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The ratio $\Phi \to K^+K^-/K^0\bar{K}^0$

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Abstract

The ratio $\Phi \to K^+K^-/K^0\bar{K}^0$ is discussed and its present experimental value is compared with theoretical expectations. A difference larger than two standard deviations is observed. We critically examine a number of mechanisms that could account for this discrepancy, which remains unexplained. Measurements at DAΦNE at the level of the per mille accuracy can clarify whether there exist any anomaly.

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1 Introduction

The ϕ-meson was discovered many years ago as a $K\bar{K}$ resonance [1]. Its decay is dominated by the two $K\bar{K}$ decay modes which proceed through Zweig-rule allowed strong interactions. The ratio $R \equiv \phi \to K^+K^-/K^0\bar{K}^0$ has been measured in a variety of independent experiments using different ϕ-production mechanisms. Among these, the cleanest one is electron-positron annihilation around the ϕ resonance peak, *i.e.* the reactions $e^+e^- \to \phi \to K^+K^-/K^0\bar{K}^0$, which have been accurately measured at Novosibirsk quite recently [2] and are the object of intense investigation at the Frascati Φ-factory [3]. With as much as 8×10^6 ϕ's on tape, the KLOE experiment at DAΦNE can be expected to measure the above ratio R with a statistical accuracy of the order of the per mille. In view of this, we wish to discuss the theoretical expectations and compare them with the most recent determinations for this ratio.

In the following we shall first review the present experimental situation, then compare it with the naïve expectations from isospin symmetry and phase space considerations thus observing that a disagreement seems to exist. Contributions arising from electromagnetic radiative corrections and $m_u - m_d$ isospin breaking effects are analyzed and shown to bring the observed discrepancy to be more than three standard deviations. Various additional theoretical improvements on our analysis, such as the use of vector-meson dominated electromagnetic form-factors, the modification of the strong vertices and the inclusion of rescattering effects through the scalar resonances $f_0(980)$ and $a_0(980)$ using the charged kaon loop model, are also examined and shown not to change in any substantial way our results which imply a clear discrepancy between theory and the available data.

The first combined measurement of the four major ϕ decay modes in a single e^+e^- dedicated experiment has been performed quite recently with the general purpose detector CMD-2 at the upgraded e^+e^- collider VEPP-2M at Novosibirsk [2]. Having a single experiment normalized to almost 100% of decay modes implies a reduction of systematic errors, and the following branching ratios (BR) and errors from VEPP-2M [2] are quoted:

$$\mathrm{BR}(\phi \to K^+K^-) = (49.2 \pm 1.2)\% \,,$$
$$\mathrm{BR}(\phi \to K^0\bar{K}^0) = (33.5 \pm 1.0)\% \,,$$

(1)

leading to

$$R_{\mathrm{exp}} \equiv \frac{\mathrm{BR}(\phi \to K^+K^-)}{\mathrm{BR}(\phi \to K^0\bar{K}^0)} = 1.47 \pm 0.06 \,.$$

(2)

All these results were in agreement with the average values quoted in the then available

PDG 1994 compilation [4]:

$$\left.\begin{aligned} \mathrm{BR}(\phi \to K^+K^-) &= (49.1 \pm 0.9)\% \\ \mathrm{BR}(\phi \to K^0\bar{K}^0) &= (34.3 \pm 0.7)\% \end{aligned}\right\} \Longrightarrow R_{\mathrm{exp}} = 1.43 \pm 0.04 \ . \qquad (3)$$

The current PDG edition [5], now including the above VEPP-2M data, quotes

$$\left.\begin{aligned} \mathrm{BR}(\phi \to K^+K^-) &= (49.1 \pm 0.8)\% \\ \mathrm{BR}(\phi \to K^0\bar{K}^0) &= (34.1 \pm 0.6)\% \end{aligned}\right\} \Longrightarrow R_{\mathrm{exp}} = 1.44 \pm 0.04 \ , \qquad (4)$$

as a result of a global fit, which appears as a very stable result, established with a 3% error. In the same PDG edition, one can also find $R_{\mathrm{direct}} = 1.35 \pm 0.06$, as the averaged result of the various experiments measuring the ratio $\phi \to K^+K^-/K^0\bar{K}^0$ directly. A reduction of these errors can be expected from DAΦNE, where the KLOE experiment has already collected 8×10^6 ϕ-mesons. Like in the case of the CMD-2 detector, all the main decay modes of the ϕ will be measured by the same apparatus and this could bring the systematic errors to a minimum, while the statistics will allow to bring the statistical error well below the 1% level. Our discussion centers around this ratio R and the possible interest in studying it with a much reduced experimental error.

