

## Dynamical $CP$ violation and FCNC processes

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## Dynamical $CP$ violation and FCNC processes

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### Abstract

We investigate the phenomenological constraints on a model where, besides the standard model Higgs sector, there is an effective new strong interaction acting on the third generation of quarks and characterized by a  $\theta$ -like term. This  $\theta$  term induces electroweak symmetry breaking and leads to dynamical spontaneous  $CP$  violation. We show that the constraints coming from  $K$  physics and the electric dipole moment of the neutron impose that the new physics scale should be of the order of 35 TeV. Contrary to naive expectations, the predictions of the model for  $B$  physics are very close to the standard model ones. The main differences appear in processes involving the up quarks such as  $D^0 - \bar{D}^0$  mixing and in the electric dipole moment of the neutron, which should be close to the experimental limit. Possible deviations from the standard model predictions for  $CP$  asymmetries in  $B$  decays are also considered.

# 1 Introduction

The fact that the top quark is much heavier than the other quarks,  $m_t = 174.3 \pm 5.1$  GeV [1], is suggestive of a new dynamics at the electroweak scale, where the third generation may be playing a special role. In particular, effective four-fermion interactions [2] can lead to the formation of quark-antiquark bound states which in turn can dynamically trigger the breaking of the electroweak symmetry [3, 4]. This is the basic idea of top-quark condensation as well as of technicolor models, i.e. the Higgs sector of the standard model is just an effective Ginzburg-Landau-type description of low-energy physics represented by a composite isodoublet scalar field (or fields) [5].

In the above framework, a particularly interesting scenario is provided by models where the top quark mass arises mostly from a  $t\bar{t}$  condensate, generated by a new strong dynamics, plus a small fundamental component, generated by an extended technicolor or Higgs sector [6]-[10]. Such a structure for the top quark mass avoids the problems usually found in pure (minimal) top-quark condensation scenarios, which assume that the  $t\bar{t}$  condensate is fully responsible for the electroweak symmetry breaking [4], thus leading to a too large  $m_t$  value ( $m_t \gtrsim 220$  GeV) and a very large scale for the new dynamics ( $\Lambda \sim 10^{15}$  GeV) with significant fine tuning.

Along this line, a dynamical scheme was proposed in Ref. [11], where it is assumed that the third generation of quarks does indeed experience new forces, symmetric in  $t$  and  $b$ , and that these new forces also generate a strong  $CP$  phase  $\theta$ . It is then possible to show that, in such a scenario, the  $\theta$  term triggers the breaking of the symmetry between  $t$  and  $b$  and induces a large  $CP$ -violating phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, due to the smallness of the  $m_b/m_t$  mass ratio [11]. In this model one expects to have a richer low-energy phenomenology when compared to the standard model (SM), which could lead to potentially interesting effects, specially in  $K$  and  $B$  physics.

The purpose of this paper is to study the low-energy phenomenological implications of the model proposed in [11] and, in particular, its implications for  $K$  and  $B$  physics. We will show that new observable effects arise due to the fact that the third generation of quarks experiences new strong forces which in turn lead to scalar flavour-changing neutral current (FCNC) interactions at tree level. These FCNC interactions result from the fact that both the up and down quark mass matrices receive contributions not only from Yukawa interactions with the standard Higgs but also from interactions involving the third generation quark-antiquark bound states.

The present and near future experiments at  $B$  factories and the large hadron collider (LHC) will certainly improve the bounds on many of the  $CP$ -violating and flavour-changing processes, which are forbidden or strongly suppressed in the SM. Therefore it is particularly interesting to determine possible experimental signatures in models involving new FCNC physics.

## 2 The model

In this section we shall briefly present the main features and physical consequences of the model in question. A more complete and detailed analysis can be found in Ref. [11].

We consider a standard model Higgs sector in combination with an effective new strong interaction acting on the third generation of quarks and characterized by a  $\theta$  term. We require that this new strong interaction conserves the isospin symmetry between  $t$  and  $b$  quarks. Moreover, if one assumes that the electroweak symmetry breaking is induced by radiative corrections due to top-quark (and possibly, bottom-quark) loops, the quartic self-interactions of the Higgs field may be neglected. In this case, the relevant classical Lagrangian for the fundamental scalar field  $H$  is given by

$$L_H = D_\mu H^\dagger D^\mu H - m_H^2 H^\dagger H + \left( h_t \bar{\psi}_L t_R H + h_b \bar{\psi}_L b_R \tilde{H} + \text{h.c.} \right), \quad (1)$$

where  $H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$ ,  $\tilde{H} = \begin{pmatrix} H^+ \\ -H^{0*} \end{pmatrix}$  and  $\psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ ;  $h_t$  and  $h_b$  are the Yukawa couplings and  $D_\mu$  is the usual covariant derivative of the SM.

