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Chiral loops and a_0(980) exchange in $\phi \to \pi^0 \eta \gamma$

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Abstract

The radiative $\phi \to \pi^0 \eta \gamma$ decay is discussed emphasizing the effects of the a_0(980) scalar resonance which dominates the high values of the $\pi^0 \eta$ invariant mass spectrum. In its lowest part, the proposed amplitude coincides with the reliable and ChPT-inspired contribution coming from chiral loops. The a_0(980) resonance is then incorporated exploiting the complementarity between ChPT and the linear sigma model for this channel. The recently reported experimental invariant mass distribution and branching ratio can be satisfactorily accommodated in our framework. For the latter, a value of $B(\phi \to \pi^0 \eta \gamma)$ in the range $(0.75–0.95) \times 10^{-4}$ is predicted.

1 Introduction

The 1 GeV energy region is a particularly challenging domain. On one side, it is far below from the perturbative QCD regime and, on the other, Chiral Perturbation Theory (ChPT) is not expected to make reliable predictions at these energy values where resonance effects are known to be present. Among the latter, those proceeding by the exchange of the scalar resonances $f_0(980)$ and $a_0(980)$ should dominate their respective channels. The controversial nature of these resonances [1] and the poor knowledge of their properties [2] adds then further complexity (and interest) to this 1 GeV energy region. Indeed, several proposals have been suggested along the years concerning the constitution of these scalars as complex $qq\bar{q}\bar{q}$ states [3], $K\bar{K}$ molecules [4] or ordinary $q\bar{q}$ mesons [5].

The Novosibirsk CMD-2 and SND Collaborations have reported very recently, among others, the branching ratio and the $\pi^0\eta$ invariant mass distribution for the $\phi \to \pi^0\eta\gamma$ decay. For the branching ratio, the CMD-2 Collaboration reports $B(\phi \to \pi^0\eta\gamma) = (0.90 \pm 0.24 \pm 0.10) \times 10^{-4}$ [6], while the SND result is, consistently, $B(\phi \to \pi^0\eta\gamma) = (0.88 \pm 0.14 \pm 0.09) \times 10^{-4}$ [7]. The observed invariant mass distribution shows a significant enhancement at large $\pi^0\eta$ invariant mass that, according to Refs. [6,7], could be interpreted as a manifestation of a sizeable contribution of the $a_0(980)\gamma$ intermediate state. This and other radiative ϕ decays are also expected to be intensively investigated at the Frascati ϕ-factory DAΦNE [8].

On the theoretical side, the $V \to P^0P^0\gamma$ decays have been considered by a number of authors [1,9–12]. In particular, it has been shown that the intermediate vector meson contributions to $\phi \to \pi^0\eta\gamma$ lead to a small $B(\phi \to \pi^0\eta\gamma)_{\rm VMD} = 5.4 \times 10^{-6}$ [13], whereas a chiral loop model closely linked to standard ChPT predicts $B(\phi \to \pi^0\eta\gamma)_\chi = 3.0 \times 10^{-5}$ [14]. Needless to say, the scalar resonance effects and, in particular, the resonance pole associated to the $a_0(980)$ were not contemplated in these two approaches. The recent experimental data from Novosibirsk —for both the branching ratio and the $\pi^0\eta$ invariant mass spectrum showing an enhancement around the $a_0(980)$ mass— seem therefore to disfavour these predictions based on vector meson exchange and/or a simple extrapolation of ChPT ideas.

If we rely on the resonance picture, it is clear that the $a_0(980)$ scalar meson —lying just below the ϕ mass and having the appropriate quantum numbers— should play an important rôle in the $\phi \to \pi^0\eta\gamma$ decay. Several theoretical attempts to describe the effects of scalars in ϕ radiative decays have appeared so far. Among others, we would like to refer to the "no structure" model [15], to the K^+K^- model [1,11], where the $\phi \to a_0\gamma$ amplitude is generated through a loop of charged kaons, and to the chiral unitary approach ($U\chi PT$) [12], where the decay $\phi \to \pi^0\eta\gamma$ occurs through a loop of charged kaons that

subsequently annihilate into $\pi^0\eta\gamma$. In the two former cases the scalar resonances are included *ad hoc* while in the latter they are generated dynamically by unitarizing the one-loop amplitudes.

