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ANALYZING ε'/ε IN THE $1/N_c$ EXPANSION

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Abstract

We present a recent analysis of ε'/ε in the $1/N_c$ expansion. We show that the $1/N_c$ corrections to the matrix element of Q_6 are large and positive, indicating a $\Delta I = 1/2$ enhancement similar to the one of Q_1 and Q_2 which dominate the CP conserving amplitude. This enhances the CP ratio and can bring the standard model prediction close to the measured value for central values of the parameters.

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1 Introduction

Direct CP violation in $K \to \pi\pi$ decays was recently observed by the KTeV and NA48 collaborations.[1,2] The present world average [2] for the parameter ε'/ε is $\mathrm{Re}\,\varepsilon'/\varepsilon = (21.2 \pm 4.6) \cdot 10^{-4}$. In the standard model CP violation originates in the CKM phase, and direct CP violation is governed by loop diagrams of the penguin type. The main source of uncertainty in the calculation of ε'/ε is the QCD non-perturbative contribution related to the hadronic nature of the $K \to \pi\pi$ decay. Using the $\Delta S = 1$ effective hamiltonian,

$$\mathcal{H}_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}\,\lambda_u \sum_{i=1}^{8} c_i(\mu)\,Q_i(\mu) \qquad (\mu < m_c)\,, \tag{1}$$

the non-perturbative contribution, contained in the hadronic matrix elements of the four-quark operators Q_i, can be separated from the perturbative Wilson coefficients $c_i(\mu) = z_i(\mu) + \tau y_i(\mu)$ (with $\tau = -\lambda_t/\lambda_u$ and $\lambda_q = V_{qs}^*\,V_{qd}$). Introducing $\langle Q_i \rangle_I \equiv \langle (\pi\pi)_I | Q_i | K \rangle$, the CP ratio can be written as

$$\frac{\varepsilon'}{\varepsilon} = \frac{G_F}{2}\frac{\omega\ \mathrm{Im}\lambda_t}{|\varepsilon|\,\mathrm{Re}A_0}\left[\left|\sum_i y_i\,\langle Q_i\rangle_0\right|\left(1-\Omega_{\eta+\eta'}\right) - \frac{1}{\omega}\left|\sum_i y_i\,\langle Q_i\rangle_2\right|\right]. \tag{2}$$

$\omega = \mathrm{Re}A_0/\mathrm{Re}A_2 = 22.2$ is the ratio of the CP conserving $K \to \pi\pi$ isospin amplitudes; $\Omega_{\eta+\eta'}$ encodes the effect of the isospin breaking in the quark masses.[3] ε'/ε is dominated by $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ which cannot be fixed from the CP conserving data.[4,5] Beside the theoretical uncertainties coming from the calculation of the $\langle Q_i\rangle_I$ and of $\Omega_{\eta+\eta'}$, the analysis of the CP ratio suffers from the uncertainties on the values of various input parameters, in particular of the CKM phase in $\mathrm{Im}\lambda_t$, of $\Lambda_{\mathrm{QCD}} \equiv \Lambda_{\overline{\mathrm{MS}}}^{(4)}$, and of the strange quark mass.

To calculate the hadronic matrix elements we start from the effective chiral lagrangian for pseudoscalar mesons which involves an expansion in momenta where terms up to $\mathcal{O}(p^4)$ are included.[6] The method we use is the $1/N_c$ expansion.[7,8] In this approach, we expand the matrix elements in powers of the momenta and of $1/N_c$. For the $1/N_c$ corrections we calculated chiral loops as described in refs. [9,10]. Especially important to this analysis are the non-factorizable corrections, which are UV divergent and must be matched to the short-distance part. They are regularized by a finite cutoff Λ_c which is identified with the short-distance renormalization scale. The definition of the momenta in the loop diagrams, which are not momentum translation invariant, is discussed in detail in ref. [9]. Other recent work on matrix elements in the $1/N_c$ approach can be found in refs. [11,12].

For the Wilson coefficients we use the leading logarithmic and the next-to-leading logarithmic values.[4] The absence of any reference to the renormalization scheme in the low-energy calculation, at this stage, prevents a complete matching at the next-to-leading order.[13] Nevertheless, a comparison of the numerical results obtained from the LO and NLO coefficients is useful as regards estimating the uncertainties and testing the validity of perturbation theory.

2 Analysis of ε'/ε

Analytical formulas for all matrix elements, at next-to-leading order in the twofold expansion in powers of momenta and of $1/N_c$, are given in refs. [9,10]. In the pseudoscalar approximation, the matching has to be done below 1 GeV. Varying Λ_c between 600 and 900 MeV, the bag factors $B_1^{(1/2)}$ and $B_2^{(1/2)}$ take the values $8.2 - 14.2$ and $2.9 - 4.6$; quadratic terms in $\langle Q_1 \rangle_0$ and $\langle Q_2 \rangle_0$ produce a large enhancement which brings the $\Delta I = 1/2$ amplitude in agreement with the data.[10] Corrections beyond the chiral limit were found to be small.

