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SINGULARITY**

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QUANTUM GRAVITY EFFECTS NEAR THE NULL BLACK HOLE SINGULARITY

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Abstract

The structure of the Cauchy Horizon singularity of a black hole formed in a generic collapse is studied by means of a renormalization group equation for quantum gravity. It is shown that during the early evolution of the Cauchy Horizon the strength of the classical divergence of the mass function is weakened when quantum fluctuations of the metric are taken into account.

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Recently much progress has been made in understanding the formation of singularities in realistic black holes. After the seminal work by Poisson and Israel [1], the outcome of several investigations with spherical models (see [2] for a general overview) was that the spacetime develops a null scalar singularity at the Cauchy Horizon (CH) whose subsequent evolution eventually stops at the final spacelike singularity at $r = 0$ [3].

In particular the Petrov type D component Ψ_2 of the Weyl curvature diverges exponentially with advanced time at this lightlike hypersurface, although the “measured” tidal distortion is bounded. The metric tensor is regular in a suitable local chart adapted to the inner horizon and the metric perturbations are small. This scenario is likely to be essentially the same in more general contexts than spherical symmetry [4], but the structure of the singularity is more complicated then since the square of the Weyl tensor $C_{\mu\nu\tau\lambda}C^{\mu\nu\tau\lambda}$ is dominated by the radiative component $\Psi_0\Psi_4$ of a Petrov type N curvature [5].

It is an interesting question whether quantum effects can modify the classical evolution of the fields in a significant way. As far as the classical evolution is concerned, causality does not permit our ignorance about the correct form of the dynamics in the inner, Planckian curvature regions of the interior to infect the description of the overlaying layers in terms of classical general relativity. The radial coordinate r is in fact timelike in the interior of a spherical hole. This picture changes in quantum field theory because in loop calculations even states localized outside the light-cone have an impact on the value of the renormalized quantities. One can then imagine that the metric fluctuations near the inner horizon modify the infrared region where the Weyl curvature is still growing but it has not yet reached Planckian levels. In particular it is interesting to see if the presence of some “self-regulator” mechanism could prevent the local curvature from diverging at the CH. An indication has been given in [6] where it has been noticed that the classical divergence of the mass function in an evaporating BH can be damped out by the contribution of the blueshifted influx of the Hawking radiation at late advanced times. More complete investigations in four dimensions [7] have been performed in the semiclassical approximation by considering a massless minimally coupled scalar field, but they were inconclusive about the “sign” of the

quantum correction, *i.e.* about whether it would lead to a stronger or to a weaker divergence.

The objective of this letter is to show that quantum fluctuations of the gravitational field indeed weaken the strength of the singularity at the inner horizon. This result is obtained by studying the running of the Newton constant at large momenta by means of the non-perturbative renormalization-group equation [8–10] which governs the scale dependence of the effective average action Γ_k for gravity [11]. Γ_k is a Wilson-type effective action with a built-in infrared (IR) cut-off at the mass scale k . The functional Γ_k is obtained by integrating out the quantum fluctuations with momenta between a fixed ultraviolet (UV) cutoff k_{UV} and the variable IR cutoff k . In this framework, a renormalizable theory with the classical action S is quantized by solving the flow equation subject to the initial condition $\Gamma_{k_{UV}} = S$ and letting then $k_{UV} \rightarrow \infty$, $k \rightarrow 0$ (after suitable renormalizations). What makes the effective average action an ideal tool for studying quantum gravity is the fact that this method can also be used in order to renormalization group-evolve (coarse grain) the actions of non-renormalizable effective field theories. In this case one assumes that there is some fundamental theory which has been “partially quantized”, *i.e.* its quantum fluctuations with momenta from infinity down to a fixed scale k_{UV} have been integrated out already. This leads to an effective action S_{eff} which, when evaluated in tree approximation, correctly describes all phenomena with typical momenta of the order k_{UV} . If we are interested in processes at smaller momenta $k < k_{UV}$ we can construct a new effective action, appropriate for the lower scale, by setting $\Gamma_{k_{UV}} = S_{\text{eff}}$ and solving the flow equation for Γ_k with this initial condition. It is clear that for effective theories the limit $k_{UV} \rightarrow \infty$ should not be performed; hence the non-renormalizability of a theory does not pose any problems in this context.

