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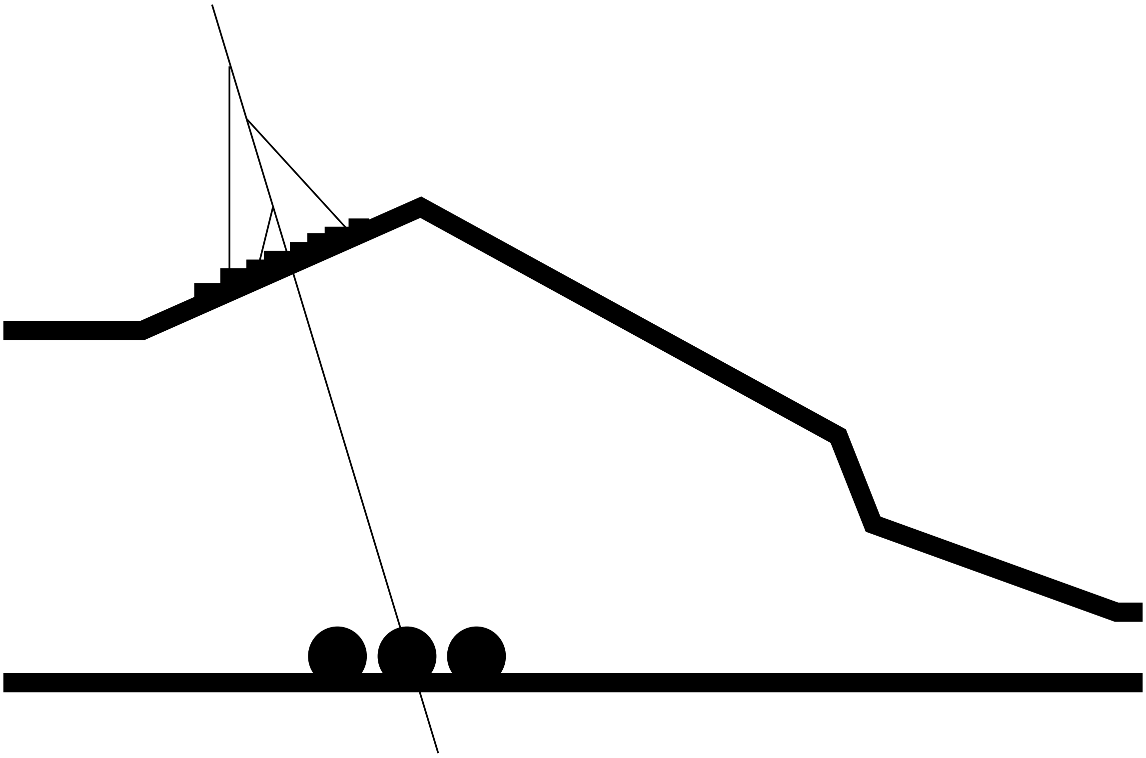

A Statistical Approach to Leptonic Mixings and Neutrino Masses

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Abstract

After an overview of what we know on massive neutrinos, we pass to argue for a peculiar structure of the neutrino mass matrix; namely, we suggest the existence of a block with relatively large elements (=dominant block). We extract predictions from this ansatz, by assuming that the $\mathcal{O}(1)$ coefficients that are needed to define the model fully are random variables. Further insights are obtained when we postulate that this structure originates from U(1) selection rules (a la Frogatt and Nielsen). A particularly interesting case emerges, when the angle θ_{13} (the mixing U_{e3}) is within reach for next generation experiments, and the large mixing angle solution for solar neutrinos is preferred.

1 What we Know on Massive Neutrinos

1.1 Snapshot

In the first table, we recall certain established facts about neutrinos,

LEP	ν_a	3 active (=interacting) ν's
Big-bang N.S.	$\nu_a,\ \nu_s$	≤ 4 ν's in thermal equilibrium

where I use the notation: ν = generically, a neutrino (or antineutrino); ν_s = a sterile (=non-interacting) ν-state; ν_μ = muon neutrino, *etc.*; ν_a = anyone among $\nu_{\rm e}, \nu_\mu, \nu_\tau$ (active state are not distinguished by neutral current interactions–NC in the following). Here a list of observed (or claimed) evidences of oscillation:

atm-ν	ν_μ	$-$	especially low E_ν, large L
	$\nu_{\rm e}$	$=$	checked at reactors
	ν_a	$+$	Super-Kamiokande (SK) NC data
sol-ν	$\nu_{\rm e}$	$-$	E_ν dependence only in total rates
	ν_a	$+$	SK+SNO
LSND-ν	$\overset{(-)}{\nu}_{\rm e}$	$+$	waiting for independent confirmation
SN1987A-ν	$\overline{\nu}_{\rm e}$	$=?$	just 19 events; theoretical uncertainties

