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Neutrino self-energy and pair creation in neutron stars

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Abstract

Massless neutrino exchange leads to a new long-range force between matter. Recently, it was claimed both that the potential energy due to this interaction i) dominates the total energy of neutron stars and ii) that it is zero. We recalculate the energy of a neutrino propagating in a classical, uniform background of neutrons and find a negligible, but non-zero contribution to the total energy of neutron stars. We estimate the neutrino pair creation rate of a neutron star caused by a density gradient of the background neutrons but found it too small to be observable.

1 Introduction

Recently Fischbach calculated the energy difference ΔE between a neutrino immersed in a neutron star and in vacuum [1]. He used perturbation theory to derive the potential energy $W^{(k)}$ of N neutrons due to the exchange of massless neutrinos, $\Delta E = \sum_k W^{(k)}$. The ratio of the contributions to ΔE from k and $k + 2$ body interactions found by him equals for $k \ll N$

$$\left| \frac{W^{(k+2)}}{W^{(k)}} \right| \sim \frac{1}{(k+2)(k+1)} \left(\frac{G_F N}{R^2} \right)^2, \quad (1)$$

where G_F is the Fermi constant and R the radius of the star. For a typical neutron star, $G_F N / R^2 = \mathcal{O}(10^{13})$ and multi-body effects become dominating. In particular, $|\Delta E|$ exceeds the mass energy of the neutron star—an obvious contradiction to the observation of neutron stars. The resolution of this paradox proposed in Ref. [1] is to consider a massive neutrino. Then the neutrino can interact only with neutrons within its Yukawa radius $1/m_\nu$, and, if m_ν is sufficiently large, the dangerous many-body effects are exponentially damped. Hereby, a lower bound for the electron neutrino mass, $m_{\nu_e} \gtrsim 0.4$ eV, was derived [1].

The method and results of Ref. [1] provoked some criticism. Smirnov and Vissani [2] argued that a neutrino sea inside the neutron star [3] reduces the potential energy $W^{(k)}$ because of Pauli blocking. They stressed also that the behaviour of the potential energy $W^{(k)}$,

$$W^{(2k)} = (-1)^k |\tilde{W}^{(2k)}|, \quad (2)$$

is unacceptable. In fact, both the increasing of $|W^{(k)}|$ and its oscillatory behaviour with $2k$ are clear signs for the breakdown of perturbation theory. Abada *et al.* [4] recalculated the energy difference ΔE taking into account non-perturbatively the interaction of the neutrino with a uniform neutron background. They obtained $\Delta E = 0$ if no neutrino sea is presented. This result would imply that neutrinos do not interact at all with the neutron background. In view of the results of Ref. [1, 4] and the possible implications for neutrino physics, we feel it appropriate to recalculate once again ΔE . Additionally, we estimate the neutrino pair creation rate of a neutron star caused by a density gradient of the background neutrons.

In our calculations we always assume that the neutron density $n_N(\mathbf{x})$ respectively its gradient $\nabla n_N(\mathbf{x})$ can be approximated locally by a constant value. This assumption looks unproblematic because of the macroscopic size of the neutron star, neglects however possible long-range effects due to massless neutrinos.

2 Neutrino energy density without neutrino sea

The energy E of a neutrino interacting with a classical background current J^μ of neutrons is given by [5, 6]

$$E = \langle P^0 \rangle = i \int d^3x \langle 0 | \psi^\dagger(x) \partial_t \psi(x) | 0 \rangle_J. \quad (3)$$

Here, ψ is the field operator in the Furry picture obtained after second quantizing the solutions of the Dirac equation

$$\left[\gamma_\mu \partial^\mu + im - \sqrt{2}iG_F a_N \gamma_\mu J^\mu P_L \right] \psi(x) = 0, \quad (4)$$

where $a_N = -0.5$ and $P_L = (1 - \gamma^5)/2$ projects out the right-handed component of the neutrinos.