In the following we shall consider Einstein gravity as an effective field theory and we identify the standard Einstein-Hilbert action with $\Gamma_{k_{\text{obs}}}$. Here k_{obs} is some typical “observational scale” at which the classical tests of general relativity have confirmed the Einstein-Hilbert action. We assume that also for $k > k_{\text{obs}}$, *i.e.* at higher energies, Γ_k is well approximated by an action of the Einstein-Hilbert form as long as k is not too close to the Planck scale. The

two parameters in this action, Newton's constant and the cosmological constant, will depend on k , however, and the flow equation will tell us how the running Newton constant $G(k)$ and the running cosmological constant $\Lambda(k)$ depend on the cutoff. Their experimentally observed values are $G(k_{\text{obs}}) = G_{\text{obs}}$ and $\Lambda(k_{\text{obs}}) = \Lambda_{\text{obs}} \simeq 0$. The Newton constant defines the Planck mass according to $m_{\text{p}} = 1/\sqrt{32\pi G_{\text{obs}}}$. We fix a scale $k_{\text{UV}} \gg k_{\text{obs}}$ in such a way that it is still sufficiently below the Planck scale so that Γ_k is not yet very different from the Einstein-Hilbert action, but is already large enough for quantum gravitational effects to play a role. We start the renormalization group evolution at the scale k_{UV} with bare parameters $G(k_{\text{UV}}) = \bar{G}$ and $\Lambda(k_{\text{UV}}) = \bar{\Lambda}$ which should be thought of as functions of G_{obs} and Λ_{obs} . We shall use our result for the function $G(k), k \in [k_{\text{obs}}, k_{\text{UV}}]$, in order to study the impact of the scale dependent Newton constant on the mass-inflation scenario. In a sense we shall “renormalization group improve” the classical metric describing the late-time behavior of the spacetime near the CH.

Considering a spherically symmetric, charged BH the metric can be conveniently expressed using the coordinates x^a ($a, b = 0, 1$) on the radial two-spaces $(\theta, \phi) = \text{const}$ and the function $r(x^a)$ that measures the area of those two-spheres whose line element is $r^2 d\Omega^2$. The metric element is then $ds^2 = g_{ab} dx^a dx^b + r^2 d\Omega^2$. By defining the scalar fields $f(x^a), m(x^a)$ and $-2\kappa(x^a) = \partial f / \partial r$ through $f = 1 - 2G_{\text{obs}}m/r + G_{\text{obs}}e^2/r^2$ the Einstein equations reduce to the two-dimensionally covariant equations

$$\begin{aligned} r_{;ab} + \kappa g_{ab} &= -4\pi G_{\text{obs}} r (T_{ab} - g_{ab} T) \\ R - 2\partial_r \kappa &= 8\pi G_{\text{obs}} (T - 2P) \end{aligned} \tag{1}$$

where the static electromagnetic field is generated by a charge of strength e and T_{ab} is the stress-energy tensor of the matter field whose two-dimensional trace is T and tangential pressure is P . From the conservation laws one finds the following two-dimensional wave equation for the mass function

$$\square m = -16\pi^2 r^3 G_{\text{obs}} T_{ab} T^{ab} + 8\pi G_{\text{obs}} f (P - T)$$

$$+4\pi r^2 G_{\text{obs}} \kappa T - 4\pi r^2 G_{\text{obs}} r_{,a} T^{,a}. \quad (2)$$