1st column, ν-experiment (symbolical); *2nd*, pertinent type of neutrino; *3rd* column, what is presumably occuring, if disappearance "$-$" of that type of neutrino, or appearance "+", or neither of them "="; *e.g.* there is no claim for disappearance of atmospheric $\nu_{\rm e}$. *4th* column, some comments. Note: Two cases for appearance are made by NC-, one by CC-events; all three are of a similar significance, $\sim 3\sigma$.

1.2 Atmospheric Neutrinos & CHOOZ

Super-Kamiokande has made a strong case for oscillations with large mixing and:

$$\Delta m^2_{atm} = (1.5 - 5) \times 10^{-3} \text{ eV}^2$$

Their results are supported by MACRO and SOUDAN2. In particular the quality of data is so high that in these experiments L/E_ν modulation is visible, and indeed, the hypothesis of oscillation of ν_μ into a sterile state can be strongly disfavored. (Few remaining doubts are related to calculated ν fluxes, constraints of new cosmic ray data, hadronic uncertainties, and Baksan results).

Few models have been concocted, that reproduce the same features of oscillations of massive neutrinos in existing detectors; but the simplest explanation of a big set of data is almost pure $\nu_\mu \to \nu_\tau$ oscillations:

$$\theta_{23} = (45 \pm 10)^\circ \quad \text{and} \quad \theta_{13} < 10^\circ$$

The result on θ_{13} is merit of the reactor experiment CHOOZ.

1.3 Solar Neutrinos

- The evidence for non-standard physics is compelling (e.g. GALLEX/GNO and SAGE are 5 σ away from expected values). It is natural to assume that this is a manifestation of ν masses, as for atmospheric ν, with $\Delta m^2_{sol} \ll \Delta m^2_{atm}$.

- Total ν-counting rates (with SSM) point to certain "solar ν solutions" (with shorthands SMA, LMA, LOW, VO, ...) Homestake is (still) crucial; for instance LOW solution is is disfavored by these results.

- Differential ν-counting rates at Super-Kamiokande give exclusion regions: this "negative evidence" is one reason why LMA (the large mixing angle solution with $\theta_{12} \in [21°, 41°]$) is favored in existing analyses. However, first SNO results reinforce this inference. Unfortunately, the day-night signal at SK is just a 1.5 σ effect.[1]

It is used to say that "neutrinos are for patient people", but it seems that SNO NC data, together with Borexino/KamLAND results on longer term will satisfy even the impatient ones...

1.4 The Scandal of LMA

This solution points to unexpected ν properties, not only because θ_{12} is $2-3$ times larger than the Cabibbo angle θ_C, but also because of the weak "hierarchy":

$$\Delta m^2_{21}/\Delta m^2_{31} \sim 1/20 - 1/100$$

(compare it with charged fermion analogues). Whence, the theoretical question arises:

WHAT ν's ARE TELLING US?

In a few pages, I will show you some guesswork on this point.

2 Arguments for a Dominant Block

2.1 Five Assumptions

Here are the ingredients we need:

1. There are 3 ν that mix among them. This explains solar and atmospheric flux deficits. By definition, $m_1 < m_2 < m_3$

2. LSND has 3.2 σ signal, but before interpretation we wait for confirmation.

[1] SN1987A electron anti-neutrino signals favor as little solar mixing as possible for LMA, together with certain values of Δm^2_{sol}.

3. There is a bunch of "small parameters",

$$\begin{cases} (\Delta m^2_{sol}/\Delta m^2_{atm})^{1/2} \\ |U_{e3}| \sim \theta_{13} \\ |U^2_{\mu 3}| - 1/2 \sim \theta_{23} - \pi/4 \end{cases}$$

let's term them collectively ε (adding a bit of prejudice).