Next one assumes that the interactions acting on the members of the third generation of quarks are strong enough to form quark-antiquark bound states at the electroweak scale. The latter can be described in terms of two complex doublet scalar fields

$$\Sigma_t = \begin{pmatrix} \Sigma_t^0 \\ \Sigma_t^- \end{pmatrix} \sim t_R \bar{\psi}_L, \quad \tilde{\Sigma}_b = \begin{pmatrix} \Sigma_b^+ \\ -\Sigma_b^{0*} \end{pmatrix} \sim b_R \bar{\psi}_L, \quad (2)$$

and the corresponding effective Lagrangian then reads:

$$L_\Sigma = D_\mu \Sigma_t^\dagger D^\mu \Sigma_t + D_\mu \Sigma_b^\dagger D^\mu \Sigma_b - m^2 (\Sigma_t^\dagger \Sigma_t + \Sigma_b^\dagger \Sigma_b) + g (\bar{\psi}_L t_R \Sigma_t + \bar{\psi}_L b_R \tilde{\Sigma}_b + \text{h.c.}) . \quad (3)$$

The effects of a new strong  $CP$  phase  $\theta$  can, in principle, be described through an arbitrary function of  $\det U$ , where

$$U \sim \begin{pmatrix} \bar{t}_L t_R & \bar{t}_L b_R \\ \bar{b}_L t_R & \bar{b}_L b_R \end{pmatrix} = \begin{pmatrix} \Sigma_t^0 & \Sigma_b^- \\ \Sigma_t^+ & -\Sigma_b^{0*} \end{pmatrix}. \quad (4)$$

In analogy with QCD [12] we shall assume the Lagrangian form<sup>1</sup>

$$L_\theta = -\frac{\alpha}{4} [i \text{Tr} (\ln U - \ln U^\dagger) + 2\theta]^2, \quad (5)$$

which typically arises as a leading term in a  $1/N$  - expansion.

The total effective Lagrangian of the model is then given by

$$L = L_H + L_\Sigma + L_\theta, \quad (6)$$

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<sup>1</sup>Another simple choice is given by the 't Hooft determinant, i.e.  $L_\theta = \alpha e^{i\theta} \det U + \text{h.c.}$ .

with  $L_H, L_\Sigma$  and  $L_\theta$  defined by Eqs. (1), (3) and (5), respectively. Notice that if  $h_t = h_b$  the Lagrangian (6) conserves an ‘‘isospin’’ symmetry. However, as shown in [11], the angle  $\theta$  provides a dynamical origin for both  $CP$  violation and isospin breaking, once the neutral components of the three doublets  $H, \Sigma_b$  and  $\Sigma_t$  acquire nonzero vacuum expectation values (VEV’s).

Denoting the VEV’s of the neutral components of the fields by

$$\langle H^0 \rangle = \frac{v}{\sqrt{2}}, \quad \langle \Sigma_t^0 \rangle = \frac{\sigma_t}{\sqrt{2}} e^{i\varphi_t}, \quad \langle \Sigma_b^0 \rangle = \frac{\sigma_b}{\sqrt{2}} e^{i\varphi_b}, \quad (7)$$

the effective potential reads

$$V = m_H^2 \frac{v^2}{2} + \frac{m^2}{2} (\sigma_t^2 + \sigma_b^2) - \beta (\mu_t^2 + \mu_b^2) + \lambda (\mu_t^4 + \mu_b^4) + \alpha (\theta - \varphi_t + \varphi_b)^2, \quad (8)$$

where

$$\mu_i^2 = \frac{1}{2} (h_i^2 v^2 + g^2 \sigma_i^2 + 2h_i v g \sigma_i \cos \varphi_i), \quad i = t, b; \quad (9)$$

$\beta$  and  $\lambda$  are some effective quadratic and quartic couplings, respectively. All couplings and parameters in the potential are assumed to be real and positive.

