Entropy Production in the Early Universe
A New Mechanism

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Abstract

We expose briefly the role of entropy in the early universe and in particular the importance of searching for new mechanisms of entropy production. We describe a mechanism [1] that shows how entropy is produced during early annihilations and under which conditions the production is not negligible. A MeV τ neutrino provides an interesting application.

This paper has been presented in the Nato School Session on "Generation of Cosmological Large-Scale Structure" (Erice, November 1996) and it will appear in the proceedings of the school (ed. D.N.Schramm).

1. Introduction

Entropy is not a fundamental quantity as e.g. energy density, but it provides an auxiliary function useful to describe the physical processes in the early universe. This is evident when thermodynamical equilibrium applies (quasi-static expansion) because entropy is conserved and thus it represents a *constant of motion*. This is a traditional result [2] that can be shown easily by applying to the cosmological expansion the usual thermodynamical relation $d(\rho R^3)/dt = TdS/dt - pdR^3/dt + \sum_i \mu_i dN_i/dt$ and imposing the energy-momentum tensor conservation equation $d(\rho R^3)/dt = -pdR^3/dt$. Then necessarily:

$$\frac{dS}{dt} = -\frac{1}{T} \sum_i \mu_i \frac{dN_i}{dt} \tag{1}$$

But as we are assuming the equilibrium also in the chemical reactions, the right sum is zero and hence $dS/dt = 0$. Entropy conservation allows for the straightforward derivation of the $R - T$ relation. If all species are in radiative equilibrium then $S \propto g_R(TR)^3$, where $g_R = const$ are the *effective degrees of freedom*, and simply $TR = const$, otherwise, the definition of g_R is generalized to the case in which some species are not ultrarelativistic by introducing a function $g_S(T)$ that can be easily calculated [3] and in this case $RT \propto [g_S(T)]^{-1/3}$. The RT factor is important because the particle numbers of any species in radiative equilibrium, and in particular the photon number N_γ, is $\propto (RT)^3$. Thus entropy conservation provides the way to know how the photon number changed during the early universe history. During the annihilations of some particle species the degrees of freedom decrease and photons are produced because the disappearing species releases its entropy to the radiative plasma including photons. Sometimes in the literature this is called entropy production while it is only entropy exchange. Photon production is an important quantity to be known because it is needed to calculate the relic abundances $(n/n_\gamma)_0$ of particle species or of the barionic number. In fact if at some time t_f a particle or charge number N_f is frozen, we are often able to calculate the abundance $(n/n_\gamma)_f$ at that time and then the relative relic abundance can be expressed as $(n/n_\gamma)_0 = (n/n_\gamma)_f (N_\gamma^f/N_\gamma^0)$. This expression shows how photon production dilutes the abundance from freezing until the present. For this reason it is convenient to define a *dilution factor* $f \equiv N_\gamma^0/N_\gamma^f$. Now it is easy to understand the effect of an entropy production: it gives a further contribution to photon production and hence

to f: $f = f_g \cdot f_S$ (where $f_g = g_S^f / g_{S0}$ and $f_S = S_0 / S_f$). We point out that the more recent the entropy production is the more effective it is, because it dilutes all densities previously frozen.

2. Entropy production calculation through the L.W. equation

Let us consider the classical problem of the freezing of particle abundances during annihilations. Some particle species h can annihilate eventually into photons through a lighter particle species l: $h + \bar{h} \longrightarrow l + \bar{l} \longrightarrow \ldots \longrightarrow \gamma$'s. Assuming that the elastic reactions rates are strong enough to support the kinetic equilibrium (thus a temperature is defined for the h and it is kept equal to photon temperature), the h distribution function will assume an equilibrium form: $f_h(\vec{p}, t) \simeq \tilde{f}(\vec{p}, t) = \{e^{\beta(t)[E_{\vec{p}} - \tilde{\mu}_h(t)]} + 1\}^{-1}$ where $\tilde{\mu}_h$ is commonly called the *pseudo-chemical* potential to underline that it does not obey, in general, the chemical equilibrium condition $\mu_h + \mu_{\bar{h}} = 0$. In this way the entropy production rate can be obtained easily from thermodynamics using (1), and in our specific case becomes:

$$\frac{dS}{dt} = -\frac{(\tilde{\mu}_h + \tilde{\mu}_{\bar{h}})}{T} \frac{dN_h}{dt} \tag{2}$$