For $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$ the leading non-factorizable loop corrections, which are of $\mathcal{O}(p^0/N_c)$, are only logarithmically divergent.[9] Including terms of $\mathcal{O}(p^0)$, $\mathcal{O}(p^2)$, and $\mathcal{O}(p^0/N_c)$, $B_6^{(1/2)}$ and $B_8^{(3/2)}$ take the values $1.10 - 0.72$ and $0.64 - 0.42$. As a result the experimental range for ε'/ε can be accommodated in the standard model only if there is a conspiracy of the input parameters.[1] However, since the leading $\mathcal{O}(p^0)$ contribution vanishes for Q_6, corrections from higher order terms beyond the $\mathcal{O}(p^2)$ and $\mathcal{O}(p^0/N_c)$ are expected to be large. In ref. [15] we investigated the $\mathcal{O}(p^2/N_c)$ contribution, i.e., the $1/N_c$ correction at the next order in the chiral expansion, because it brings about, for the first time, quadratic corrections on the cutoff. From counting arguments and more generally from the fact that the chiral limit is assumed to be reliable, the quadratic terms (which are not chirally suppressed) are expected to be dominant. It is still desirable to check that explicitly by calculating the corrections beyond the chiral limit, from logarithms and finite terms, as done for Q_1 and Q_2. Numerically, we observe a large positive correction from the quadratic term in $\langle Q_6 \rangle_0$. This point was already emphasized in ref. [16]. The slope of the correction is qualitatively consistent and welcome since it compensates for the logarithmic decrease at $\mathcal{O}(p^0/N_c)$. Varying Λ_c between 600 and 900 MeV, the $B_6^{(1/2)}$ factor takes the values $1.50 - 1.62$. Q_6 is a $\Delta I = 1/2$ operator, and the enhancement of $\langle Q_6 \rangle_0$ indicates that at the level of the $1/N_c$ corrections the dynamics of the $\Delta I = 1/2$ rule applies to Q_6 as to Q_1 and Q_2.

Using the quoted values for $B_6^{(1/2)}$ together with the full leading plus next-to-leading order B factors for the remaining operators [15] we calculated ε'/ε. The results for the three sets of Wilson coefficients LO, NDR, and HV and for Λ_c between 600 and 900 MeV

[1] For supersymmetric contributions to ε'/ε see ref. [14] and references therein.

Table 1: Numerical values for ε'/ε (in units of 10^{-4}) as explained in the text.

LO	$14.8 \leq \varepsilon'/\varepsilon \leq 19.4$	$6.1 \leq \varepsilon'/\varepsilon \leq 48.5$
NDR	$12.5 \leq \varepsilon'/\varepsilon \leq 18.3$	$5.2 \leq \varepsilon'/\varepsilon \leq 49.8$
HV	$7.0 \leq \varepsilon'/\varepsilon \leq 14.9$	$2.2 \leq \varepsilon'/\varepsilon \leq 38.5$

are given in Tab. 1. The numbers are close to the measured value for central values of the parameters (first column). They are obtained by assuming zero phases from final state interactions. This approximation is very close to the results we would get if we used the small imaginary part obtained at the one-loop level.[15]

Performing a scanning of the parameters [$125\,\text{MeV} \leq m_s(1\,\text{GeV}) \leq 175$ MeV, $0.15 \leq \Omega_{\eta+\eta'} \leq 0.35$, $1.04 \cdot 10^{-4} \leq \text{Im}\lambda_t \leq 1.63 \cdot 10^{-4}$, and $245\,\text{MeV} \leq \Lambda_{\text{QCD}} \leq 405\,\text{MeV}$] we obtain the numbers in the second column of Tab. 1. They can be compared with the results of refs. [5,17–20]. The values of $B_6^{(1/2)}$ can also be compared with ref. [11] and those of $B_8^{(3/2)}$ with refs. [12,21]. The large ranges reported in the table can be traced back to the large ranges of the input parameters. This can be seen by comparing them with the relatively narrow ranges obtained for central values of the parameters. The parameters, to a large extent, act multiplicatively, and the large range for ε'/ε is due to the fact that the central value(s) for the ratio are enhanced roughly by a factor of two compared to the results obtained with B factors for Q_6 and Q_8 close to the VSA. More accurate information on the parameters, from theory and experiment, will restrict the values for ε'/ε.

To estimate the uncertainties due to higher order final state interactions we also calculated ε'/ε using the real part of the matrix elements and the phenomenological values of the phases [22], $\delta_0 = (34.2 \pm 2.2)^\circ$ and $\delta_2 = (-6.9 \pm 0.2)^\circ$, i.e., we replaced $|\sum_i y_i \langle Q_i \rangle_I|$ in Eq. (2) by $\sum_i y_i \text{Re}\langle Q_i \rangle_I / \cos\delta_I$. The corresponding results are given in Tab. 2. They are enhanced by $\sim 25\,\%$ compared to the numbers in Tab. 1. We would like to emphasize that this $\sim 25\,\%$ error should be taken into account by any analysis which either does not include final state interactions or cannot reproduce the numerical values of the phases. To reduce the uncertainties in the $1/N_c$ approach it would be interesting to investigate the two-loop imaginary part and/or to combine our calculation with a dispersive calculation along the lines of refs. [23–25]. In order to reduce the scheme dependence in the result, appropriate subtractions would be necessary.[11,26] Finally, it is reasonable to assume that the effect of the pseudoscalar mesons is the most important one. Nevertheless, the incorporation of vector mesons and higher resonances would be desirable in order to im-

Table 2: Same as in Tab. 1, but now with the phenomenological values for the phases as explained in the text.

LO	$19.5 \leq \varepsilon'/\varepsilon \leq 24.7$	$8.0 \leq \varepsilon'/\varepsilon \leq 62.1$
NDR	$16.1 \leq \varepsilon'/\varepsilon \leq 23.4$	$6.8 \leq \varepsilon'/\varepsilon \leq 63.9$
HV	$9.3 \leq \varepsilon'/\varepsilon \leq 19.3$	$2.8 \leq \varepsilon'/\varepsilon \leq 49.8$

prove the treatment of the intermediate region around the rho mass and to show explicitly that the large enhancement we find at low energy at the level of the pseudoscalars remains up to the scale $\sim m_c$, where the matching with the short-distance part can be done more safely.

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