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# Non Standard Electroweak Contributions to the muon g-2 from Anomalous Gauge Boson Couplings

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**Abstract**

Anomalous trilinear couplings between the gauge bosons produce virtual effects on the muon anomalous magnetic moment. The size of the contributions arising from the most general dimension six Lagrangian, preserving the Minimal Standard Model gauge symmetry and even under C and P, is evaluated and compared with the precision in the $g-2$ determination provided by the expected future direct measurements and improved theoretical evaluation of the hadronic contributions.

# 1 Introduction

The anomalous magnetic moments of the light charged leptons are measured with extreme precision. The discrepancy ($a_\mu = \frac{g-2}{2}$) between the value of the gyromagnetic ratio dictated by the Dirac equation for a fermion in an electromagnetic field ($g = 2$) and the experimental one arises from the interaction of the particles with virtual fields. Therefore, $a_\mu^{\text{leptons}}$ has represented a powerful testing ground of the quantum field theories, describing fundamental fields and their interactions, and can still effectively constrain models of new physics.

In this respect, the muon anomaly is a more powerful tool than the electron one, since the relevant virtual effects are, in general, proportional to the squared mass of the lepton.

A forthcoming experiment [1] plans to improve the current precision on the measurement of the muon $g-2$ by more than a factor 20 which entails a purely experimental uncertainty of 0.35ppm. As a consequence of this large improvement, the dominant error (of approximately 1.5ppm) in the value of the muon $g-2$ would be of theoretical origin, in particular it would be related to the evaluation of the hadronic contribution to $a_\mu$.

The opportunity of measuring with high statistics and, hopefully, low systematic error, the total hadronic cross section in the process of $e^+e^-$ annihilation from threshold to 1.4 GeV at DA$\Phi$NE [2, 3] would allow to lower significantly this theoretical error.

The resulting overall increased precision is the reason for considering the sensitivity of the muon anomaly to new physics. In this note the contributions to $a_\mu$ from anomalous gauge boson couplings (AGC) have been studied in the framework of a gauge invariant effective Lagrangian [4, 5], which enriches the Standard Model by terms accounting, at low energy, for the residual effects of the underlying physics related to an energy scale $\Lambda$ higher than the electroweak scale. The large contribution of the standard $WW\gamma$ vertex to the $g-2$ (roughly double in size with respect to the total e.w. effect) suggests, in fact, that sizable effects could arise from new kinds of $WW\gamma$ interactions. Actually, previous attempts to evaluate these anomalous contributions to $a_\mu$ [6, 7, 8] led to potentially appreciable contributions. In general, though, different results were obtained working in the unitary gauge when the computational scheme was varied. The aim of this note is that of proposing a different evaluation performed in the Feynmann 't Hooft $R_\xi$ gauge using the dimensional regularization approach proposed by 't Hooft and Veltmann.

After a summary of the status of the experimental knowledge of the $g-2$ and of the theoretical expectations (section 2), with special care to the contributions of the electroweak sector (section 3), a brief description of a general Lagrangian for $WW\gamma$ interactions will be done in section 4. Then the contributions to $a_\mu$ from the anomalous interaction will be presented (section 5), together with some details of the calculation. Finally, the bounds on the anomalous couplings which can be derived by the future measurement of the $g-2$ will be discussed.

# 2 Measurements and theoretical estimates

The most recent and precise measurement of the muon anomaly [9] produced the result

$$a_\mu^{\text{exp}} = 1\,165\,923(8.5) \times 10^{-9} \tag{1}$$

as an average of $a_{\mu^-}$ and $a_{\mu^+}$. The goal of the next experiment, which is going to measure the muon $g-2$ at the AGS of Brookhaven, is to lower the error below $0.4 \cdot 10^{-9}$, allowing to observe the electroweak contribution with a 30% accuracy.

The theoretical prediction, which is in excellent agreement with the experimental value, comes from the computation of contributions from QED, hadronic and electroweak virtual processes. The QED contribution accounts for about 99.99% of the total value of $a_\mu$. It is known analytically up to the order $\alpha^3$ [10] and numerical evaluations are available for the orders $\alpha^4$ and $\alpha^5$. The precision of the prediction is now depending on the accuracy in the experimental determination of the coupling constant $\alpha$. With the value of $\alpha$ measured in the quantum Hall effect the QED contribution is

$$a_\mu^{\mathrm{QED}} = 1\,165\,847\,070(18)(28) \times 10^{-12} \tag{2}$$

where the total error (accounting, respectively, for theoretical uncertainties and experimental error on $\alpha$) is well below the future precision on $a_\mu^{\mathrm{exp}}$.

The electroweak contribution, known at one loop since 1972 [11, 12] and recently computed at two loops [13], is smaller than the present error

$$a_\mu^{\mathrm{EW}} = 1.54(0.04) \times 10^{-9} \tag{3}$$

Finally, the hadronic contribution can be split into three kinds of effects: the vacuum polarization of the photon, which represents the leading term and the second order contributions involving either iterations of the photon vacuum polarization or the light by light scattering process.