This expression can be obtained from statistical mechanics as well. It can be noticed that in order to have $dS/dt \neq 0$ two conditions must be satisfied: the first one is departure from chemical equilibrium ($\tilde{\mu}_h + \tilde{\mu}_{\bar{h}} \neq 0$) and the second is a decreasing particle number ($dN_h/dt \neq 0$) which means the presence of annihilation processes not balanced by pair production processes. If we want to perform a quantitative calculation of the entropy production, we must use some model to calculate $\tilde{\mu}_h$ and dN_h/dt. The simplest way to describe the annihilations is represented by the special Lee-Weinberg equation:

$$\frac{d\bar{N}}{dy} = A_0 \left[\bar{N}^2(y) - \bar{N}_{eq}^2(y) \right] \tag{3}$$

($y = T/m$, $\bar{N} = N_h/N_{in}$, $N_{in} = N_h(T \gg m_h)$, $A_0 = 0.055(g_h/\sqrt{g_\rho^{in}})\sigma_0 m m_{PL}$). In the modern literature this equation is usually written using the variable $Y = n/s$ (s is the entropy density) where $Y \propto N$ if entropy is conserved. But as we are relaxing entropy conservation assumption we must use a variable like $\bar{N}$. In this equation particle-antiparticle symmetry is assumed.

3

This is not restrictive because it means only that the departure from equilibrium and the consequent freezing must happen for temperatures when $(N_h - N_{\bar{h}})/N_h \ll 1$. Otherwise it is well known [4] that departure from equilibrium would be greatly delayed and therefore entropy production would be negligible. Thus we can set $\tilde{\mu}_h = \tilde{\mu}_{\bar{h}} = \tilde{\mu}$ and the chemical equilibrium condition becomes $\tilde{\mu} = 0$. Moreover, it is an implicit approximation of the equation that $\tilde{z} \equiv \tilde{\mu}/m = y\ln[\bar{N}(y)/\bar{N}_{eq}(y)]$. In Fig. 1 we show the solutions $\bar{N}$ of Lee-Weinberg equation for different values of A_0 and the correspondent $\tilde{z}$.
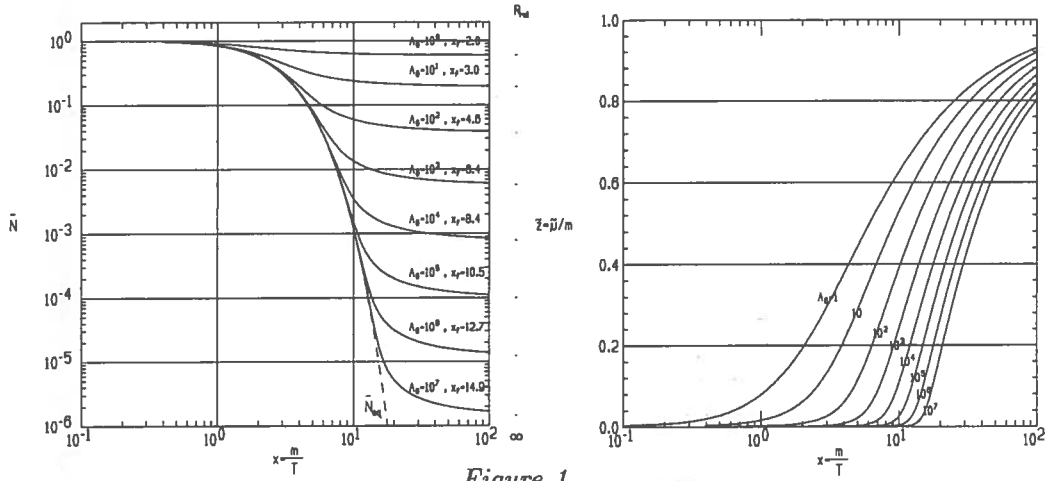

Figure 1.

