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Photon splitting above the pair creation threshold in very strong magnetic fields

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Abstract

We reanalyze the strong-field limit of the photon splitting amplitude in the presence of very strong magnetic fields $B \gg B_{\text{cr}} = m^2/e \approx 4.14 \times 10^{13}$ G. The known approximation valid below the electron-positron pair creation threshold, $k_{\perp}^2 = 4m^2$, is extended above $4m^2$ if the restriction $2eB \gg k_{\perp}^2$ is fulfilled. In particular, the obtained photon splitting rate applies approximately for photons polarized perpendicularly to the magnetic field B and with momenta $2m < k_{\perp} < m + (m + 2eB)^{1/2}$, for which photon splitting above the threshold is the only process which limits their lifetime.

Photon splitting, i.e. the decay of one photon into two photons, is forbidden in vacuum by the Furry theorem. Therefore this process is only allowed in the presence of an external field. From the theoretical point of view, photon splitting is of particular interest because it is induced by the nonlinearity of the quantum action of the electromagnetic field and, consequently, is a genuine quantum phenomenon. However, it still awaits experimental observation.

In astrophysics, photon splitting has been used for modelling spectra of neutron stars, gamma ray burst sources and, more recently, of soft gamma ray repeaters (SGRs) [1]. Especially in the case of SGRs, photon splitting has the potential of explaining the particular softness of their spectra. A field estimate by Thompson and Duncan [2] for the SGR of March 5, 1979, yields a magnetic field $B \sim 6 \times 10^{14}$ G for this repeater. This exceeds by a factor of 15 the critical field of the electron $B_{\text{cr}} = m^2/e \sim 4.14 \times 10^{13}$ G. For such strong fields, the gap in the evaluation of the possible attenuation processes for photons considered so far becomes important: While photons with polarization vector parallel (\parallel) to the plane formed by their momentum \vec{k} and the magnetic field \vec{B} can produce an electron-positron pair for energies $\omega/\sin\theta > 2m$, where $\theta = \angle(\vec{B}, \vec{k})$, perpendicularly (\perp) polarized photons require an energy $\omega/\sin\theta > m + \sqrt{m^2 + 2eB}$ to produce an electron-positron pair [3]. Consequently, photon splitting above the pair creation threshold $k_{\perp} = \omega/\sin\theta = 2m$ is the only process capable of limiting the lifetime of such photons.

The investigation of photon splitting in an external magnetic field started soon after it had been proposed that strong magnetic fields $\mathcal{O}(B) = 10^{13}$ G exist in pulsars. In first attempts to calculate the photon splitting rate the lowest term of an expansion of the Heisenberg-Euler (HE) effective Lagrangian was used [4]. In this way, the polarization selection rules and the photon splitting rate for $\omega \ll 2m$ and $B \ll B_{\text{cr}}$ were derived. The first full calculation of the photon splitting process below the pair creation threshold was performed by Adler [5]. He used the exact Green's function of the electron in an external magnetic field in the Schwinger proper-time representation to derive the rate for $\omega < 2m$ and arbitrary B . Additionally, he calculated the splitting rate for $\omega \ll 2m$ and arbitrary magnetic fields using the full HE Lagrangian.

Recently, Mentzel *et al.* [6] recalculated the photon splitting rate using as representation for the electron propagator in a magnetic field an infinite sum over the solutions of the Dirac equation. They calculated numerical values of splitting rates that were in contradiction to all earlier results, and in fact turned out to be incorrect. In this way, ref. [6] and the critique in ref. [7] initiated several new investigations of photon splitting [8, 9, 10, 11]. In one of these, Baier *et al.* [8] presented a new asymptotic formula for the photon splitting rate valid in the limit of very high magnetic fields $B \gg B_{\text{cr}}$ and for $\omega < 2m$. Subsequently, it was shown [10] that this formula represents a good approximation to the exact one already for $B \sim 5 B_{\text{cr}}$.

In this note, we use the formula given by Baier *et al.* for $\omega < 2m$ to compute the splitting rate of perpendicularly polarized photons for $4m^2 < k_{\perp}^2 \ll 2eB$. Although the splitting rate was derived by Baier *et al.* [12] for arbitrary ω , and Adler's result [5] for $\omega < 2m$ can be modified to also cover the case $\omega > 2m$, the resulting expressions are too complicated as to lend themselves to immediate evaluation for arbitrary B .

