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Large N_c, chiral approach to $M_{\eta'}$ at finite temperature

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Abstract

We study the temperature dependence of the η and η' meson masses within the framework of $U(3)_L \times U(3)_R$ chiral perturbation theory, up to next-to-leading order in a simultaneous expansion in momenta, quark masses and number of colours. We find that both masses decrease at low temperatures, but only very slightly. We analyze higher order corrections and argue that large N_c suggests a discontinuous drop of $M_{\eta'}$ at the critical temperature of deconfinement T_c, consistent with a first order transition to a phase with approximate $U(1)_A$ symmetry.

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1 Introduction

The fate of the $U(1)_A$ symmetry of QCD at finite temperature is a fascinating problem [1–3] which could also have interesting consequences for the ongoing heavy ion collisions program and, possibly, for cosmology [4–8]. Even a partial restoration of $U(1)_A$ symmetry in the vicinity of the critical temperature of chiral symmetry breaking ($T_c \sim 200$ MeV) could dramatically change the mass and mixing pattern of the lightest neutral mesons (π^0, η and η'), with signals including enhanced strangeness production or the more speculative possibility of forming parity violating disoriented η' condensates in heavy ion collisions [7,9,10].

Our aim in the present paper is rather modest: we will study the shift of the mass of the η and η' mesons at low temperatures, in a regime in which the hadronic gas is mostly composed of pions. We will work in the framework of $U(3)_L \times U(3)_R$ chiral perturbation theory (χPT), in a simultaneous expansion in momenta, quark masses, number of colours N_c, and temperature T.

Our motivation for doing this investigation was threefold. First, the predictions of χPT in a pion thermal bath, although limited in scope to $T \lesssim$ few f_π, are essentially model independent (see for instance, the review of Smilga [11] and references therein). Given the phenomenological success of the large N_c expansion in vacuum, one might perhaps hope that the predictions of the present work are as robust. Next, we wanted to see to which extent the results derived in Ref. [12] could be amended. As us, the authors have computed the shift of M_η and $M_{\eta'}$ at low temperatures using the Di Vecchia-Veneziano-Witten effective lagrangian (DVW) [13–15], but only to leading order $\Delta M^2_{\eta'} \sim T^2$. However, it was not clear to us whether the leading order DVW lagrangian was a good approximation for this problem. Although the parameters of the lagrangian can be fitted to the observed mass and mixing pattern of the η and η' mesons to within 10% [16,17], the decay rates predicted for $\eta' \to \eta\pi\pi$ are off the experimental values by a factor of about 40. This issue, which is obviously relevant in order to determine the shift of $M_{\eta'}$ in a pion bath, is however easily cured at next-to-leading order in the large N_c expansion [18, 19]. As we will show, next-to-leading order corrections are also quite important at finite temperature, but not to the point of dramatically changing the conclusion of Ref. [12]: at low temperatures, $M_{\eta'}$ stays essentially constant. Finally, we wanted to see what the large N_c expansion could teach us about the fate of the $U(1)_A$ symmetry at finite temperature. At zero temperature, in the confined phase of QCD, large N_c arguments predict that $M^2_{\eta'} \propto 1/N_c$. On the other hand, at very high temperatures, in the quark-gluon plasma phase of QCD ($T \gg \mu_{\rm hadr} \sim 200$ MeV), instanton calculus is reliable and predicts an effective restoration of $U(1)_A$ symmetry. Because of screening, instanton effects are suppressed at very large temperatures $\propto \exp(-8\pi^2/g(T)^2)$. At large N_c, the suppression is more important, as $1/g^2 \to N_c/\lambda$ with fixed 't Hooft coupling $\lambda = g^2 N_c$, and the exponential tends to vanish $\exp(-N_c/\lambda) \to 0$ as N_c increases. Because of asymptotic freedom, λ growths at lower temperatures and the instanton argument breaksdown. However, for N_c large enough, a natural assumption is that the exponential suppression holds all the way down to the critical temperature of deconfinement $T_c \sim \mu_{\rm hadr}$ [7]. Although we have no proof of this statement, such a behaviour seems natural given the large release of entropy $\propto N_c^2$ at T_c and is actually known to occur in models in two dimensions [20]. With this assumption, $M_{\eta'}$ can be taken as an order parameter for $U(1)_A$ symmetry restoration at T_c. In Ref. [7], some information on the behaviour of $M_{\eta'}$ near T_c could be extracted assuming that the *deconfining* phase transition could be of second order at large N_c [21]. We will argue here that large N_c favours a sharp drop of $M_{\eta'}$ at T_c, consistent with first order transition to the phase with (approximate) $U(1)_A$ symmetry.

Our paper is organized as follows. In the next section, we briefly review $U(3)_L \times U(3)_R$ χPT, which extends the framework of the large N_c DVW effective lagrangian beyond leading order. For definitiveness, we refer to the recent analysis of Herrera-Siklódy *et al.* [22]. We then discuss the implications of these corrections at low temperature, in presence of a pion thermal bath. Most formulas are relegated to the appendix. In the last section, we speculate on the effect of higher order corrections in the large N_c expansion and draw the conclusions.

2 Sketch of $U(3)_L \times U(3)_R$ chiral perturbation theory

At low energies and temperatures, the dynamics of QCD is governed by an approximate $SU(3)_L \times SU(3)_R$ chiral symmetry which is spontaneously broken to the diagonal $SU(3)$ in vacuum. If the mass of the up, down and strange quarks were vanishing, the symmetry would become exact and there would be eight massless Goldstone bosons. Phenomenological lagrangians, which treat the mass of the quarks as small perturbations, provide a powerful framework, known as Chiral Perturbation Theory (χPT), to study the properties of the light π, K, and η mesons [23,24]. The η' meson doesn't *a priori* fit in this frame. It is substantially heavier than the other eight light mesons, and, in vacuum, would stay so even in the chiral limit of zero quark masses, because it receives most of its mass from the $U(1)_A$ anomaly through non-perturbative instanton-like effects [25]. The effect of the axial anomaly can however be conveniently turned off by going to the limit of large number of colours N_c [26–28]. At infinite N_c and in the chiral limit, the global symmetry becomes $U(3)_L \times U(3)_R$, spontaneously broken in vacuum to $U(3)$, with nine massless Goldstone bosons. Like chiral symmetry breaking effects by finite quark masses, $1/N_c$

suppressed contributions can be systematically introduced as perturbations in an effective lagrangian, an approach which has been quite fruitful [13,14,19,29]. A systematic analysis of next-to-leading corrections, including $\mathcal{O}(p^4)$ operators, has been initiated in the recent [18,22,30]. We refer to these latter works for more details and follow their conventions for ease of reference. We will work in Euclidean spacetime with metric $g_{\mu\nu} \equiv \delta_{\mu\nu}$ and use the imaginary time formalism to compute the thermal corrections.

