a value of ε'/ε smaller than the data is therefore not justified. One should note that the main methods used to calculate the $1/N_c$ corrections [7,12,17] all find them large and positive.

- Dispersive techniques as proposed in ref. [26] do not help in the calculation of many of these corrections (i.e. of the scale dependent terms).

- Errors are large. A value between $\sim$ few $\cdot\ 10^{-4}$ and $\sim\ 5 \cdot 10^{-3}$ appears perfectly plausible without going beyond the reasonable ranges of the parameters.

- Therefore ε'/ε cannot be used at present to investigate new physics. Even if Imλ_t was found as small as $0.6 \cdot 10^{-4}$ as could be suggested by recent measurements [31], we could still not exclude that the standard model reproduces the data.

- At low energy there is a significant enhancement of $B_6^{(1/2)}$ [7]. It is quite reasonable that below $500\ -\ 600\,$MeV the model independent lagrangian of Eq. (3) gives the bulk of the result. This is an important indication for a large value of ε'/ε in accordance with the data. The results of refs. [12,17] also point towards this direction.

- The effects of dimension-eight operators could possibly change the results largely [32] and should certainly be investigated.

- Despite the recent progress in the calculation of $\Omega_{\rm IB}$ [3,28,29] the problem of an accurate calculation of this parameter is still relevant in the same way as for $B_6^{(1/2)}$.

Questions (E. Pallante, Univ. Barcelona):
1) *Can you comment on the one-loop estimate of $B_8^{(3/2)}$ in ref. [19]?* The results of ref. [19] (which have been obtained in the chiral limit) and our results for $B_8^{(3/2)}$ are not incompatible within their respective errors especially for moderate values of Λ_c.
2) *The value of $\Omega_{\rm IB} = 0.16 \pm 0.03$ is actually not a pure large-N_c calculation. There is no reason to expect large $1/N_c$ corrections in this case.* There is a priori no reason for the $1/N_c$ corrections from irreducible one-loop diagrams beyond the reducible ones calculated in ref. [3] to be negligible (see also refs. [28,29]).

Acknowledgements
We warmly thank our collaborators G.O. Köhler and E.A. Paschos. We acknowledge partial support from BMBF, 057D093P(7), Bonn, FRG, DFG Antrag PA-10-1 and EEC, TMR-CT980169.

References

[1] A. Alavi-Harati *et al.*, Phys. Rev. Lett. **83**, 22 (1999); B. Hsiung, these proceedings.

[2] V. Fanti *et al.*, Phys. Lett. **B465** (1999) 335; G. Tatishvili, these proceedings.

[3] G. Ecker *et al.*, Phys. Lett. **B477**, 88 (2000), and references therein.

[4] A.J. Buras *et al.*, Nucl. Phys. **B370**, 69 (1992), ibid. **B400**, 37 (1993), **B400**, 75 (1993), **B408**, 209 (1993). See also: M. Ciuchini *et al.*, Phys. Lett. **B301**, 263 (1993); Nucl. Phys. **B415**, 403 (1994).

[5] S. Bosch *et al.*, Nucl. Phys. **B565**, 3 (2000).

[6] J. Gasser, H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).

[7] T. Hambye *et al.*, Nucl. Phys. **B564**, 391 (2000); hep-ph/0001088; T. Hambye, P. Soldan, hep-ph/9908232.

[8] T. Hambye *et al.*, Phys. Rev. **D58**, 014017 (1998).

[9] T. Hambye, G. Köhler, P. Soldan, Eur. Phys. J. **C10** (1999) 27.

[10] W.A. Bardeen, A.J. Buras, J.-M. Gérard, Phys. Lett. **B180**, 133 (1986); ibid. **B192**, 138 (1987); Nucl. Phys. **B293**, 787 (1987).

[11] J. Bijnens, J. Prades, JHEP 9901:023 (1999), ibid. 0001:002 (2000); J. Bijnens, hep-ph/ 9907514; J. Prades, hep-ph/9909245.

[12] J. Prades, these proceedings; J. Bijnens, J. Prades, JHEP 0006:035 (2000).

[13] M. Knecht, S. Peris, E. de Rafael, hep-ph/9910396; Phys. Lett. **B457**, 227 (1999), ibid. **B443**, 255 (1998); S. Peris, M. Perrottet, E. de Rafael, JHEP **9805**, 11 (1998).

[14] A.J. Buras, Les Houches lectures (1998), hep-ph/9806471.

[15] P. Soldan, hep-ph/9608281.

[16] A.J. Buras et al., hep-ph/0007313.

[17] S. Bertolini, J.0. Eeg, M. Fabbrichesi, hep-ph/0002234; Rev. Mod. Phys. **72**, 65 (2000); Nucl. Phys. **B476**, 225 (1996); S. Bertolini *et al.*, Nucl. Phys. **B514**, 93 (1998).

[18] M. Ciuchini, G. Martinelli, hep-ph/0006056; M. Ciuchini *et al.*, hep-ph/9910237; Nucl. Phys. **B573**, 201 (2000).

[19] J.F. Donoghue, E. Golowich, Phys. Lett. **B478**, 172 (2000); V. Cirigliano, E. Golowich, Phys. Lett. **B475**, 351 (2000).

[20] A.A. Bel'kov *et al.*, hep-ph/9907335.

[21] E.A. Paschos, hep-ph/9912230.

[22] S. Narison, these proceedings; hep-ph/0004 247.

[23] E. Chell, M.G. Olsson, Phys. Rev. **D48**, 4076 (1993).

[24] J. Gasser, U.G. Meissner, Phys. Lett. **B258**, 219 (1991).

[25] W.A. Bardeen, Nucl. Phys. Proc. Suppl. **7A**, 149 (1989).

[26] E. Pallante, I. Scimemi, these proceedings; E. Pallante, A. Pich, hep-ph/0007208; Phys. Rev. Lett. **84**, 2568 (2000).

[27] A.J. Buras *et al.*, Phys. Lett. **B480**, 80 (2000).

[28] S. Gardner, G. Valencia, Phys. Lett. **B466**, 355 (1999).

[29] C.E. Wolfe, K. Maltman, hep-ph/0007319; Phys. Lett. **B482**, 77 (2000).

[30] Y. Nir, hep-ph/0008226.

[31] Talks presented by D. Hitlin (Babar Collaboration) and H. Aihara (Belle Collaboration) at ICHEP2000, Osaka (July 2000).

[32] J.F. Donoghue, these proceedings; V. Cirigliano, J.F. Donoghue, E. Golowich, hep-ph/0007196.