The minimization of the potential implies the following system of equations:

$$\begin{aligned} A_H v &= gh_t I_t \sigma_t \cos \varphi_t + gh_b I_b \sigma_b \cos \varphi_b, \\ A_t \sigma_t &= gh_t I_t v \cos \varphi_t, \\ A_b \sigma_b &= gh_b I_b v \cos \varphi_b, \\ gh_t I_t v \sigma_t \sin \varphi_t &= -gh_b I_b v \sigma_b \sin \varphi_b = 2\alpha (\theta - \varphi_t + \varphi_b), \end{aligned} \quad (10)$$

where

$$\begin{aligned} A_H &= m_H^2 - h_t^2 I_t - h_b^2 I_b, \\ A_i &= m^2 - g^2 I_i, \\ I_i &= \beta - 2\lambda \mu_i^2. \end{aligned} \quad (11)$$

The mass parameters  $m_H$  and  $m$  are chosen such that the quantities  $A_H, A_t$  and  $A_b$  defined in Eqs. (11) are always positive.

If the parameter  $\alpha$  is large,  $\alpha \gg \beta m_i^2$ , then the last equation in (10) implies the constraint

$$\theta \simeq \varphi_t - \varphi_b. \quad (12)$$

Furthermore, if  $\theta = 0$ , it is easy to show that  $\varphi_t = \varphi_b = 0$  is the only solution of the equations and therefore  $CP$  is conserved.

A simple analytical solution can be given for the isospin symmetric case  $h_t = h_b \neq 0$  and assuming  $\beta \gg 2\lambda m_t^2$ . In this case  $I_t \simeq I_b$ ,  $A_t \simeq A_b$  and therefore  $\sin 2\varphi_t \simeq -\sin 2\varphi_b$ .

Clearly the large splitting between the physical values of the bottom and top masses ( $m_b \ll m_t$ ) requires  $\sigma_b \ll \sigma_t$  and thus  $\varphi_t \simeq 0, \varphi_b \simeq -\pi/2$ , which in turn demands that the  $CP$ -violating phase  $\theta$  be close to  $\pi/2$ . In other words, the presence of a phase  $\theta$  close to  $\pi/2$  induces both isospin breaking and  $CP$  violation with

$$\sigma_b \ll \sigma_t \neq 0, \quad v \neq 0, \quad \varphi_t \simeq \sigma_b/\sigma_t, \quad \varphi_b \simeq -\pi/2 + \sigma_b/\sigma_t. \quad (13)$$

The actual values of the VEV's can be determined from the physical values of the masses  $m_b, m_t$  and  $m_W$ . For small values of  $v$ , i.e  $v \ll \sigma_b, \sigma_t$ , one has the simple expressions

$$\sigma_b \simeq \frac{m_b v_0}{\sqrt{m_t^2 + m_b^2}}, \quad \sigma_t \simeq \frac{m_t v_0}{\sqrt{m_t^2 + m_b^2}}, \quad \tan \varphi_t \simeq \frac{m_b}{m_t}, \quad (14)$$

where  $v_0 = \sqrt{v^2 + \sigma_t^2 + \sigma_b^2} = (\sqrt{2}G_F)^{-1/2} \simeq 246$  GeV,  $G_F$  is the Fermi coupling constant.

The mass spectra of the neutral and charged (pseudo) scalars are easily found. In the neutral sector, it is straightforward to find the linear combination corresponding to the Goldstone boson eaten up by the  $Z^0$  gauge boson. For  $\alpha$  very large, one of the eigenvalues of the mass matrix will be proportional to  $\sqrt{\alpha}$  and therefore the corresponding linear combination of the fields will decouple from the theory. The remaining  $4 \times 4$  mass matrix can be easily diagonalized. One finds that the standard Higgs scalar  $h$  has a mass given by  $m_h \simeq 2g\sqrt{\lambda} m_t$ , two of the remaining masses are proportional to  $\sqrt{\beta}$  and thus are quite large. Finally the mass which corresponds mainly to a  $\bar{b}\gamma_5 b$  bound state is very sensitive to the difference  $h_t - h_b$ , but as soon as  $h_t$  and  $h_b$  differ (as expected from higher order corrections) it will also get a contribution proportional to  $\sqrt{\beta}$ . In the charged sector one of the eigenstates is eaten up by the  $W$  gauge boson through the usual Higgs mechanism. For the isospin symmetric case  $h_b = h_t$ , we find that one of the charged Higgs masses is very small, i.e. a new pseudo-Goldstone boson appears as it happens in the neutral sector. Nevertheless, radiative corrections yield  $h_b \neq h_t$  and therefore this mass will get a large contribution proportional to  $\sqrt{\beta}$ .