4. The neutrino mass spectrum "resembles" the usual ones, namely it is of the type  , not of the type[2]  (inverted spectrum) $\Rightarrow \Delta m^2_{21} = \Delta m^2_{sol}$ and $\Delta m^2_{31} = \Delta m^2_{atm}$.

5. The mass m_1 is not large[3] in comparison with the smallest oscillation scale, $\sqrt{\Delta m^2_{sol}}$

Admittedly, this is quite a heavy mix of solid information and prejudice–though, all assumptions seem, at least, defensible.

2.2 Inference

Let us begin by including only the biggest mass scale $m_3 \sim (\Delta m^2_{atm})^{1/2}$ in $\mathbf{M}_\nu$:

$$\mathbf{M}_\nu = m_3 \; v_3 \otimes v_3 \quad \text{with } v_3 \approx (\varepsilon, s^{-1}, s^{-1}) \text{ where } s = \sqrt{2}$$

This, taken literally, implies:

$$\mathbf{M}_\nu \propto \begin{pmatrix} \varepsilon^2/2 & \varepsilon/s & \varepsilon/s \\ \varepsilon/s & 1 & 1 \\ \varepsilon/s & 1 & 1 \end{pmatrix} \tag{1}$$

Here is the "dominant block"! Adding $m_2 \; v_2 \otimes v_2$ and $m_1 \; v_1 \otimes v_1$ modifies the elements of the matrix by terms order ε and lifts the determinant of the "dominant block" from 0.

Actually, it might be that the element $(\mathbf{M}_\nu)_{ee}$ remains $\mathcal{O}(\varepsilon^2)$, if the two little contributions tend to compensate each other, due to Majorana phases.

2.3 Can we Weaken the Assumptions?

What if we use only ν_{atm} (that is, the strongest and most reliable evidence for oscillations)? We could tell very little on mass matrix.[4] This is quite evident, after trying to imagine what these mass matrices (in eV) have in common:

$$10^{-2} \times \begin{pmatrix} 2.36 & 2.71 \\ 2.71 & 3.12 \end{pmatrix}, \quad 10^{-2} \times \begin{pmatrix} 0 & 7.27 \\ 7.27 & 2.04 \end{pmatrix},$$

[2] This hypothesis saves us from the need of operating a fine-tuning on a certain mixing. In fact, SN1987A $\overline{\nu}_\mu$, $\overline{\nu}_\tau$ were probably not converted to $\overline{\nu}_e$, since the measured energy is already quite low when compared with expectations.

[3] This hypothesis saves us from the need of fine-tunings: if we play to increase m_1, we have to tune more and more the mass differences, since $m_j - m_i \sim \Delta m^2_{ji}/(2m_1)$.

[4] Of course, we could still get the dominant block if we renounce to explain solar neutrinos, but maintain the assumptions that the spectrum is not "inverted" there is no m_1 offset, and m_3 is the biggest mass scale.

$$\begin{pmatrix} -9/65 & 91/92 \\ 91/92 & 7/50 \end{pmatrix}, \quad \mathbb{1} + 10^{-3} \times \begin{pmatrix} 20/31 & 20/27 \\ 20/27 & 29/34 \end{pmatrix};$$

you may check that they all have $\Delta m^2 = 3 \times 10^{-3}$ eV^2 and $\theta = 41°$. Thus, they would be un-distinguishable by even an ideal atmospheric neutrino experiment, and no doubt that we are not in the ideal situation.[5]

In other words, we are quite far from complete information!!! Or, from another point of view, there is space for speculations (theory).

3 Neutrino Mass Matrix with a Dominant Block

3.1 An "Electronic" Selection Rule

We assume that the structure of mass matrix (1) is dictated by a selection rule, that requires that the elements with electron flavor have to pay some suppression factor ε :

$$\mathbf{M}_\nu \overset{\mathcal{O}(1)}{=} \frac{\langle H \rangle^2}{M_X} \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} \tag{2}$$

where $M_X = (0.8 - 1.6) \times 10^{14}$ GeV, $\langle H \rangle = 174$ GeV, and there is a bunch of $\mathcal{O}(1)$ coefficients. There is a number of important remarks:

- This is a *class* of mass matrices.
- Mass scale is fixed by hand; but *adimentional quantities* can be predicted.
- We use $\mathcal{O}(1)$ coefficients with random phase, and modulus=1 ± 20 % .