This latter equation is the key to understanding the phenomenon of the mass-inflation. The late time behavior of the external gravitational field produced during the collapse of a star is that of a (Kerr-Newman, in general) black hole of external mass m_0 perturbed by a tail of gravitational waves whose flux decays as $\sim v^{-p}$ with $p = 4(l + 1)$ for a multipole of order l . As a consequence of the boundary conditions set at the event horizon, the $T_{ab}T^{ab}$ interaction term between the influx and outflux of gravitational waves scattered from the inner potential barrier triggers a divergent source term for the local mass function $m(u, v)$. The outflow can be modelled as a radial stream of lightlike material particles because of the infinite blueshift near the Cauchy Horizon. It is possible to show [1] that near the CH $m(v, r) \sim v^{-p} e^{\kappa_0 v}$ ($v \rightarrow \infty$), where v is the standard advanced time Eddington-Finkelstein coordinate. (κ_0 denotes the surface gravity of the Reissner-Nordström static black hole that characterizes the external field configuration.)

It must be observed that in Eq.(2) the strength of the gravitational interactions between outflux and influx is proportional to the Newton constant. Hence small changes in G due to renormalization effects are then exponentially amplified by the mass function like in a magnifying lens! In particular if gravity is asymptotically free the classical divergence of the mass function can be weakened by the decreasing of the Newton constant at small distances. In order to discuss this phenomenon in the mass-inflation scenario we consider the model analyzed in [3] for the scalar field collapse although our result should not depend on this particular framework. We are interested in the asymptotic portion of the spacetime at late retarded times (the “corner” region near the H point in Fig.(1) of [3]) before the strong focussing region where $r \rightarrow 0$. The null Kruskal coordinates U, V are thus introduced, being $\kappa_0 U = -\exp(\kappa_0 u)$, $\kappa_0 V = -\exp(\kappa_0 v)$ were (u, v) are the retarded and advanced time coordinates. In a neighborhood of $(U = -\infty, V = 0)$ an approximate analytical solution of the Einstein equations and the wave equation for a massless minimally coupled scalar field Φ can be found [3]. The explicit asymptotic expression for the metric is

$$ds^2 = -2\frac{r_0}{r}dUdV + r^2d\Omega^2 \quad (3)$$

$$r^2 = r_0^2 - 2G_{\text{obs}}[A(U) + B(V)] \quad (4)$$

where r_0 is the location of the CH in the static BH spacetime configuration. The dimensionless functions $A(U)$ and $B(V)$ are regular at the CH, $A(-\infty) = B(0) = 0$, but \dot{B} diverges like $1/V(-\ln(-\kappa_0 V))^{(p+2)}$ as $V \rightarrow 0^-$ while A is positive definite and \dot{A} is bounded. Even though the metric coefficients and the scalar field Φ are both regular at the CH, the mass function is divergent for $V \rightarrow 0^-$ being

$$m(U, V) \simeq \frac{G_{\text{obs}}}{r_0} \dot{A} \dot{B} \quad (5)$$

We consider the evolution of the above geometry in the mass-inflating regime starting from a value of the coordinate $V = V_{\text{IR}}$ for which the mass function is already exponentially growing $m(U, V)/m_0 \gg 1$ (we assume $r_0^{-1} \sim m_0 \gg m_p$) but the curvature has not yet reached Planckian values.

At this point we need an explicit expression for the running Newton constant. We use the result obtained in [11] where, for pure gravity, the evolution of Γ_k has been obtained in the ‘‘Einstein-Hilbert approximation’’ where only the $\sqrt{-g}$ and $\sqrt{-g}R$ operators are considered in the renormalization group flow. For this truncation the flow equation reads

$$\begin{aligned} k\partial_k \Gamma_k[g, \bar{g}] &= \frac{1}{2} \text{Tr} \left[(\Gamma_k^{(2)}[g, \bar{g}] + \mathcal{R}_k^{\text{grav}}[\bar{g}])^{-1} k\partial_k \mathcal{R}_k^{\text{grav}}[\bar{g}] \right] \\ &- \text{Tr} \left[(-\mathcal{M}[g, \bar{g}] + \mathcal{R}_k^{\text{gh}}[\bar{g}])^{-1} k\partial_k \mathcal{R}_k^{\text{gh}}[\bar{g}] \right] \end{aligned} \quad (6)$$