To conclude this section let us comment on the origin of  $CP$  violation in the present model. As shown in Ref. [11] the new interaction characterized by a  $\theta \neq 0$  term induces a  $CP$ -violating effect which filters down to the SM only if  $m_b/m_t \neq 0$ . Moreover, this new source of  $CP$  violation can be in principle responsible for what is observed in the  $K^0 - \bar{K}^0$  system, since it leads indeed to a sizeable  $CP$ -violating phase in the CKM matrix,  $\delta_{KM} \simeq -(\varphi_t + \varphi_b) \simeq \pi/2$ .

### 3 The structure of flavour-changing interactions

In general, the presence of more than one Higgs doublet in the SM leads to FCNC interactions at the tree level, which are mediated by the physical neutral scalars. Such interactions are severely constrained by the smallness not only of the  $CP$ -violating parameter  $\varepsilon_K$  but also of the  $K^0 - \bar{K}^0$  and  $B^0 - \bar{B}^0$  mixing. The model we are considering is

effectively equivalent to a three Higgs doublet model with a specific structure for Yukawa couplings. It is therefore straightforward to determine the form of the induced FCNC interactions by generalizing the results obtained in the two Higgs doublet case [13].

Let us consider 3 Higgs doublets  $\Phi_j$  and make the decomposition:

$$\Phi_j = e^{i\alpha_j} \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + R_j + iI_j) \end{pmatrix}, \quad j = 1, 2, 3, \quad (15)$$

where  $R_j, I_j$  are real fields and  $v_j e^{i\alpha_j}$  denote the VEV's of the Higgs fields. The Yukawa couplings of the Higgs fields to the quark weak eigenstates are given by

$$L_Y = -(\bar{u}_L \bar{d}_L) \Phi_1 g_1^d d_R - (\bar{u}_L \bar{d}_L) \Phi_2 g_2^d d_R - (\bar{u}_L \bar{d}_L) \tilde{\Phi}_1 g_1^u u_R - (\bar{u}_L \bar{d}_L) \tilde{\Phi}_3 g_3^u u_R, \quad (16)$$

where  $\tilde{\Phi} \equiv i\sigma_2 \Phi^*$  and  $g_i^{u,d}$  ( $i = 1, 2, 3$ ) are the Yukawa coupling matrices. The quark mass matrices are easily obtained,

$$M_u = \frac{1}{\sqrt{2}} v_1 g_1^u + \frac{1}{\sqrt{2}} e^{-i\alpha_3} v_3 g_3^u, \quad (17)$$

$$M_d = \frac{1}{\sqrt{2}} v_1 g_1^d + \frac{1}{\sqrt{2}} e^{i\alpha_2} v_2 g_2^d, \quad (18)$$

where the phase  $\alpha_1$  has been put equal to zero by an appropriate redefinition of the fields.

To single out the pseudo-Goldstone boson  $G^0$  we introduce the new fields  $\phi^0, R, R', G^0, I$  and  $I'$  defined through the transformation

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = O \begin{pmatrix} \phi^0 \\ R \\ R' \end{pmatrix}, \quad \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = O \begin{pmatrix} G^0 \\ I \\ I' \end{pmatrix}, \quad (19)$$

with

$$O = \begin{pmatrix} v_1/v_0 & v_2/v' & v_1 v_3/v_0 v' \\ v_2/v_0 & -v_1/v' & v_2 v_3/v_0 v' \\ v_3/v_0 & 0 & -v'/v_0 \end{pmatrix} \quad (20)$$

and  $v_0^2 = v_1^2 + v_2^2 + v_3^2$ ,  $v'^2 = v_1^2 + v_2^2$ . In terms of the new fields, the scalar couplings to the down quarks can be written as

$$\begin{aligned} L_Y^d = & -\frac{1}{v_0} \bar{d}_L M_d d_R (\phi^0 + iG^0) - \bar{d}_L \left( g_1^d \frac{v_2}{v'} - g_2^d \frac{v_1}{v'} e^{i\alpha_2} \right) d_R \frac{R + iI}{\sqrt{2}} \\ & - \frac{v_3}{v_0 v'} \bar{d}_L M_d d_R (R' + iI') + \text{h.c.} . \end{aligned} \quad (21)$$

We notice that the couplings to the fields  $\phi^0, G^0, R'$  and  $I'$  are flavour-conserving while the couplings to  $R$  and  $I$  are flavour-violating. Similarly, for the couplings to the up quarks one obtains

$$L_Y^u = -\frac{1}{v_0} \bar{u}_L M_u u_R (\phi^0 - iG^0) - \bar{u}_L \left( g_1^u \frac{v_2}{v'} \right) u_R \frac{R - iI}{\sqrt{2}} - \bar{u}_L \left( g_1^u \frac{v_1 v_3}{v_0 v'} - g_3^u \frac{v'}{v_0} e^{-i\alpha_3} \right) u_R \frac{R' - iI'}{\sqrt{2}} + \text{h.c.} , \quad (22)$$

and thus the couplings of  $\phi^0, G^0$  conserve flavour while the couplings of  $R, R', I, I'$  do violate flavour.