In this letter, we are mainly interested in incorporating scalar resonances and their pole effects into a ChPT inspired context [16]. While vector and axial-vector resonances can be included in a transparent and successful way, offering some theoretical basis to conventional vector meson dominance (VMD) ideas [17], the incorporation of scalar resonances has been more ambiguous and less successful up to now [16]. In order to take explicitly into account scalar resonances and their pole effects, we propose to use the linear sigma model (LσM). This will allow us to take advantage of the common origin of ChPT and the LσM to improve the chiral loop predictions for $V \rightarrow P^0P^0\gamma$ exploiting the complementarity of both approaches for these specific processes. On one side, ChPT is the established theory of the pseudoscalar interactions at low energy. However, it is not reliable at energies of a typical vector meson mass and, as just stated, scalar resonance poles are not explicitly included. As a consequence, ChPT inspired loop models can give rough estimates for $B(V \rightarrow P^0P^0\gamma)$ but will hardly be able to reproduce the observed enhancements in the invariant mass spectra. On the other side, the LσM is a much simpler model dealing similarly with pseudoscalar interactions but incorporating *scalar* resonances in a systematic and definite way. Thanks to this, the LσM should be able to reproduce the resonance peaks in the spectra and, although it does not provide a systematical framework for the pseudoscalar meson physics, this model could be of relevance in describing the scalar resonances when linked to a well established ChPT context. In order to show in detail the proposed framework, we will focus our attention on the $\phi \rightarrow \pi^0\eta\gamma$ decay mode. Other decay modes are somewhat more involved and will be analyzed in forthcoming work.

2 $\phi \rightarrow \pi^0\eta\gamma$ and chiral loops

The vector meson initiated $V \rightarrow P^0P^0\gamma$ decays cannot be treated in strict Chiral Perturbation Theory (ChPT). This theory has to be extended to incorporate on-shell vector meson fields. At lowest order, this may be easily achieved by means of the $\mathcal{O}(p^2)$ ChPT Lagrangian:

$$\mathcal{L}_2 = \frac{f^2}{4}\langle D_\mu U^\dagger D^\mu U + U^\dagger\chi + \chi^\dagger U\rangle\,, \tag{1}$$

where $f = f_\pi = 92.4$ MeV at this order, $U = \exp(i\sqrt{2}P/f)$ with P being the usual pseudoscalar nonet matrix, and $\chi = 2B_0\mathcal{M}$ with $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$. The covariant derivative, now enlarged to include vector mesons, is defined as $D_\mu U = \partial_\mu U -$

$ieA_\mu[Q, U] - ig[V_\mu, U]$, with $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ being the quark charge matrix and V_μ the additional matrix containing the nonet of ideally mixed vector meson fields. The diagonal elements of V are $(\rho^0 + \omega)/\sqrt{2}, (-\rho^0 + \omega)/\sqrt{2}$ and ϕ, thus following the same conventional normalization as for the pseudoscalar nonet matrix P.

There is no tree-level contribution from this Lagrangian to the $\phi \to \pi^0\eta\gamma$ amplitude and at the one-loop level one needs to compute the set of diagrams shown in Ref. [14]. A straightforward calculation leads to the following *finite* amplitude for $\phi(q^*, \epsilon^*) \to \pi^0(p)\eta(p')\gamma(q, \epsilon)$ (see Ref. [14] for further details):

$$
\begin{aligned}
\mathcal{A}(\phi \to \pi^0\eta\gamma)_\chi &= \tfrac{eg}{2\pi^2 m_{K^+}^2}(\epsilon^*\epsilon\, q^*q - \epsilon^*q\, \epsilon q^*)L(m_{\pi^0\eta}^2) \\
&\times \mathcal{A}(K^+K^- \to \pi^0\eta)_\chi ,
\end{aligned}
\tag{2}
$$

where $\epsilon^*\epsilon\, q^*q - \epsilon^*q\, \epsilon q^*$ makes the amplitude Lorentz- and gauge-invariant, $m_{\pi^0\eta}^2 \equiv s \equiv (p + p')^2 = (q^* - q)^2$ is the invariant mass of the final pseudoscalar system and $L(m_{\pi^0\eta}^2)$ is the loop integral function defined as

$$
\begin{aligned}
L(m_{\pi^0\eta}^2) &= \tfrac{1}{2(a-b)} - \tfrac{2}{(a-b)^2}\left[f\left(\tfrac{1}{b}\right) - f\left(\tfrac{1}{a}\right)\right] \\
&+ \tfrac{a}{(a-b)^2}\left[g\left(\tfrac{1}{b}\right) - g\left(\tfrac{1}{a}\right)\right] ,
\end{aligned}
\tag{3}
$$