Since we are interested in the infrared regime, we can assume in the following J^μ as static, $J^\mu = (n_N, \mathbf{0})$. Moreover, the neutron number density n_N is also nearly homogenous. Therefore, the solutions of Eq. (4) can still be characterized by the four-momentum $p^\mu = (E, \mathbf{p})$ of the neutrino. Then, as it is well known, the only change compared to the vacuum case is the modification of the dispersion relation of the neutrino [7, 8],

$$E = (m^2 + |\mathbf{p}|^2)^{1/2} \pm V \quad (5)$$

$$V = -\frac{G_F}{\sqrt{2}} n_N. \quad (6)$$

Here, the upper sign in Eq. (5) corresponds to neutrinos and the lower sign to antineutrinos.

Equation (3) can be rewritten as

$$E = \int d^3x \partial_t \text{tr} \left\{ \gamma^0 S_F(x, x') \right\}_{x' \searrow x}, \quad (7)$$

where $S_F(x, x')$ is the Greens function of Eq. (4). Now we evaluate the energy difference

$$\Delta E = \int d^3x \int \frac{d^4p}{(2\pi)^4} (-ip^0) e^{-ip(x-x')} \text{tr} \left\{ \gamma^0 [S_F(p) - S_F^{(0)}(p)] \right\}_{x' \searrow x} \quad (8)$$

between a neutrino propagating in a neutron background [10],

$$S_F(p) = \frac{1}{(p^0 - V)\gamma^0 - \mathbf{p} \cdot \boldsymbol{\gamma} - m + i\varepsilon} P_L, \quad (9)$$

and a neutrino propagating in the vacuum,

$$S_F^{(0)}(p) = \frac{1}{\not{p} - m + i\varepsilon} P_L. \quad (10)$$

Performing first the \mathbf{x} and then the \mathbf{p} integral results in

$$\Delta E = -\frac{i}{2\pi} \int dp^0 p^0 e^{-ip^0(t-t')} \text{tr} \left\{ \gamma^0 [S_F(p^0, \mathbf{0}) - S_F^{(0)}(p^0, \mathbf{0})] \right\}_{t' \searrow t}. \quad (11)$$

Although each of the two contributions to the energy difference ΔE is UV-divergent, a finite final result for ΔE can be obtained combining the two fractions before integrating.

Performing first the subtraction regularizes the integral, because then its leading term in p^0 vanishes,

$$\Delta E = -\frac{iV}{\pi} \int dp^0 p^0 e^{-ip^0(t-t')} \frac{(p^0)^2 - p^0 V + m^2}{[(p^0 - V)^2 - m^2 + i\varepsilon][(p^0)^2 - m^2 + i\varepsilon]}_{t' \searrow t}. \quad (12)$$

The final integral can be done with the help of the residue theorem (cf. Fig. 1) and gives independently of the neutrino mass m

$$\Delta E = V \approx -19 \text{ eV} \frac{n_N}{0.3 \text{ fm}^{-3}}. \quad (13)$$

This result could be expected in virtue of Eq. (5). Since ΔE is the energy difference of one neutrino, we obtain the total contribution ΔM of neutrinos to the self-energy of a neutron star with radius $R \approx 10 \text{ km}$ as

$$\Delta M = \frac{4\pi}{3} R^3 n_\nu \Delta E \approx -42 \text{ kg} \frac{n_\nu}{3 \times 10^{-22} \text{ fm}^{-3}}. \quad (14)$$

Obviously, ΔM is negligible for a realistic value (cf. next sections) of the neutrino density n_ν .

Finally, we want to comment on the possible source of error in the results of Ref. [1, 4]. In Ref. [1], the 8th order term of perturbation theory $W^{(8)}$ was used to estimate ΔE . Since $\sum_k W^{(k)}$ is divergent and alternating, any result obtained in a finite order of perturbation theory is meaningless. By contrast, the authors of Ref. [4] used the same non-perturbative method as we did. However, they made – as apparent from their Eq. (8) – the limit $x' \searrow x$ *before* integrating over x . Then, they argued that after regularization their d^4p integral vanishes due to the antisymmetry of the integrand. However, this is not true because of the presence of the factor $p^0 e^{-ip^0(t-t')}$.

