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and B. Choosing the most stable periods, and correcting for the linear drift, the mean value for the shift is found to be $\Delta = 0.005 \pm 0.013$ nsec.

The result for the velocity measurement of γ rays of energy $\geq$ 6 GeV from a source with $\beta = 0.99975$ (according to special relativity) as given by eq. (2) is

$$c' = (2.9977 \pm 0.0004) \times 10^{10} \text{ cm/sec} .$$

If we interpret this result using eq. (1) with $c = 2.9979 \times 10^{10}$ cm/sec, the corresponding value for k is:

$$k = (-3 \pm 13) \times 10^{-5}.$$

Note that the γ rays observed in this experiment pass through some beryllium, a thin mylar window and about 60 m of air before their velocity is measured. As this material is refractive, the extinction theorem implies that the original γ rays from the moving source will be slowly absorbed and replaced by similar radiation re-emitted by the stationary medium, thus invalidating the experiment [7]. This effect becomes important if the phase delay due to the medium exceeds say $\lambda/2\pi$, where λ is the wavelength of the γ rays. Deriving the refractive index for γ rays from the forward scattering amplitude per electron $A = e^2/mc^2$, the maximum allowable distance becomes

$$d_{\max} = (\lambda n A)^{-1} \approx 5 \text{ km of air}$$

for γ rays of 6 GeV, where n is the number of electrons per cm^3 of the medium. We conclude that reradiation by stationary material is a negligible effect in this experiment.

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RADIATIVE CORRECTIONS TO $e^+ + e^- \rightarrow \mu^+ + \mu^-$

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In this note we give an evaluation of the radiative corrections to the colliding beam reaction $e^+ + e^- \rightarrow \mu^+ + \mu^-$. We use Eriksson's work on the renormalization group in electrodynamics [1] to obtain a tentative estimate of the higher order corrections in α. Our results show that the higher order terms play a minor role for colliding beam experiments in the energy range of interest for the various projects now under development [2].

Our calculation of the two-photon contribu-

Table 1
Radiative corrections to $e^{+}+e^{-} \rightarrow \mu^{+}+\mu^{-}$. The cross section is written as $d\sigma = d\sigma_0(1+\delta)$, where $d\sigma_0$ is the lowest order cross section. The total c.m. energy is W and the c.m. angle is θ. The cut-off ϵ for the emitted photons is isotropic in c.m. and is chosen as $\frac{1}{10}$ of W.

W (GeV)	θ	$\delta \times 10^2$	W (GeV)	θ	$\delta \times 10^2$
0.5	30°	- 9.42	10.0	30°	-12.09
	60°	- 8.03		60°	-10.73
	90°	- 7.04		90°	- 9.77
	120°	- 6.05		120°	- 8.81
	150°	- 4.67		150°	- 7.47
1.0	30°	-10.04	20.0	30°	-12.69
	60°	- 8.66		60°	-11.35
	90°	- 7.67		90°	-10.39
	120°	- 6.69		120°	- 9.44
	150°	- 5.33		150°	- 8.11
2.0	30°	-10.66	30.0	30°	-13.05
	60°	- 9.29		60°	-11.71
	90°	- 8.31		90°	-10.75
	120°	- 7.33		120°	- 9.80
	150°	- 5.98		150°	- 8.49
4.0	30°	-11.28			
	60°	- 9.91			
	90°	- 8.94			
	120°	- 7.97			
	150°	- 6.62			

tions is carried out, using the method of Yennie et al. [3]. The results are reported in table 1 for a c.m. isotropic energy cut-off ϵ, for the emitted photons, of $\frac{1}{10}$ of the total c.m. energy.