where $\bar{g}_{\mu\nu}$, $\Gamma_k^{(2)}$ and \mathcal{M} denote the background metric, the Hessian of Γ_k with respect to the ‘‘ordinary’’ metric argument $g_{\mu\nu}$, and the Faddeev-Popov ghost operator, respectively. The operators $\mathcal{R}_k^{\text{grav}}$ and $\mathcal{R}_k^{\text{gh}}$ are the IR cutoffs in the graviton and the ghost sector, respectively. They are defined in terms of an arbitrary smooth function $\mathcal{R}_k(p^2)$ (interpolating between zero for $p^2 \rightarrow \infty$ and a constant $\propto k^2$ at $p^2 = 0$) by replacing p^2 with the graviton and ghost kinetic operator, respectively. Inside loops, they suppress the contribution from modes with covariant momenta $p < k$. Upon projecting the renormalization group flow on a two

dimensional space spanned by the operators $\sqrt{-g}$ and $\sqrt{-g}R$ the functional flow equation becomes two ordinary differential equations for $G(k)$ and $\Lambda(k)$. The equation for the scale-derivative of the running dimensionless Newton constant $g(k) = k^2 G(k)$ is found to be $k\partial_k g(k) = [2 + \eta(k)]g(k)$ where $\eta(k) \equiv gB_1/(1 - gB_2)$ is an anomalous dimension involving two known functions [11] of the cosmological constant, B_1 and B_2 , which depend on the choice for $\mathcal{R}_k(p^2)$. The beta-function describing the running of the Newton constant is not universal. In our framework this is reflected by the \mathcal{R}_k -dependence of η . To lowest order of an expansion in powers of k/m_p one may ignore the impact of the running cosmological constant on $\eta(k)$ and set $\Lambda(k) \simeq 0$. Thus, returning to physical units and retaining only the leading term of the k/m_p -expansion the solution of the flow equation reads

$$G(k) = G_{\text{obs}}[1 - \omega G_{\text{obs}} k^2 + O(k^4/m_p^4)]. \quad (7)$$

For pure gravity one obtains $\omega = \omega_G \equiv -B_1(\Lambda = 0)/2 > 0$ which assumes the numerical value $\omega_G = 4(1 - \pi/144)/\pi$ for a standard exponential cutoff [11]. While ω depends on the shape of \mathcal{R}_k , it can be shown that ω is positive for any choice of this function. Consequently pure gravity is “antiscreening”: Newton’s constant decreases as k increases, *i.e.* it is large in the IR and becomes smaller in the UV. Eq.(7) is believed to be reliable as long as k_{UV} is still below the Planck mass. If $k < k_{\text{UV}} \ll m_p$ then the effect of the higher curvature invariants such as R^2 or $R_{\mu\nu}R^{\mu\nu}$ which were omitted from our ansatz is small since these invariants are suppressed by additional powers of k/m_p . We shall assume that $k_{\text{UV}} = m_p/a$ with a a fixed number well above unity.

It is straightforward to include matter fields. In our model it might appear natural to keep the electromagnetic field classical but quantize the full scalar field. The only effect on the running of G is to shift the parameter ω . Using the same cut-off as above, one finds [12] $\omega = \omega_{GS} = 4/\pi - 3\pi/72$ which, again, is positive and leads to the same qualitative features as pure gravity.

The running of G has dramatic consequences for the mass-inflation scenario. The leading quantum correction of the metric is obtained by replacing G_{obs} in eq.(5) for the mass function

by the running Newton constant $G(k)$ with an appropriately chosen scale k . Since $G(k) < G_{\text{obs}}$ for any value of $k > k_{\text{obs}}$ we conclude that the quantum corrections tend to weaken the divergence of the mass function. It is not difficult to implement this idea in a self-consistent calculation. For the sake of simplicity let us consider the simpler case of the cross-flow model discussed in [3]. We define a position dependent IR cutoff by $k^2(V) = \max_U \{b^2 |\Psi_2|\} = \max_U \{b^2 G_{\text{obs}} m(U, V) / r_0^3\}$ with b another fixed number much larger than unity and the maximum performed over the region near $U \rightarrow -\infty$. Here we are invoking a kind of adiabatic approximation where the use of a position dependent cutoff is justified because the mass function $m(U, V)$ is almost constant on the length scales at which the eigenmodes integrated out are varying. (A similar approximation has already been used in [7] in a semiclassical calculation.) From $k(V)$ one obtains a running Newton constant as a function of the V coordinate

