Limiting SUSY-QCD spectrum and its application for decays of superheavy particles

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Abstract

The supersymmetric generalization of the limiting and Gaussian QCD spectra is obtained. These spectra are valid for $x \ll 1$, when the main contribution to the parton cascade is given by gluons and gluinos. The derived spectra are applied to decaying superheavy particles with masses up to the GUT scale. These particles can be relics from the Big Bang or produced by topological defects and could give rise to the observed ultrahigh energy cosmic rays. General formulae for the fluxes of protons, photons and neutrinos due to decays of superheavy particles are obtained.

1   Introduction

The spectra of hadrons produced in deep-inelastic scattering and e^+e^- annihilation are formed due to QCD cascading of the partons. In the Leading Logarithmic Approximation (LLA) which takes into account $\ln(Q^2)$ terms this cascade is described by the Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD) equation [1]. The Modified Leading Logarithmic Approximation (MLLA) takes into account additionally $\ln(x)$ terms, where $x = k_\parallel / k_\parallel^{max}$ and $k_\parallel$ is the longitudinal momentum of the produced hadron. Color coherence effects are described in MLLA. Two approximate analytic solutions to the MLLA evolution equations have been obtained. These are the *limiting spectrum* [2] and the *Gaussian spectrum* [3, 4], in which we include the *distorted Gaussian spectrum* [5, 6, 7]. The limiting spectrum is the most accurate one among them. In fact, it describes well the experimental data at large x, too, and this is natural, though accidental (for an explanation see Ref. [6]). The limiting spectrum has a free normalization constant, $K_{\rm lim}$, which cannot be calculated theoretically and is found from comparison with experimental data. This constant has to be considered as a basic parameter of the theory, and it can be used at all energies, where the physical assumptions, under which the limiting spectrum is derived, are valid. For detailed calculations of hadron spectra in e^+e^- annihilation and comparison with experimental data see Ref. [8]. Up to energies of existing e^+e^- colliders, $\sqrt{s} \lesssim 183$ GeV, the limiting spectrum and the distorted Gaussian spectrum describe well the available data. At large energies $\sqrt{s} \gtrsim 1$ TeV the production of supersymmetric particles might substantially change the QCD spectra. Apart from future experiments at LHC, supersymmetry (SUSY) might strongly reveal itself in the decays of superheavy particles. They can appear as relics of the Big Bang, or be produced by topological defects (TD), and can be the sources of the observed ultrahigh energy cosmic rays (UHECR) at $E \gtrsim 1 \cdot 10^{10}$ GeV. The range of masses, m_X, of interest for UHECR goes from the GUT scale ($m_X \sim 10^{16}$ GeV) or less down to $m_X \sim 10^{12} - 10^{14}$ GeV.

In this paper we obtain the generalization of the limiting and Gaussian QCD spectra for the SUSY case. Although the influence of SUSY on, e.g., the evolution of parton distributions or the running coupling constant was considered in many works in the 80's [9], this is, to the best of our knowledge, the first time that fragmentation spectra of hadrons are examined for large $\sqrt{s}$ up to the GUT scale in SUSY-QCD. As application we use these spectra for calculations of the fluxes of UHECR produced in the decays of superheavy particles.

2   Limiting spectrum in SUSY-QCD

The GLAPD equation [1] describes in LLA the evolution of the parton distributions $D_A^B(x, \xi)$ with ξ. Here D_A^B is the distribution of partons B inside the parton A dressed by QCD interactions with coupling constant $\alpha(k_\perp^2)$, where $k_\perp$ is the transverse momentum and $x = k_\parallel / k_\parallel^{max}$ is the longitudinal momentum fraction of the parton B. The variable ξ

characterizes the maximum value of $k_\perp^2$ available in the considered process $(k_\perp^2 < Q^2)$,

$$\xi(Q^2) = \int_{\Lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \frac{\alpha_s(k_\perp^2)}{4\pi} \approx \frac{\alpha_s(Q^2)}{4\pi} \ln\left(\frac{Q^2}{\Lambda^2}\right) , \tag{1}$$

with $\Lambda \sim 0.25$ GeV as phenomenological parameter.

In LLA, when terms with $\alpha_s(Q^2) \ln(Q^2)$ are kept and terms proportional to $\alpha(Q^2)$ are neglected, the GLAPD equation can be written as [6, Eq. (1.79)]

$$\frac{\partial}{\partial\xi} D_A^B(x,\xi) = \sum_C \int_0^1 \frac{dz}{z} \Phi_A^C(z) D_C^B(x/z,\xi) - \sum_C \int_0^1 dz\, z\Phi_A^C(z) D_A^B(x,\xi) , \tag{2}$$

where $\Phi_A^B(z)$ is the splitting function characterizing the decay $A \to B + C$.