2.1 Leading order

The leading order effective lagrangian is well-known [13–15]. In the notation of [22] it is written as

$$\mathcal{L}_{\rm LO} = \frac{f^2}{4}\Big(v_{02}X^2 + \langle \partial_\mu U^\dagger \partial_\mu U\rangle - \langle U^\dagger \chi + \chi^\dagger U\rangle\Big) , \tag{1}$$

where U is the $U(3)$ matrix

$$U \equiv e^{i\sqrt{2}\Phi/f} , \tag{2}$$

with Φ the pseudoscalar meson matrix

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix} , \tag{3}$$

and $f = f_\pi = 92.4$ MeV at leading order. The mass matrix is

$$\chi = 2\,B\,\mathrm{diag}(m_u, m_d, m_s) , \tag{4}$$

but we shall neglect isospin breaking effects ($m_u = m_d \equiv m$). The constant B is related to the value of the $\langle\bar{q}q\rangle$ condensate, $M_\pi^2 = 2mB \equiv -2m\langle\bar{q}q\rangle/f_\pi^2$ at leading order. The combination

$$X(x) \equiv \langle \log U(x)\rangle + i\theta_{\rm QCD} = i\,\frac{\sqrt{6}}{f}\eta_0 + i\theta_{\rm QCD} , \tag{5}$$

is invariant under $U(3)_L \times U(3)_R$ transformations, $\langle \log U\rangle \to \langle \log(g_R U g_L^\dagger)\rangle + 2i\langle\alpha\rangle$ and $\theta_{\rm QCD} \to \theta_{\rm QCD} - 2\langle\alpha\rangle$. Because of this, any arbitrary function of X can *a priori* enter in the construction of the effective lagrangian, with thus little predictive power. This is where the large N_c expansion comes to the rescue by limiting the number of operators that can contribute at each level of approximation. In the chiral limit ($m, m_s \to 0$), Eq. (1) gives

$$M_{\eta'}^2 = -3v_{02} , \tag{6}$$

which is the celebrated Veneziano-Witten relation for three massless flavours [27,28], with $v_{02} \equiv -2\tau/f^2 \sim 1/N_c$, where τ is the topological susceptibility of pure Yang-Mills theory. The rationale of $U(3)_L \times U(3)_R$ χPT is to count powers of p^2, m_q, and $1/N_c$ on the same level $\mathcal{O}(\delta)$ [22,29]:

$$\mathcal{O}(\delta) \sim p^2 \sim m_q \sim 1/N_c \; . \tag{7}$$

According to this counting rule, the leading order lagrangian (1) is $\mathcal{O}(\delta^0)$ because $f^2 \sim \mathcal{O}(N_c)$ [1].

At leading order, there are four unknown parameters in the lagrangian: f, v_{02}, and the combinations mB and m_sB (or $x \equiv m_s/m - 1$). On the other hand, we have at our disposal seven observables: f_π, f_K, the four masses of the light mesons, and the η–η' mixing angle θ. Using $M_{\eta'}$ as input and the formulæ given for reference in the appendix, one obtains

$$\begin{aligned} f^2 &= f_\pi^2 = f_K^2 \; , \\ 2mB &= M_\pi^2 \; , \\ x &\simeq 24.1 \; , \\ v_{02} &\simeq -0.22 \;\mathrm{GeV}^2 \; , \end{aligned} \tag{8}$$

which predict that $\theta \simeq -20^\circ$ and

$$M_\eta \simeq 494.4 \;\mathrm{MeV} \; . \tag{9}$$

Remarkably, the latter number is only 10% off the experimental value $M_\eta = 547.3$ MeV. It is however known that adjusting the parameters cannot improve the prediction because the ratio $M_\eta^2/M_{\eta'}^2$ has an upper bound[2] [16]. One has to take into account next-to-leading order corrections to reach agreement [17].

At leading order, the only coupling between η' and the pions is from the quark mass term in the lagrangian (1) and is thus chirally suppressed. The amplitude for $\eta' \to \eta\pi\pi$ is

[1] Note that the field expansion of U brings further powers of $1/f \sim 1/\sqrt{N}_c$. The $\mathcal{O}(\delta)$ counting is to be understood to hold at the operator level.

[2] Assuming $M_\pi = 0$ to simplify, Eq. (1) gives

$$\frac{M_\eta^2}{M_{\eta'}^2} = \frac{3 - y - \sqrt{9 + 2y + y^2}}{3 - y + \sqrt{9 + 2y + y^2}} \; , \tag{10}$$

where $y \equiv 9v_{02}/2(M_K^2 - M_\pi^2)$. This ratio reaches a maximum at $y = -3$ (note that $v_{02} < 0$) corresponding to

$$\frac{M_\eta}{M_{\eta'}} < 0.518 \; , \tag{11}$$

to be compared with the measured ratio $M_\eta/M_{\eta'} \simeq 0.571$. Taking into account $M_\pi \neq 0$ improves things, but not enough.

then

$$\mathcal{A} = \frac{M_\pi^2}{6 f_\pi}\Big(2\sqrt{2}\cos(2\theta) - \sin(2\theta)\Big) \; . \tag{12}$$

The corresponding decay rates

$$\begin{aligned} \Gamma(\eta' \to \eta\pi^0\pi^0) &= 1.0\ \mathrm{keV}\ , \\ \Gamma(\eta' \to \eta\pi^+\pi^-) &= 1.9\ \mathrm{keV} \approx 2\times\Gamma(\eta' \to \eta\pi^0\pi^0)\ , \end{aligned} \tag{13}$$

are however much smaller than the experimental ones,

$$\begin{aligned} \Gamma_{\mathrm{exp}}(\eta' \to \eta\pi^0\pi^0) &= 42.0 \pm 4.2\ \mathrm{keV}\ , \\ \Gamma_{\mathrm{exp}}(\eta' \to \eta\pi^+\pi^-) &= 88.9 \pm 7.6\ \mathrm{keV}\ . \end{aligned} \tag{14}$$

We will not speculate on the reasons for this well-known discrepancy (see Refs. [19,31] for a more recent discussion), but simply note that within the present framework, this issue can also be resolved at next-to-leading order [18,19][3].

2.2 Next-to-leading order

In our case, at next-to-leading order, $\mathcal{O}(\delta)$, only a few more terms can be added to the lagrangian (1) [22]:

$$\begin{aligned} \mathcal{L}_{NLO} &= \mathcal{L}_{LO} \\ &+ \tfrac{f^2}{4}\Big(-v_{31}\, X\langle U^\dagger\chi - \chi^\dagger U\rangle + v_{40}\,\langle U^\dagger\partial_\mu U\rangle\langle U^\dagger\partial_\mu U\rangle \\ &+ i\, v_{50}\,\langle U^\dagger\partial_\mu U\rangle\partial_\mu\theta_{\mathrm{QCD}} + v_{60}\,\partial_\mu\theta_{\mathrm{QCD}}\partial_\mu\theta_{\mathrm{QCD}}\Big) \\ &- M_0\, O_0 - M_3\, O_3 + L_5\, O_5 - L_8\, O_8\ , \end{aligned} \tag{15}$$

where the $O_{0,3,5,8}$ are $\mathcal{O}(p^4)$ operators whose coupling constants are $\mathcal{O}(N_c)$:

$$\begin{aligned} O_0 &= \langle\partial_\mu U\partial_\nu U^\dagger\partial_\mu U\partial_\nu U^\dagger\rangle\ , \\ O_3 &= \langle\partial_\mu U^\dagger\partial_\mu U\partial_\nu U^\dagger\partial_\nu U\rangle\ , \\ O_5 &= \langle\partial_\mu U^\dagger\partial_\mu U(U^\dagger\chi + \chi^\dagger U)\rangle\ , \\ O_8 &= \langle\chi^\dagger U\chi^\dagger U + U^\dagger\chi U^\dagger\chi\rangle\ . \end{aligned} \tag{16}$$

The couplings v_{40}, v_{50}, and v_{60} are not independent and either one of them can be set to zero by an appropriate change of variables, $\eta_0/f \to \eta_0/f + \kappa\,\theta_{\mathrm{QCD}}$. We shall

[3] Note that the amplitude (12) is constant and vanishes in the limit $m_u = m_d = 0$, for any m_s. However, general arguments [19] (and a fit to experimental data) indicate that for $m_u = m_d = 0$, the amplitude should behave like $\mathcal{A} = \mathrm{const} \times p_\pi^{(1)} \cdot p_\pi^{(2)}$, where $p_\pi^{(1,2)}$ are the momenta of the outgoing pions, and where the constant is vanishing as the strange quark mass goes to zero. As shown in Ref. [19], this behaviour can be easily accommodated by introducing higher-order terms, an approach that is systematized by the δ expansion [18]. The smallness of the leading order contribution is then considered as a mere accident, related to the smallness of the ratio M_π^2/M_K^2.

choose $v_{40} = 0$. Moreover, v_{50} and v_{60} will not appear in our calculations and can be discarded. At $\mathcal{O}(\delta)$, the only coupling related to the breaking of $U(1)_A$ symmetry is thus $v_{31} \sim \mathcal{O}(1/N_c)$. Note that the corresponding operator is also chirally suppressed $\propto m_q$.