where

$$
f(z) = \begin{cases} -\left[\arcsin\left(\tfrac{1}{2\sqrt{z}}\right)\right]^2 & z > \tfrac{1}{4} \\ \tfrac{1}{4}\left(\log\tfrac{\eta_+}{\eta_-} - i\pi\right)^2 & z < \tfrac{1}{4} \end{cases}
$$

$$
g(z) = \begin{cases} \sqrt{4z-1}\arcsin\left(\tfrac{1}{2\sqrt{z}}\right) & z > \tfrac{1}{4} \\ \tfrac{1}{2}\sqrt{1-4z}\left(\log\tfrac{\eta_+}{\eta_-} - i\pi\right) & z < \tfrac{1}{4} \end{cases}
\tag{4}
$$

and $\eta_\pm = \tfrac{1}{2}(1 \pm \sqrt{1-4z})$, $a = \tfrac{m_\phi^2}{m_{K^+}^2}$ and $b = \tfrac{m_{\pi^0\eta}^2}{m_{K^+}^2}$. The coupling constant g comes from the strong amplitude $\mathcal{A}(\phi \to K^+K^-) = g\epsilon^*(p_+ - p_-)$ with $|g| = 4.59$ to agree with $\Gamma(\phi \to K^+K^-)_{\exp} = 2.19$ MeV. The latter is the part beyond standard ChPT which we have fixed phenomenologically. The four-pseudoscalar amplitude is instead a standard ChPT amplitude[1] which is found to depend linearly on the variable $s = m_{\pi^0\eta}^2$:

$$
\mathcal{A}(K^+K^- \to \pi^0\eta)_\chi = \frac{1}{\sqrt{6}f_\pi^2}\left(m_{\pi^0\eta}^2 - \frac{10}{9}m_K^2 + \frac{1}{9}m_\pi^2\right) .
\tag{5}
$$

---

[1] $\mathcal{A}(K^+K^- \to \pi^0\eta_8)_\chi = \frac{\sqrt{3}}{4f_\pi^2}\left(m_{\pi^0\eta}^2 - \frac{4}{3}m_K^2\right)$ if only the η_8 contribution is taken into account as in Ref. [14].

In the calculation of the decay amplitudes (2) and (5) we have introduced η-η' mixing effects. As it is well known, a rigorous and general extension of $SU(3)$ ChPT to include the ninth pseudoscalar meson η_0 is not straightforward and requires the introduction of new terms in the chiral Lagrangian [18]. However, if one relies on classical arguments based on nonet symmetry, a phenomenologically successful description of the η-η' system is achieved [19]. The η-η' mixing angle is then found to be compatible with $\theta_P = \arcsin(-1/3) \simeq -19.5°$, quite in agreement with recent phenomenological estimates [20]. In Sect. 3, it will be shown that this choice for the η-η' mixing angle greatly simplifies the calculation of the $\phi \to \pi^0\eta\gamma$ amplitude in the LσM, and reduces up to a minimum the number of free-parameters.

The invariant mass distribution for the $\phi \to \pi^0\eta\gamma$ decay is predicted to be given by the following spectrum (see Fig. 1):

$$\frac{d\Gamma(\phi\to\pi^0\eta\gamma)_\chi}{dm_{\pi^0\eta}} = \frac{\alpha}{192\pi^5}\frac{g^2}{4\pi}\frac{m_\phi^4}{m_{K^+}^4}\frac{m_{\pi^0\eta}}{m_\phi}\left(1 - \frac{m_{\pi^0\eta}^2}{m_\phi^2}\right)^3\sqrt{1 - 2\frac{m_{\pi^0}^2+m_\eta^2}{m_{\pi^0\eta}^2} + \left(\frac{m_\eta^2-m_{\pi^0}^2}{m_{\pi^0\eta}^2}\right)^2}$$
$$\times |L(m_{\pi^0\eta}^2)|^2|\mathcal{A}(K^+K^- \to \pi^0\eta)_\chi|^2 . \tag{6}$$

Integrating Eq. (6) over the whole physical region one obtains for the branching ratio:

$$B(\phi \to \pi^0\eta\gamma)_\chi = 0.47 \times 10^{-4} . \tag{7}$$

As expected, Fig. 1 shows that our chiral loop approach gives a reasonable prediction for the lower part of the spectrum but fails to reproduce the observed enhancement in its higher part, where $a_0(980)$-resonance effects (ignored up to this point of our approach) should manifest. As a consequence, the predicted branching ratio turns out to be below the experimental value by about a factor of 2.

