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SUPERWORLDVOLUME DYNAMICS OF SUPERBRANES FROM NONLINEAR REALIZATIONS

S. Bellucci [a,1], E. Ivanov[b,c,2], S. Krivonos[c,3]

[a] *INFN-Laboratori Nazionali di Frascati, Via E. Fermi 40, I-00044 Frascati, Italy*

[b] *Laboratoire de Physique Théorique et des Hautes Energies, Université Paris 7, 2 Place Jussieu, 75251 Paris Cedex 05, France*

[c] *Bogoliubov Laboratory of Theoretical Physics, JINR 141 980 Dubna, Moscow region, Russian Federation*

Abstract

Based on the concept of the partial breaking of global supersymmetry (PBGS), we derive the worldvolume superfield equations of motion for *N=1, D=4* supermembrane, as well as for the space-time filling D2– and D3–branes, from nonlinear realizations of the corresponding supersymmetries. We argue that it is of no need to take care of the relevant automorphism groups when being interested in the dynamical equations. This essentially facilitates computations. As a by-product, we obtain a new polynomial representation for the *d=3,4* Born-Infeld equations, with merely a cubic nonlinearity.

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E-Mail:
[1] bellucci@lnf.infn.it
[2] eivanov@lpthe.jussieu.fr, eivanov@thsun1.jinr.ru
[3] krivonos@thsun1.jinr.ru

1. Introduction. During last few years there was a considerable interest in applying the general method of nonlinear realizations to systems with partial breaking of global supersymmetries (PBGS), first of all to the superbranes as a notable example of such systems (see, e.g., [1, 2, 3] and refs. therein). On this path one meets two problems. The first one is purely computational. Following the general prescriptions of nonlinear realizations, one is led to include into the coset, alongside with the spontaneously broken translation and supertranslation generators, also the appropriate part of generators of the automorphism group for the given supersymmetry algebra (including those of the Lorentz group). This makes the computations beyond the linearized approximation rather complicated. Moreover, sometimes these additional symmetries which we should take into account at the step of doing the coset routine appear to be explicitly broken at the level of the invariant action (see, e.g., refs. [4, 5, 6]), with no clear reasons for this. The second, closely related difficulty is lacking of a systematic procedure for constructing the PBGS actions. In all the cases elaborated so far, the PBGS Lagrangians cannot be constructed in a manifestly invariant way from the relevant Cartan forms: under the broken supersymmetry transformations they are shifted by the spinor or x-derivatives (like the WZNW or Chern-Simons Lagrangians).

In the present note we argue, on several instructive examples, that the automorphism symmetries can be ignored if we are interested only in the equations of motion for the given PBGS system. This radically simplifies the calculations, resulting in rather simple manifestly covariant equations in which all nonlinearities are hidden inside the covariant derivatives.

2. $N=1, D=4$ **supermembrane and D2-brane.** To clarify the main idea of our approach, let us start from the well known systems with partially broken global supersymmetries [7, 8]. Our goal is to get the corresponding superfield equations of motion in terms of the worldvolume superfields starting from the nonlinear realization of the global supersymmetry group.

The supermembrane in $D=4$ spontaneously breaks half of four $N=1, D=4$ supersymmetries and one translation. Let us split the set of generators of $N=1$ $D=4$ Poincaré superalgebra (in the $d=3$ notation) into the unbroken $\{Q_a, P_{ab}\}$ and broken $\{S_a, Z\}$ ones $(a, b=1,2)$. The $d=3$ translation generator $P_{ab}=P_{ba}$ together with the generator Z form the $D=4$ translation generator. The basic anticommutation relations read [1]

$$\{Q_a, Q_b\} = P_{ab} \ , \quad \{Q_a, S_b\} = \epsilon_{ab} Z \ , \quad \{S_a, S_b\} = P_{ab} \ . \tag{1}$$

In contrast to our previous considerations [8, 1, 2], here we prefer to construct the nonlinear realization of the superalgebra (1) itself, ignoring all generators of the automorphisms of (1) (the spontaneously broken as well as unbroken ones), including those of $D=4$ Lorentz group $SO(1,3)$. Thus, we put all generators into the coset and associate the $N=1$, $d=3$ superspace coordinates $\left\{\theta^a, x^{ab}\right\}$ with Q_a, P_{ab}. The remaining coset parameters are Goldstone superfields, $\psi^a \equiv \psi^a(x,\theta)$, $q \equiv q(x,\theta)$. A coset element g is defined by

$$g = e^{x^{ab}P_{ab}} e^{\theta^a Q_a} e^{qZ} e^{\psi^a S_a} \ . \tag{2}$$

[1] Hereafter, we consider the spontaneously broken supersymmetry algebras modulo possible extra central-charge type terms which should be present in the full algebra of the corresponding Noether currents to evade the no-go theorem of ref. [9] along the lines of ref. [10].