At next-to-leading order, seven unknown parameters enter in the definition of the meson mass matrix: f, v_{02}, v_{31}, $L_{5,8}$, together with the quark masses m and m_s (see appendix for details). These can be expressed in terms of seven independent observables: f_π, f_K, M_π, M_K, M_η, M'_η, and the η–η' mixing angle θ [30]. At this level, large N_c χPT is thus not predictive. The strategy adopted in Ref. [30] was to impose that $\mathcal{O}(\delta)$ corrections are not too large so that the large N_c expansion makes sense. For mixing angle in the range $20^\circ < \theta < 24^\circ$, the fit gives [30]

$$\begin{array}{rcl} 0.980 \leq & 2mB/M_\pi^2 & \leq 0.988 \, , \\ 18.3 \leq & x & \leq 20.9 \, , \\ 0.214 \text{ GeV}^2 \leq & |v_{02}| & \leq 0.239 \text{ GeV}^2 \, , \\ 1.35 \; 10^{-3} \leq & L_8 & \leq 1.57 \; 10^{-3} \, , \\ -0.164 \leq & v_{31} & \leq -0.161 \, . \end{array} \tag{17}$$

together with $f = 90.8$ MeV and $L_5 = 2.0 \; 10^{-3}$ which are fixed by f_π and f_K. Note that if v_{02} does only change by about 10%, the shift in m_s is quite large, $\sim$ 20–25.

Because they have four derivatives, the operators O_0 and O_3 do not contribute to the meson mass matrix in vacuum. However, they give the dominant contributions to the decay $\eta' \to \eta\pi\pi$ [18]. This is essentially because the extra derivatives introduce large amplification factors, $\propto (M_{\eta'}/M_\pi)^2$, with respect to the leading order amplitude[4]. The observed decay rates are well reproduced with

$$\begin{array}{rcl} M_0 & \simeq & 1.2 \; 10^{-3} \, , \\ M_3 & \sim & -0.4 \; 10^{-3} \, , \end{array} \tag{18}$$

values which can be independently inferred from the known L_1, L_2, and L_3 of $SU(3) \times SU(3)$ χPT (in the nomenclature of Gasser and Leutwyler [24])[5].

Thus all the parameters of the next-to-leading order effective lagrangian are fixed by low-energy phenomenology.

[4]This may actually casts some doubts on the validity of χPT for such processes as one could expect higher-order effects to give non-negligible contributions to the decay $\eta' \to \eta\pi\pi$. One may nevertheless hope that the large N_c expansion is still reliable and that these corrections are $1/N_c$ suppressed. Whether this is true is unfortunately hard to check as we would evidently have too few hadronic data to completely fit the parameters of the effective lagrangian at higher orders in the δ expansion. Of course, this is precisely why the large N_c expansion is invoked in the first place.

[5]According to Ref. [18], $M_0 = \frac{2}{3}(L_1 + L_2) + \mathcal{O}(N_c^0)$ and $M_3 = L_3 + 2M_0$.

3 M_η' in a pion thermal bath

In Ref. [12], the leading order lagrangian (1) has been used to study the shift of M_η and $M_{\eta'}$ at one-loop in a pion thermal bath. The effect they found is very tiny, as at $T \sim 200$ MeV the relative mass shifts are only $\mathcal{O}(0.1\%)$. The reason for this is easy to understand. The η and η' mesons receive most of their mass from the topological susceptibility term $\propto v_{02}$ and/or from the strange quark mass, while the pion thermal corrections only modifies the tiny contribution from the pion mass term $\propto M_\pi^2$. Thermal kaons could give a larger effect, $\propto M_K^2$, but the density of these is exponentially suppressed at low temperatures, $\propto \exp(-M_K/T)$. One might wonder whether next-to-leading order corrections could directly affect the contribution of the leading order $U(1)_A$ breaking term v_{02}. As we have seen in the previous section, five extra operators appear at next-to-leading order in the large N_c expansion and, of these, only the one with coupling v_{31} is related to $U(1)_A$ symmetry breaking. Unfortunately, this term is also chirally suppressed, $\propto m$, and its contribution is only $\mathcal{O}(v_{31}M_\pi^2T^2/f_\pi^2)$. At temperatures of interest, this is small compared to v_{02}, but of the same magnitude as the leading order thermal correction. The other four operators will also contribute, but in a less interesting way, as they are invariant under $U(1)_A$. Furthermore, their effects are also $\propto M_\pi^2$.

We have computed the shift of the mass of η and η' at one-loop, at next-to-leading order in the expansion in δ. We have *not* taken into account two-loop corrections from the leading order lagrangian. Although it is not clear whether this is legitimate numerically speaking, neglecting these is however consistent with the rules of large N_c chiral perturbation theory. Indeed, the natural extension of δ power-counting to finite temperature is

$$\mathcal{O}(\delta) \sim p^2 \sim m_q \sim 1/N_c \sim T^2 \; . \tag{19}$$

At leading order, $M_{\eta'}^2 = \mathcal{O}(\delta) \sim 1/N_c$ and the one-loop thermal correction is $\propto M_\pi^2T^2/f^2 \sim \delta^3$. At two-loop, using the leading order lagrangian, the shift is $\propto M_\pi^2T^4/f^4 \sim \delta^5$, while at one-loop using the next-to-leading order lagrangian, the shift is typically $v_{31}M_\pi^2T^2/f^2 \sim \delta^4$ (using $v_{31} \sim 1/N_c$) and thus dominant. Consistency thus requires to neglect the two-loop contributions. This greatly simplifies the calculations which are a bit cumbersome, but otherwise straightforward.

The relevant diagrams are those of Fig. 1, where the loops contain only pions. At next-to-leading order there are two related thermal loops:

$$I_1(T) = T \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{K^2 + M_\pi^2} \; , \tag{20}$$

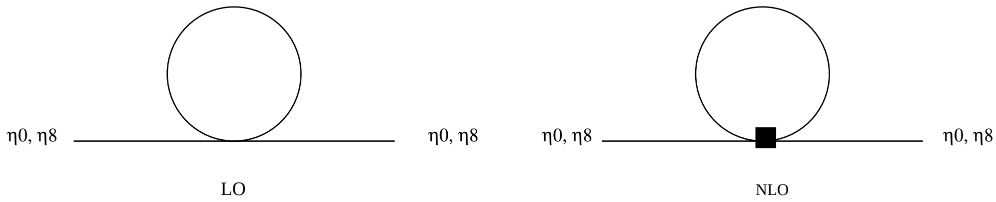

Figure 1: One-loop pion corrections to M_η and $M_{\eta'}$ at low temperature. The black box represent insertions of next-to-leading order operators.