3 Improved approach to $\phi \to \pi^0\eta\gamma$

To analyze the scalar resonance effects in the $V \to P^0P^0\gamma$ decay amplitudes, the linear sigma model (LσM) [21] will be shown to be particularly appropriate. It is a well-defined $U(3) \times U(3)$ chiral model which incorporates *ab initio* both the nonet of pseudoscalar mesons together with its chiral partner, the scalar mesons nonet. Recently, the model has been resurrected as a framework to study the implications of chiral symmetry for the controversial scalar sector of QCD, and some variations of the basic LσM Lagrangian have shown to be phenomenologically rather successful [22–24].

In this context, the $V \to P^0P^0\gamma$ decays proceed through a loop of charged pseudoscalar mesons emitted by the initial vector. Due to the additional emission of a photon, these charged pseudoscalar pairs with the initial $J^{PC} = 1^{--}$ quantum numbers can rescatter into $J^{PC} = 0^{++}$ pairs of charged or neutral pseudoscalars. The scalar resonances are
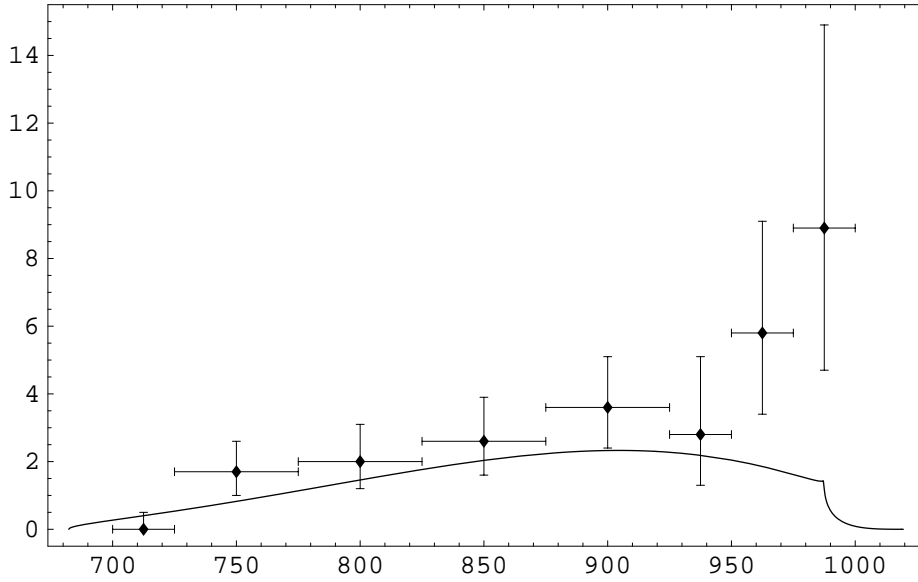
Figure 1: $dB(\phi \to \pi^0\eta\gamma)/dm_{\pi^0\eta} \times 10^7$ MeV^{-1} as a function of the $m_{\pi^0\eta}$ invariant mass in a chiral loop model. Experimental data are taken from Ref. [7].

expected to play an essential rôle in this rescattering process and the LσM seems mostly appropriate to fix the corresponding amplitudes.

Several simplifications happen when one considers the $\phi \to \pi^0\eta\gamma$ decay mode. As in the analysis of Sect. 2, contributions from charged pions in the loops are highly suppressed because they involve the isospin violating and OZI–rule forbidden $\phi\pi\pi$ coupling; hence, the dominant contributions arise exclusively from loops of charged kaons. The subsequent rescattering of these charged kaon pairs into the final $J^{PC} = 0^{++}$ $\pi^0\eta$ state is then quite simple. Indeed, the LσM amplitude for $K^+K^- \to \pi^0\eta$ contains a contact term, a term with an a_0 exchanged in the s-channel and two terms with a κ (*i.e.* the strange $I = 1/2$ scalar resonance) exchanged in the t- and u-channels. However, the latter κ-exchange contributions are absent for an η-η' mixing angle $\theta_P = \arcsin(-1/3) \simeq -19.5°$ since the $g_{\kappa K\eta}$ coupling constant appearing in one of the κ vertices vanishes. The calculation is then reduced to the diagrams shown in Fig. 2, and it is thus much simpler for this particular (but phenomenologically acceptable [20]) choice of the η-η' mixing angle. Moderate departures from this value translate into weak $g_{\kappa K\eta}$ couplings[2] appearing in the $\phi \to \pi^0\eta\gamma$ amplitude. Their effects seem to be small (see below) for the present and expected levels of experimental accuracy and for this kind of processes governed by poorly known scalar resonances. More importantly, the absence of κ contributions makes