As the next step of the coset formalism, one constructs the Cartan 1-forms

$$g^{-1}dg = \omega_Q^a Q_a + \omega_P^{ab} P_{ab} + \omega_Z Z + \omega_S^a S_a, \tag{3}$$

$$\begin{aligned} \omega_Z &= dq + \psi_a d\theta^a \ , \ \omega_P^{ab} = dx^{ab} + \frac{1}{4}\theta^{(a}d\theta^{b)} + \frac{1}{4}\psi^{(a}d\psi^{b)} \ , \\ \omega_Q^a &= d\theta^a \ , \ \omega_S^a = d\psi^a; . \end{aligned} \tag{4}$$

and define the covariant derivatives

$$\mathcal{D}_{ab} = (E^{-1})_{ab}^{cd}\,\partial_{cd} \ , \quad \mathcal{D}_a = D_a + \frac{1}{2}\psi^b D_a \psi^c\,\mathcal{D}_{bc} = D_a + \frac{1}{2}\psi^b \mathcal{D}_a \psi^c\,\partial_{bc} \ , \tag{5}$$

where

$$D_a = \frac{\partial}{\partial\theta^a} + \frac{1}{2}\theta^b \partial_{ab} \ , \quad \{D_a, D_b\} = \partial_{ab} \ , \tag{6}$$

$$E_{ab}^{cd} = \frac{1}{2}(\delta_a^c\delta_b^d + \delta_a^d\delta_b^c) + \frac{1}{4}(\psi^c\partial_{ab}\psi^d + \psi^d\partial_{ab}\psi^c) \ . \tag{7}$$

They obey the following algebra

$$\begin{aligned} &[\mathcal{D}_{ab}, \mathcal{D}_{cd}] = -\mathcal{D}_{ab}\psi^f \mathcal{D}_{cd}\psi^g\,\mathcal{D}_{fg} \ , \\ &[\mathcal{D}_{ab}, \mathcal{D}_c] = \mathcal{D}_{ab}\psi^f \mathcal{D}_c\psi^g\,\mathcal{D}_{fg} \ , \\ &\{\mathcal{D}_a, \mathcal{D}_b\} = \mathcal{D}_{ab} + \mathcal{D}_a\psi^f \mathcal{D}_b\psi^g\,\mathcal{D}_{fg} \ . \end{aligned} \tag{8}$$

Not all of the above Goldstone superfields $\{q(x,\theta), \psi^a(x,\theta)\}$ must be treated as independent. Indeed, ψ_a appears inside the form ω_Z *linearly* and so can be covariantly eliminated by the manifestly covariant constraint (inverse Higgs effect [11])

$$\omega_Z|_{d\theta} = 0 \Rightarrow \psi_a = \mathcal{D}_a q \ , \tag{9}$$

where $|_{d\theta}$ means the ordinary $d\theta$-projection of the form. Thus the superfield $q(x,\theta)$ is the only essential Goldstone superfield needed to present the partial spontaneous breaking $N = 1\,,\ D = 4\ \Rightarrow\ N = 1\,,\ d = 3$ within the coset scheme.

Now we are ready to put additional, manifestly covariant constraints on the superfield $q(x,\theta)$, in order to get dynamical equations. The main idea is to covariantize the "flat" equations of motion. Namely, we simply replace the flat covariant derivatives in the standard equation of motion for the bosonic scalar superfield in $d = 3$

$$D^a D_a q = 0 \tag{10}$$

by the covariant ones (5)

$$\mathcal{D}^a \mathcal{D}_a q = 0 \ . \tag{11}$$

The equation (11) coincides with the equation of motion of the supermembrane in $D = 4$ as it was presented in [8]. Thus, we conclude that, at least in this specific case, additional superfields-parameters of the extended coset with all the automorphism symmetry generators included are auxiliary and can be dropped out if we are interested in the equations of motion only.