The last point is the most important. It means that we do not pretend to understand the details of the underlying theory; we concentrate on the "gross" structure.

3.2 Underlying Mass Mechanisms

Previous mass matrix might be due to the *vev* of a scalar triplet Δ, with family dependent couplings to leptons, or even to seesaw mechanism:

$$\mathbf{M}_\nu = \langle H \rangle^2 \mathbf{Y}_\nu \mathbf{M}_R^{-1} \mathbf{Y}_\nu^t$$

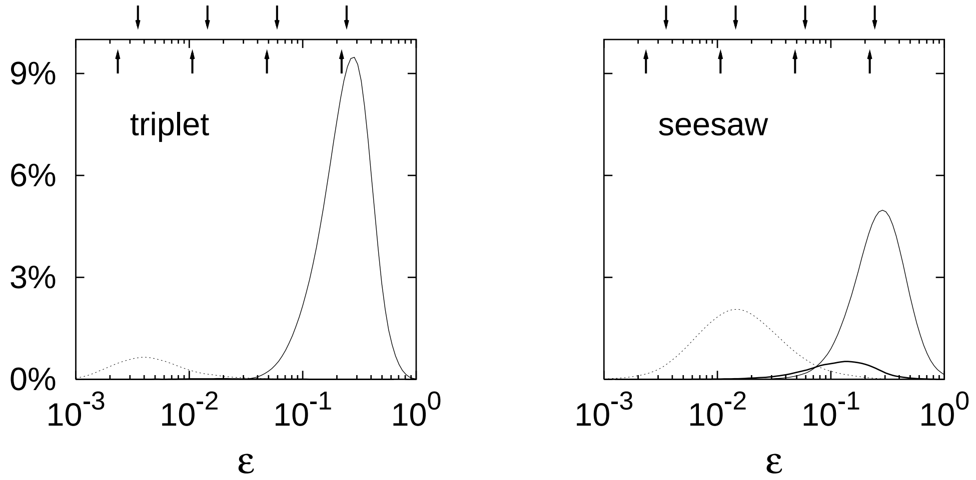
Proof: The hypothesis of family dependent couplings reads:
$\mathbf{Y}_\nu = \mathrm{diag}(\varepsilon, 1, 1)\, \mathcal{O}(1)\, \mathrm{diag}(\varepsilon^{n_1}, \varepsilon^{n_2}, \varepsilon^{n_3})$ and $\mathbf{M}_R = \mathrm{diag}(\varepsilon^{n_1}, \varepsilon^{n_2}, \varepsilon^{n_3})\, \mathcal{O}(1)\, \mathrm{diag}(\varepsilon^{n_1}, \varepsilon^{n_2}, \varepsilon^{n_3})$
$\Rightarrow$ the powers of ε attached to "right-handed" neutrinos cancel in the light ν mass matrix (not in all observables).

Beware of $\mathcal{O}(1)$ matrices! $\mathcal{O}(1)^{-1} \neq \mathcal{O}(1) \Rightarrow$ "triplet" and "seesaw" yield different outcomes.

[5] They differ because of parameters that are irrelevant to oscillations: m_1 and the Majorana phases.

3.3 Check with Phenomenology

Now, what remains to be done is, basically, to toss the dices and wish that the model is successful. In next plot, we show the percentage of success as a function of ε.



Dashed line corresponds to SMA region, continuous thin line to LMA, thick one to LOW. We emphasize certain special values of ε : $\varepsilon = (m_\mu/m_\tau)^{0.5,1,1.5,2}$, *arrows pointing downward;* $\varepsilon = (\sin\vartheta_C)^{1,2,3,4}$, *arrows pointing upward.*

Let us comment upon this result:

⋆ For certain ε's, there are many successful mass matrices.

⋆ The value $\varepsilon = 1$ is not particularly good (especially for triplet case). Decreasing ε, the cut on θ_{13} (CHOOZ) becomes ineffective, and an LMA peak arises.