The leading contribution is usually calculated by means of the measured total hadronic cross section in $e^+e^-$ annihilation $\sigma(q^2)$ through the dispersion relation [14]

$$a_\mu^{\mathrm{hadr(1\,loop)}} = \frac{1}{4\pi^2\alpha}\int_{4m_\pi^2}^{\infty} \mathrm{d}s K(s)\sigma(s) \tag{4}$$

where the kernel function $K(s)$ is given by the following expression:

$$K(s) = \frac{\alpha}{\pi}\int_0^1 \mathrm{d}x \frac{x^2(1-x)}{x^2 + s(1-x)/m_\mu^2}. \tag{5}$$

This approach to the calculation allows to overcome the difficulties related to the computation of QCD effects in a low energy regime which would lead to model dependent results. The unavoidable drawback is an error on the theoretical forecast of experimental nature coming from the error on the measurement of the cross section, which at the moment is the main contribution to the error on $a_\mu^{\mathrm{th}}$. In fact, recent analysis of the $e^+e^- \to$ hadrons data [15] produced the result

$$a_\mu^{\mathrm{hadr(1\,loop)}} = 70.23(0.58)(1.41) \times 10^{-9} \tag{6}$$

where most of the error (statistical and systematic, respectively) comes from the energy region between threshold for hadron production up to 1.4 GeV that is responsible for 99% of the statistical error and 85% of the systematic error as a consequence of the steep increase of the kernel function at low energies.

The other hadronic contributions are higher order corrections and, even though the light by light scattering contribution, which requires a computation in some effective model, is still somehow controversial [1], it is generally assumed that they introduce a small total uncertainty [16, 17, 18].

[1]Two recent evaluations of the light by light contribution, coming from different authors, are $-0.92(0.32)\times10^{-9}$ [16] and $-0.52(0.18)\times10^{-9}$ [17]. The remaining two loop hadronic corretions contribute the $g-2$ with $-0.90(0.05) \times 10^{-9}$ [18].

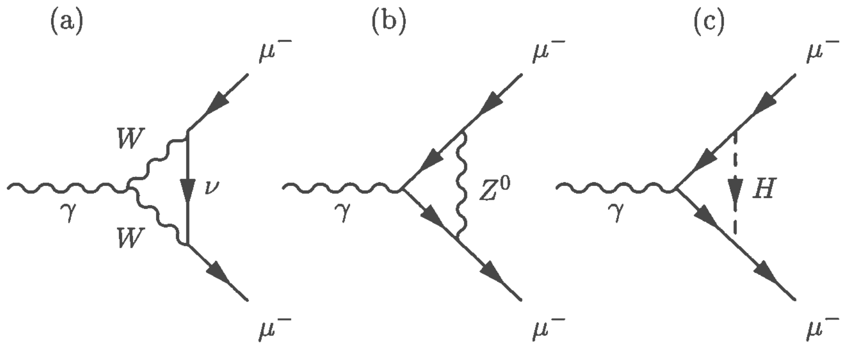


Figure 1: Electro-weak processes contributing the muon anomaly at one loop.

If one uses for the hadronic contributions the results in [15, 17, 18] the total theoretical prediction for $a_\mu$ is

$$a_\mu^{\text{th}} = 1165\,917.42(1.54) \times 10^{-9} \tag{7}$$

where the error, which consists, essentially, of the uncertainty coming from the hadronic cross section, is higher than the future experimental accuracy and comparable with the electroweak effects.

# 3 The standard Electroweak contributions at one loop

In the Minimal Standard Model (MSM) of electroweak interactions the processes contributing the anomalous magnetic moment of the muon at one loop are the ones shown in fig. 1. The graph with the exchanged Higgs boson is suppressed with respect to the others by a factor $m_\mu^2$ and, hence, it will be neglected in the following. The processes in fig. 1a and 1b respectively produce the following contributions:

$$a_\mu^W = \left(\frac{g-2}{2}\right)^W = \frac{G_F}{8\sqrt{2}\pi^2}\, m_\mu^2\, \frac{10}{3} \tag{8}$$

$$a_\mu^Z = \left(\frac{g-2}{2}\right)^Z = \frac{G_F}{8\sqrt{2}\pi^2}\, m_\mu^2\, \frac{4}{3}\,(g_V^2 - 5g_A^2) = \frac{G_F}{8\sqrt{2}\pi^2}\, m_\mu^2\, \frac{1}{3}\left[(3-4c^2)^2 - 5\right] \tag{9}$$

in the normalization $\mid g_A \mid = \frac{1}{2}$. The total effect gives a numerical value of

$$\left(\frac{g-2}{2}\right)_{EW}^{1\ \text{loop}} = 1.95 \cdot 10^{-9}. \tag{10}$$

The high level of agreement between the theoretical expectations and the experimental yields concerning the couplings of the gauge bosons to the fermions assures that no room is left to non standard $Z^0 - \mu$ coupling effects in the $Z^0$ exchange process. Alternatively, one might advocate a heavy partner ($Z^{0'}$) of the $Z^0$, but its mass is already strongly constrained by present data, for reasonable values of its couplings [19], and, therefore, it couldn't produce any sizable effect on the muon $g-2$[2].