Actually, in [8] eq. (11) was deduced, proceeding from the $D = 4$ Lorentz covariant coset formalism with preserving all initial symmetries. This means that (11), having been now reproduced from the coset involving only the translations and supertranslations generators, possesses the hidden covariance under the full $D = 4$ Lorentz group. On the other hand, one more automorphism symmetry of the $N = 1$, $D = 4$ supersymmetry algebra, "γ_5" symmetry, is explicitly broken in eq. (11), and there is no way to keep it. In the $d = 3$ notation this symmetry is realized as an extra $SO(2)$ with respect to which the generators Q_a and S_b and, respectively, the coset parameters θ^a, ψ^a form a 2-vector. This symmetry is spontaneously broken at the level of the transformation laws, with the auxiliary field of $q(x, \theta)$ being the relevant Goldstone field. From eq. (11) we conclude that it cannot be preserved even in this spontaneously broken form when q is subjected to the dynamical equation: one can preserve the spontaneously broken $D = 4$ Lorentz symmetry at most. This $U(1)$ is explicitly broken in the off-shell PBGS action of ref. [8], as well as in the corresponding Green-Schwarz action [7]. A similar phenomenon was observed in refs. [4, 5] for the $N = (1, 0)$, $D = 6$ 3-brane. There, the auxiliary fields of the basic worldvolume $N = 1$, $d = 4$ Goldstone chiral supermultiplet are the Goldstone fields parameterizing the coset $SU(2)_A/U(1)_A$ of the automorphism $SU(2)_A$ group of $N = (1, 0)$, $D = 6$ Poincaré superalgebra, and the coset part of $SU(2)_A$ is realized as nonlinear shifts of these fields. In the superfield equations of motion of the 3-brane and the corresponding off-shell action this $SU(2)_A$ is *explicitly* broken down to $U(1)_A$, though the spontaneously broken $D = 6$ Lorentz symmetry is still preserved.

As a straightforward application of the idea that the automorphism symmetries are irrelevant when deducing the equations of motion, let us consider the case of the "space-time filling" D2-brane (i.e. having no scalar fields in its worldvolume multiplet the field content of which is that of $N = 1$, $d = 3$ vector multiplet). The main problem with the description of D-branes within the standard nonlinear realization approach is the lack of the coset generators to which one could relate the gauge fields as the coset parameters [2]. So we do not know how interpret the gauge fields as coset parameters in this case [3]. Let us show how these difficulties can be circumvented in the present approach.

The superalgebra we start with is the same algebra (1), but now without the central charge

$$Z = 0 .$$

The coset element g contains only one Goldstone superfield ψ^a which now must be treated as the essential one, and the covariant derivatives coincide with (5). Bearing in mind to end up with the irreducible field content of $N = 1$, $d = 3$ vector multiplet, we are led to treat ψ^a as the corresponding superfield strength and to find the appropriate covariantization of the flat irreducibility constraint and the equation of motion. In the flat case the $d = 3$ vector multiplet is represented by a $N = 1$ spinor superfield strength μ_a subjected to the Bianchi

[2]For the covariant field strengths as Goldstone fields such generators can still be found in the automorphism symmetry algebras [3, 12].

[3]It seems that the existing interpretation of gauge fields as the coset fields [13] can be generalized to the PBGS case only on the way of non-trivial unification of the gauge group algebra with that of supersymmetry, so that the gauge group transformations appeared in the closure of supersymmetries before any gauge-fixing as a sort of tensorial central charges.

identity [14]:

$$D^a \mu_a = 0 \ \Rightarrow \ \left\{ \begin{array}{l} D^2 \mu_a = -\partial_{ab} \mu^b \ , \\ \partial_{ab} D^a \mu^b = 0 \ . \end{array} \right\} . \tag{12}$$

This leaves in μ_a the first fermionic (Goldstone) component, together with the divergenceless vector $F_{ab} \equiv D_a \mu_b|_{\theta=0}$ (i.e., just the gauge field strength). The equation of motion reads

$$D^2 \mu_a = 0 \ . \tag{13}$$

In accordance with our approach, we propose the following equations which should describe the D2-brane:

$$(a) \ \ \mathcal{D}^a \psi_a = 0 \ , \quad (b) \ \ \mathcal{D}^2 \psi_a = 0 \ . \tag{14}$$

The equation (a) is a covariantization of the irreducibility constraint (12) while (b) is the covariant equation of motion.