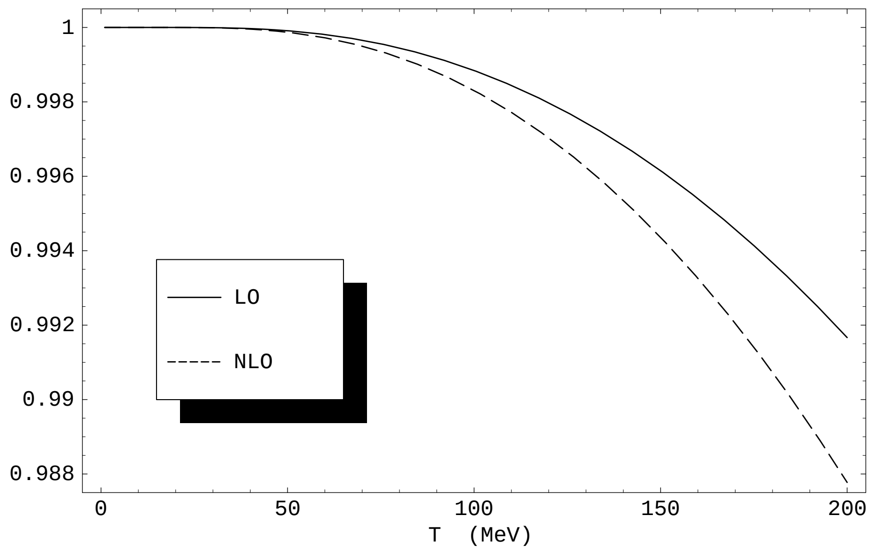

Figure 2: Leading order (solid line) and next-to-leading order (dashed line) contributions to $M_\eta(T)$. Both curves are normalized to $M_\eta(0)$.

with $K^2 = k_0^2 + \vec{k}^2$, where $k_0 = 2\pi n T$, with n integer, are the Matsubara frequencies and

$$I_2(T) = T \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{K^2}{K^2 + M_\pi^2} \; , \tag{21}$$

with $I_2(T) = -M_\pi^2 I_1(T)$. As usual, we drop the ultraviolet divergent part of the pion loops as these can in principle be reabsorbed in vacuum parameters, including next-to-next-to-leading order counter-terms. The sum over n can then be readily evaluated using standard techniques [32],

$$I_1(T) = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{\omega} \frac{1}{\exp(\omega/T) - 1} = \frac{M_\pi T}{2\pi^2} \sum_{n=0}^{\infty} \frac{1}{n} K_1\Big(\frac{n M_\pi}{T}\Big) \; , \tag{22}$$

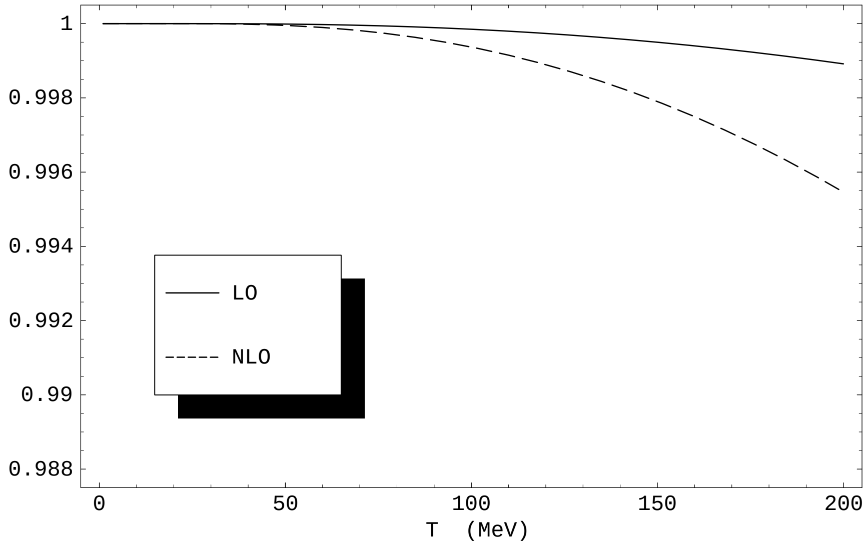

Figure 3: Leading order (solid line) and next-to-leading order (dashed line) contributions to $M_{\eta'}(T)$. Both curves are normalized to $M_{\eta'}(0)$.

where $\omega^2 = \vec{k}^2 + M_\pi^2$. For instance, for $T \gtrsim M_\pi$

$$I_1(T) \approx \frac{T^2}{12} \, . \tag{23}$$

In the sequel we simply compute (22) numerically. As the relevant formula are not particularly transparent, we have relegated them to the appendix. Fig. 2 and Fig. 3 show the shift of M_η' and M_η at low temperature both at leading and next-to-leading order. To be definite we have chosen the set of parameters corresponding to $\theta \simeq -20°$. The net thermal effects are not dramatic: both masses decrease, but only slightly. As expected, the shift of the mass M_η' is more pronounced at next-to-leading order, but the effect is not very significant. Again, this is because, both at leading and next-to-leading orders, the thermal corrections are chirally suppressed, $\propto M_\pi^2 T^2/f_\pi^2$. For completeness, we have also plotted in Fig. 4 the shift of the mixing angle at low temperature. As both the η and η' masses diminish, the angle is not very much affected. It decreases a bit (toward ideal mixing?), consistent with the relatively larger shift of $M_{\eta'}$.

4 Lessons from large N_c?

As we have seen in the previous section, the mass of η' is almost not affected at low temperatures in a pion bath. This is because, at this order, the pion thermal corrections are chirally suppressed, smaller than $M_\pi^2 \simeq 0.02$ GeV^2, and thus essentially negligible

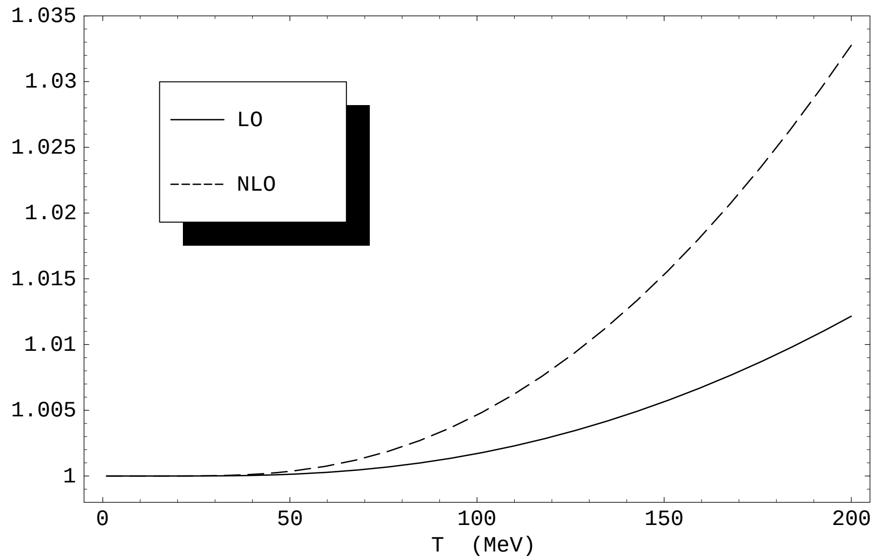

Figure 4: Leading order (solid line) and next-to-leading order (dashed line) contributions to $\tan 2\theta(T)$. Both curves are normalized to $\tan 2\theta(0)$.