We consider a photon with energy ω that splits into two photons with energies ω_1 and ω_2 . Since a Lorentz transformation along the field direction does not change \vec{B} , it is enough to consider photons propagating perpendicularly to the field. It was shown [4] that, out of the six transitions possible with respect to the photon polarizations, only the transition $\perp \rightarrow \parallel\parallel$ is allowed by CP -invariance and dispersion effects of the magnetized vacuum. For $B \gg B_{\text{cr}}$, Baier *et al.* found that the amplitude $T(\perp \rightarrow \parallel\parallel)$ is independent of the magnetic field. They obtained in this limit

$$T(\perp \rightarrow \parallel\parallel) = \frac{2(4\pi\alpha)^{3/2}m^2}{\pi^2} \left[\frac{\omega_1\Omega_2}{\omega_2^2} \arctan \Omega_2 + \frac{\omega_2\Omega_1}{\omega_1^2} \arctan \Omega_1 - \frac{\omega}{4m^2} \right] \quad (1)$$

with

$$\Omega_i = \frac{\omega_i}{\sqrt{4m^2 - \omega_i^2}}, \quad i = 1, 2. \quad (2)$$

From now on, we use the abbreviation T for $T(\perp \rightarrow \parallel\parallel)$. The photon splitting rate R is given by

$$R = \frac{1}{32\pi\omega^2} \int_0^\omega d\omega_1 |T|^2. \quad (3)$$

We now make use of the fact that, although eq. (1) was derived under the assumption $\omega < 2m$, the amplitude T can be *analytically continued* into the complex $\omega_{1/2}^2$ planes. The discontinuities $\text{Disc}(T)$ across the branching cuts starting at $\omega_{1/2}^2 = 4m^2$ give rise to an imaginary part of the amplitude, viz. $2i\Im(T) = \text{Disc}(T) = T(\omega_{1/2}^2 + i\varepsilon) - T(\omega_{1/2}^2 - i\varepsilon)$. The imaginary part then follows as

$$\Im(T) = \frac{(4\pi\alpha)^{3/2}m^2}{\pi} \left[\frac{\omega_1\Phi_2}{\omega_2^2} \theta(\omega_2 - 2m) + \frac{\omega_2\Phi_1}{\omega_1^2} \theta(\omega_1 - 2m) \right], \quad (4)$$

where

$$\Phi_i = \frac{\omega_i}{\sqrt{\omega_i^2 - 4m^2}}, \quad i = 1, 2 \quad (5)$$

and $\theta(\omega)$ is the step function. Note the strong formal similarity between the terms $\Omega_i \arctan \Omega_i$ that give rise to the imaginary part of photon splitting and the corresponding term appearing in the vacuum polarization amplitude without external fields [13]. For the real part of T we find

$$\begin{aligned} \Re(T) = & -\frac{2(4\pi\alpha)^{3/2}m^2}{\pi^2} \left\{ \frac{\omega}{4m^2} + \frac{\omega_1}{\omega_2^2} \left[\Phi_2 \text{arctanh} \Phi_2 \theta(\omega_2 - 2m) \right. \right. \\ & \left. \left. - \Omega_2 \arctan \Omega_2 \theta(2m - \omega_2) \right] \right. \\ & \left. + \frac{\omega_2}{\omega_1^2} \left[\Phi_1 \text{arctanh} \Phi_1 \theta(\omega_1 - 2m) - \Omega_1 \arctan \Omega_1 \theta(2m - \omega_1) \right] \right\}. \quad (6) \end{aligned}$$

The amplitude contains only the lowest lying singularity at $\omega_{1/2} = 2m$, but not singularities due to excited Landau levels at $\omega_{1/2} = \sqrt{m + 2NeB} + \sqrt{m + 2N'eB}$. Therefore, eq. (4) and (6) are only valid for $2eB \gg k_{\perp}^2$. As a test for this method, we have derived the imaginary part of the vacuum polarization and have verified that it agrees with the 1γ -pair creation rate in the strong field limit $2eB \gg k_{\perp}^2$.

The differential splitting rate

$$\frac{dR}{d\omega_1} = \frac{1}{32\pi\omega^2} |T|^2 \quad (7)$$

can now be computed for $\omega_i^2 \neq 4m^2$. However, as the singularities of $|T|^2$ at $\omega_i^2 = 4m^2$ are not integrable, the total splitting rate R above the pair creation threshold is still ill-defined. Near the threshold, higher order corrections become important. Therefore the vacuum dispersion relation $\omega^2 - k_{\perp}^2 - k_{\parallel}^2 = 0$ of the two parallel polarized photons has to be replaced by

$$n_{\parallel}^2 \omega^2 - k_{\perp}^2 - k_{\parallel}^2 = 0, \quad (8)$$

where n_{\parallel} is the refractive index of the parallel polarized photons in the magnetized vacuum [14, 15]. In the strong field limit, the real part of the refractive index n_{\parallel} is given by $n_{\parallel}^2 = 1 + \frac{\alpha}{3\pi} \sin^2 \theta \frac{B}{B_{\text{cr}}}$. Although in most applications of astrophysical interest the deviations of the refractive indices n_{\parallel} and n_{\perp} from one are small and can be safely discarded, the real part of n_{\parallel} plays an essential role: It displaces the poles from $k_{\perp,1/2}^2 = 4m^2$ to $k_{\perp,1/2}^2 = 4m^2 n_{\parallel}^2 > 4m^2$. For $k_{\perp,1/2}^2 > 4m^2$, the imaginary part of the refractive index is non-zero and pushes the singularities from the real axis¹. In this way, the splitting rate becomes finite and slightly dependent on B .