3 Neutrino energy density with neutrino sea

Loeb proposed first that neutrinos¹ with energies below $\sim 50 \text{ eV}$ are bounded inside neutron stars, while antineutrinos are repelled [3]. Qualitatively, this result follows directly from Eq. (5) considering a neutron star as potential wall with depth V and radius R , and simply assuming that all the levels with energy $[0, V]$ are occupied. However, one should note that neutrino states with energy $V < E < -m_\nu$ do not have an exponentially damped wave function for $r > R$, and are resonances instead of true bound states.

To account for a possible neutrino sea inside an neutron star, we can apply either the imaginary or the real-time formalism of finite-temperature field theory. We use the latter since it is easier in this formalism to separate the medium from the vacuum effects. Then

¹In Ref. [9], it is argued that only Dirac neutrinos are trapped while Majorana neutrinos can leave freely the medium.

the thermal neutrino propagator $S_F(p, \beta)$ is given by

$$S_F(p, \beta) = \left[(p^0 - V)\gamma^0 - \mathbf{p} \cdot \boldsymbol{\gamma} \right] \left[\frac{1}{(p^0 - V)^2 - |\mathbf{p}|^2} + 2\pi i \delta((p^0 - V)^2 - |\mathbf{p}|^2) f(p^0) \right] P_L, \quad (15)$$

where $\beta = 1/T$ is the inverse temperature and $f(p^0)$ is the distribution function of the neutrinos. To simplify the notation, we have set $m = 0$. In the case of thermal equilibrium, $f(p^0)$ is the Fermi-Dirac distribution function

$$f_D(p^0) = f_{D,\nu}(p^0) + f_{D,\bar{\nu}}(-p^0) = \frac{\theta(p^0 - V)}{e^{\beta(p^0 - \mu)} + 1} + \frac{\theta(-p^0 - V)}{e^{-\beta(p^0 - \mu)} + 1}. \quad (16)$$

Note that the edge between particles and antiparticles is shifted by V . Inserting the thermal propagator into Eq. (7), performing first the \mathbf{x} integration restricted to the volume \mathcal{V} of the neutron star and then the p_0 integral results in

$$\Delta E_\beta / \mathcal{V} = \int \frac{d^3 p}{(2\pi)^3} [(|\mathbf{p}| + V) f_\nu(|\mathbf{p}| + V) + (-|\mathbf{p}| + V) f_{\bar{\nu}}(|\mathbf{p}| - V)] \quad (17)$$

$$= \int \frac{d^3 p}{(2\pi)^3} [E_\nu f_\nu(E_\nu) - E_{\bar{\nu}} f_{\bar{\nu}}(E_{\bar{\nu}})] \quad (18)$$

$$= \langle E_\nu \rangle_\beta - \langle E_{\bar{\nu}} \rangle_\beta. \quad (19)$$

In the last step, we denoted the thermal average with the (anti-)particle distribution functions by $\langle \dots \rangle_\beta$. Also in this case, the result obtained has a very plausible form.

To obtain a numerical estimate for ΔE_β , we follow Ref. [3] and assume a degenerated neutrino sea with $n_\nu \sim 3 \times 10^{-22} \text{ fm}^{-3}$ and Fermi momentum $p_F \sim 50 \text{ eV}$. Furthermore we set $n_{\bar{\nu}} = 0$ and obtain

$$\Delta E_\beta / \mathcal{V} \sim \left(V + \frac{3}{4} p_F \right) n_\nu \sim 5.5 \times 10^{-21} \frac{\text{eV}}{\text{fm}^3} \frac{n_\nu}{3 \times 10^{-22} \text{ fm}^{-3}}. \quad (20)$$

For a neutron star, the thermal contribution ΔE_β is dominated by the contribution of the kinetic energy of the neutrinos and therefore positive. However, since the number density of neutrinos is much smaller than the number density of neutrons, $n_\nu/n_N \sim 10^{-21}$, the thermal contribution ΔE_β is irrelevant compared to ΔE . Finally, we want to remind that – as mentioned in the introduction – the results obtained in section 2 and 3 are only valid for an uniform neutron background, *i.e.* in the limit of an infinite neutron star.