Finally we can write the entropy production rate through $\bar{N}$ obtaining:

$$\frac{1}{S_h^{in}}\frac{dS(y)}{dy} = -\frac{135\zeta(3)}{4\pi^4}\frac{\tilde{z}(y)}{y}\frac{d\bar{N}(y)}{dy} \tag{4}$$

where $S_h^{in} = g_h S_{in}/g_S^{in}$. By integration one can obtain the entropy production $\Delta S(y, y_{in})$. The results are shown in Fig. 2.

The rates present a maximum because entropy production is zero before the system goes out of equilibrium (i.e. $\tilde{\mu} = 0$) and after it tends again to zero because the annihilation rate decreases ($dN/dt \to 0$, while $\tilde{\mu}$ reaches its asymptotical value m). Moreover, one can notice the existence of a value for A_0 for which entropy production is maximum. In fact both for $A_0 \to 0$ and for $A_0 \to \infty$ entropy production also tends to zero: in the first case because particles would not annihilate at all, though the system is out of equilibrium,

4

and in the second because the system never goes out of equilibrium. Thus there must exist a maximum. It is reached when $A_0 = 3.65$ and $x_f = 2.5$, where we defined x_f as that value for which $\bar{N} - \bar{N}_{eq} = 0.5 \bar{N}_{eq}$. The maximum production of entropy occurs for a freezing during which h-particles are semi-relativistic.
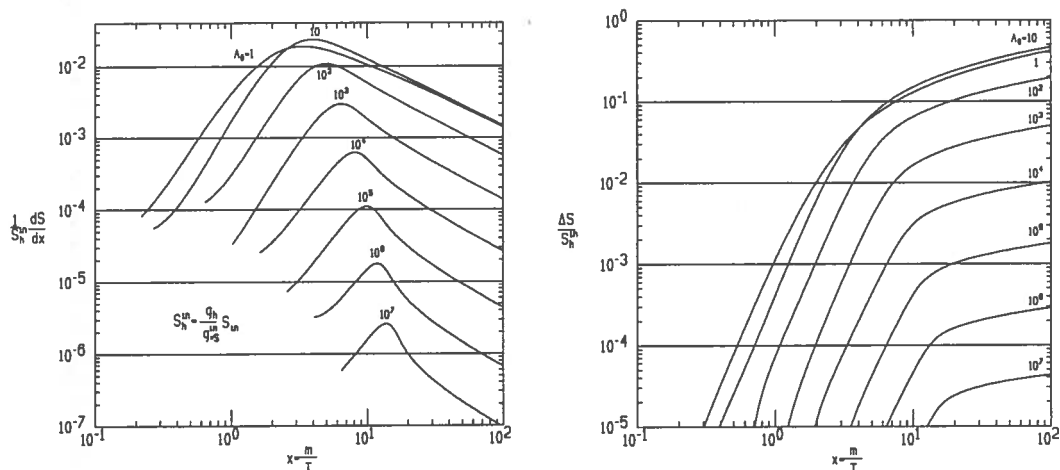

Figure 2.

In the *asymptotical limit* ($x = m/T \to \infty$) all expressions become simpler: $d\bar{N}(x)/dx \longrightarrow -A_0 \bar{N}_0^2/x^2$, $\tilde{z}(x) \longrightarrow 1$ and $dS(x)/dx \to \propto A_0 \bar{N}_0^2/x$: hence asymptotically the entropy production $\Delta S = \int dx (dS/dx)$ does not stop but it increases logarithmically, thanks to the relic annihilation.