Let us now consider the photon splitting rate R in the high energy limit $k_{\perp} \gg m$ but $k_{\perp}^2 \ll 2eB$. Then we can neglect the terms $\mathcal{O}(m^2)$ in T and obtain a total splitting rate R independent of n_{\parallel} ,

$$R = \frac{1}{32\pi\omega^2} \int_0^{\omega} d\omega_1 \frac{(4\pi\alpha)^3}{4\pi^4} \omega^2 = \frac{\alpha^3}{2\pi^2} \omega. \quad (9)$$

Since in this limit the only energy scale associated with photon splitting is ω (remember that T is independent of B), the dependence $R \propto \alpha^3 \omega$ follows already from a naive dimensional analysis and Lorentz invariance.

We now discuss our numerical evaluation of the differential photon splitting rate, which were computed with $\theta = 90^\circ$ and for the channel $\perp \rightarrow \parallel\parallel$. In the figures, the differential attenuation coefficient per cm and energy unit m are shown for $\omega = 5m$ and $B = 25B_{\text{cr}}$ as a function of the energy ω_1/m of one of the two produced photons. In Fig. 1, the rate was calculated assuming the vacuum dispersion relation $k^2 = 0$ for the photons. Here the two resonances at $\omega_1 = 2m$, $\omega - 2m$ can be clearly recognized. In Fig. 2, the frequency shift due to the (real) refractive index $n \neq 1$ as well as the imaginary part due to the finite life

¹See ref. [15] for a detailed discussion how the refractive index is connected with the polarization operator and the 1γ -pair creation rate calculated on-shell.

time of the perpendicularly polarized photons were taken into account. The imaginary part was calculated using the exact 1γ -pair creation rate of ref. [3]. As a consequence, the two resonances have practically disappeared. Between the position of the two resonances, the plateau observed by Baier *et al.* in the case $\omega < 2m$ is visible. Note that the height of the plateau is approximately fixed according to Eq. (9).

In conclusion, we have presented the first analysis of photon splitting attenuation coefficients for photon energies *above* the pair creation threshold in the limit of very strong magnetic fields $2eB \gg k_{\perp}^2$. We have derived a simple formula for the total splitting rate in this limit, which applies approximately for those photons for which photon splitting above the threshold is the *only* process which limits their lifetime.

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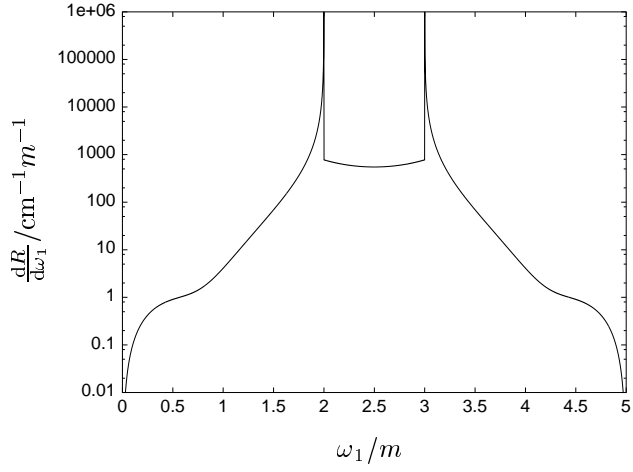


Figure 1: Differential attenuation coefficient for photon splitting $\frac{dR}{d\omega_1}/\text{cm}^{-1}m^{-1}$ for $2eB \gg k_{\perp}^2$ as function of ω_1/m for $\omega = 5m$ with $k^2 = 0$.

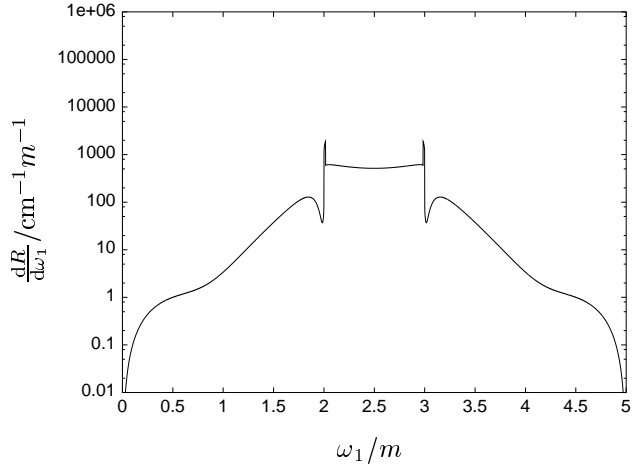


Figure 2: Differential attenuation coefficient for photon splitting $\frac{dR}{d\omega_1}/\text{cm}^{-1}m^{-1}$ for $2eB \gg k_{\perp}^2$ as function of ω_1/m for $\omega = 5m$ with $n_{\parallel}^2\omega^2 - |\mathbf{k}|^2 = 0$.