On the other hand, the MSM formal expression of the $WW\gamma$ vertex which appears in fig. 1a has not undergone, until now, experimental tests and only loose bounds come from the analysis of virtual effects in precision electroweak measurements [21, 22] and from

[2] A much better sensitivity to this kind of new physics is expected at LEP2[20].

present data of direct boson pair production [23]. Therefore, one could ask which kind of effect on the observable $g-2$ would arise from a more general structure of the $WW\gamma$ interaction.

## 4 An effective Lagrangian for WW$\gamma$ interactions

The general Lorentz invariant Lagrangian for the interaction of two charged vector bosons with a neutral vector boson consists, in the most general case, of seven terms related to the helicity states allowed to the pair $W^+W^-$ when coupled to a state of spin one [4]. The usual choice of extending the requirement coming from electric charge conservation at generic $q^2$ and of preserving the C and P symmetry of the photon couplings reduces the number of possible $WW\gamma$ and $WWZ$ couplings from 14 to 9. Then, the total number of free couplings can be reduced in different ways by assuming further symmetry properties of the theory [4, 5].

In the following we will adopt the framework depicted in [5], where the local $SU(2)_W \times U(1)_Y$ invariance of the MSM is fully preserved together with C and P symmetry. Under these hypotheses, the number of free anomalous couplings becomes two, for $WW\gamma$, and three, for $WWZ$ [5]

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{WWV} = ig_{WWV} \quad [ \quad & g_{1V}(W^+_{\mu\nu}W^{-\mu} - W^{+\mu}W^-_{\mu\nu})V^\nu \\ + \quad & k_V W^+_\mu W^-_\nu V^{\mu\nu} \\ + \quad & \frac{\lambda_V}{M_W^2} W^{+\nu}{}_\mu W^{-\rho}{}_\nu V^\mu_\rho \quad ]\end{aligned} \tag{11}$$

where $g_{WW\gamma} = -e$ and $g_{WWZ} = -e\cot\theta_W$ ($\theta_W$ is the Weinberg-Salam angle). While $g_{1\gamma}$ cannot assume values different from one because of e.m. gauge invariance, the other couplings (which in the MSM have the values $g_{1Z} = k_\gamma = k_Z = 1$ and $\lambda_\gamma = \lambda_Z = 0$) can deviate from the standard setting if one assumes that the physics beyond the MSM, supposedly valid at an energy scale $\Lambda \gg M_W$, manifests itself at low energies ($\sqrt{s} \ll \Lambda$) through an enrichment of the SM Lagrangian by terms of the form

$$\mathcal{L}_{\text{eff}} = \sum_{n=1}^{\infty}\sum_i \frac{f_i^{(n)}}{\Lambda^n}\mathcal{O}_i^{(n+4)}. \tag{12}$$

According to the study in [5], if one restricts oneself to dimension six operators involving only boson fields and a doublet scalar field (whose vacuum expectation value breaks the gauge SM symmetry), the operators contributing $\mathcal{L}_{\text{eff}}^{WW\gamma}$ are three ($\mathcal{O}_{WWW}$, $\mathcal{O}_W$ and $\mathcal{O}_B$) and the trilinear anomalous couplings are related to the coefficients into $\mathcal{L}_{\text{eff}}$ (see eq. 12) by the following relationships:

$$g_{1Z} = 1 + f_W\frac{M_W^2}{2\Lambda^2} \tag{13}$$

$$k_\gamma = 1 + (f_B + f_W)\frac{M_W^2}{2\Lambda^2} \tag{14}$$

$$k_Z = 1 + [f_W - s^2(f_B + f_W)]\frac{M_Z^2}{2\Lambda^2} \tag{15}$$

$$\lambda_\gamma = \lambda_Z = \frac{3M_W^2 g^2}{2\Lambda^2} f_{WWW}. \tag{16}$$

For the two parameters that are relevant in our case, the deviation from the MSM in the Lagrangian (12) will be separated in two terms

$$\mathcal{L}_{\Delta k_\gamma} = -ie\,\Delta k_\gamma W^+_\mu W^-_\nu V^{\mu\nu} \tag{17}$$

$$\mathcal{L}_{\lambda_\gamma} = -ie\,\frac{\lambda_\gamma}{M_W^2} W^{+\nu}{}_\mu W^{-\rho}{}_\nu V^\mu_\rho \tag{18}$$

where $\Delta k_\gamma = k_\gamma - k_\gamma^{SM} = k_\gamma - 1$.