We shall approach this discussion by starting with the most naïve result for the above ratio R, *i.e.* $R = 1$, which follows from assuming that these $\phi \to K\bar{K}$ decay modes proceed exclusively via the strong interaction dynamics in the good isospin limit $m_u = m_d$ and ignoring phase space differences. The mass difference between neutral and charged kaons —which includes both isospin breaking effects ($m_u \neq m_d$) and electromagnetic (photonic) contributions— considerably increases this too-naïve prediction via the (purely kinematical) phase-space factor. Assuming now perfect isospin symmetry only for the strong interaction dynamics (equal couplings for ϕK^+K^- and $\phi K^0\bar{K}^0$) and knowing that $\phi \to K\bar{K}$ are P-wave decay modes of a narrow resonance, one necessarily has

$$R = \frac{\left(1 - \frac{4m_{K^+}^2}{M_\phi^2}\right)^{3/2}}{\left(1 - \frac{4m_{K^0}^2}{M_\phi^2}\right)^{3/2}} = 1.528 \ , \qquad (5)$$

with negligible errors coming from the mass values quoted in the PDG. The phase-space correction thus pushes the ratio R two standard deviations above its experimental value (4). This kinematical correction is exceptionally large because of the vicinity of the ϕ mass to the $K\bar{K}$ thresholds, which translates into considerably large differences between the charged and neutral kaon momenta (or velocities, $v_+/v_0 = 0.249/0.216 = 1.152$), a difference which is further increased to its third power in such P-wave decay modes.

This two-σ discrepancy between experiments and the theoretical tree level predictions obviously claims for further corrections. The most immediate of such corrections is due to electromagnetic radiative effects on the ratio R, which affect the numerator but not the denominator, and which will be discussed in the next section.

2 Electromagnetic radiative corrections

Electromagnetic radiative corrections are frequently ignored when dealing with strong decays. In our case, they could be relevant since, although small, they affect the charged decay mode but not the neutral one, and, in order to solve the discrepancy in the ratio R under consideration, only a few per cent correction is needed. Many years ago they were already considered by Cremmer and Gourdin [6] who found a positive correction of the order of 4% to the prediction in Eq. (5), thus enlarging that discrepancy. The dominant contribution was found to arise from the so-called Coulomb term which is positive for $\phi \to K^+ K^-$ and rather large because of the small kaon velocities $v_\pm = 0.249$. A similar increase of the ratio R (some 5%) by radiative corrections is expected by the experimentalists at VEPP-2M [2], whose quoted result is inclusive of any vertex correction. If we include this correction in the theoretically predicted ratio, the final result for the radiatively corrected ratio is then $R \simeq 1.59$ [6], in agreement with still another independent analysis by Pilkuhn leading to R in the range 1.52–1.61 [7]. To better qualify these statements, we shall now examine in detail the contribution of such corrections to the ratio R.

We have recalculated the electromagnetic radiative corrections to $\phi \to K^+ K^-$ along the lines of Ref. [6]. For the charged amplitude we start with the usual and simplest tree level expression $A_0(\phi \to K^+ K^-) = g_0 \epsilon_\mu (p_+ - p_-)^\mu$, where g_0 is the uncorrected strong coupling constant for $\phi K \bar{K}$, ϵ_μ is the ϕ polarization and $p_\pm$ are the kaon four-momenta. As is well known, the various contributions to the radiative corrections can be grouped in two parts. The first part comprises one-loop corrections to the uncorrected amplitude $A_0(\phi \to K^+ K^-)$. This part contains three vertex diagrams with one virtual photon exchanged between the two charged-kaons or between the $\phi K^+ K^-$ vertex and each charged-kaon. In addition, it also contains wave-function renormalization of external kaon lines that render the whole amplitude ultraviolet finite[1]. The second part is needed to cancel the infrared divergence. It contains three real-photon emission diagrams which are order $\sqrt{\alpha}$. Adding these two parts we find the complete order α corrective

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[1]Notice that Eq. (19) in Ref. [6] contains a small imaginary part while it is supposed to be the real part of the one-loop amplitude.