$$G(V) = G_{\text{obs}} [1 - \mathcal{A} \dot{B}(V)] \quad (8)$$

where $\mathcal{A} = \omega b^2 \max_U \{G_{\text{obs}}^3 \dot{A}(U) / r_0^4\}$. It is then possible to evolve the classical geometry in eq.(3) by considering the running Newton's constant in the Einstein equations. Within our approximation, the improvement amounts to replacing $G_{\text{obs}} T_{ab} = G_{\text{obs}} T_{ab}^{\text{in}}(V) + G_{\text{obs}} T_{ab}^{\text{out}}(U)$ with $T_{ab}^{\text{imp}} = G(V) T_{ab}^{\text{in}}(V) + G_{\text{obs}} T_{ab}^{\text{out}}(U)$. This modified stress-energy tensor is then covariantly conserved since it satisfies $(T_{ab}^{\text{imp}} r^2)^{;b} = 0$.

From the Bianchi identities one obtains the following wave equation for the mass function

$$\square(Gm) = -16\pi^2 r^3 G^2 T_{ab} T^{ab} + e^2 \left(\frac{G_{,a}}{2r}\right)^{;a}. \quad (9)$$

The general solution is uniquely determined once the value of the fields along the characteristic $U = U_{\text{IR}}$ and $V = V_{\text{IR}}$ are given. Asymptotically the improved metric is still of the form (3) but with r^2 replaced by $r_0^2 - 2G(V)B(V) - 2G_{\text{obs}}A(U)$. By noticing that G approaches its bare value very rapidly one finds that the leading term (as $V \rightarrow 0^-$) on the right hand side is given by the classically divergent $T_{ab} T^{ab}$ contribution. Thus, near the CH, one obtains

$$m(U, V) \simeq \frac{G_{\text{obs}}}{r_0} (1 - \mathcal{A}\dot{B})\dot{A}\dot{B} - \frac{e^2 G_{\text{obs}}}{4r_0^3} \mathcal{A}\dot{B}A \quad (10)$$

which confirms our result when the back-reaction of the metric is taken into account.

A priori it might have appeared equally natural to perform the substitution $G_{\text{obs}} \rightarrow G(k)$ directly in the Einstein equations rather than in its solution but then it would have been problematic to identify k with some geometrical quantity. However it is important to observe that in this case a decreasing G leads to the additional effect of lowering the value of the surface gravity at the inner horizon which, too, renders the mass function less divergent. It should also be stressed that the above results were obtained by integrating out only the field modes with momenta between $b\sqrt{|\Psi_2|}$ and m_p/a . While lowering the IR cutoff even further is difficult from the technical point of view (the adiabatic approximation is not available any longer) the monotonicity of the function in eq.(7) suggests that taking additional modes into account will lead to an even stronger damping of the classical divergence. On the other hand, by adding further matter fields the antiscreening nature of the gravitational interaction could be destroyed in principle. (In ref. [13] a condition on the number of the various species of fields implying $\omega > 0$ can be read off.) We nevertheless believe that the qualitative features of our discussion will hold for arbitrary matter systems with $\omega > 0$. In particular for the matter system consisting of a massless minimally coupled scalar field considered in this investigation the quantum-corrected geometry is less singular than its classical counterpart. One is then lead to speculate that if $G \rightarrow 0$ at the CH the geometry of the spacetime near the late-time portion of the CH is regular with a sub-planckian Weyl curvature and that this is a general feature that would appear in a more general context than what we have discussed. The further evolution down to the stronger $r = 0$ singularity could in principle be followed with similar means and we hope to address this problem in the future.

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