In order to see which kind of dynamics is encoded in (14), we considered it in the bosonic limit. We found that it amounts to the following equations for the vector $V_{ab} \equiv \mathcal{D}_a \psi_b|_{\theta=0}$:

$$\left(\partial_{ac} + V_a^m V_c^n \partial_{mn}\right) V_b^c = 0 \ . \tag{15}$$

One can wonder how these nonlinear but polynomial equations can be related to the nonpolynomial Born-Infeld theory which is just the bosonic core of the superfield D2-brane theory as was explicitly demonstrated in [8]. The trick is to rewrite the parts of the equation (15), respectively antisymmetric and symmetric in the indices $\{a, b\}$, as follows:

$$\partial_{ab} \left(\frac{V^{ab}}{2 - V^2} \right) = 0 \ , \tag{16}$$

$$\partial_{ac} \left(\frac{V_b^c}{2 + V^2} \right) + \partial_{bc} \left(\frac{V_a^c}{2 + V^2} \right) = 0 \ , \tag{17}$$

where $V^2 \equiv V^{mn} V_{mn}$. After passing to the "genuine" field strength

$$F^{ab} = \frac{2 V^{ab}}{2 - V^2} \Rightarrow \partial_{ab} F^{ab} = 0 \ , \tag{18}$$

the equation of motion (17) takes the familiar Born-Infeld form

$$\partial_{ac} \left(\frac{F_b^c}{\sqrt{1 + 2F^2}} \right) + \partial_{bc} \left(\frac{F_a^c}{\sqrt{1 + 2F^2}} \right) = 0 \ . \tag{19}$$

Thus we have proved that the bosonic part of our system (14) indeed coincides with the Born-Infeld equations. One may explicitly show that the full equations (14) are equivalent to the worldvolume superfield equation following from the off-shell D2-brane action given in [8] (augmented with the Bianchi identity (12)). An indirect proof is based on the fact that (14) is an $N = 1$ extension of the bosonic $d = 3$ Born-Infeld equations, such that it possesses one more nonlinearly realized supersymmetry completing the explicit one to $N = 2$, $d = 3$ superalgebra (1) with $Z = 0$. On the other hand, the $N = 1$, $d = 3$ superfield action of [8] is uniquely specified by requiring it to possess this second supersymmetry. Hence both types of equations should be equivalent.

In closing this Section, it is worth mentioning that the equations (15) which equivalently describe the bosonic Born-Infeld dynamics in $d=3$, look much simpler than the standard ones (18), (19).

3. D3-brane. As another interesting application of the proposed approach, we shall consider the space-time filling D3-brane in $d=4$. This system amounts to the PBGS pattern $N=2,\, d=4 \;\to\; N=1,\, d=4$, with a nonlinear generalization of $N=1$, $d=4$ vector multiplet as the Goldstone multiplet [15, 6]. The off-shell superfield action for this system and the related equations of motion are known [15], but the latter have never been derived directly from the coset approach.

Our starting point is the $N=2$, $d=4$ Poincaré superalgebra *without* central charges:

$$\left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = 2P_{\alpha\dot\alpha}\;,\; \left\{S_\alpha, \bar{S}_{\dot\alpha}\right\} = 2P_{\alpha\dot\alpha}\;. \tag{20}$$

Assuming the $S_\alpha, \bar{S}_{\dot\alpha}$ supersymmetries to be spontaneously broken, we introduce the Goldstone superfields $\psi^\alpha(x,\theta,\bar\theta)$, $\bar\psi^{\dot\alpha}(x,\theta,\bar\theta)$ as the corresponding parameters in the following coset (we use the same notation as in [15])

$$g = e^{ix^{\alpha\dot\alpha}P_{\alpha\dot\alpha}} e^{i\theta^\alpha Q_\alpha + i\bar\theta_{\dot\alpha}\bar{Q}^{\dot\alpha}} e^{i\psi^\alpha S_\alpha + i\bar\psi_{\dot\alpha}\bar{S}^{\dot\alpha}}\;. \tag{21}$$