factor to the $\phi \to K^+ K^-$ decay width

$$1 + C_f + \beta_f \log \frac{2\Delta E}{m_{K^+}} \equiv 1 + \frac{\alpha}{\pi} \left\{ \frac{1+v^2}{2v}\pi^2 - 2\left(1 + \log \frac{2\Delta E}{m_{K^+}}\right)\left(1 + \frac{1+v^2}{2v}\log \frac{1-v}{1+v}\right) \right.$$

$$-\frac{1}{v}\log \frac{1-v}{1+v} - \frac{1+v^2}{4v}\log \frac{1-v}{1+v}\log \frac{1-v^2}{4} - \frac{1+v^2}{2v}\left[\text{Li}_2\left(\frac{2v}{1+v}\right) - \text{Li}_2\left(\frac{-2v}{1-v}\right)\right] \qquad (6)$$

$$\left. +\frac{1+v^2}{2v}\left[\text{Li}_2\left(\frac{1+v}{2}\right) - \text{Li}_2\left(\frac{1-v}{2}\right)\right] - \frac{1+v^2}{v}\left[\text{Li}_2(v) - \text{Li}_2(-v)\right] \right\} ,$$

where $v = \sqrt{1 - 4m_{K^+}^2/M_\phi^2}$ is the kaon velocity and ΔE stays for the photon energy resolution. For $\Delta E = 1$ MeV the correction (6) amounts to a 4.2% increase. Taking for ΔE the maximal available photon energy (32.1 MeV, not far from the energy resolution in the KLOE detector at DAΦNE, which is ≈ 20 MeV) makes no substantial difference as the main contribution comes from the Coulomb term, the first one inside the brackets.

The above discussion ignores the fact that what is actually measured at VEPP-2M and at DAΦNE is the ratio

$$R_{e^+e^-} \equiv \frac{\sigma(e^+e^- \to \phi \to K^+K^-)}{\sigma(e^+e^- \to \phi \to K^0\bar{K}^0)} , \qquad (7)$$

and that radiative corrections to R correspond to consider the ratio of the radiatively corrected cross-sections which appear at the numerator and denominator of $R_{e^+e^-}$. In addition to consider both initial and final state corrections, a complete treatment also requires to discuss the presence of the ϕ resonance and the associated distortion of the cross-sections [8]. At the numerator, radiative corrections include virtual corrections as well as emission of soft unobserved photons, both from the initial and final states, with no interference between initial and final state radiation for an inclusive measurement (*i.e.* in a measurement that does not distinguish the charges of the kaons) [9]. For the cross-section at the denominator, there are only initial state radiative corrections since the final kaons are neutral. In the absence of final state radiation, the presence of a narrow resonance like the ϕ in the intermediate state introduces large double logarithms which can be resummed [8,10] and factorized in an expression like

$$\left(\frac{\Gamma_\phi}{M_\phi}\right)^{\beta_i}(1 + C_i) , \qquad (8)$$

where $\beta_i = \frac{2\alpha}{\pi}\left(\log \frac{s}{m_e^2} - 1\right)$ is the initial state radiation factor and C_i is the finite part of the initial virtual and soft photon corrections, which survives after the cancellation of the infrared divergence and the exponentiation of the large resonant dependent factors. The same factor for initial state radiation appears both at numerator and denominator, and since there is no interference between initial and final state radiation, the real soft-photon radiative corrections to the initial state cancel out in the ratio (7). In principle, one should

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also resum the contributions coming from final state radiation but the final state radiative factor $\beta_f = \frac{2\alpha}{\pi}\left(\frac{1+v^2}{2v}\log\frac{1+v}{1-v} - 1\right) \simeq 3.9\times 10^{-4}$ is very small and resummation in this case is irrelevant. One then obtains the following expression for the ratio $R_{e^+e^-}$ as defined in Eq. (7):

$$R_{e^+e^-} = \frac{\Gamma(\phi\to K^+K^-)}{\Gamma(\phi\to K^0\bar{K}^0)}\,\frac{1+C_i+C_f+\beta_f\log\frac{2\Delta E}{m_{K^+}}}{1+C_i}\,. \tag{9}$$

Since $C_i \approx \frac{\alpha}{\pi}\left(\frac{3}{2}\log\frac{s}{m_e^2} + \frac{\pi^2}{3} - 2\right) \simeq 5.6\times 10^{-2}$ [11], one can expand the denominator in Eq. (9), canceling the C_i term and remaining with the final state correction terms C_f and β_f given explicitly in Eq. (6). We thus conclude that one is justified in using the expressions as above and that the conventional treatment of radiative corrections increases the previous two-σ discrepancy between experiment and theory for the ratio R to the level of three standard deviations.