compared to the contribution from the $U(1)_A$ symmetry breaking term $v_{02} \simeq 0.22$ GeV^2. In particular, in the chiral limit, $M_\pi = 0$, all the corrections vanish and $M_{\eta'}$ is temperature independent up to next-to-leading order in χPT. In the chiral limit, the leading contribution from pions to the shift of $M_{\eta'}$ presumably arise from $\mathcal{O}(p^4)$ operators like[6]

$$\mathcal{L} \sim \frac{1}{N_c} X^2 \langle \partial_\mu U \partial_\nu U^\dagger \partial_\mu U \partial_\nu U^\dagger \rangle \, . \tag{25}$$

The coupling is $\mathcal{O}(1/N_c)$ because there is a factor of $1/N_c^2$ coming with X^2 and one of N_c from the coupling of the $\mathcal{O}(p^4)$ operator. The best way to see this is to replace the coupling $M_0 \sim N_c$ (or M_3) from the next-to-leading order lagrangian by a function of X, $M_0 \to M_0(X)$ and expand to second order in X, which brings down a factor of $1/N_c^2$. Because there are at least four pions in the expansion of the operator (25), the leading pion thermal correction to $M_{\eta'}$ in the chiral limit is a two-loop effect,

$$\delta M_{\eta'}^2(T) \sim \frac{1}{N_c} \frac{T^8}{f^6} \sim \frac{1}{N_c^4} \frac{T^8}{\mu_{\rm hadr}^6} \, . \tag{26}$$

[6]The $O(p^2)$ operator

$$\mathcal{L} \sim f^2 \frac{X^2}{N_c^2} \langle \partial_\mu U^\dagger \partial_\mu U \rangle \, , \tag{24}$$

can contribute at one-loop if and only if $M_\pi^2 \neq 0$. It could contribute at two-loop order in the chiral limit $\propto T^4$, but doesn't because pion interactions are too soft. This is a well-known feature of pion thermal corrections which is for instance manifest in the absence of T^6 terms in the free energy of a pion gas in the chiral limit [33], or in the fact that massless thermal pions move at the speed of light to order T^2 [34].

We have made the large N_c dependence of the pion decay constant f^2 manifest by defining $f^2 \sim N_c \mu_{\rm hadr}$. Of course, the sign of the correction is not known and $M_{\eta'}$ could go up or down. Also, if we compare with $M^2_{\eta'}(0) \sim \mu^2_{\rm hadr}/N_c$, we infer that $M_{\eta'}$ is quasi-constant for temperatures

$$T \ll T_* \sim N_c^{3/8} \mu_{\rm hadr} \ . \tag{27}$$

In the large N_c framework, the natural scale for deconfinement is $T_c \sim \mu_{\rm hadr}$, which is also the temperature at which the pions from the hadronic gas overlap. It is natural to assume that chiral symmetry restoration takes place at the same temperature, driven by the release of $\mathcal{O}(N_c^2)$ gluon degrees of freedom [11,35]. The estimate in Eq. (27) then seems to imply that $M_{\eta'}$ is essentially constant up to the temperature of deconfinement, since $T_* \gg T_c \sim \mu_{\rm hadr}$ for N_c large. This conclusion is however premature because the low momentum expansion breaks down near T_c and we must take into account the contribution of operators with arbitrary number of derivatives. We claim that the dominant operators at large N_c are of the form

$$\mathcal{L} \sim \frac{1}{N_c\, \mu_{\rm hadr}^{2k-4}} X^2 \left\langle \partial_{\mu_1} U \partial_{\mu_2} U^\dagger \ldots \partial_{\mu_k} U \partial_{\mu_1} U^\dagger \partial_{\mu_2} U \ldots \partial_{\mu_k} U^\dagger \right\rangle \ . \tag{28}$$

These operators are irrelevant at low energies but become marginal for $\partial \sim T_c$. A six-derivative operator, for instance, first contribute at three loops $\delta M^2_{\eta'} \propto T^{12}/(N_c^5 \mu^{10}_{\rm hadr})$. For comparison, the contribution of a three-loop diagram with a four-derivatives (NLO) and a two-derivatives vertices (LO) is $\propto 1/N_c\, T^{10}/f^8 \sim 1/N_c^5\, T^{10}/\mu^8_{\rm hadr}$ and is subdominant at large N_c. For generic k, the operators of Eq. (28) give $\delta M^2_{\eta'} \sim T^{4k}/(N_c^{k+2} \mu_{\rm hadr}^{4k-2})$. The ratio of two consecutive terms k and $k+1$ becomes $\mathcal{O}(1)$ at $T' \sim N_c^{1/4} \mu_{\rm hadr}$, independent of k. At large N_c, $T_c \ll T' \ll T^*$ and the perturbative expansion still breaks down above the temperature of deconfinement.

Another set of operators could be relevant at large N_c because the η' is then rather light, $M_{\eta'} \sim \mu_{\rm hadr}/N_c^{1/2} < T_c$. Thus one should include operators that involve arbitrary powers of the η' field, like

$$\begin{aligned} \mathcal{L} &\sim f^2 F\left(\tfrac{X}{N_c}\right) \left\langle \partial_\mu U^\dagger \partial_\mu U \right\rangle \\ &= \left(\tfrac{\eta_0^2}{\mu^2 N_c^2} + \tfrac{\eta_0^4}{\mu^4 N_c^5} + \ldots\right) (\partial_\mu \eta^0)^2 + \text{pion terms} \ , \end{aligned} \tag{29}$$

which contributes to the wave-function renormalization of η',

$$\delta Z_{\eta'} \sim T^2/(N_c^2 \mu^2_{\rm hadr}) + T^4/(N_c^5 \mu^4_{\rm hadr}) + \ldots \ , \tag{30}$$

or terms of the form

$$\mathcal{L} \sim N_c^2 \mu^4_{\rm hadr} G\left(\frac{X}{N_c}\right) \sim \frac{\mu^2_{\rm hadr}}{N_c} \eta_0^2 + \#_1 \frac{1}{N_c^4}\, \eta_0^4 + \#_2 \frac{1}{\mu^2_{\rm hadr} N_c^7}\, \eta_0^6 + \ldots \ . \tag{31}$$

However, a common feature of these operators is that they are very suppressed at large N_c. They become important only for $T \sim N_c^{3/2}\mu_{\rm hadr}$, much higher than T' so that their contribution is subleading compared to operators like in Eq. (28).

Can we conclude anything from these considerations? In all the cases discussed above, the leading thermal corrections to $M_{\eta'}$, in the chiral limit and for N_c large, become important for temperature which are higher than the critical temperature of deconfinement $T_c \sim \mu_{\rm hadr}$ by a factor of N_c^γ. Although the value of γ is hard to guess, as various corrections can get mixed up, we believe it is reasonable to conjecture that γ is strictly positive. This implies that just below T_c, $M_{\eta'}(T) = M_{\eta'}(0)$ to a very good approximation. The standard lore is that the deconfining phase transition at T_c is of first order for N_c large [35][7]. Because the temperature at which *hadronic* interactions can affect $M_{\eta'}$ is (very much) larger than the temperature of deconfinement, we expect that changes in $M_{\eta'}$ will be instead triggered by the release of the large number of gluons and will thus drop *discontinuously* at T_c, *i.e.* that there is a first order transition to a phase with (approximate) $U(1)_A$ symmetry.