The supersymmetrization of Eq. (2) is simple: each parton A should be substituted by the supermultiplet which contains A and its superpartner $\tilde{A}$. We shall generalize here the limiting spectrum of QCD given e.g. in [6] to SUSY-QCD. This spectrum is valid for small x, where gluons strongly dominate the other partons. Therefore, we restrict ourselves to calculations taking into account only two partons, namely gluons g and gluinos λ, in the tree diagrams. However, in the loop diagrams which govern the running of $\alpha_s(k_\perp^2)$ we take into account also quarks and squarks.

Multiplying Eq. (2) by x^{j+1} and integrating it over x, we obtain an equation for the moments $D_A^B(j,\xi)$,

$$\frac{\partial}{\partial\xi} D_A^B(j,\xi) = \sum_C \Phi_A^C(z) D_C^B(j,\xi) - D_A^B(j,\xi) \sum_C \int_0^1 dz\, z\Phi_A^C(z) , \tag{3}$$

where

$$D_A^B(j,\xi) = \int_0^1 dx\, x^{j-1} D_A^B(x,\xi) \tag{4}$$

and the indices A, B, C run through g and λ.

The splitting functions are given e.g. in [9] as

$$\Phi_g^g(z) = 4N_c \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right] \tag{5}$$

$$\Phi_g^\lambda(z) = 2N_c \left[z^2 + (1-z)^2 \right] \tag{6}$$

$$\Phi_\lambda^g(z) = 2N_c \frac{1 + (1-z)^2}{z} \tag{7}$$

$$\Phi_\lambda^\lambda(z) = 2N_c \frac{1 + z^2}{1-z} . \tag{8}$$

Equation (3) can be rewritten in matrix form choosing as basis (g, λ),

$$\frac{\partial}{\partial\xi} D(j,\xi) = H(j) D(j,\xi) \tag{9}$$

3

where

$$H(j) = \begin{pmatrix} \nu_g(j) & \Phi_g^\lambda(j) \\ \Phi_\lambda^g(j) & \nu_\lambda(j) \end{pmatrix} \tag{10}$$

$$\nu_g(j) = \int_0^1 dz \left[\left(z^{j-1} - z \right) \Phi_g^g(z) - \Phi_g^\lambda(z) \right] \tag{11}$$

$$\nu_\lambda(j) = \int_0^1 dz \left(z^{j-1} - 1 \right) \Phi_\lambda^\lambda(z) . \tag{12}$$

The procedure of solving Eq. (9) is identical to that given in Ref. [6]. After diagonalization of $H(j)$ with the help of (D^+, D^-), the eigenvalues of H are

$$\nu_\pm = \frac{1}{2} \left(\nu_g + \nu_\lambda \pm \left[(\nu_g - \nu_\lambda)^2 + 4\Phi_g^\lambda \Phi_\lambda^g \right]^{1/2} \right) . \tag{13}$$

In the limit $\omega = j - 1 \to 0$, the leading term ν_+ is given by

$$\nu_+ = \frac{4N_c}{\omega} - a + O(\omega) \tag{14}$$

with $a = \frac{11}{3} N_c = 11$. ($N_c = 3$ is the number of colours.)

Up to now we have considered the LLA approximation. This approximation is not correct for $x \ll 1$, when colour coherence effects become important. A better description of distributions at small x is given by the MLLA approximation [2]. It takes into account both $\ln(Q^2)$ and $\ln(x)$ terms as well as angular ordering. The MLLA evolution equation results in Eq. (9) for the moments, with ξ replaced by $\xi_{\mathrm{MLLA}} = \alpha_s(Y)/(4\pi) \ln(Y^2)$ with $Y = \ln(E\theta/Q_0)$, where E is the energy and θ the opening angle of the jet (see [6, Eq. (7.4)]). For its analytic solution at small x, the limiting spectrum, the eigenvalues (13) derived above in LLA are still valid. Properly normalized, the limiting spectrum $D_{\mathrm{lim}}(l, Y) = x D_{\mathrm{lim}}(x, Y)$ gives $\sigma^{-1} d\sigma/dl$ in the case of $e^+ e^-$ annihilation and the decay spectrum of X particles, where $l = \ln(1/x)$. This spectrum is given by [10]

$$D_{\mathrm{lim}}(l, Y) = K_{\mathrm{lim}} \frac{4C_F}{b} , (B) \int_{-\pi/2}^{\pi/2} \frac{d\tau}{\pi} e^{-B\alpha} \left(\frac{b}{8N_c} \frac{\sinh\alpha}{\alpha} \frac{Y}{y} \right)^B I_B(y) . \tag{15}$$