The most natural way of looking for manifestations of $\mathcal{L}_{\Delta k_\gamma}$ or$\mathcal{L}_{\lambda_\gamma}$ is provided by the direct production of the vertex $\gamma W^+W^-$ at the second generation of the LEP experiments [24, 25]. Alternatively, the possibility exists of observing a not vanishing $\Delta k_\gamma$ and $\lambda_\gamma$ in virtual effects. The muon $g-2$, thanks to the remarkable precision of the future measurement, represents, in this spirit, an interesting probe for AGC, which, in principle, could provide confirmation or complementation of the results from the direct investigation about the trilinear AGC.

# 5 Contributions to the muon $g-2$ from the anomalous WW$\gamma$ couplings

## 5.1 Outline of the calculation

The general structure of the vertex $\mu\mu\gamma$, dictated by electric charge conservation and CP invariance, is the following:

$$V_\mu = (-ie)\bar{u}(k_2)\left[F_1(q^2)\gamma_\mu + \frac{i}{2m}F_2(q^2)\sigma_{\mu\nu}q^\nu + F_3(q^2)(q^2\gamma_\mu - \not{q}q_\mu)\gamma_5\right]u(k_1) \tag{19}$$

where $k_1$ and $k_2$ are the momenta of the incoming and of the outgoing muon and $q = k_2 - k_1$ is the momentum of the incoming photon. According to usual conventions the muon anomalous magnetic moment is defined as

$$\frac{g-2}{2} = F_2(0).$$

All the existing evaluations of the contributions of AGC to the muon $g-2$ [7, 8] have been carried on in the unitary gauge, since a gauge invariant treatment of the anomalous gauge boson Lagrangian was not available before the work in ref. [5]. Here, we stick to the gauge invariant Lagrangian described in § 4 which allows us to perform the calculations of the standard and not standard contributions at one loop from electroweak diagrams in the Feynmann 't Hooft gauge, where the order of the divergences is sensibly reduced.

This corresponds to the $\xi = 1$ choice in the general $R_\xi$ gauge, where the propagator for the gauge boson of mass $M_G$ and momentum $p$ has the form

$$-\frac{i}{p^2 - M_G^2 + i\epsilon}\left(g^{\sigma\rho} + (\xi_G - 1)\frac{p^\sigma p^\rho}{p^2 - \xi_G M_G^2}\right). \tag{20}$$

The higher dimension of the unitary propagator, which entails more divergent integrals, is traded with the survival of unphysical particles, the would-be-Goldston bosons (wbGB), which represent additional degrees of freedom and contribute every process through new Feynmann diagrams with a wbGB of propagator

$$\frac{i}{p^2 - \xi_G M_G^2 + i\epsilon}. \tag{21}$$

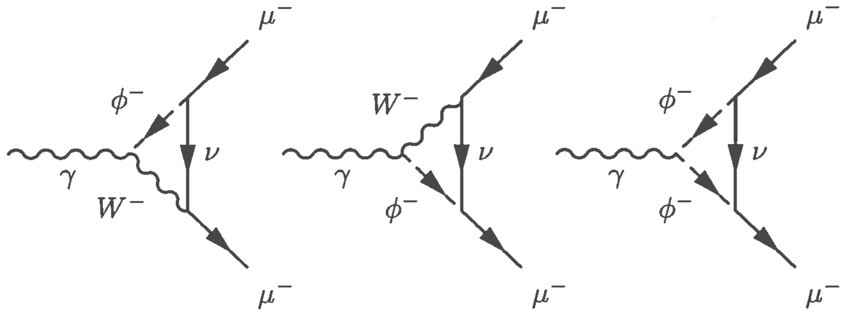


Figure 2: Diagrams with unphysical bosons contributing $a_\mu^W$ in the $R_\xi$ gauge.

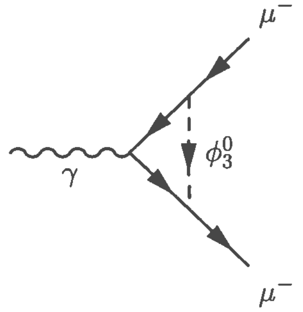


Figure 3: Diagram with the unphysical partner of the $Z^0$ boson contributing $a_\mu^Z$ in the $R_\xi$ gauge.

The divergent integrals have been handled by the dimensional regularization procedure [26]. Therefore, in the vertex amplitudes the integration over the loop internal momentum $k$ is performed in an enlarged space-time of dimension $n$. The $x$ and $y$ Feynmann parameters [27] are introduced in order to express the product of propagators in terms of an integral over the $x - y$ plane of a more convenient function of $k$. A subsequent shift of the momentum allows to give a well defined parity symmetry to each term to be integrated and, hence, to retain only the even ones by benefitting of the symmetrical domain of integration.

Once the integration over the momentum space and over the Feynmann parameters have been carried on, the magnetic form factor $F_2(q^2)$ is extracted, in the simplest cases, by means of the Gordon decomposition rule and the on-shell condition for the muons or, alternatively, by applying to the vertex amplitude a projection operator [11, 28, 29] which suppresses all the term with a Lorentz structure different from $\sigma_{\mu\nu}q^\nu$.