With the help of the Cartan forms

$$\begin{aligned}
g^{-1}dg &= i\omega^{\alpha\dot\alpha}P_{\alpha\dot\alpha} + i\omega_Q^\alpha Q_\alpha + i\bar\omega_{Q\,\dot\alpha}\bar{Q}^{\dot\alpha} + i\omega_S^\alpha S_\alpha + i\bar\omega_{S\,\dot\alpha}\bar{S}^{\dot\alpha}\;,\\
\omega^{\alpha\dot\alpha} &= dx^{\alpha\dot\alpha} - i\left(\theta^\alpha d\bar\theta^{\dot\alpha} + \bar\theta^{\dot\alpha}d\theta^\alpha + \psi^\alpha d\bar\psi^{\dot\alpha} + \bar\psi^{\dot\alpha}d\psi^\alpha\right)\;,\\
\omega_Q^\alpha &= d\theta^\alpha\;,\; \bar\omega_Q^{\dot\alpha} = d\bar\theta^{\dot\alpha}\;,\; \omega_S^\alpha = d\psi^\alpha\;,\; \bar\omega_S^{\dot\alpha} = d\bar\psi^{\dot\alpha}\;,
\end{aligned} \tag{22}$$

one can define the covariant derivatives

$$\begin{aligned}
\mathcal{D}_{\alpha\dot\alpha} &= \left(E^{-1}\right)_{\alpha\dot\alpha}^{\beta\dot\beta}\partial_{\beta\dot\beta}\;,\\
\mathcal{D}_\alpha &= D_\alpha - i\left(\bar\psi^{\dot\beta}D_\alpha\psi^\beta + \psi^\beta D_\alpha\bar\psi^{\dot\beta}\right)\mathcal{D}_{\beta\dot\beta}\;,\\
\overline{\mathcal{D}}_{\dot\alpha} &= \bar{D}_{\dot\alpha} - i\left(\bar\psi^{\dot\beta}\bar{D}_{\dot\alpha}\psi^\beta + \psi^\beta\bar{D}_{\dot\alpha}\bar\psi^{\dot\beta}\right)\mathcal{D}_{\beta\dot\beta}\;,
\end{aligned} \tag{23}$$

where

$$E_{\alpha\dot\alpha}^{\beta\dot\beta} = \delta_\alpha^\beta\delta_{\dot\alpha}^{\dot\beta} - i\psi^\beta\partial_{\alpha\dot\alpha}\bar\psi^{\dot\beta} - i\bar\psi^{\dot\beta}\partial_{\alpha\dot\alpha}\psi^\beta\;, \tag{24}$$

and the flat covariant derivatives are defined as follows

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\bar\theta^{\dot\alpha}\partial_{\alpha\dot\alpha}\;,\quad \bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\alpha\partial_{\alpha\dot\alpha}\;. \tag{25}$$

Now we are ready to write the covariant version of the constraints on ψ^α, $\bar\psi^{\dot\alpha}$ which define the superbrane generalization of $N=1$, $d=4$ vector multiplet, together with the covariant equations of motion for this system.

As is well-known [16], the $N=1$, $d=4$ vector multiplet is described by a chiral $N=1$ field strength W_α,

$$\overline{D}_{\dot\alpha}W_\alpha = 0\;,\quad D_\alpha\overline{W}_{\dot\alpha} = 0\;, \tag{26}$$

which satisfies the irreducibility constraint (Bianchi identity)

$$D^\alpha W_\alpha + \overline{D}_{\dot\alpha} \overline{W}^{\dot\alpha} = 0 \ . \tag{27}$$

The free equations of motion for the vector multiplet read

$$D^\alpha W_\alpha - \overline{D}_{\dot\alpha} \overline{W}^{\dot\alpha} = 0 \ . \tag{28}$$

It was shown in [15] that the chirality constraints (26) can be directly covariantized

$$\overline{\mathcal{D}}_{\dot\alpha} \psi_\alpha = 0 \ , \quad \mathcal{D}_\alpha \bar\psi_{\dot\alpha} = 0 \ . \tag{29}$$