Here, Y now is $Y = \ln[\sqrt{s}/(2\Lambda)]$, $\alpha = \alpha_0 + i\tau$, $\alpha_0 = \mathrm{arctanh}(2\zeta - 1)$, $\zeta = 1 - l/Y$, and $\sqrt{s}$ is the c.m. energy of an $e^+ e^-$ pair or the mass m_X of the superheavy decaying particle. The parameters depending on the structure of the theory are $a = \frac{11}{3} N_c$, the constant b of evolution of $\alpha_s(k_\perp^2)$ in one-loop approximation, $k_\perp^2 d\alpha_s(k_\perp^2)/dk_\perp^2 = -b\alpha_s^2/(4\pi)$, and $C_F = (N_c^2 - 1)/(2N_c) = 4/3$. When the masses of the superheavy coloured Higgses are larger than $Q^2 = k_{\perp,\mathrm{max}}^2$, $b = b_{\mathrm{SUSY}} = 9 - n_f$, where $n_f = 6$ is the number of quark flavours. Finally, I_B is the modified Bessel function of order $B = a/b$, and argument

$$y(\tau) = \left(\frac{16N_C}{b} \frac{\alpha}{\sinh\alpha} [\cosh\alpha + (1 - 2\zeta)\sinh\alpha] Y \right)^{1/2} . \tag{16}$$

A convenient way for the numerical evaluation of $I_B(y)$ is the use of its series expansions, given for example by (8.445) and (8.451.5) in Ref. [11].

In Fig. 1 the SUSY-QCD limiting spectrum is shown in comparison with the QCD limiting spectrum, for the case of three colours, $N_c = 3$, and six quarks flavours, $n_f = 6$. For the sake of comparison, we normalize both spectra by the condition

$$\int_0^1 dx\, x D_{\text{lim}}(x, Y) = 2 \ . \tag{17}$$

The maximum of the spectra is at $l_m = Y(0.5 + \sqrt{c/Y} - c/Y)$ with $c \approx 0.39$ in QCD and $c \approx 0.84$ in SUSY-QCD. Therefore, the SUSY spectra are shifted to the right, and since they are also narrower than the QCD spectra (see Eq. (19) below), the SUSY maxima are dramatically higher (by a factor of 30) than the QCD ones. We remind the reader that the value of the maximum is given by the multiplicity.

We compare these spectra also with the Gaussian approximation obtained for the MLLA solution [5, 7]. This approximation can be easily generalized to the SUSY case [12] and has for $x \ll 1$ as function of $x = 2E/\sqrt{s}$ the form,

$$D_G(x, Y) = \frac{K_G}{x}\, \exp\left(-\frac{\ln^2 x/x_m}{2\sigma^2}\right) \tag{18}$$

Assuming one-loop SUSY evolution of $\alpha_s(k_\perp^2)$ with $b = b_{\text{SUSY}}$ and $\Lambda = \Lambda_{\text{QCD}}$, one has

$$\sigma^2 = \frac{1}{24}\sqrt{\frac{b_{\text{SUSY}}}{6}}\, \ln^{3/2}\left(\frac{s}{\Lambda^2}\right) \tag{19}$$

and

$$x_m = \left(\frac{\Lambda}{\sqrt{s}}\right)^{1/2} \ . \tag{20}$$

Since $b_{\text{SUSY}} = 3$ (for $n_f = 6$) is less than $b_{\text{QCD}} = 7$ (for the same n_f), the SUSY-QCD peak is narrower than the QCD one.

The Gaussian spectrum given by Eq. (18) has its maximum at $x = x_m$. It gives a less precise description than the limiting spectrum. One might expect that at large Y, when the higher momenta, skewness s and kurtosis κ, become small ($s \sim Y^{-3/2}$ and $\kappa \sim Y^{-1/2}$) the agreement improves. We have found that both for the case of ordinary QCD and SUSY-QCD the shape of the spectra differs substantially in the interesting range $10^{-6} \lesssim x \lesssim 10^{-2}$ (Fig. 2).

3   Applications

We shall apply now the limiting spectrum in SUSY-QCD to the calculation of the spectrum of ultrahigh energy cosmic rays (UHECR) generated by the decay of superheavy particles with masses $10^{12} - 10^{15}$ GeV.

Let us discuss first the problem of the normalization of spectrum. We remind the reader that the limiting spectrum has been derived for $x \ll 1$, though at least for small

s it describes well the experimental data for x up to $x \sim 1$ (see [6] for discussion). In Ref. [10], the normalization constant was fixed by a comparison with experimental data on e^+e^- annihilation to $K_{\lim} = 2.6$. This value is fixed by $D_{\lim}(l, Y)$ at maximum (see Fig. 1), i.e. by multiplicity.