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[2]The coupling constant $g_{\kappa K\eta}$ is proportional to $\sin[\theta_P - \arcsin(-1/3)]$ and its dependence on the η-η' mixing angle for values around $\theta_P = \arcsin(-1/3) \simeq -19.5°$ is soft.
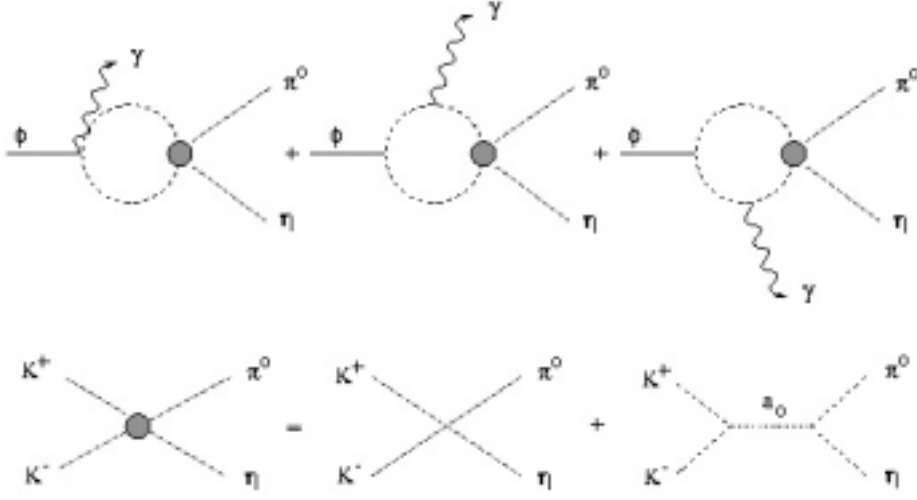
Figure 2: One-loop Feynman diagrams for $\phi \to \pi^0 \eta \gamma$ in the LσM for a η-η' mixing angle $\theta_P = \arcsin(-1/3) \simeq -19.5°$.

our predictions more definite and solid since we avoid one of the major uncertainties affecting the scalar nonet dynamics, namely, the mass of its strange members. Indeed, recent analysis by various authors require a light $\kappa(900)$ [3,22,25–27], while other authors deny the existence of a low mass pole [28] and identify the PDG $K_0^*(1430)$ state with the strange member of the lowest lying scalar nonet [23,24].

A straightforward calculation of the $\phi \to \pi^0 \eta \gamma$ decay amplitude leads to an expression identical to that in Eq. (2) but with the four-pseudoscalar amplitude now computed in a LσM context. In this case, the amplitude is just

$$\mathcal{A}(K^+ K^- \to \pi^0 \eta)_{\mathrm{L}\sigma\mathrm{M}} = g_{K^+ K^- \pi^0 \eta} - \frac{g_{a_0 K^+ K^-} g_{a_0 \pi^0 \eta}}{D_{a_0}(m_{\pi^0 \eta}^2)} \ , \tag{8}$$

where $D_{a_0}(m_{\pi^0 \eta}^2)$ is the a_0 propagator and the coupling constants are[3]

$$
\begin{aligned}
g_{K^+ K^- \pi^0 \eta} &= \frac{g_{a_0 \pi^0 \eta}}{2 f_K} = -\sqrt{\frac{2}{3}} \frac{m_{a_0}^2 - m_\eta^2}{2 f_K f_\pi} \ , \\
g_{a_0 K^+ K^-} &= -\frac{m_{a_0}^2 - m_K^2}{2 f_K} \ .
\end{aligned}
\tag{9}
$$

This amplitude can then be rewritten as

$$\mathcal{A}(K^+ K^- \to \pi^0 \eta)_{\mathrm{L}\sigma\mathrm{M}} = \frac{1}{\sqrt{6} f_K f_\pi}(m_{\pi^0 \eta}^2 - m_K^2) \times \frac{m_\eta^2 - m_{a_0}^2}{D_{a_0}(m_{\pi^0 \eta}^2)} \ . \tag{10}$$

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[3]See Ref. [22] for a detailed calculation of these coupling constants in the LσM.