These conditions are compatible with the algebra of the covariant derivatives (23). This algebra, with the constraints (29) taken into account, reads [15]

$$\begin{aligned}
&\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\overline{\mathcal{D}}_{\dot\alpha}, \overline{\mathcal{D}}_{\dot\beta}\} = 0 \ , \\
&\{\mathcal{D}_\alpha, \overline{\mathcal{D}}_{\dot\beta}\} = 2i\, \mathcal{D}_{\alpha\dot\beta} - 2i\, (\mathcal{D}_\alpha \psi^\gamma \overline{\mathcal{D}}_{\dot\beta} \bar\psi^{\dot\gamma})\, \mathcal{D}_{\gamma\dot\gamma} \ , \\
&\{\mathcal{D}_\alpha, \mathcal{D}_{\gamma\dot\gamma}\} = -2i\, (\mathcal{D}_\alpha \psi^\beta \mathcal{D}_{\gamma\dot\gamma} \bar\psi^{\dot\beta})\, \mathcal{D}_{\beta\dot\beta} \ .
\end{aligned} \tag{30}$$

The first two relations in (30) guarantee the consistency of the above nonlinear version of $N=1,\ d=4$ chirality. They also imply, like in the flat case,

$$(\mathcal{D})^3 = (\overline{\mathcal{D}})^3 = 0 \ . \tag{31}$$

The second flat irreducibility constraint, eq. (27), is not so simple to covariantize. The straightforward generalization of (27),

$$\mathcal{D}^\alpha \psi_\alpha + \overline{\mathcal{D}}_{\dot\alpha} \overline{\psi}^{\dot\alpha} = 0 \ , \tag{32}$$

is contradictory. Let us apply the square $(\mathcal{D})^2$ to the left-hand side of (32). When hitting the first term in the sum, it yields zero in virtue of the property (31). However, it is not zero on the second term. To compensate for the resulting non-vanishing terms, and thus to achieve compatibility with the algebra (30) and its corollaries (31), one should modify (32) by some higher-order coorrections [15].

Let us argue that the constraints (27) *together* with the equations of motion (28) can be straightforwardly covariantized as

$$\mathcal{D}^\alpha \psi_\alpha = 0 \ , \quad \overline{\mathcal{D}}_{\dot\alpha} \overline{\psi}^{\dot\alpha} = 0 \ . \tag{33}$$

Firstly, we note that no difficulties of the above kind related to the compatibility with the algebra (30) arise on the shell of eqs. (33). As a consequence of (33) and the first two relations in (30) we get

$$\mathcal{D}^2\, \psi_\alpha = 0, \quad \overline{\mathcal{D}}^2\, \bar\psi_{\dot\alpha} = 0 \ . \tag{34}$$

This set is a nonlinear version of the well-known reality condition and the equation of motion for the auxiliary field of vector multiplet. Then, applying, e.g., $\mathcal{D}_\alpha$ to the second equation in

(33) and making use of the chirality condition (29), we obtain the nonlinear version of the equation of motion for photino

$$\mathcal{D}_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} - (\mathcal{D}_\alpha\psi^\gamma\overline{\mathcal{D}}_{\dot\alpha}\bar\psi^{\dot\gamma})\,\mathcal{D}_{\gamma\dot\gamma}\bar\psi^{\dot\alpha} = 0\ . \tag{35}$$

Acting on this equation by one more $\mathcal{D}_\alpha$ and taking advantage of the equations (34), we obtain:

$$[\mathcal{D}^\alpha,\mathcal{D}_{\alpha\dot\alpha}]\,\bar\psi^{\dot\alpha} - \mathcal{D}_\alpha\psi^\gamma\{\mathcal{D}^\alpha,\overline{\mathcal{D}}_{\dot\alpha}\}\bar\psi^{\dot\gamma}\mathcal{D}_{\gamma\dot\gamma}\bar\psi^{\dot\alpha} - \mathcal{D}_\alpha\psi^\gamma\overline{\mathcal{D}}_{\dot\alpha}\bar\psi^{\dot\gamma}\,[\mathcal{D}^\alpha,\mathcal{D}_{\gamma\dot\gamma}]\,\bar\psi^{\dot\alpha} = 0\ . \tag{36}$$

After substituting the explicit expressions for the (anti)commutators from (30), we observe that (36) is satisfied identically, i.e. it does not imply any further restrictions on $\psi^\alpha,\bar\psi^{\dot\alpha}$. It can be also explicitly checked, in a few lowest orders in $\psi^\alpha,\bar\psi^{\dot\alpha}$, that the higher-order corrections to (32) found in [15] are vanishing on the shell of eqs. (33).