Since the shape of the spectrum and the position of its peak change dramatically, if one goes from QCD and $\sqrt{s} \sim 100$ GeV to SUSY-QCD and $\sqrt{s} \sim 10^{12} - 10^{15}$ GeV, we cannot use this value of $K_{\lim}$. Instead we use as normalization condition Eq. (17) replacing the factor 2 by $2f_i$, where i runs through N (all nucleons) and $\pi^\pm$ and π^0 (charged and neutral pions), while f_i is the fraction of energy carried by the hadron i. Note that the main contribution to the integral in Eq. (17) comes from large values $x \sim 1$, where the limiting spectrum might have large uncertainties. However, this is, in our opinion, the most physical way of normalization. The numerical values of f_i are unknown at large s. One can assume that $f_\pi \approx 1 - f_{\rm LSP}$, where $f_{\rm LSP}$ is the energy fraction taken away by the lightest supersymmetric particle (LSP). According to the simplified Monte-Carlo simulation of [14], $f_{\rm LSP} \sim 0.4$. For the ratio f_N/f_π we use ~ 0.05 inspired by Z^0 decay.

Let us assume that the decay rate of X particles $\dot{n}_X$ in the extragalactic space does not depend on distance and time. Then taking into account the energy losses of UHE protons and the absorption of UHE photons due to pair production ($\gamma + \gamma \to e^+ + e^-$) on the radio and microwave background, the diffuse flux of UHE protons and antiprotons is

$$I_{p+\bar{p}}(E) = \frac{1}{2\pi} \frac{\dot{n}_X}{m_X} \int_0^\infty dt_g \, D_N(x_g, Y) \frac{dE_g(E, t_g)}{dE} \, , \tag{21}$$

where $E_g(E, t_g)$ is the energy at generation time t_g of a proton which has at present the energy E and $x_g = 2E_g/m_X$. Denoting the proton energy looses on microwave radiation by $dE/dt = b(E, z)$, dE_g/dE is given by [15]

$$\frac{dE_g(E, z_g)}{dE} = (1 + z_g) \exp \left[\int_0^{z_g} \frac{dz}{H_0} (1 + z)^{1/2} \left(\frac{\partial b(E, 0)}{\partial E} \right)_{E = E_g(z)} \right] \, , \tag{22}$$

where H_0 is the Hubble constant and z the redshift. In the case that the energy looses on microwave radiation are much larger than the adiabatic ones, Eq. (22) reduces to

$$\frac{dE_g(E, z_g)}{dE} \approx \frac{b(E_g, 0)}{b(E, 0)} \, . \tag{23}$$

The diffuse spectrum of UHE photons can be calculated as

$$I_\gamma(E) = \frac{\dot{n}_X}{\pi} \lambda_\gamma(E) \frac{1}{m_X} \int_{2E/m_x}^1 \frac{dx}{x} D_{\pi^0}(x, Y) \, , \tag{24}$$

where $\lambda_\gamma(E)$ is the absorption length of a photon. For the numerical evaluation of I_γ, we use λ_γ from [16].

The neutrino flux depends generally on the evolution of the sources,

$$\dot{n}_X(t) = \dot{n}_X(t_0) \left(\frac{t_0}{t} \right)^{3+p} \, . \tag{25}$$

The number of neutrinos with energy E produced per decay of one X particle can be approximately calculated as

$$N_\nu(E) \approx \frac{12}{m_X} \int_{4E/m_x}^1 \frac{dx}{x} D_{\pi^\pm}(x, Y) .$$ (26)

Combining Eqs. (25) and (26), we obtain for the diffuse neutrino flux

$$I_\nu(E) = \frac{3\dot{n}_X(t_0)}{\pi H_0 m_X} \int_0^{z_{\max}} dz \, (1+z)^{\frac{3}{2}p} \int_{\frac{4E(1+z)}{m_x}}^1 \frac{dx}{x} D_{\pi^\pm}(x, Y) ,$$ (27)

where $1 + z_{\max}(E) \approx m_X/(4E)$.

In Fig. 3, the spectra of UHE protons, photons, and neutrinos are shown together with experimental data [17] for the model of cosmic necklaces – topological defects which consists of monopoles connected by strings [18]. We use the model of Ref. [13], where

$$\dot{n}_X = \frac{r^2\mu}{m_X t^3}$$ (28)

with $r^2\mu = 5 \cdot 10^{27}$ GeV^2 and $m_X = 1 \cdot 10^{14}$ GeV. The proton flux is suppressed at the highest energies as compared with the calculations of Ref. [13], where the Gaussian SUSY-QCD spectrum was used. The model corresponds to $p = 0$, or effectively to the absence of evolution for the integral over z in Eq. (27).

In conclusion, we have calculated the SUSY-QCD limiting spectrum of partons in a jet. This spectrum considerably differs from that of ordinary QCD: the maximum of the Gaussian peak is shifted towards smaller x and the peak is narrower and higher. The limiting spectrum has been applied for calculations of the UHE fluxes of protons, photons and neutrinos from decays of superheavy particles in the Universe. The fluxes of these particles are different from those calculated with the ordinary QCD parton spectra.

Acknowledgments

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