3 $SU(2)$-breaking in $\phi K\bar{K}$ vertices

The ϕK^+K^- and $\phi K^0\bar{K}^0$ vertices are not equal (and thus do not cancel in the ratio R) once $SU(2)$-breaking effects are taken into account. The way $SU(2)$-breaking is usually introduced in the effective lagrangians is the same as for $SU(3)$-breaking, namely, via quark mass differences. In the latter $SU(3)$ case, an improved description of the vector-meson couplings to two pseudoscalar-mesons can easily be achieved as shown, for example, in Refs. [12,13]. But the situation is by far less convincing when turning to the much smaller $SU(2)$-breaking effects [14]. The essential feature —common to most models— is that the dynamics of these flavour symmetry breakings suppress the creation of heavier $q\bar{q}$ pairs. In the ϕK^+K^- and $\phi K^0\bar{K}^0$ vertices, one needs to produce a $u\bar{u}$ and a $d\bar{d}$ pair, respectively. Since the latter is heavier, the $\phi\to K^0\bar{K}^0$ decay is further suppressed and then the ratio R is further increased. To be somewhat more precise, we will consider two recent and independent models dealing quite explicitly with such kind of effects [12,15].

In the $SU(3)$-breaking treatment of VP_1P_2 vertices by Bijnens *et al.* [15], these decays proceed through two independent terms containing the relevant vector and pseudoscalar masses (M_V and $m_{1,2}$) and thus incorporating quark-mass breaking effects. In the notation of Ref. [15], to which we refer for details, these VP_1P_2 couplings are then proportional to

$$M_V^2\left(g_V + 2\sqrt{2}f_\chi\frac{m_1^2+m_2^2}{M_V^2}\right)\,. \tag{10}$$

For the ϕK^+K^- and $\phi K^0\bar{K}^0$ coupling constants, the uncorrected strong coupling con-

stant g_0 becomes, respectively,

$$\frac{M_\phi^2}{2\sqrt{2}g_0 f^2}\left(1 + 4\sqrt{2}\frac{f_\chi}{g_V}\frac{m_{K^+,K^0}^2}{M_\phi^2}\right) , \tag{11}$$

with the pion decay constant $f \simeq 92$ MeV. One then obtains the ratio

$$\frac{g_{\phi K^+ K^-}}{g_{\phi K^0 \bar{K}^0}} \simeq 1 + 4\sqrt{2}\frac{f_\chi}{g_V}\frac{m_{K^+}^2 - m_{K^0}^2|_{m_u \neq m_d}}{M_\phi^2} \simeq 1.01 , \tag{12}$$

where we have used $m_{K^+}^2 - m_{K^0}^2|_{m_u \neq m_d} \simeq -6\ 10^{-3}$ GeV^2 for the non-photonic kaon mass difference [16] and the estimate $\frac{f_\chi}{g_V} \simeq -\frac{1}{3}$ obtained in Ref. [15] when fitting the $\rho \to \pi\pi$ and $K^* \to K\pi$ decay widths.

Similarly, in the independent treatment of $SU(3)$ symmetry breaking [12], some relevant VP_1P_2 couplings are given by

$$\begin{aligned}
g_{\rho\pi\pi} &= \sqrt{2}g , \\
g_{\phi K^+ K^-} &= g_{\phi K^0 \bar{K}^0} = -g(1 + 2c_V)(1 - c_A) ,
\end{aligned} \tag{13}$$

with $c_V \simeq 0.28$ and $c_A \simeq 0.36$ (see Ref. [12] for notation and details) mimicking the $SU(3)$ mass difference effects discussed in the previous approach [15]. The transition from $SU(3)$- to $SU(2)$-breaking offers no difficulties. One now obtains

$$\frac{g_{\phi K^+ K^-}}{g_{\phi K^0 \bar{K}^0}} \simeq 1 - \frac{m_{K^+}^2 - m_{K^0}^2|_{m_u \neq m_d}}{m_K^2 - m_\pi^2}c_A \simeq 1.01 . \tag{14}$$

As in the approach of Ref. [15], these $SU(2)$-breaking corrections work in the undesired direction and the discrepancy between theory and experiment for the ratio R increases by an additional 2%.