⋆ In the triplet case, the success takes place for well separated set of values of ε; LMA is rather prominent.

⋆ In the seesaw case, solutions like LOW are often found. Note that this solution needs a big hierarchy, namely a little $(\Delta m^2_{sol}/\Delta m^2_{atm})^{1/2}$.

⋆ Why there is a correlation between ε and the solar ν solutions? Let us perform an approximate diagonalization in the dominant block. The ν mass matrix becomes:

$$\mathbf{M}_\nu \propto \begin{pmatrix} \varepsilon^2 & \varepsilon & 0 \\ \varepsilon & \delta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

δ depends on the the dominant block: it can be little especially for seesaw mass mechanism, because $\mathcal{O}(1)^{-1} \neq \mathcal{O}(1)$ for matrices. Given δ, SMA prefers certain small values of ε; similarly there is an optimal value of $\varepsilon \sim \delta$ where LMA and LOW arise.

The question becomes then: What is the value of ε?

3.4 More Guesswork & Some Theory

(Let us make a step back.) Froggatt and Nielsen suggested that gross structure of quark mass matrices is "explained" by a small[6] ratio v/M, and a *flavor and field dependent* set of charges $Q(q_i)$ and $Q(q_i^c)$ such that:

$$\mathcal{L}_{mass} \in q_i q_j^c \times \mathcal{O}(1)_{ij} \times \left(\frac{v}{M}\right)^{Q(q_i)+Q(q_j^c)} + h.c.$$

It seems we are doing just the same for leptons! Let us buy U(1) selection rules.
Since the charges of left leptons are ~fixed, only few choices for the right leptons charges reproduce the correct mass hierarchies. Optimal values for $v/M = (m_\mu/m_\tau)^{1/2} \sim \sin\theta_C$ are:

$Q(\mathrm{e})$	$Q(\mu)$	$Q(\tau)$	$Q(\mathrm{e}^c)$	$Q(\mu^c)$	$Q(\tau^c)$	ε (degrees)
3	0	0	3	2	0	.83°
2	0	0	4	2	0	3.4°
1	0	0	5	2	0	14.°

The value of $Q(\mu^c)$ is needed for m_μ/m_τ; the sum rule of $Q(\mathrm{e})+Q(\mathrm{e}^c) = 3\times Q(\mu^c)$ is needed for $m_\mathrm{e}/m_\tau \approx (m_\mu/m_\tau)^3$. Thus, in these assumptions, we arrive at the striking conclusion that:

$\varepsilon = (v/M)^{Q(\mathrm{e})}$ comes in quantised values !!!

Note $Q(\nu_\mu)$ and $Q(\nu_\tau)$ are the same–degenerate charges– that formally is licit, but a bit odd in the *spirit* of the approach (maybe, neutrinos are really a bit odd).

3.5 Implications

If one takes the point of view of Froggatt and Nielsen, there is a big simplification (in that only certain values of ε are expected to arise), but there is a part of the analysis above that has to be redone.

In fact, the ν mixing matrix receives a contribution from the charged lepton mass matrix:

$$(\mathbf{M_E})_{ij} = (v/M)^{Q(l_i)}\ \mathcal{O}(1)_{ij}\ (v/M)^{Q(l_j^c)} \tag{3}$$

(in other words, in these assumptions the charged lepton mass matrix is not diagonal in the flavor basis from which we start). This new contribution to the ν mixing matrix is similar in size to the one of the neutrino mass matrix itself; this is not irrelevant, since, for instance, $\theta_{13} \sim \varepsilon$ and the probability of survival of electron neutrinos in vacuum is:

$$P_{\nu_\mathrm{e}\to\nu_\mathrm{e}} \propto \theta_{13}^2$$

[6]However, the parameter v/M cannot be too small, if one wants to explain $\sin\theta_C$ itself.

Actually, we calculate this new mixing only if the ratios $R_\ell = m_\ell/m_\tau$ are sufficiently well reproduced $\sum_{\ell=e,\mu}(R_\ell(th.)/R_\ell(exp.)-1)^2 < (30\%)^2$; this happens in ~ 20 % of the cases and permits to avoid patological situations.