### 5.2 Rederivation of the standard Electroweak effect

Before applying the full formalism to the computation of the anomalous coupling effects on the muon anomaly, the standard electroweak contributions have been calculated in the same framework in order to check that known results are reproduced.

In the $\xi = 1$ gauge the contribution of the $W$ to the $g - 2$ comes from the sum of four vertex amplitudes

$$V_\mu^W = \mathcal{V}_\mu^{W^-W^-} + \mathcal{V}_\mu^{\phi^-W^-} + \mathcal{V}_\mu^{W^-\phi^-} + \mathcal{V}_\mu^{\phi^-\phi^-} \tag{22}$$

where $\mathcal{V}_\mu^{W^-W^-}$ corresponds to the diagram in fig. 1a and the other three terms account for the associated diagrams with unphysical charged Higgs (fig. 2). The vertex amplitudes,

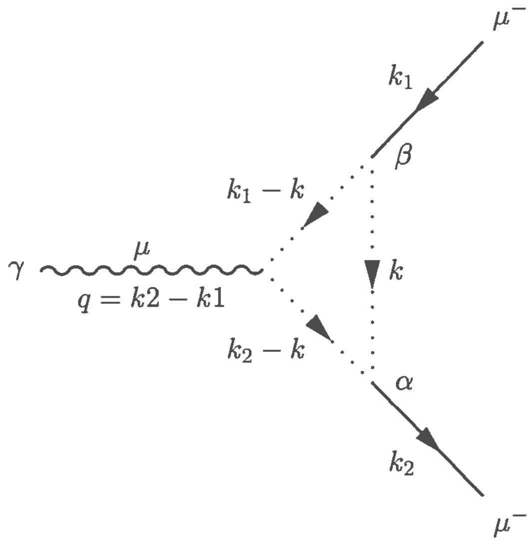


Figure 4: Conventions about the momentum assignment and the vertex labelling used in the calculations. The dotted lines represent internal boson or fermion lines depending on the particular graph. In the case of internal vector bosons, the Lorentz index $\sigma$ is associated to the line of momentum $k_1 - k$ and $\rho$ to the line of momentum $k_2 - k$.

resulting from the momentum assignments and the vertex labelling in fig. 4, are the following:

$$\mathcal{V}_\mu^{W^-W^-} = \frac{g^2 e}{8} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\bar{u}(k_2)\gamma_\alpha(1-\gamma_5)\not{k}\gamma_\beta(1-\gamma_5)u(k_1) t_{\sigma\mu\rho} g^{\alpha\rho} g^{\beta\sigma}}{k^2[(k-k_2)^2 - M_W^2][(k-k_1)^2 - M_W^2)]} \tag{23}$$

$$\mathcal{V}_\mu^{\phi^-W^-} = -\frac{g^2 e m_\mu}{8} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\bar{u}(k_2)\gamma_\alpha(1-\gamma_5)\not{k}(1+\gamma_5)u(k_1) g^{\alpha\rho} g_{\rho\mu}}{k^2[(k-k_2)^2 - M_W^2][(k-k_1)^2 - M_W^2)]} \tag{24}$$

$$\mathcal{V}_\mu^{W^-\phi^-} = -\frac{g^2 e m_\mu}{8} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\bar{u}(k_2)(1-\gamma_5)\not{k}\gamma_\beta(1-\gamma_5)u(k_1) g^{\beta\rho} g_{\rho\mu}}{k^2[(k-k_2)^2 - M_W^2][(k-k_1)^2 - M_W^2)]} \tag{25}$$

$$\mathcal{V}_\mu^{\phi^-\phi^-} = \frac{g^2 e m_\mu^2}{8 M_W^2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\bar{u}(k_2)(1-\gamma_5)\not{k}(1+\gamma_5)u(k_1) f_\mu}{k^2[(k-k_2)^2 - M_W^2][(k-k_1)^2 - M_W^2)]} \tag{26}$$

where

$$t_{\sigma\mu\rho} = (2k_2 - k_1 - k)_\sigma g_{\mu\rho} + (2k - k_1 - k_2)_\mu g_{\sigma\rho} + (2k_1 - k_2 - k)_\rho g_{\sigma\mu} \tag{27}$$

$$f_\mu = (k_1 + k_2 - 2k)_\mu \tag{28}$$

The last amplitude, eq. (26), is strongly suppressed by the factor $m_\mu^2/M_W^2$ and, therefore, can be neglected, while the second and third diagrams generate important contributions of size comparable with that of the first amplitude:

$$\left(\frac{g-2}{2}\right)^{W^-W^-} = \frac{G_F}{8\sqrt{2}\pi^2} m_\mu^2 \frac{7}{3} \tag{29}$$

$$\left(\frac{g-2}{2}\right)^{\phi^-W^-} + \left(\frac{g-2}{2}\right)^{W^-\phi^-} = \frac{G_F}{8\sqrt{2}\pi^2} m_\mu^2 \tag{30}$$