An independent $SU(2)$-breaking effect can arise from ρ-ϕ mixing. This is both isospin and Zweig-rule violating, and should therefore lead to rather tiny corrections. Indeed, in this context one can immediately obtain the following relation among coupling constants[2]: $g_{\phi K^+ K^-} - g_{\phi K^0 \bar{K}^0} = g_{\phi\pi^+\pi^-}$, with a small value for the $g_{\phi\pi^+\pi^-}$ coupling coming from the observed smallness of the $\phi \to \pi^+\pi^-$ branching ratio ($\mathcal{O}(10^{-4})$ [5]) in spite of its much larger phase space. A more quantitative estimate is now possible thanks to the recent data on $e^+e^- \to \phi \to \pi^+\pi^-$ coming from VEPP-2M [17]. These data describe the pion form factor around the ϕ peak, $F(s \simeq M_\phi^2)$, in terms of the complex parameter Z by the expression

$$F(s)\left(1 - \frac{ZM_\phi\Gamma_\phi}{M_\phi^2 - s - iM_\phi\Gamma_\phi}\right) . \tag{15}$$

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[2]Notice that this isospin relation not only accounts for $\rho(770)$–ϕ mixing effects but also for those between ϕ and any other higher mass isovector ρ-like resonance.

This Z, in turn, can be easily related to $\epsilon_{\phi\rho}$, the complex parameter describing the amount of ρ-like (or $(u\bar{u} - d\bar{d})/\sqrt{2}$) contamination in the ϕ wave function. One finds

$$\epsilon_{\phi\rho} \simeq -\frac{f_\phi}{f_\rho}\frac{\Gamma_\phi}{M_\phi}F(s = M_\phi^2)Z \ , \tag{16}$$

where the first coefficient $\frac{f_\phi}{f_\rho} \simeq -\frac{3}{\sqrt{2}}$ is the well-known ratio of ϕ-γ to ρ-γ couplings. One finally obtains

$$\frac{g_{\phi K^+ K^-}}{g_{\phi K^0 \bar{K}^0}} \simeq 1 - \sqrt{2}\Re(\epsilon_{\phi\rho}) \simeq 1.001 \ , \tag{17}$$

where an average of the values for Z in Ref. [17] and the parametrization of $F(s = M_\phi^2)$ from Ref. [18] have been used in the final step. This time the correction is tiny and the accuracy of our estimate is rather rough, but again it tends to increase the discrepancy on the ratio R.

4   Further attempts

Since the discrepancy between the theoretical and experimental value for R remains (or has even been increased by some additional 2% due to the $SU(2)$-breaking effects just discussed), we have tried to improve our analysis in different aspects. First, we have taken into account that the couplings of photons to kaons, rather than being point-like (as assumed in our previous and conventional treatment of radiative corrections), are known to be vector-meson dominated [19]. Accordingly, we have redone the calculation performed in Sec. 2 including the corresponding electromagnetic (vector-meson dominated) kaon form-factors. Now, not only the decay mode $\phi \to K^+ K^-$ can be affected but also the $\phi \to K^0 \bar{K}^0$ one due to the ρ, ω and ϕ mass differences. For the $\phi \to K^+ K^-$ case, the contribution of the charged kaon form-factor modifies the point-like result for $\Gamma(\phi \to K^+ K^-)$ by $\approx 2\ 10^{-3}$. For the case of $\phi \to K^0 \bar{K}^0$, a vanishing effect will be obtained in the limit of exact $SU(3)$ symmetry, and a fraction of the preceding one if $SU(3)$-broken masses are used. In both cases, the effect of kaon form-factors on real-photon emission diagrams is null. So then, the additional net effect of electromagnetic kaon form-factors on the ratio R leads to a modification of the point-like radiative corrections result of Sec. 2 by some per mille and is thus fully negligible.