Thus the full set of equations describing the dynamics of the D3-brane supposedly consists of the generalized chirality constraint (29) and the equations (33). To prove its equivalence to the $N=1$ superfield description of D3-brane proposed in [15], recall that the latter is the $N=1$ supersymmetrization [17] of the $d=4$ Born-Infeld action with one extra nonlinearly realized $N=1$ supersymmetry. So, let us consider the bosonic part of the proposed set of equations. Our superfields $\psi,\bar\psi$ contain the following bosonic components:

$$V^{\alpha\beta} = V^{\beta\alpha} \equiv \mathcal{D}^\alpha\psi^\beta|_{\theta=0}\ ,\quad \bar V^{\dot\alpha\dot\beta} = \bar V^{\dot\beta\dot\alpha} \equiv \overline{\mathcal{D}}^{\dot\alpha}\bar\psi^{\dot\beta}|_{\theta=0}\ , \tag{37}$$

which, owing to (33), obey the following simple equations

$$\partial_{\alpha\dot\alpha}V^{\alpha\beta} - V^\gamma_\alpha\bar V^{\dot\gamma}_{\dot\alpha}\,\partial_{\gamma\dot\gamma}V^{\alpha\beta} = 0\ ,\quad \partial_{\alpha\dot\alpha}\bar V^{\dot\alpha\dot\beta} - V^\gamma_\alpha\bar V^{\dot\gamma}_{\dot\alpha}\,\partial_{\gamma\dot\gamma}\bar V^{\dot\alpha\dot\beta} = 0\ . \tag{38}$$

Like in the D2-brane case, in the equations (38) nothing reminds us of the Born-Infeld equations. Nevertheless, it is possible to rewrite these equations in the standard Born-Infeld form.

The first step is to rewrite eqs.(38) as

$$\left(1-\frac14 V^2\bar V^2\right)\partial_{\beta\dot\alpha}V^\beta_\alpha + \frac14\bar V^2 V^\beta_\alpha\partial_{\beta\dot\alpha}V^2 + \frac12\bar V^{\dot\beta}_{\dot\alpha}\partial_{\alpha\dot\beta}V^2 = 0\ , \tag{39}$$

$$\left(1-\frac14 V^2\bar V^2\right)\partial_{\alpha\dot\beta}\bar V^{\dot\beta}_{\dot\alpha} + \frac14 V^2\bar V^{\dot\beta}_{\dot\alpha}\partial_{\alpha\dot\beta}\bar V^2 + \frac12 V^\beta_\alpha\partial_{\dot\alpha\beta}\bar V^2 = 0\ . \tag{40}$$

After some algebra, one can bring them into the following equivalent form

$$\partial_{\beta\dot\alpha}\left(fV^\beta_\alpha\right) - \partial_{\alpha\dot\beta}\left(\bar f\bar V^{\dot\beta}_{\dot\alpha}\right) = 0\ ,\quad \partial_{\beta\dot\alpha}\left(gV^\beta_\alpha\right) + \partial_{\alpha\dot\beta}\left(\bar g\bar V^{\dot\beta}_{\dot\alpha}\right) = 0\ , \tag{41}$$

where

$$f = \frac{\bar V^2 - 2}{1-\frac14 V^2\bar V^2}\ ,\quad g = \frac{\bar V^2+2}{1-\frac14 V^2\bar V^2}\ . \tag{42}$$

After introducing the "genuine" field strengths

$$F^\beta_\alpha \equiv \frac{1}{2\sqrt2}\,f\,V^\beta_\alpha\ ,\quad \bar F^{\dot\beta}_{\dot\alpha} \equiv \frac{1}{2\sqrt2}\,\bar f\,\bar V^{\dot\beta}_{\dot\alpha}\ , \tag{43}$$

first of eqs. (41) is recognized as the Bianchi identity

$$\partial_{\beta\dot\alpha} F_\alpha^\beta - \partial_{\alpha\dot\beta} \bar F_{\dot\alpha}^{\dot\beta} = 0 \; , \tag{44}$$

while the second one acquires the familiar form of the Born-Infeld equation

$$\begin{aligned} &\partial_{\beta\dot\alpha}\left(\frac{1+F^2-\bar F^2}{\sqrt{(F^2-\bar F^2)^2 - 2(F^2+\bar F^2)+1}} F_\alpha^\beta \right) \\ &+ \partial_{\alpha\dot\beta}\left(\frac{1-F^2+\bar F^2}{\sqrt{(F^2-\bar F^2)^2 - 2(F^2+\bar F^2)+1}} \bar F_{\dot\alpha}^{\dot\beta} \right) = 0 \; . \end{aligned} \tag{45}$$