The graphs contributing to the lowest order radiative corrections to $e^{+}+e^{-} \rightarrow \mu^{+}+\mu^{-}$ are reported in fig. 1. We are interested in the asymptotic expressions of the corrections for large W, total centre-of-mass energy. For the vertex corrections δ_V (graphs V_i and V_f) and the self-energy corrections δ_{SE} (graphs SE_i and SE_f), we have derived exact asymptotic expressions, including constant terms in W, by adapting known results [4]. For the two-photon-exchange corrections (graphs 2γ) we have used the technique of Yennie et al. [3]. In the terminology of these authors, we have calculated the dominant convection contribution (neglecting the dependence of the basic matrix element from the soft photon momentum k), which gives a logarithmic contribution. We call this contribution δ_2. We have also calculated the spin convection contribution and the correction to the convection contribution due to first order terms in k. We call these contributions δ_2'. They only give constant terms in the asymptotic limit (for a fixed scattering angle θ different from 0 or π). The corrections from initial and final emission of real soft photons (graphs B_i and B_f, respectively) were calculated exactly. These corrections are called δ_B. The dominant convection contri-

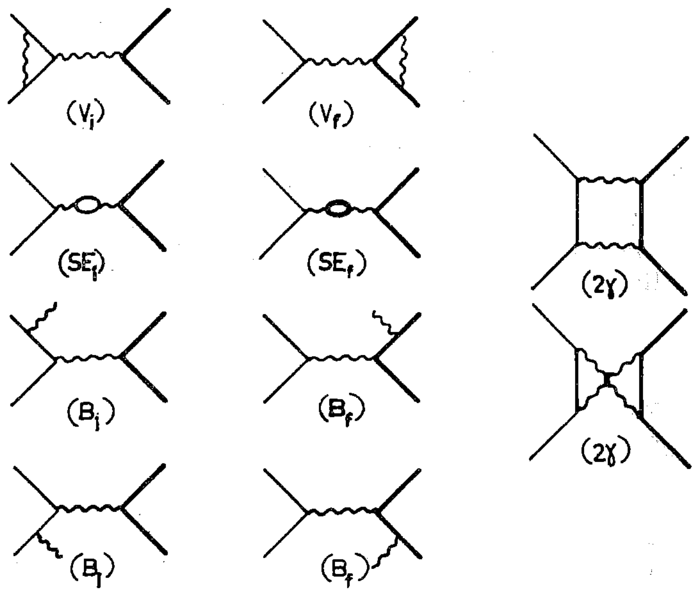

Fig. 1. Lowest order radiative corrections to $e^{+}+e^{-} \rightarrow \mu^{+}+\mu^{-}$ (the heavy line denotes the muon). Initial and final vertex graphs are denoted by (V_i) and (V_f) respectively, self-energy graphs by (SE_i) and (SE_f), bremsstrahlung graphs by (B_i) and (B_f), and the two-photon exchange graphs by (2γ).

butions δ_B to the interference terms between initial and final bremsstrahlung, were consistently calculated with the technique of Yennie et al. These terms give in fact compensating terms in λ (photon mass) to the two-photon-exchange correction. Spin convection contributions to these interference terms should not contribute appreciably for small energy cut-off ϵ to the emitted real photons. The expressions we give here are also corrected for the Coulomb interaction between the particles in the initial and final states, which affects the constant term in δ_V.

We distinguish between initial or final correction by an index i or f. The final corrections can be obtained from the initial ones by the substitution of m, initial fermion mass, by μ, the mass of the final fermion. The initial radiative correction is the following:

$$\delta_R^{(i)}(\epsilon) = \delta_V^{(i)}(\lambda) + \delta_B^{(i)}(\lambda, \epsilon) + \delta_{SE}^{(i)}$$
$$= \frac{\alpha}{\pi}\left\{-\frac{2}{3}\pi^2 - \frac{28}{9} + 2\ln\frac{\epsilon}{W}\left(2\ln\frac{W}{m} - 1\right) - 2\ln 2 + \left(\frac{13}{3} + 4\ln 2\right)\ln\frac{W}{m}\right\}. \quad (1)$$