A second and independent possibility consists in adopting a different framework for VPP decays. This is usually done in terms of more general effective lagrangians with VPP vertices containing two derivatives of the pseudoscalar fields instead of a single one as in our previous discussion. The radiative decay $\rho \to \pi^+ \pi^- \gamma$ —quite similar to the processes we are considering— has been quite recently analyzed in this modern context in Ref. [20]. The two relevant coupling constants (F_V and G_V, in the notation of Ref. [21])

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and their relative sign can be fixed to the canonical values $F_V = 2G_V = \sqrt{2}f_\pi$ [22] thanks to the experimental data for $\rho \to \pi^+\pi^-\gamma$ and other ρ meson processes [5]. As discussed in Ref. [20], a good description of these data is then achieved in terms of an amplitude that coincides with the one previously introduced in Ref. [23], and which originated from the simple one-derivative VPP vertices used by Ref. [6] as well as in our recalculation in Sec. 2. In other words, both types of effective lagrangians lead to exactly the same real-photon emission amplitudes once the coupling constants are properly fixed. This is also true for the other corrections concerning one-loop effects: for the canonical value $F_V = 2G_V$ one reobtains precisely our previous expression in Eq. (6).

A third attempt includes the effect of final $K\bar{K}$ rescattering through scalar resonances. It is well known that the charged kaons emitted in $\phi \to K^+K^-$ are always accompanied by soft photons. In the case of single photon emission, the K^+K^- system is found to be in a $J^{PC} = 0^{++}$ or 2^{++} state with an invariant mass just below the ϕ mass. The presence of the $J^{PC} = 0^{++}$ scalar resonances $f_0(980)$ and $a_0(980)$, with masses and decay widths that cover the invariant mass range of interest (from the $K\bar{K}$ threshold to the ϕ mass) [5], would suggest that rescattering effects could be important[3]. We have computed these rescattering effects through the exchange of the f_0 and a_0 using the charged kaon loop model [24–26]. In this model, the ϕ decays into a K^+K^- system that emits a photon (from the charged kaon internal lines and from the ϕK^+K^- vertex) before rescattering into a final K^+K^- or $K^0\bar{K}^0$ state through the propagation of f_0 and a_0 resonances. If the emitted soft photon is unobserved, the process $\phi \to K^+K^-(\gamma) \to f_0/a_0(\gamma) \to K^+K^-(\gamma)$ or $K^0\bar{K}^0(\gamma)$ contributes to the ratio R, both at the numerator and denominator. In order to calculate these effects, one needs an estimate of the coupling constant $g_{SK\bar{K}}$, where S is either the f_0 or the a_0. Recent measurements of the $\phi \to f_0\gamma$ and $a_0\gamma$ decay modes at VEPP-2M [27] are consistent with the predictions of the charged kaon loop model for values of the above couplings given by

$$\frac{g^2_{f_0 K\bar{K}}}{4\pi} = (1.48 \pm 0.32)\,\text{GeV}^2\,, \qquad \frac{g^2_{a_0 K\bar{K}}}{4\pi} = (1.5 \pm 0.5)\,\text{GeV}^2\,. \tag{18}$$

We have then found that the contribution of these kaon loops to the $\text{BR}(\phi \to K^+K^-(\gamma))$ is $\mathcal{O}(10^{-7})$, while for $\text{BR}(\phi \to K^0\bar{K}^0(\gamma))$ is $\mathcal{O}(10^{-9})$. For charged kaons in the final state, there is an additional contribution from the interference between the soft-bremsstrahlung

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[3]Rescattering effects from 2^{++} states are suppressed because the nearest tensorial resonances, $f_2(1270)$ and $a_2(1310)$, are well above the ϕ mass [5].

and the scalar amplitudes. This contribution is given by

$$
\Gamma_{\text{int}}(\phi \to K^+ K^- (\gamma)) = -\tfrac{4}{3}\, \alpha\, \frac{g^2_{\phi K^+ K^-}}{4\pi}\, \frac{g^2_{f_0 K^+ K^-}}{4\pi}\, \frac{M_\phi}{2\pi^2 m^2_{K^+}}\, \int_0^{\Delta E} d\omega\, \omega \times
$$

$$
\Re \left(I(a,b) \left[\frac{1}{D_{f_0}(m)} + \frac{g^2_{a_0 K^+ K^-}}{g^2_{f_0 K^+ K^-}}\, \frac{1}{D_{a_0}(m)} \right] \right) \left(w + \tfrac{1-v^2}{2}\, \log \tfrac{1-w}{1+w} \right) , \tag{19}
$$

where $I(a,b)$ is the kaon loop integral defined in Refs. [25,26] (with $a \equiv M_\phi^2/m_{K^+}^2$ and $b \equiv m^2/m_{K^+}^2$), $D_{f_0/a_0}(m)$ are the scalar propagators and $w = \sqrt{1 - 4m_{K^+}^2/m^2}$, with $m = M_\phi \sqrt{1 - 2\omega/M_\phi}$ being the invariant mass of the $K\bar{K}$ system and ω the photon energy. Using the values in Eq. (18) for the scalar couplings, we find that the interference term, which contributes to R only in the numerator, is positive and $\mathcal{O}(10^{-5})$, *i.e.* completely negligible in spite of being the dominant one.