Thus, in this new basis the action for our bosonic system is the Born-Infeld action:

$$S = \int d^4x \sqrt{(F^2-\bar F^2)^2 - 2(F^2+\bar F^2)+1} \; . \tag{46}$$

Now the equivalence of the system (33) to the equations corresponding to the action of ref. [15], like in the D2-brane case, can be established proceeding from the following two arguments: (i) It is $N=1$ supersymmetrization of the $d=4$ Born-Infeld equations; (ii) It possesses the second hidden nonlinearly realized supersymmetry lifting $N=1$, $d=4$ to $N=2$, $d=4$. The action given in [15] provides the unique extension of the $d=4$ Born-Infeld action with both these requirements satisfied. Hence, both representations should be equivalent to each other.

Note that at the full superfield level the redefinition (43) should correspond to passing from the Goldstone fermions ψ_α, $\bar\psi_{\dot\alpha}$ which have the simple transformation properties in the nonlinear realization of $N=1$, $d=4$ supersymmetry but obey the nonlinear irreducibility constraints, to the ordinary Maxwell superfield strength W_α, $\bar W_{\dot\alpha}$ defined by eqs. (26), (27). The nonlinear action in [15] was written just in terms of this latter object. The equivalent form (33) of the equations of motion and Bianchi identity is advantageous in that it is manifestly covariant under the second (hidden) supersymmetry, being constructed out of the covariant objects.

4. Conclusions. In this Letter we demonstrated that in many cases one can simplify the analysis of the equations of motion which follow from the coset approach by taking no account of the automorphism group at all. We showed that the equations of motion for the $N=1$, $D=4$ supermembrane, D2- and D3-branes in a flat background have a very simple form when written in terms of Goldstone superfields of nonlinear realizations and the corresponding nonlinear covariant derivatives. As a by-product, we got a new simple form for the $d=3$ and $d=4$ Born-Infeld theory equations of motion combined with the appropriate Bianchi identities. The remarkable property of this representation is that it involves only a third order nonlinearity in the gauge field strength.

Note that the idea to use the geometric and symmetry principles to derive the dynamical equations is not new, of course. For instance, the completely integrable $d=2$ equations admit the geometrical interpretation as the vanishing of some curvatures. In the superembedding approach (see [18] and refs. therein) the equations of motion for superbranes in a number of important cases amount to the so-called "geometro-dynamical" constraint which, in the

PBGS language, is just a kind of the inverse Higgs constraints. For instance, this applies to the $N = 1$, $D = 10$ 5-brane [1, 2]. In this case the condition like (9), besides eliminating the Goldstone fermion superfield in terms of the appropriate analog of the $d = 3$ superfield q ($d = 6$ hypermultiplet superfield), also yields the equation of motion for the latter [4]. However, as we saw in the above examples, in other interesting cases the inverse Higgs (or geometro-dynamical) constraints do not imply any dynamics which, however, can still be implemented in a manifestly covariant way using the approach proposed here.

It still remains to fully understand why in the PBGS scheme the dynamical worldvolume superfield equations are not sensitive to the presence or absence of the automorphism generators in the initial coset construction. This is in contrast with the case of purely bosonic p-branes. For the self-consistent description of them in terms of nonlinear realizations one should necessarily make use of the cosets of the full target Minkowski space Poincaré group including the Lorentz (automorphism) part of the latter [3, 12]. A possible explanation of this apparent disagreement is that the Goldstone fermion superfields or Goldstone superfields associated with the central charges (and/or with the transverse components of the full momenta) already accommodate the Lorentz and other automorphism groups Goldstone fields. These come out as component fields in the θ -expansion of the Goldstone superfields. So the automorphism groups Goldstone fields are implicitly present in the superbrane superfield equations of motion.

The most interesting practical application of the approach exemplified here is the possibility to construct, more or less straightforwardly, the equations for the $N = 4$ and $N = 8$ supersymmetric Born-Infeld theory.This work is in progress now [19].

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