4 Spontaneous neutrino pair creation

Let us consider in more detail the analogy between a neutron star and a potential wall. We keep now again the neutrino mass m_ν finite and consider Dirac neutrinos. The limit

for the electron neutrino mass is $m_{\nu_e} \lesssim 5$ eV. Hence the potential² $V \sim -50$ eV is overcritical, $V < -m_{\nu_e}$, and therefore able to produce $\nu_e \bar{\nu}_e$ -pairs at its interface. The quantity characterizing this process, $m_\nu^2/|\nabla V|$, is not well-defined in the simple picture of the potential wall. We assume instead that $n_N(\mathbf{x})$ can be approximated by $n_N(\mathbf{x}) = n_N(\mathbf{x}_0) + (\mathbf{x} - \mathbf{x}_0)\nabla n_N(\mathbf{x})$. Then we can treat locally the “electric” field $\mathbf{E} = -\nabla V - \partial_t \mathbf{A}$, where $A = (V, \mathbf{0})$, as uniform and derive the spontaneous neutrino pair creation rate due to a density gradient of the neutron background.

The action $S(A)$ describing the vacuum with a potential A is given by [11, 12]

$$\ln S(A) = \text{Tr} \ln \left\{ S_F^{-1} S_F^{(0)} \right\}. \quad (21)$$

Here, Tr means the trace over Dirac indices and integration over the continuous variable x . Furthermore, S_F denotes the operator with matrix element $\langle x | S_F | x' \rangle = S_F(x, x')$. Adding the transposed version of its RHS to Eq. (21), inserting $CC^{-1} = \mathbf{1}$ and using $C\gamma^5 C^{-1} = \gamma^{5,t}$, $C\gamma_\mu C^{-1} = -\gamma_\mu^t$, we obtain

$$2 \ln S(A) = \text{Tr} \ln \left\{ P_L (\not{P} - \not{A} + m_\nu - i\varepsilon) (\not{P} - \not{A} - m_\nu + i\varepsilon) P_L \right. \quad (22)$$

$$\times \left. \frac{1}{\not{P} - m_\nu + i\varepsilon} P_L \frac{1}{\not{P} + m_\nu - i\varepsilon} \right\}$$

$$= \frac{1}{2} \text{Tr} \ln \left\{ \left[(P - A)^2 - m_\nu^2 + i\varepsilon + \frac{1}{2} \sigma_{\mu\nu} F^{\mu\nu} \right] \frac{1}{P^2 - m_\nu^2 + i\varepsilon} \right\} \quad (23)$$

This is 1/2 of the corresponding result for the pair production of fermions by an uniform electric field. Therefore, we can borrow the final QED result [12] and obtain for the pair creation probability per unit time and volume

$$w = \frac{E^2}{8\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m_\nu^2}{|E|}\right). \quad (24)$$

To obtain an order-of-magnitude estimate for w , we choose a density profile that models roughly a neutron star with a soft Reidl equation-of-state [13]: we assume as radius of the star $R = 10$ km, a homogenous core and a crust with thickness $L = 2$ km, in which the density decreases linearly to zero,

$$n_N(r) = \begin{cases} n_0 & \text{for } r < R - L \\ n_0(R - r)/L & \text{for } R - L < r < R. \end{cases} \quad (25)$$