For more details please check the following table:

$.83^\circ$	$45-\theta_{23}$	θ_{13}	θ_{12}	h	$m_{ee}/10^{-4}$
t,w/o	±12	.37±.19	1.0±1.4	.35±.26	1.4±3.3
t,w	±23	.70±.33	1.2±1.4	.35±.26	2.9±1.7
s,w/o	±17	.52±.29	1.3±1.7	.12±.16	1.4±1.3
s,w	±21	.79±.41	1.5±1.7	.12±.16	2.9±2.4
3.4°					$m_{ee}/10^{-3}$
t,w/o	±12	1.5±0.8	3.8±3.8	.35±.26	2.4±0.6
t,w	±23	2.9±1.4	4.6±3.8	.35±.26	4.9±2.9
s,w/o	±17	2.1±1.2	5.0±5.0	.12±.16	2.3±2.1
s,w	±21	3.3±1.7	5.7±5.1	.12±.16	4.9±4.0
$14.^\circ$					$m_{ee}/10^{-2}$
t,w/o	±12	6.2±3.2	12.5±8.4	.36±.26	4.0±0.9
t,w	±23	11.8±5.6	16.3±9.3	.36±.26	7.9±4.6
s,w/o	±17	8.7±4.6	17.1±12.3	.13±.17	3.7±3.1
s,w	±21	13.1±6.6	20.0±12.6	.13±.17	7.6±5.9

We show the calculated neutrino properties assuming triplet or seesaw (`t` and `s` resp.) mass mechanism, and with or without the account of the lepton mixing matrix U_E (`w` and `w/o` resp.). Note that:
• The 3 parts of the table correspond to the models defined in previous table (in the left-upper corners, the values of ε in degrees are recalled).
• Here, $h = \Delta m^2_{sol}/\Delta m^2_{atm}$ and $m_{ee} = |(\mathbf{M}_\nu)_{ee}|/(\Delta m^2_{atm})^{1/2}$; the angles θ_{ij} are those of neutrino mixing matrix in the most common (PDG) parameterization. All angles in the table are in degrees.

4 Summary and Discussion

$\star$ We studied ν mass matrices with a dominant block and a free parameter, ε. This ansatz is motivated by a variety of considerations (in particular, the large value of θ_{23}).
$\star$ Using random number generators, we scanned the various possibilities and emphasised the most likely outcomes.
$\star$ The **triplet** mass mechanism wants little hierarchy and thence disfavors LOW and (less strongly) SMA solutions. It is more predictive than the **seesaw** mechanism, and it likes LMA.
$\star$ There is an interesting class of mass matrices with $\varepsilon \sim \sin\theta_C$ (see especially last table). They have large θ_{13} and give some chance of success for next generation $0\nu2\beta$ experiments, due to the scaling $(\mathbf{M}_\nu)_{ee} \propto \varepsilon^2$.

⋆ Rotations operating on charged leptons (due to U(1) selection rules) increase *(1)* the spread of θ_{23} around 45° (unfortunately) *(2)* the expected θ_{13} *(3)* and $|(\mathbf{M}_\nu)_{\rm ee}| = m_{\rm ee} \times (40-70)$ meV.

In conclusion, let us stress that what we presented is an appealing framework for massive neutrinos, more than a compelling theory, that however–theoretically modest as it is–is able to give hints for future experiments. Probably, one should not take these considerations *too* seriously; we have a rather limited experimental information and this makes all too easy to find a successful model at present. However, present data certainly point to important features of massive neutrinos; simple and motivated theoretical proposals may help us to delimit the field of what is known, and may perhaps suggest useful new views.

Note to Bibliography

The present study is about theoretical speculations, but Section 1 is mostly based on experimental facts: [1, 2, 3, 4, 5] and little theoretical ingredients [6]. The simple minded argument of Section 2 are taken from [7]; it subtends also [8]. Together with the seminal paper [9], the works in [8] form the conceptual basis of Section 3, which reproduces several results of [10] (this is the main reference for more details). Other relevant works are [11] (the case $\varepsilon = 0$ and the use of random number generators) [12] (the case $\varepsilon = m_\mu/m_\tau$), and [13]. A similar (but different) class of mass matrices is the one denoted as "lopsided" [14], where is is assumed that the neutrino mixing comes mostly from $\mathbf{M_E}$.

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