Admittedly, this estimate of the $K\bar{K}$ rescattering effects is model dependent and affected by large uncertainties. Before concluding, we would thus like to make a few comments on possible variations on the magnitude of the scalar coupling constants and the expressions for the scalar propagators $D_{f_0/a_0}(m)$ which enter into our evaluation in the preceding paragraph. The values of the couplings $g_{SK\bar{K}}$ depend on the nature of the scalar mesons, *i.e.* whether they are two- or four-quark states, or $K\bar{K}$ molecules. The results of the $K\bar{K}$ molecule model, in addition to the couplings $g_{SK\bar{K}}$, depend upon the spatial extension of the scalar $K\bar{K}$ bound state, and the predictions for $\text{BR}(\phi \to f_0/a_0 \gamma)$ (for the same $g_{SK\bar{K}}$) are always smaller than in the purely point-like case, *i.e.* the effects on R tend to vanish for more extended objects [26]. The two-quark model, irrespectively of the $s\bar{s}$ *vs.* $(u\bar{u} + d\bar{d})/\sqrt{2}$ quark content of the f_0, predicts too small values (see, for example, Refs. [26,28]) for the branching ratios $\text{BR}(\phi \to f_0/a_0 \gamma)$ [27], and is unable anyway to account for the near mass degeneracy of the isoscalar f_0 and isovector a_0. On the other hand, such mass degeneracy is well understood in the four-quark model, critically reexamined very recently in Refs. [29,30]. The four-quark model also predicts values for $g_{SK\bar{K}}$ that seem to be in agreement with the available measurements of $\text{BR}(\phi \to f_0/a_0 \gamma)$ [27,28]. In all cases, the different possibilities are found to modify the previously quoted sizes of the $K\bar{K}$ rescattering effects by at most one order of magnitude. Something similar happens with the lack of consensus on the specific form for the scalar propagators to be used in these estimates. Here the uncertainties arise because of the opening of the $K\bar{K}$ channels quite close to the nominal scalar masses. This translates into sharp modifications of the conventional Breit-Wigner curves and changes the size of the $K\bar{K}$ rescattering effects again by one order of magnitude. Although affected by large uncertainties, the contributions coming from final-state $K\bar{K}$ rescattering are thus found to be negligible and their effects on the ratio R irrelevant.

5 Conclusions

In this letter, we have performed a discussion of the ratio $R \equiv \phi \rightarrow K^+ K^- / K^0 \bar{K}^0$. From the experimental point of view, the value $R_{\mathrm{exp}} = 1.44 \pm 0.04$ seems to be firmly established [5]. However, in our present theoretical analysis of this ratio R we have failed to reproduce the value R_{exp} quoted above. In a first and conservative attempt, including isospin symmetry for the strong vertices and the appropriate phase-space factor, one obtains $R = 1.53$ which is two σ's above R_{exp}. In a second step, we have also included conventional electromagnetic radiative corrections to order α, thus obtaining $R = 1.59$ and increasing the previous discrepancy up to three σ's. This value confirms some existing results and has been checked to be quite independent from the details of the relevant vertices. In a third step, we have tried to correct our predictions for R introducing various isospin breaking corrections to the $\phi K \bar{K}$ coupling constants. As a result, the ratio R is found to be further increased by some 2%, an estimate affected by rather large errors reflecting our poor knowledge on the $SU(2)$-breaking details. In view of all this, we have introduced final-state rescattering effects which should be dominated by the almost on-shell formation of the $f_0(980)$ and $a_0(980)$ resonances in the S-wave $K \bar{K}$ channel. The controversial nature of these scalar resonances allows for quite disparate estimates of their effects, but one can safely conclude that they are well below those previously mentioned. The disagreement on the ratio R persists well above two (experimental) standard deviations. Higher statistics from DAΦNE are expected in order to settle definitively whether the discrepancy on R is a real problem, or final agreement between theory and experimental data can be achieved.

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