The contribution of the $Z^0$ exchange can be computed in the $R_\xi$ gauge by the vertex amplitudes ($\mathcal{V}_\mu^{Z^0}$ and $\mathcal{V}_\mu^{\phi^z}$) associated to the diagram in fig. 1b and to the diagram with the unphysical Higgs partner of the $Z^0$ replacing the gauge boson (fig. 3):

$$\mathcal{V}_\mu^{Z^0} = -\frac{g^2 e}{4c^2}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\bar{u}(k_2)\gamma_\alpha(g_V+g_A\gamma_5)\Gamma_\mu\gamma_\beta(g_V+g_A\gamma_5)u(k_1)g^{\alpha\beta}}{[k^2-M_Z^2][(k-k_2)^2-M_Z^2][(k-k_1)^2-M_Z^2)]} \tag{31}$$

$$\mathcal{V}_\mu^{\phi^z} = -\frac{g^2 e m^2}{4M_W^2}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\bar{u}(k_2)\gamma_5\Gamma_\mu\gamma_5 u(k_1)}{[k^2-M_Z^2][(k-k_2)^2-M_Z^2][(k-k_1)^2-M_Z^2)]} \tag{32}$$

where $\Gamma_\mu = (\not{k}_2 - \not{k} + m)\gamma_\mu(\not{k}_1 - \not{k} + m)$. Also in this case the contribution from the second amplitude can be disregarded because of the suppressing ratio of the muon to the $W$ squared masses. The first amplitude, hence, is responsible for the total $Z^0$ contribution (eq. 9) which is, roughly, one half of the $W$ contribution and opposite in sign.

## 5.3 The contribution from $\Delta k_\gamma$

In order to evaluate the contribution to the $g-2$ from the term $\mathcal{L}_{\Delta k_\gamma}$ of the effective Lagrangian we need to take into account the Feynmann diagram in fig. 1a and to associate to the $WW\gamma$ vertex the term

$$ie\Delta k_\gamma\,[q_\rho g_{\mu\sigma} - q_\sigma g_{\mu\rho}] \tag{33}$$

instead of the standard one $-iet_{\sigma\mu\rho}$ (see eq. 27).

When using an unphysical gauge, like the Feynmann 't Hooft one, all the contributions to the process from every degree of freedom existing in the theory must be taken into account. Therefore, since the parameter $\Delta k_\gamma$ is a function of the AGC $f_B$ and $f_W$, one has to check whether anomalous couplings between $W^-\phi^-\gamma$ are also introduced by the operators $\mathcal{O}_B$ and $\mathcal{O}_W$. Actually, by isolating into the anomalous Lagrangian the terms which are trilinear in the fields of interest at the lowest order in $\alpha$, one comes out with the following Lagrangian:

$$\mathcal{L}_{W^-\phi^+\gamma} = -\frac{eM_W}{2}\left(\frac{f_B+f_W}{\Lambda^2}\right)W_\mu^-\partial_\nu\phi^+(\partial^\mu A^\nu - \partial^\nu A^\mu) \tag{34}$$

from which the Feynmann rule

$$i\frac{e}{M_W}\Delta k_\gamma\,[\,q_\sigma(k-k_2)_\mu - g_{\mu\sigma}q\cdot(k-k_2)\,] \tag{35}$$

for the $W^-\phi^-\gamma$ vertex (second diagram in fig. 2) is derived. This means that the amplitudes related to the first two graphs in fig. 2 have to be calculated using eq. (35) and added, like in the standard case, to the vertex amplitude described by the diagram with the physical fields. The third diagram in fig. 2 should also be taken into account, for completeness, but the amplitude related to it is suppressed by a factor $m_\mu^2/M_W^2$ and hence it can be safely neglected.

The total gauge invariant amplitude $V_\mu'^W$ is split into four terms, like in eq.s (23-26). For the three which are relevant, the result of the computation gives:

$$\mathcal{V'}_\mu^{W^-W^-} \to \frac{G_F}{8\sqrt{2}\pi^2}m_\mu^2\,2\,\Delta k_\gamma \tag{36}$$

$$\mathcal{V'}_\mu^{\phi^-W^-} + \mathcal{V'}_\mu^{W^-\phi^-} \to -\frac{G_F}{8\sqrt{2}\pi^2}m_\mu^2\,2\,\Delta k_\gamma\left(\frac{2}{4-n}+\ln\frac{\mu^2}{M_W^2}+\frac{1}{2}\right) \tag{37}$$

The contributions arising from the longitudinal degrees of freedom of the $W$ bosons are divergent in the limit $n \to 4$ and, moreover, the observable anomalous magnetic moment seems to depend on the arbitrary mass $\mu$ which restores the proper dimensionality of the vertex amplitudes but does not have any physical meaning. It is well known that results obtained with dimensional regularization and with cutoff procedures can be compared, at the leading order, by identifying the infinities by means of the rule:

$$\frac{2}{4-n} = \ln \frac{\Lambda^2}{\mu^2}. \tag{38}$$

By applying the described substitution it turns out that the leading contribution from $\mathcal{L}_{\Delta k_\gamma}$ to the $g-2$ is

$$\left(\frac{g-2}{2}\right)_{\Delta k_\gamma} = -\frac{G_F}{8\sqrt{2}\pi^2}\, m_\mu^2\, 2\Delta k_\gamma \, \ln \frac{\Lambda^2}{M_W^2} \tag{39}$$

in agreement with the previous computations [7, 8].