3. Estimation of maximum entropy dilution factor

In order to estimate the maximum dilution that the mechanism can produce we must integrate the entropy production over the entire early universe history, until matter-radiation decoupling. Another consideration to be made is that during the early universe the degrees of freedom decrease, and this requires the generalization of the calculation of dN/dt done with the simple Lee-Weinberg equation which assumes degrees of freedom and entropy constant. At the same time we can also include the correction due to the variation of entropy. It is possible to introduce these modifications only in the asymptotical regime (which gives the major contribution to the entropy production). Hence during this phase the decoupled equations (3) and (4)

5

can be replaced by two coupled equations:

$$\frac{d\bar{S}}{dy} = -\frac{135}{4}\frac{\xi(3)}{\pi^4}\frac{g_h}{g_S^{in}}\frac{1}{y}\frac{d\bar{N}}{dy} \qquad \frac{d\bar{N}}{dy} = A_0 \cdot \frac{\bar{g}_{Si}}{\sqrt{\bar{g}_{\rho i}}}\frac{1}{\bar{S}(y)}\bar{N}_0^2 \qquad (5)$$

($\bar{S} = S/S_{in}$, $\bar{g}_{Si} = g_{Si}/g_S^{in}$, $\bar{g}_{\rho i} = g_{\rho i}/g_\rho^{in}$) that yield the solution:

$$\bar{S}_{dec} = \bar{S}_1 = \bar{S}_* \sqrt{1 + 2\frac{a}{\bar{S}_*^2}\frac{g_h}{g_S^{in}}\sum_{i=*,N,...,1}^{1}\frac{\bar{g}_{Si}}{\sqrt{\bar{g}_{\rho i}}}\ln\frac{x_{i-1}}{x_i}} \qquad a = \frac{135}{4}\frac{\xi(3)}{\pi^4}A_0\bar{N}_0^2 \quad (6)$$

If the second term in the square root is much smaller than 1 then, expanding at first order, the logarithmic behaviour is restored. Until now we supposed that all components are coupled to photons and therefore $f_S = \bar{S}_{dec}$. If there is a decoupled component it must be considered that some fraction of the entropy can be given to the decoupled component and thus does not contribute to the dilution.

Now we must find the maximum value that the expression (6) can assume. It depends on two parameters: A_0 and T_{in} (g_h can be set equal 2). We have already indicated that $A_0 = 3.65$ yields the maximum value for $A_0\bar{N}_0^2$. To find the appropriate T_{in} there are two opposite considerations: to get the greatest temperature range requires annihilations as early as possible, but to have the greatest fractional weight of h-particles ($\equiv g_h/g_S^{in}$) requires annihilations that happen when as few particle species as possible have survived. The second consideration suggests to take a time interval such that g_S^{in} is minimum, and the first suggests the insertion of annihilations as early as possible inside this interval. Thus the best situation is when freezing starts soon after electron-positron annihilations so that $m \lesssim m_e/2 \simeq 0.25MeV$ and from (6) $f_S = 1.43$ is obtained. The second possibility is that freezing starts soon after muon annihilations ($m_h \lesssim m_\mu/2 \simeq 50MeV$) and in this case $f_S = 1.31$ is obtained.

4. Application: Mev τ-neutrino

In this section we want to consider a realistic case that produces a dilution factor as much as possible close to the order of magnitude found earlier. This time we must substitute $\sigma_0 = const$ with the thermally averaged cross section $\langle\sigma_{ann}v_{Mo}\rangle(y)$, that can be calculated with a single-integral formula [5] valid for any value of x_f. Unfortunately it is not realistic an hundred Kev particle