We would like to make a few remarks on the four-pseudoscalar amplitude in Eq. (10) and compare it with the ChPT amplitude in Eq. (5):

i) For $m_{a_0} \to \infty$ and ignoring $SU(3)$-breaking in the pseudoscalar masses and decay constants, the LσM amplitude (10) reduces to the ChPT one (5). The former consists of a constant four-pseudoscalar vertex plus a second term whose s-dependence is generated by the $s \equiv m_{\pi^0\eta}^2$ piece in the a_0 propagator $D_{a_0}(s = m_{\pi^0\eta}^2)$, as shown in Eq. (8). Their sum (see Eq. (10)) in the good $SU(3)$ and $m_{a_0} \to \infty$ limits ends up with an amplitude which is linear in $s = m_{\pi^0\eta}^2$ and mimics perfectly the effects of the derivative and massive terms in the ChPT Lagrangian (1) leading respectively to the two terms in the ChPT amplitude (5). This, we believe, is the main virtue of our approach and makes the use of the LσM reliable at least for amplitudes like ours where s-channel exchange plays the main rôle and (t, u)-channel exchange can be ignored.

ii) The LσM and ChPT yield slightly different amplitudes in the $m_{a_0} \to \infty$ limit because of the way $SU(3)$-symmetry is broken in the two approaches. In the case of the LσM [22–24], a non $SU(3)$ symmetric choice of the vacuum expectation values makes simultaneously $m_\pi^2 \neq m_K^2$ and $f_\pi \neq f_K$, whereas in ChPT $m_\pi^2 \neq m_K^2$ is already present in the lowest order Lagrangian while $f_\pi \neq f_K$ is only achieved at higher orders.

iii) In the $\phi \to \pi^0\eta\gamma$ decay, the threshold for $\pi^0\eta$ production is not far from the mass of the $a_0(980)$ and that makes crucial the incorporation of the a_0 in an explicit way. Due to the presence of the full propagator $D_{a_0}(s)$, as in Eq. (10), such an amplitude —closely linked to that from ChPT and thus expected to be able to account for the lowest part of the $\pi^0\eta$ mass spectrum— should also be able to reproduce the effects of the a_0 pole at higher $\pi^0\eta$ invariant mass values.

iv) The need for the a_0 propagator introduces, however, some uncertainties in our treatment. Indeed, the opening of the $K\bar{K}$ channel near the $a_0(980)$ mass has motivated the use of different expressions for $D_{a_0}(s)$. A first possibility consists in using a Breit-Wigner propagator with an energy dependent width (to incorporate the known kinematic corrections):

$$D_{a_0}(s) = s - m_{a_0}^2 + i\sqrt{s}\,\Gamma_{a_0}(s)\,, \tag{11}$$

where

$$\begin{aligned}
\Gamma_{a_0}(s) &= \frac{g^2_{a_0\pi^0\eta}}{16\pi\sqrt{s}}\sqrt{\left[1 - \frac{(m_{\pi^0}+m_\eta)^2}{s}\right]\left[1 - \frac{(m_{\pi^0}-m_\eta)^2}{s}\right]}\,\theta(\sqrt{s} - (m_{\pi^0}+m_\eta)) \\
&+ \frac{g^2_{a_0K^+K^-}}{16\pi\sqrt{s}}\sqrt{1 - \frac{4m^2_{K^+}}{s}}\,\theta(\sqrt{s} - 2m_{K^+}) \\
&+ \frac{g^2_{a_0K^0\bar{K}^0}}{16\pi\sqrt{s}}\sqrt{1 - \frac{4m^2_{K^0}}{s}}\,\theta(\sqrt{s} - 2m_{K^0})\,.
\end{aligned} \tag{12}$$

Another interesting and widely accepted option was proposed by Flatté time ago specifically for the two-channel a_0 resonance [29]. The relative narrowness of the observed $\pi\eta$ peak around 980 MeV is then explained by the action of unitarity and analyticity at the $K\bar{K}$ threshold. This amounts to extend the preceding formulae below the $K\bar{K}$ threshold to include the now purely imaginary kaon contributions.