This contribution could produce a sizable effect on the $g-2$ for values of energy scale $\Lambda \geq 1$ TeV that are usually assumed in the philosophy of the model, as it will be discussed in the final section. Note that in the $R_\xi$ calculation a finite term survives that was not present in [7, 8]. The origin of this apparent discrepancy is, in fact, known since in the unitary gauge different computational prescriptions can lead to different non leading terms. It is gratifying to notice that for $\Lambda$ values of 1 TeV these finite terms are completely negligible. This might be not so obvious for small energy scales.

### 5.4 The contribution from $\lambda_\gamma$

The term $\mathcal{L}_{\lambda_\gamma}$ in the effective Lagrangian comes from the operator $\mathcal{O}_{WWW}$, which does not involve the doublet scalar field $\Phi$. Therefore, no diagrams of the kind shown in fig. 2 arise from this anomalous Lagrangian and the only amplitude which can give a contribution to the muon anomaly is related to the graph in fig. 1a with the following rule for the $WW\gamma$ vertex

$$\begin{aligned} -ie\frac{\lambda_\gamma}{M_W^2} \quad \{ \quad & q_\sigma(k_1-k)_\rho(k-k_2)_\mu - q_\rho(k_1-k)_\mu(k-k_2)_\sigma \\ + \quad & g_{\mu\sigma}[(k_1-k)\cdot(k-k_2)q_\rho - (k-k_2)\cdot q(k_1-k)_\rho] \\ + \quad & g_{\mu\rho}[(k_1-k)\cdot q(k-k_2)_\sigma - (k_1-k)\cdot(k-k_2)q_\sigma] \\ + \quad & g_{\sigma\rho}[q\cdot(k-k_2)(k_1-k)_\mu - (k_1-k)\cdot q(k-k_2)_\mu] \quad \} \end{aligned} \tag{40}$$

for an incoming $W^-$ of momentum $k_1 - k$, an outgoing $W^-$ of momentum $k_2 - k$ and an incoming photon of momentum $q = k_2 - k_1$. The calculation has been performed, as usual, in the Feynmann 't Hooft gauge. In spite of the fact that the energy dimension of $\mathcal{O}_{WWW}$ is six and, hence, the operator is not renormalizable, an exact cancellation of the divergent integrals occurring in the coefficient of $\sigma_{\mu\nu}q^\nu$ makes the result to be finite. The resulting contribution to $a_\mu$

$$\left(\frac{g-2}{2}\right)_{\lambda_\gamma} = -\frac{G_F}{8\sqrt{2}\pi^2}\, m_\mu^2\, 2\lambda_\gamma \tag{41}$$

is in agreement with the previous computations which were performed in the same regularization framework that has been used in this work [8].

## 5.5 Improving the bounds on anomalous WW$\gamma$ couplings with the muon magnetic anomaly

The overall relevant contributions to the muon anomaly from AGC give the following deviation from the SM prescription:

$$\delta a_\mu^{AGC} = \frac{G_F}{8\sqrt{2}\pi^2} m_\mu^2 2 \left[ \Delta k_\gamma \left( 1 - \ln \frac{\Lambda^2}{M_W^2} - \frac{1}{2} \right) - \lambda_\gamma \right]. \qquad (42)$$

As already pointed out, the new physics energy scale is usually supposed to be of the order of 1 TeV and, hence, all the finite terms proportional to $\Delta k_\gamma$ in eq. (42) can be safely neglected compared to the logarithmically divergent contribution.

With this assumption, when expressing the effective couplings $\Delta k_\gamma$ and $\lambda_\gamma$ as a function of the parameters $f_B$, $f_W$ and $f_{WWW}$ of the Lagrangian in eq. (12), the anomalous contributions to $a_\mu$ are

$$\delta a_\mu^{AGC} = -\frac{G_F}{8\sqrt{2}\pi^2} m_\mu^2 \frac{M_W^2}{\Lambda^2} \left[ (f_B + f_W) \ln \frac{\Lambda^2}{M_W^2} + f_{WWW} \frac{12\pi\alpha}{s^2} \right] \qquad (43)$$

From this expression it is clear that the $g-2$ of the muon has an interesting sensitivity to the coupling $f_B + f_W$, while it cannot be significantly affected by $f_{WWW}$. For this reason this coupling will be neglected, for simplicity, in the following discussion.

It is worth, now, to investigate whether there is any chance of constraining $\Delta k_\gamma$ by comparing the expression of $\delta a_\mu^{AGC}$ with a realistic estimate of the future uncertainty on the knowledge of the muon anomalous magnetic moment.