that undergo the semirelativistic freezing ($x_f \simeq 2$) we need, though further investigations are in progress considering Majoron or Axino. An Mev τ-neutrino is a particle that would freeze with the right x_f and it is light enough to have a good fractional weight. This then seems to be the best application of the mechanism we described and also because of the possible consequences that it could have on its mass constraints. In last six years constraints from nucleosynthesis [6, 7], SN1987A [8], have been greatly improved, reducing the possibilities for an MeV neutrino from an astrophysical point of view. At the same time the laboratory experiment lower limit has been lowered to 24 MeV [9] and thus in future years there could be an important test for astroparticle physics. This is why we think that it is worthwhile to continue investigations on MeV τ neutrino. Here, however, we want mainly to provide an application for the general mechanism we described.
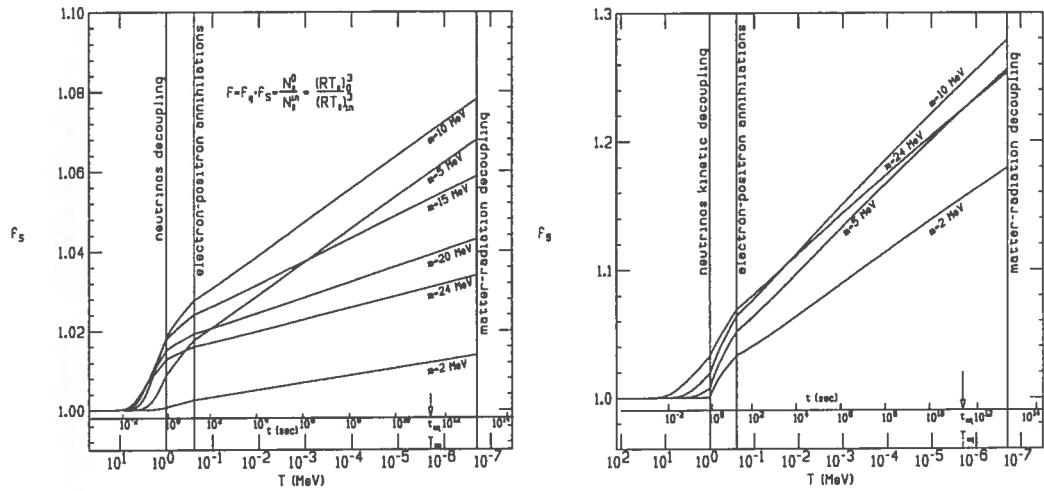

Figure 3. Entropy dilution factor: MeV Dirac ν_τ (left); MeV Giudice ν_τ (right).

We performed the calculations for a Dirac neutrino and for a neutrino with a magnetic moment $\mu \gtrsim 2 \cdot 10^{-9}(m_\nu/10MeV)\mu_B$ [10]. In the first case it must be considered that ν_τ can annihilate into e^+e^-, $\nu_\mu\bar{\nu}_\mu$, $\nu_e\bar{\nu}_e$ and that for $T \lesssim 1MeV$ the neutrinos are decoupled: this reduces of a factor $\simeq 1/3$ the dilution factor from the maximum and the result is $f_S = 1.01 \div 1.08$ for masses $m = 2 \div 24MeV$ (see Fig.3 left). In the second case the annihilation through photon exchange in e^+e^- is dominant. In this case the factor 1/3 is

avoided and the dilution factor can approach the maximum value permitted ($f_S = 1.1 \div 1.3$ for masses $m = 2 \div 24 MeV$ as shown in Fig.3 right).

5. Conclusions

In the introduction we clarified the difference between the simple entropy transfer from the annihilating particle species to the radiative plasma and an effective entropy production. We presented here the possibility to have a significant entropy production in the late phase of the early universe if the annihilating particle species undergoes a semirelativistic freezing. This mechanism can dilute all relic abundances in the early universe by a factor 1.5 as maximum value. In the last section we presented an application that realizes this possibility: an MeV ν_τ. In the general cases of nonrelativistic and ultrarelativistic freezing, entropy production is a negligible effect as usually assumed.

8

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