Due to these distinct possibilities to deal with the a_0 propagator, as well as to other differences introduced when implementing and fitting the basic LσM Lagrangian by several authors, a set of predictions can be obtained for the four-pseudoscalar amplitude (10). In turn, these various amplitudes have to substitute the four-pseudoscalar ChPT amplitude in Eq. (6) to finally obtain the corresponding invariant mass distributions of the $\phi \rightarrow \pi^0\eta\gamma$ decay mode[4]. Our purpose hereafter is to briefly discuss a few of these treatments in order to show that the observed properties for this specific decay can be accommodated in our ChPT- and LσM-inspired approach.

We start our discussion along the lines of Ref. [22] taking for the a_0 propagator the simple Breit-Wigner prescription in Eq. (11). The use of this propagator for the LσM amplitude Eq. (10) and its insertion in Eq. (6) predicts the $m_{\pi^0\eta}$ invariant mass spectrum shown by the dotted line in Fig. 3. Integrating over the whole physical region leads to the branching ratio

$$B(\phi \rightarrow \pi^0\eta\gamma)_{\mathrm{L}\sigma\mathrm{M}[22]} = 0.80 \times 10^{-4}\,, \tag{13}$$

in agreement with the experimental data [6,7]. However, since the simple expression used for the a_0 propagator implies a large a_0-width ($\Gamma_{a_0\rightarrow\pi\eta} \simeq 460$ MeV [22]), the desired enhancement in the invariant mass spectrum appears in its central part rather than around the a_0 peak.

This unpleasant feature is easily corrected when turning to the proposal by Törnqvist [23]. Indeed, a Gaussian form factor related to the finite size of physical mesons and depending on the final CM-momentum is introduced to describe the decays of scalar resonances in this approach. As a result, the decay width of $a_0(980)$ into $\pi^0\eta$ is reduced

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[4]Since all the four-pseudoscalar amplitudes we are considering depend only on the variable s, they factorize out of the loop integration and the structure of Eq. (6) is fully preserved.
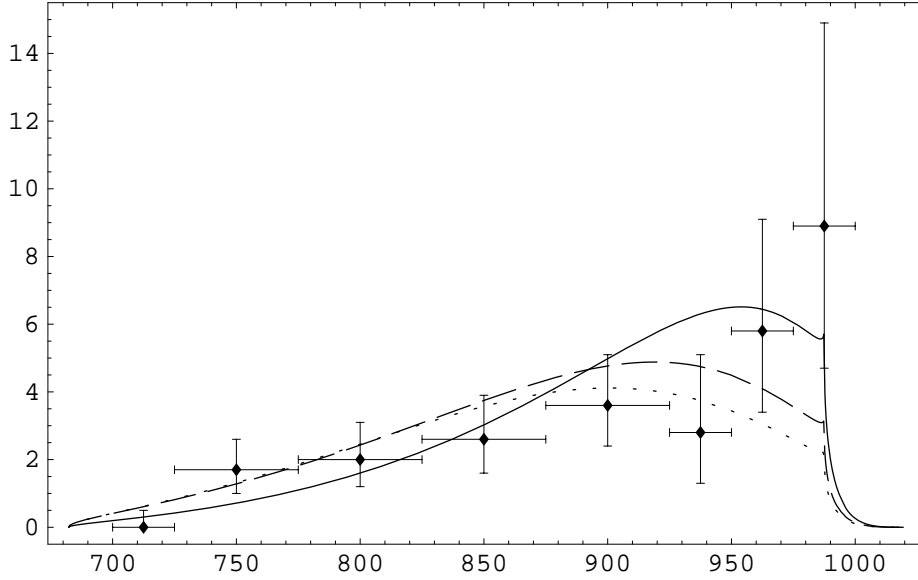
Figure 3: $dB(\phi \to \pi^0 \eta \gamma)/dm_{\pi^0 \eta} \times 10^7$ MeV^{-1} as a function of the $m_{\pi^0 \eta}$ invariant mass in the LσM. The dotted, dashed and solid lines correspond to the versions of the LσM proposed by Refs. [22–24] respectively. Experimental data are taken from Ref. [7].