A measurement of the total hadronic cross section at DA$\Phi$NE, in the range $2m_\pi \div 1.4$ GeV, with the precision required in order to lower the corresponding contribution to the error on $a_\mu^{\text{hadr(1 loop)}}$ to $1.5{\cdot}10^{-10}$, seems to be a quite easy task for KLOE [2, 3]. In this case, according to [15], the total hadronic error (at one loop) would become $0.75 \cdot 10^{-9}$. When the uncertainty on the higher order hadronic contributions is added, the error becomes $0.82 \cdot 10^{-9}$ with the most pessimistic estimate of the error on the contribution from light by light scattering. Finally, by combining this number with the claimed future experimental error, one ends up with a total error on the muon $g-2$ of

$$\Delta a_\mu^{\text{exp+th}} = 0.9 \times 10^{-9}. \qquad (44)$$

If the observed discrepancy between the measured $a_\mu$ and the corresponding SM prediction is below $n_\sigma$ standard deviations, i.e.

$$\mid a_\mu^{\text{exp}} - a_\mu^{SM} \mid \leq n_\sigma \, \Delta a_\mu^{\text{exp+th}} \qquad (45)$$

the combination of the couplings $f_B$ and $f_W$, which appears in the expression of $\delta a_\mu^{AGC}$ can be constrained, at the same C.L., in the following way

$$\mid \Delta k_\gamma \mid = \mid \frac{M_W^2}{2\Lambda^2} (f_B + f_W) \mid \leq n_\sigma \frac{0.43 \cdot 10^9}{\ln \frac{\Lambda^2}{M_w^2}} \Delta a_\mu^{\text{exp+th}} \qquad (46)$$

By assuming for $\Delta a_\mu^{\text{exp+th}}$ the value in eq. (44) and setting $\Lambda = 1$ TeV, one immediately realizes that at 95% C.L. the bound coming from $a_\mu$ is

$$\mid \Delta k_\gamma \mid = \mid \frac{M_W^2}{2\Lambda^2} (f_B + f_W) \mid \leq 0.15 \qquad (47)$$

This result is quite remarkable when compared with the existing bounds (one order of magnitude higher than in eq. 47), derived from the analysis of virtual effects induced by the AGC on precision e.w. data [22] and from direct effects in boson pair production [23]. Note that, since the effects of the different couplings on the low energy observables are often correlated, the constraints, in that case, are obtained for each coupling in the optimistic hypothesis on absence of mutual cancellations [5](and they also depend on the value of the Higgs boson mass).

A better picture will be provided by the forthcoming data from LEP2. The analysis of all the observables, which can be accessed in the process $e^+e^- \to W^+W^-$ at the center of mass energy of 190 GeV, has been shown [25] to lead to the constraint $-0.4 \leq \Delta k_\gamma \leq 0.9$ at 95% C.L. when only three parameters ($\Delta k_\gamma$, $\lambda_\gamma$ and $g_{1Z}$) are free in the fit to the simulated data.

Therefore, with the generally accepted prescription $\Lambda \simeq 1$ TeV, the sensitivity of the muon $g-2$ to $f_B + f_W$ turns out to be competitive with the direct production of the vertex $WW\gamma$. Moreover, it has to be pointed out that the muon $g-2$ allows to disentangle one specific anomalous coupling.

## 6 Conclusions

The precision on the knowledge of the muon anomalous magnetic moment is going to increase in the next years both from the experimental point of view, thanks to the planned precision of the forthcoming measurement at the BNL $g-2$ experiment [1], and from the theoretical side. In fact, the future measurements of the total hadronic cross section at the existing low energy $e^+e^-$ colliders [2, 30] would allow to lower significantly the main theoretical uncertainty related to the hadronic contribution to $a_\mu$. A set of realistic assumptions about these future improved measurements allows to rely on a value of the total error $\Delta a_\mu = 0.9 \cdot 10^{-9}$ on the muon anomaly.

As a consequence of this increased precision, the sensitivity of $a_\mu$ to new physics effects is enhanced.

In this note, the contributions to the muon $g-2$ from a certain kind of anomalous gauge boson couplings (those which survive the requirement of local $SU(2) \times U(1)$ invariance, together with C and P parity, of an effective theory embedding the SM) have been computed in the Feynmann 't Hooft gauge within the dimensional regularization computational scheme. The results of the calculation confirm the previous claims [6, 7, 8], coming from calculations performed in the unitary gauge, about a significant reactivity of $a_\mu$ to the coupling $\Delta k_\gamma$, originated by a logarithmically divergent contribution.

The constraint on this coupling which can be derived from the future determination of $a_\mu$ is $\mid \Delta k_\gamma \mid \leq 0.15$. It represents a remarkable improvement with respect to the current bounds from precision e.w. measurements and a useful, and competitive, complementation to the constraints which will come from the direct production of $W$ pairs at LEP2 [24, 25].

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