The dominant convection contribution from two-photon-exchange δ_2 and the dominant convection contribution $\tilde{\delta}_B$ in the interference between initial and final bremsstrahlung give a correction (always for $\theta \neq 0, \pi$):

$$\delta_2 + \tilde{\delta}_B = \frac{\alpha}{\pi}\,(4\ \ln \frac{W}{2\epsilon} - 1)\ \ln \frac{1-\cos\theta}{1+\cos\theta}\,. \qquad (2)$$

In these formulae α is the fine structure constant, W the total centre-of-mass energy, ϵ is the soft photon energy cut-off, and θ is the centre-of-mass scattering angle. Spin convection contributions and terms arising from the k dependence in the matrix element of the basic process contribute the constant term

$$\delta_2' = -\frac{\alpha}{\pi}\ln\frac{1-\cos\theta}{1+\cos\theta}\,. \qquad (3)$$

We have included δ_2' in the radiative corrections, even if other similar terms may come from other neglected contributions (as, for instance, spin convection interferences from bremsstrahlung graphs). In any case, θ independent terms cannot appear in δ_2, $\tilde{\delta}_B$, and δ_2'; these contributions have to be odd in $\cos\theta$ as they must vanish for any observation symmetrical with respect to the final charges. Numerically, the energy independent correction δ_2' turns out to be negligible with respect to the energy independent terms of δ_V, δ_{SE}, and δ_B.

We now try to get from these first order corrections an indication on the contribution of the higher order terms in α. To this end we use Eriksson's results [1], which allow us to obtain from the first order corrections an approximate expression for the differential cross section of the reaction including terms of all orders in α. Actually, the original Eriksson's results are given for the case of charged particles of equal masses, but the generalization is straightforward. We obtain:

$$\mathrm{d}\sigma = \mathrm{d}\sigma_0\Big\{1 - \frac{\alpha}{\pi}[2(1+2\ \ln\ 2)]\ \ln\frac{1-\cos\theta}{1+\cos\theta} + 4\ \ln\ 2 + \frac{56}{9} + \frac{4\pi^2}{3}\Big\}\exp\Big\{\ln\frac{\epsilon}{W}\Big[-\frac{4\alpha}{\pi}\Big(1+\ln\frac{1-\cos\theta}{1+\cos\theta}\Big) - 6\ \ln\ \Big(1-\frac{\alpha}{3\pi}\Big[\ln\frac{W^2}{m^2}+\ln\frac{W^2}{\mu^2}\Big]\Big)\Big] - \frac{3}{2}\Big(\frac{13}{3}+4\ \ln\ 2\Big)\ln\ \Big(1-\frac{\alpha}{3\pi}\Big[\ln\frac{W^2}{m^2}+\ln\frac{W^2}{\mu^2}\Big]\Big)\Big\}\,, \qquad (4)$$