($\Gamma_{a_0 \to \pi \eta} \simeq 273$ MeV [23]) without affecting that of $a_0(980)$ into $K\bar{K}$. This fact produces an enhancement in the spectrum for the higher values of the $m_{\pi^0 \eta}$ invariant mass, as shown by the dashed line in Fig. 3. The integrated branching ratio is then predicted to be

$$B(\phi \to \pi^0 \eta \gamma)_{\text{L}\sigma\text{M}[23]} = 0.90 \times 10^{-4} \; , \tag{14}$$

in good agreement with the experimental data [6,7]. A possible difficulty of this approach at the phenomenological level is that it predicts an η-η' mixing angle of $\theta_P \simeq -5.0°$ [23], considerably less negative than the usually accepted value (θ_P between $-20°$ and $-15°$ [20]) and the value required in our simplified analysis $\theta_P \simeq -19.5°$. This allows for an estimate of the typical errors introduced when neglecting κ-exchange in the t- and u-channels as compared to a_0-exchange in s-channel. The significant factor is the ratio of coupling constants $r \equiv (g_{\kappa^\pm K^\pm \eta} \, g_{\kappa^\pm K^\pm \pi^0})/(g_{a_0 \pi^0 \eta} \, g_{a_0 K^+ K^-})$, which vanishes in the ideal situation where $\theta_P \simeq -19.5°$ but takes the value $r \simeq -1/3$ for $\theta_P \simeq -5.0°$ if one ignores $SU(3)$-breaking corrections. To the smallness of $r \simeq -1/3$ one has to add the fact that the amplitude for the almost on-shell a_0-exchange in the s-channel is mainly imaginary and does not interfere with the almost real amplitude for off-shell κ-exchange in the t- and u-channels. As a result, the error in Eq. (14) introduced by neglecting this latter contribution can be estimated to be below some 10% even for such unusual values of $\theta_P \simeq -5°$.

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None of these drawbacks are encountered when turning to the treatment proposed by Shabalin [24]. In the fitting procedure adopted by this author no attempt is made to fix the η-η' mixing angle within the model. Thanks to this, one minimizes the uncertainties associated with the incorporation of the ninth pseudoscalar meson η_0 via the axial anomaly term. The value of the η-η' mixing angle is then fixed outside the model to its phenomenologically preferred value $\theta_P \simeq -19.5°$. Another relevant feature of Shabalin's approach is the introduction of the well-known Flatté corrections to the $a_0(980)$ propagator. The a_0-width is then drastically reduced from the uncorrected value $\Gamma_{a_0} \simeq 304$ MeV to a more acceptable visible width of $\Gamma_{a_0} \simeq 65$ MeV. With all this information taken from Ref. [24], our approach predicts the $m_{\pi^0\eta}$ invariant mass spectrum shown by the solid line in Fig. 3 and the integrated branching ratio

$$B(\phi \to \pi^0\eta\gamma)_{\text{L}\sigma\text{M}[24]} = 0.93 \times 10^{-4} \ . \tag{15}$$

Both the spectrum and the branching ratio are in nice agreement with the experimental data [6,7]. The fact that Shabalin's model incorporates the Flatté corrections to the a_0 resonant shape [29] has played a substantial rôle in this achievement.

4   Conclusions

The main aim of the present letter has been to propose and discuss an amplitude for the radiative $\phi \to \pi^0\eta\gamma$ decay exploiting the complementarity between ChPT and LσM ideas. Thanks to the latter, our amplitude contains the full propagator of the $a_0(980)$ scalar resonance which dominates the higher part of the $\pi^0\eta$ invariant mass spectrum. In the low invariant mass region, where ChPT is expected to work quite reliably, the proposed amplitude is shown to coincide with that coming from a chiral loop calculation. This, we believe, makes reliable our approach to the $V \to P^0 P^0 \gamma$ dynamics and, in particular, to the $\phi \to \pi^0\eta\gamma$ decay mode for which some simplifying conditions hold and lead to a simple and well-defined amplitude. Then our predictions depend only on a reduced number of parameters which, in principle, can be extracted from independent data. Some of these data refer to scalar meson properties which are not well established and thus affect the accuracy of our predictions although by no means in a drastic way.

We can safely conclude that all the reported properties for the $\phi \to \pi^0\eta\gamma$ decay mode can be accommodated in our approach. The branching ratio is predicted to be in the range $BR(\phi \to \pi^0\eta\gamma) = (0.75\text{–}0.95) \times 10^{-4}$, compatible with the present available data. Similarly, the measured $\pi^0\eta$ invariant mass spectrum is reproduced by our amplitude in a reasonable way. The uncertainties affecting these predictions suggest that further tests and more refined analyses are needed, particularly when the higher accuracy data from

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ongoing experiments will be available. This should contribute to clarify one of the most controversial aspects of hadron physics: the scalar states around 1 GeV.

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