This behaviour is not inconsistent with various other expectations. For three light quark flavours, $N_f \geq 3$, the transition to the chirally symmetric phase is probably first order while for $N_f = 2$, the phase transition is supposed to be of second order, in the universality class of $O(4)$ [2]. It has been argued by Smilga that the latter behaviour is not inconsistent with a first order deconfining phase transition at large N_c [11]. The reason is that, unlike for $M_{\eta'}$, there is an infinite subset of thermal corrections that contribute to the *same order* in N_c to the shift of the quark condensate $\Sigma \equiv \langle \bar{q} q \rangle$,

$$\Sigma(T) = \Sigma \left(1 - \# \frac{T^2}{N_c^2 \mu_{\rm hadr}^2} F\left(\frac{T}{\mu_{\rm hadr}} \right) \right) . \tag{32}$$

Even though thermal corrections are suppressed like $1/N_c^2$, the (unknown) function $F(x)$ may be singular near, but below $T_c \sim \mu_{\rm hadr}$. If, for instance, F has a simple pole at $T = T_0 < T_c$, $F \sim \mu_{\rm hadr}^2/(T^2 - T_0^2)$ and the chiral phase transition is second order with a critical region near T_c, that is of order

$$\frac{\Delta T}{T_c} \sim \frac{1}{N_c^2} . \tag{33}$$

Alternatively, if $U(1)_A$ symmetry is effectively restored at T_c, the large N_c behaviour (32) is also consistent with a fluctuation induced first order phase transition. For $N_f = 1$

[7] Various arguments, including recent developments in string theory (see Sect. 6.2.2 in Ref [36]) and lattice simulations of $N_c = 4$ pure Yang-Mills theory [37], favour a first order deconfining phase transition. A case for a second order phase transition has been made in Ref. [21], in light of the structure of the Columbia diagram.

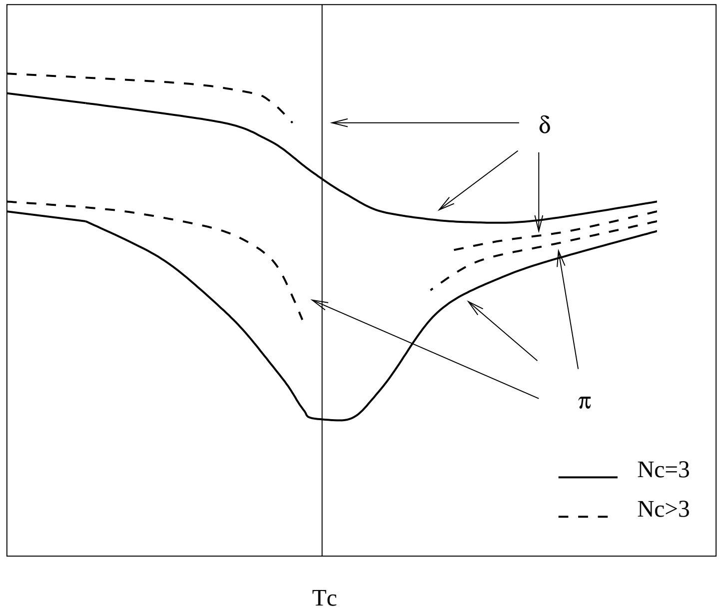

Figure 5: Plot of the π and δ (a_0) near the critical temperature for $N_c = 3$ (continuous lines) and possible changes for $N_c \gg 3$ (dashed lines).

finally, chiral symmetry is broken by the anomaly at all temperature and there is no chiral phase transition. However, if instanton transitions are strongly suppressed just above T_c, chiral symmetry can be effectively restored and the phase transition is presumably first order.

5 Conclusions

We have studied the behaviour of the mass of the η' pseudoscalar meson at finite temperature using constraints from chiral symmetry and large N_c power counting. The main conclusion to be drawn from this work is that $M_{\eta'}$ is essentially unchanged at low temperatures. A tentative analysis of the effect of leading higher order corrections at large N_c *suggests* that $M_{\eta'}$ changes discontinuously at the temperature of deconfinement. The implications of these considerations for the real world, *i.e.* $N_c = 3$, are not quite clear as we would expect the suppression of instanton effects only at asymptotically high temperatures. It is however striking that recent lattice simulations, with $N_f = 2$ staggered [38] and domain wall [39] fermions, both show a strong suppression of $U(1)_A$ breaking effects at low temperatures $T \sim 1.2\,T_c$. Because this temperature is outside the critical region, the order of the chiral phase transition is probably not affected. It could be of interest to consider doing simulations with $N_c > 3$, although this would probably be time consuming, or maybe with one flavour and various N_c. Consider for example a plot of

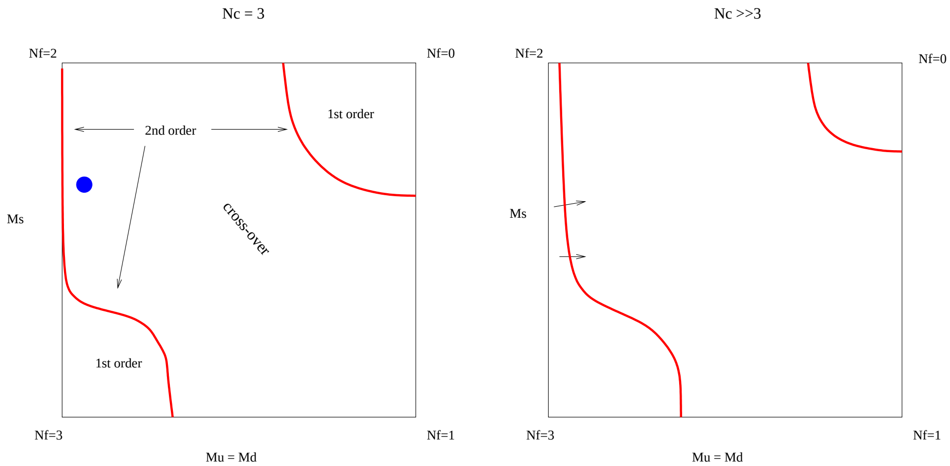

Figure 6: Columbia phase diagram as function of $M_u = M_d$ (horizontal axis) and M_s (vertical axes) for $N_c = 3$ (left, the blue dot is where QCD stands) and how it could evolve at large N_c (right).

the π and δ susceptibilities near the critical temperature as computed on the lattice [38]. Large N_c arguments suggest that the curves of the susceptibilities would be flatter below T_c —because the confined phase is colder— and that splitting between π and δ (*a.k.a.* a_0) should be narrower above T_c —because $U(1)_A$ breaking is more suppressed, maybe like in Fig. 5—. The Columbia diagram could change accordingly: the critical line around the region of small $M_u = M_d$ masses would move as in Fig. 6.

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We would like to thank P. Herrera-Siklódy and F. Karsch for useful discussions and S. Peris and J. Taron for a careful reading of the manuscript. Work partly supported by the EEC, TMR-CT98-0169, EURODAPHNE network.

Appendix

5.1 Useful formulæ

5.1.1 Leading order

$$2mB = M_\pi^2 \,, \tag{34}$$

$$x = 2\frac{M_K^2}{M_\pi^2} - 2 \,, \tag{35}$$

$$f = f_\pi \,, \tag{36}$$

$$-3v_{02} = M_{\eta'}^2 - \frac{2M_K^2 + M_\pi^2}{3} + \frac{2\sqrt{2}}{3}(M_K^2 - M_\pi^2)\tan\theta \ . \tag{37}$$