where $\mathrm{d}\sigma_0$ is the cross section at lowest order. We write $\mathrm{d}\sigma = \mathrm{d}\sigma_0(1+\delta)$. Numerical values of δ are given in table 1 for different values of W, total centre-of-mass energy, of the centre-of-mass scattering angle θ, and for the choice of $\epsilon/W = 10^{-1}$. The expression (4) is derived by combining the renormalization group techniques to sum up the finite (not infra-red) part of the matrix element and the well-known summation formula for its intra-red part and for the soft photon contribution. Consequently, it has definite limits of validity that we want now to discuss. The equations of the renormalization group are a strict consequence of very general properties of the theory, so that it is difficult to get from them very explicit consequences, just owing to their too general character. Thus, if we assume that the quantities of interest can be expanded as a power series in the charge, it is not possible - in general - to improve upon perturbation results, as the group equations do not give unique solutions without imposing particular conditions. The discussion has then to be focused on the additional requirements - of physical character - one has to introduce to get unique results, at least for the dominant terms [5]. The investigation of the asymptotic region of large momenta seems particularly favourable. This is the region where the kinematical variables (s, t and u for a scattering amplitude) are all large, as compared to the involved masses, and all of the same order. Moreover (and this is really the crux of the matter) one has to assume the existence of the limit of vanishing masses for quantities which are the generalization (in the sense of the renormalization group) of the physical Green functions. As a consequence of these hypotheses the equations finally depend on one variable only and can be more easily handled. An improvement of the perturbative expansion can then be achieved in the following sense. The series can be rearranged according to the different "logarithmicity" of its terms (that is through combinations of the form (e^{2i} $\ln^j(W/m)$) and in particular once the lowest order corrections are known, it is possible to sum up the maximally logarithmic terms, which always turn out to be of the form $(e^2\ \ln\ (W/m))^n$. It is this last result which heavily rests on the possibility of neglecting the mass dependence. Such a possibility can be verified for the photon propagator but, for instance, for matrix elements the problem seems to be quite open. Since we use the Eriksson's formula, we take this hypothesis as granted: anyway its validity can be verified in the limit of our perturbative results. We have still to recall the limitations on ϵ, which are characteristic of the soft photon summation procedure. The presence of ϵ limits the result only to the c.m. frame with, a rigori, $\epsilon \ll m$. However, as it can be verified explicitly by evaluation of the hard photon contribution [6], the soft photon formula is still quite good also for large ϵ. To summarize eq. (4) gives in the asymptotic region ($\ln\ (W/m) \gg 1$, θ not too close to $0, \pi$) a summation of the maximally logarithmic parts, i.e., it reproduces correctly the terms $e^{2n} \ln^n(W/m)$ while the neglected ones are of the form $e^{2n} \ln^i(W/m)$ $(i < n)$. In addition, one has to remember that we are working in the frame of perturbation theory $(\alpha \ln (W/m) < 1)$ so that the previous result has to be meant as an expansion in α, where - to avoid the problem of the convergence of the series - only a finite number of terms is taken into account.

In particular eq. (4), expanded in a power series of α, reproduces at first order the calculated lowest order correction δ_1:

$$\delta_1 = \frac{\alpha}{\pi}\left\{2 \ln \frac{\epsilon}{W}\left(2 \ln \frac{W}{m} - 1\right) - 2 \ln 2 - \frac{28}{9} - \frac{2\pi^2}{3} + \left(\frac{13}{3} + 4 \ln 2\right) \ln \frac{W}{m} + (\text{same terms with } m \to \mu) - 2\left(1 + 2 \ln 2 + 2 \ln \frac{\epsilon}{W}\right) \ln \frac{1-\cos\theta}{1+\cos\theta}\right\}.$$

Numerically, δ_1 differs from δ only at most by 6% in all cases considered in table 1. We remark that the presence of the μ meson, whose mass is rather larger than the electron mass, shifts the asymptotic region of at least an order of magnitude. So at the lowest energies the evaluation of δ gives really only an order of magnitude. It is clear that increasing the energy we came closer and closer to the region where it would be possible to limit ourselves to the higher logarithmic terms only. In this limit it would become really unimportant to use the electron or the muon mass in our final expressions.

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ON A THEORY OF HIGH-ENERGY PROCESSES BASED ON THE HYPOTHESES OF INDEPENDENT EMISSION AND GAUGE-INVARIANCE

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We wish to report here on a theoretical investigation, the details of which will be given elsewhere. We will limit ourselves here to a very simple example, without any claim for generality, in order to illustrate as clearly as possible the underlying physical ideas.

We will show that the existence of vector bosons coupled to conserved currents *necessarily* leads to the exponential damping of the transverse momenta * of those secondaries that are involved in conserved currents, provided the dominant mechanism of multiple production obeys the rule of independent emission. The importance of the smallness of transverse momenta and inelasticity for the understanding of high-energy interactions has been repetidly emphasized [1-4]. Let us stress here that starting from these two facts, one can explain practically all the most important features of particle collisions at high energies.

We assume, therefore:

i) There exist vector mesons (V-mesons) uni-

* The smallness of the inelasticity can be explained by the same argument.