Then, in a volume $\mathcal{V} \sim 2 \cdot 10^{18}$ cm³ of the neutron star exists a field $|E| = V/L \sim 5 \cdot 10^{-9}$ eV² in radial direction. If $m_\nu^2 \gtrsim |E|/(n\pi)$, the creation of a $\nu\bar{\nu}$ -pair by n field quanta is exponentially suppressed. But even if $m_\nu \ll (|E|/\pi)^{1/2} = 4 \cdot 10^{-5}$ eV, the total luminosity \mathcal{L} is only

$$\mathcal{L} = 2m_\nu w \mathcal{V} \sim 2.1 \cdot 10^{11} \text{ erg/s} \left(\frac{m_\nu}{10^{-6} \text{ eV}} \right) \quad (26)$$

²Since the absolute value of the potential V has no physical meaning, a more correct statement is that the potential difference $\Delta V = V(r < R) - V(r > R)$ is overcritical.

and has therefore practically no influence on the energy budget of the star. However, $\nu_e\bar{\nu}_e$ -pair creation by neutron stars is probably the only case in the present Universe that massive particles are spontaneously created and therefore it is interesting in its own.

5 Summary

We have recalculated the energy of a neutrino in the uniform background of classical neutrons, both without and with a neutrino sea. In the first case, we found that the energy is changed by the small amount $V \sim -20$ eV, in agreement with the well-known result of Wolfenstein [7, 8]. Moreover, the influence of a possible neutrino sea inside the neutron star has an even smaller effect on the neutrino energy. Therefore, the contribution of neutrinos to the total energy of a neutron star seems to be negligible. However, to settle definitely the question if many-body effects become important in neutron stars it is necessary to calculate ΔE not only non-pertubatively but to take into account also the finite geometry of the star [14]. We have estimated the energy-loss of an neutron star due to spontaneous $\nu\bar{\nu}$ -pair production but found it too small to be observable.

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After submission of this work, a preprint of As. Abada, O. Pène and J. Rodrigues-Quintero (hep-ph/9712266) following the line of arguments of Ref. [4] appeared. Their main conclusion is that the border of a finite neutron star (modelled as a square wall) automatically generates the neutrino sea inside the star.

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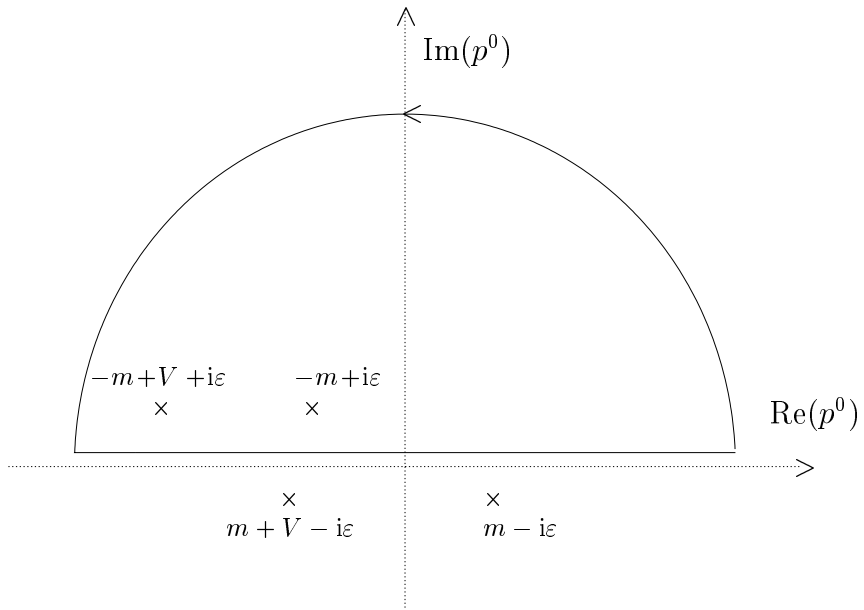


Figure 1: Residues and contour of integration for the evaluation of ΔE , Eq. (12).