5.1.2 Next-to-leading order

Some definitions: Δ_M, Δ_N are defined as

$$\Delta_M \equiv \frac{8}{f^2}(M_K^2 - M_\pi^2)(2L_8 - L_5) \ , \tag{38}$$

$$\Delta_N \equiv 3v_{31} - 12\frac{L_5}{f^2}v_{02} \ . \tag{39}$$

Next-to-leading order parameters: The next-to-leading order parameters can be expressed in terms of observables through

$$2mB = M_\pi^2\left(1 - \frac{M_\pi^2}{M_K^2 - M_\pi^2}\Delta_M\right) \ , \tag{40}$$

$$x = 2\frac{M_K^2}{M_\pi^2}(1 - \Delta_M) - 2 \ , \tag{41}$$

$$f = f_\pi\left(1 - 4\frac{L_5}{f^2}M_\pi^2\right) \ , \tag{42}$$

$$\begin{aligned} -3v_{02} &= M_{\eta'}^2 - \frac{2M_K^2 + M_\pi^2}{3} \\ &+ \frac{2\sqrt{2}}{3}(M_K^2 - M_\pi^2)(1 + \Delta_M - \Delta_N)\tan\theta \\ &- \frac{2}{3}\left[(M_K^2 - M_\pi^2)\Delta_M - (2M_K^2 + M_\pi^2)\Delta_N\right] \ , \end{aligned} \tag{43}$$

$$\frac{L_5}{f^2} = \frac{1}{4(M_K^2 - M_\pi^2)}\Delta_P \ , \tag{44}$$

where

$$\Delta_M = \frac{M_\pi^2 + 3M_\eta^2 - 4M_K^2 + 3(M_{\eta'}^2 - M_\eta^2)\sin^2\theta}{4(M_K^2 - M_\pi^2)} \ , \tag{45}$$

$$\Delta_N = 1 + \frac{3}{4\sqrt{2}}\frac{(M_{\eta'}^2 - M_\eta^2)\sin 2\theta}{M_K^2 - M_\pi^2} + \Delta_M \ , \tag{46}$$

$$\Delta_P = \frac{f_K}{f_\pi} - 1 \ . \tag{47}$$

5.2 Results at $T = 0$

5.2.1 Leading order

Mass matrix:

$$m_{88}^2 = \frac{1}{3}(4M_K^2 - M_\pi^2)\,, \tag{48}$$

$$m_{80}^2 = -\frac{2\sqrt{2}}{3}(M_K^2 - M_\pi^2)\,, \tag{49}$$

$$\begin{aligned} m_{00}^2 &= \frac{1}{3}(2M_K^2 + M_\pi^2) - 3v_{02} \\ &= \frac{1}{3}(2M_K^2 + M_\pi^2) - \frac{2}{3}y(M_K^2 - M_\pi^2)\,, \end{aligned} \tag{50}$$

where

$$y \equiv \frac{9v_{02}}{2(M_K^2 - M_\pi^2)}\,. \tag{51}$$

Mixing angle:

$$\tan 2\theta \equiv \frac{2m_{80}^2}{m_{00}^2 - m_{88}^2} = \frac{2\sqrt{2}}{1+y}\,. \tag{52}$$

Physical masses:

$$M_\eta^2 = M_K^2 - \frac{M_K^2 - M_\pi^2}{3}(y + \sqrt{9 + 2y + y^2})\,, \tag{53}$$

$$M_{\eta'}^2 = M_K^2 - \frac{M_K^2 - M_\pi^2}{3}(y - \sqrt{9 + 2y + y^2})\,, \tag{54}$$

with

$$M_\eta^2 + M_{\eta'}^2 \equiv m_{88}^2 + m_{00}^2 = 2M_K^2 - \frac{2}{3}y(M_K^2 - M_\pi^2)\,. \tag{55}$$

5.2.2 Next-to-leading order

Mass matrix:

$$m_{88}^2 = \frac{1}{3}(4M_K^2 - M_\pi^2) + \frac{4}{3}(M_K^2 - M_\pi^2)\Delta_M\,, \tag{56}$$

$$m_{80}^2 = -\frac{2\sqrt{2}}{3}(M_K^2 - M_\pi^2)(1 + \Delta_M - \Delta_N)\,, \tag{57}$$

$$
\begin{aligned}
m_{00}^2 &= \frac{1}{3}(2M_K^2 + M_\pi^2) - 3v_{02} \\
&+ \frac{2}{3}(M_K^2 - M_\pi^2)\Delta_M - \frac{2}{3}(2M_K^2 + M_\pi^2)\Delta_N \\
&= \frac{1}{3}(2M_K^2 + M_\pi^2)(1 - 2\Delta_N) + \frac{2}{3}(M_K^2 - M_\pi^2)\Delta_M \\
&- \frac{2}{3}y(M_K^2 - M_\pi^2)\,. \qquad (58)
\end{aligned}
$$

Mixing angle:

$$
\tan 2\theta = \frac{2\sqrt{2}}{1+y}\left(1 + \frac{y}{1+y}\Delta_M - \left(1 + \frac{1}{1+y}\frac{2M_K^2 + M_\pi^2}{M_K^2 - M_\pi^2}\right)\Delta_N\right)\,. \qquad (59)
$$

Physical masses:

$$
\begin{aligned}
M_\eta^2 &= M_K^2 - \frac{M_K^2 - M_\pi^2}{3}(y + \sqrt{9 + 2y + y^2}) \\
&+ \left(1 - \frac{9+y}{3\sqrt{9 + 2y + y^2}}\right)(M_K^2 - M_\pi^2)\Delta_M \\
&- \frac{1}{3}\left(2M_K^2 + M_\pi^2 - \frac{3(2M_K^2 - 3M_\pi^2) - y(2M_K^2 + M_\pi^2)}{\sqrt{9 + 2y + y^2}}\right)\Delta_N\,, \qquad (60) \\
M_{\eta'}^2 &= M_K^2 - \frac{M_K^2 - M_\pi^2}{3}(y - \sqrt{9 + 2y + y^2}) \\
&+ \left(1 + \frac{9+y}{3\sqrt{9 + 2y + y^2}}\right)(M_K^2 - M_\pi^2)\Delta_M \\
&- \frac{1}{3}\left(2M_K^2 + M_\pi^2 + \frac{3(2M_K^2 - 3M_\pi^2) - y(2M_K^2 + M_\pi^2)}{\sqrt{9 + 2y + y^2}}\right)\Delta_N\,, \qquad (61)
\end{aligned}
$$

with

$$
\begin{aligned}
M_\eta^2 + M_{\eta'}^2 &= 2M_K^2 - \frac{2}{3}y(M_K^2 - M_\pi^2) \\
&+ 2(M_K^2 - M_\pi^2)\Delta_M - \frac{2}{3}(2M_K^2 + M_\pi^2)\Delta_N\,. \qquad (62)
\end{aligned}
$$

5.3 Results at $T \neq 0$

5.3.1 Leading order

Mass matrix:

$$m_{88}^2(T) = m_{88}^2(0) - \frac{M_\pi^2}{2f_\pi^2} I(T) \ , \qquad (63)$$

$$m_{80}^2(T) = m_{80}^2(0) - \frac{M_\pi^2}{\sqrt{2} f_\pi^2} I(T) \ , \qquad (64)$$

$$m_{00}^2(T) = m_{00}^2(0) - \frac{M_\pi^2}{f_\pi^2} I(T) \ , \qquad (65)$$

where

$$\begin{aligned} I(T) &\equiv \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{\omega} \frac{1}{e^{\beta\omega} - 1} \ ; \ \omega = \sqrt{\vec{k}^2 + M_\pi^2}, \ \beta = \frac{1}{T} \\ &= \frac{M_\pi T}{2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} K_1 \left(\frac{n M_\pi}{T} \right) \stackrel{M_\pi \to 0}{\longrightarrow} \frac{T^2}{12} \ . \end{aligned} \qquad (66)$$

Mixing angle:

$$\tan 2\theta(T) = \tan 2\theta(0) + \frac{2\sqrt{2}}{1+y} \frac{y}{1+y} \frac{3}{4} \frac{M_\pi^2}{M_K^2 - M_\pi^2} \frac{1}{f_\pi^2} I(T) \ . \qquad (67)$$

Physical masses:

$$M_\eta^2(T) = M_\eta^2(0) - \frac{3}{4} \frac{M_\pi^2}{f_\pi^2} I(T) \left(1 + \frac{9+y}{3\sqrt{9+2y+y^2}} \right) \ , \qquad (68)$$

$$M_{\eta'}^2(T) = M_{\eta'}^2(0) - \frac{3}{4} \frac{M_\pi^2}{f_\pi^2} I(T) \left(1 - \frac{9+y}{3\sqrt{9+2y+y^2}} \right) \ , \qquad (69)$$

with

$$M_\eta^2(T) + M_{\eta'}^2(T) = M_\eta^2(0) + M_{\eta'}^2(0) - \frac{3}{2} \frac{M_\pi^2}{f_\pi^2} I(T) \ . \qquad (70)$$

5.3.2 Next-to-leading order

Mass matrix:

$$m_{88}^2(T) = m_{88}^2(0) - \frac{M_\pi^2}{2f_\pi^2} I(T) \left(1 + \frac{2M_\pi^2}{M_K^2 - M_\pi^2} \left(\Delta_P + \frac{3}{2} \Delta_M \right) \right.$$

$$
\begin{aligned}
&+ \quad 24\frac{M_\pi^2}{f_\pi^2}(M_0+M_3)\Big) \ , \qquad (71)\\
m_{80}^2(T) &= m_{80}^2(0) - \frac{M_\pi^2}{\sqrt{2}f_\pi^2} I(T)\left(1+\frac{2M_\pi^2}{M_K^2-M_\pi^2}\left(\Delta_P+\frac{3}{2}\Delta_M\right)\right.\\
&- \quad \left.\Delta_N + 24\frac{M_\pi^2}{f_\pi^2}(M_0+M_3)\left(1-\frac{y}{3}\frac{M_K^2-M_\pi^2}{M_\pi^2}\right)\right) \ , \qquad (72)\\
m_{00}^2(T) &= m_{00}^2(0) - \frac{M_\pi^2}{f_\pi^2} I(T)\left(1+\frac{2M_\pi^2}{M_K^2-M_\pi^2}\left(\Delta_P+\frac{3}{2}\Delta_M\right)\right.\\
&- \quad \left.2\Delta_N + 24\frac{M_\pi^2}{f_\pi^2}(M_0+M_3)\left(1-\frac{2y}{3}\frac{M_K^2-M_\pi^2}{M_\pi^2}\right)\right) \ . \qquad (73)
\end{aligned}
$$

Mixing angle:

$$
\begin{aligned}
\tan 2\theta(T) &= \tan 2\theta(0) + \frac{2\sqrt{2}}{1+y}\frac{y}{1+y}\frac{3}{4}\frac{M_\pi^2}{M_K^2-M_\pi^2}\frac{1}{f_\pi^2} I(T) \times \\
(1 &+ \frac{2M_\pi^2}{M_K^2-M_\pi^2}\left(\Delta_P+\frac{3}{2}\Delta_M\right) - \frac{2}{1+y}\Delta_M \\
&+ \frac{1-y}{1+y}\Delta_N + \frac{3}{(1+y)y}\frac{2M_K^2-(1+y)M_\pi^2}{M_K^2-M_\pi^2}\Delta_N \\
&+ 24\frac{M_\pi^2}{f_\pi^2}(M_0+M_3)\left(\frac{M_K^2-\frac{y}{3}(M_K^2-M_\pi^2)}{M_\pi^2}\right)\Big) \ . \qquad (74)
\end{aligned}
$$

Physical masses:

$$
\begin{aligned}
M_\eta^2(T) &= M_\eta^2(0) - \frac{3}{4}\frac{M_\pi^2}{f_\pi^2} I(T)\left(1+\frac{9+y}{3\sqrt{9+2y+y^2}}\right.\\
&+ \frac{2M_\pi^2}{M_K^2-M_\pi^2}\left(\left(1+\frac{9+y}{3\sqrt{9+2y+y^2}}\right)\Delta_P\right.\\
&\quad +\frac{3}{2}\left(1+\frac{27(3+y)+(9+y)y^2}{3(9+2y+y^2)^{3/2}}\right.\\
&\quad \left.\left. +\frac{\frac{2}{3}y^2}{3(9+2y+y^2)^{3/2}}\frac{4M_K^2-M_\pi^2}{M_\pi^2}\right)\Delta_M\right)\\
&- \frac{4}{3}\left(1+\frac{(3+y)((9+y)+(3+y)y)}{(9+2y+y^2)^{3/2}}\right.\\
&\quad \left.+\frac{6y}{(9+2y+y^2)^{3/2}}\frac{M_\pi^2}{M_K^2-M_\pi^2}\right)\Delta_N
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 24\frac{M_\pi^2}{f_\pi^2}(M_0+M_3)\left(1+\frac{9+y}{3\sqrt{9+2y+y^2}}\right. \\
&\quad \left.-\frac{4}{9}y\left(1+\frac{3+y}{\sqrt{9+2y+y^2}}\right)\frac{M_K^2-M_\pi^2}{M_\pi^2}\right)\right) \ , \qquad (75)
\end{aligned}
$$

$$
\begin{aligned}
M_{\eta'}^2(T) \;=\;& M_{\eta'}^2(0)-\frac{3}{4}\frac{M_\pi^2}{f_\pi^2}I(T)\left(1-\frac{9+y}{3\sqrt{9+2y+y^2}}\right. \\
&+ \quad \frac{2M_\pi^2}{M_K^2-M_\pi^2}\left(\left(1-\frac{9+y}{3\sqrt{9+2y+y^2}}\right)\Delta_P\right. \\
&\quad +\frac{3}{2}\left(1-\frac{27(3+y)+(9+y)y^2}{3(9+2y+y^2)^{3/2}}\right. \\
&\quad \left.\left.-\frac{\frac{2}{3}y^2}{3(9+2y+y^2)^{3/2}}\frac{4M_K^2-M_\pi^2}{M_\pi^2}\right)\Delta_M\right) \\
&- \quad \frac{4}{3}\left(1-\frac{(3+y)((9+y)+(3+y)y)}{(9+2y+y^2)^{3/2}}\right. \\
&\quad \left.-\frac{6y}{(9+2y+y^2)^{3/2}}\frac{M_\pi^2}{M_K^2-M_\pi^2}\right)\Delta_N \\
&+ \quad 24\frac{M_\pi^2}{f_\pi^2}(M_0+M_3)\left(1-\frac{9+y}{3\sqrt{9+2y+y^2}}\right. \\
&\quad \left.-\frac{4}{9}y\left(1-\frac{3+y}{\sqrt{9+2y+y^2}}\right)\frac{M_K^2-M_\pi^2}{M_\pi^2}\right)\right) \ , \qquad (76)
\end{aligned}
$$

with

$$
\begin{aligned}
M_\eta^2(T)+M_{\eta'}^2(T) \;=\;& M_\eta^2(0)+M_{\eta'}^2(0)-\frac{3}{2}\frac{M_\pi^2}{f_\pi^2}I(T)\times \\
(1 \quad +& \quad \frac{2M_\pi^2}{M_K^2-M_\pi^2}\left(\Delta_P+\frac{3}{2}\Delta_M\right)-\frac{4}{3}\Delta_N \\
+& \quad 24\frac{M_\pi^2}{f_\pi^2}(M_0+M_3)\left(1-\frac{4}{9}y\frac{M_K^2-M_\pi^2}{M_\pi^2}\right)\right) \ . \qquad (77)
\end{aligned}
$$

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