It is useful to obtain the scalar-quark couplings in terms of the quark mass eigenstates. In the down quark sector we find

$$L_Y^d = -\frac{1}{v_0} \bar{d}_L D_d d_R (\phi^0 + iG^0) - \bar{d}_L N_d d_R \frac{R + iI}{\sqrt{2}} - \frac{v_3}{v_0 v'} \bar{d}_L D_d d_R (R' + iI') + \text{h.c.} , \quad (23)$$

where  $D_d = U_{dL}^\dagger M_d U_{dR} = \text{diag}(m_d, m_s, m_b)$  and

$$N_d = U_{dL}^\dagger \left( g_1^d \frac{v_2}{v'} - g_2^d \frac{v_1}{v'} e^{i\alpha_2} \right) U_{dR} = \frac{\sqrt{2} v_2}{v' v_1} D_d - \frac{v'}{v_1} e^{i\alpha_2} G_2^d , \quad (24)$$

$$G_2^d \equiv U_{dL}^\dagger g_2^d U_{dR} . \quad (25)$$

In the up quark sector the couplings to the scalars in the quark mass eigenstate basis are given by

$$L_Y^u = -\frac{1}{v_0} \bar{u}_L D_u u_R (\phi^0 - iG^0) - \bar{u}_L N_u u_R \frac{R - iI}{\sqrt{2}} - \bar{u}_L N'_u u_R \frac{R' - iI'}{\sqrt{2}} + \text{h.c.} , \quad (26)$$

where  $D_u = U_{uL}^\dagger M_u U_{uR} = \text{diag}(m_u, m_c, m_t)$  and

$$N_u = \frac{v_2}{v'} U_{uL}^\dagger g_1^u U_{uR} , \quad (27)$$

$$N'_u = U_{uL}^\dagger \left( \frac{v_1 v_3}{v_0 v'} g_1^u - \frac{v'}{v_0} g_3^u e^{-i\alpha_3} \right) U_{uR} , \quad (28)$$

which can be rewritten as

$$N_u = \frac{v_2}{v_1^2} \sqrt{2} D_u - \frac{v_2 v_3}{v_1^2} e^{-i\alpha_3} G_3^u , \quad (29)$$

$$N'_u = \frac{v_3}{v_0 v'} \sqrt{2} D_u - \left( \frac{v_3^2}{v_0 v'} + \frac{v'}{v_0} \right) e^{-i\alpha_3} G_3^u , \quad (30)$$

$$G_3^u \equiv U_{uL}^\dagger g_3^u U_{uR} . \quad (31)$$



The Yukawa coupling matrices  $g_2^d, g_3^u$  have a very simple form in the present model, namely,

$$g_2^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g_b \end{pmatrix}, \quad g_3^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g_t \end{pmatrix}, \quad (32)$$

and therefore the matrices  $G_2^d, G_3^u$  defined in Eqs. (25) and (31) are given by

$$(G_2^d)_{ij} = g_b (U_{dL}^*)_{3i} (U_{dR})_{3j}, \quad (33)$$

$$(G_3^u)_{ij} = g_t (U_{uL}^*)_{3i} (U_{uR})_{3j}. \quad (34)$$

These matrices completely determine the structure of tree-level FCNC interactions in the model. Without further assumptions we cannot predict the size of such interactions. We shall assume that the quark mass matrices  $M_{u,d}$  are hermitian<sup>2</sup> and that the CKM mixing matrix  $V \equiv U_{uL}^\dagger U_{dL}$  is dominated by  $U_{dL}$ , i.e.  $U_{uL} \simeq \mathbb{1}$ , as favoured phenomenologically. Under the above “reasonable” assumptions, the off-diagonal elements of  $N_d$  in Eq. (24) are entirely predicted in terms of  $V$  since from Eq. (33) we obtain

$$(G_2^d)_{ij} = g_b V_{3i}^* V_{3j}. \quad (35)$$

Finally, new contributions to flavour-changing processes will be also induced by the couplings of the heavy charged Higgs fields to the quarks. Such contributions correspond to Feynman box diagrams with  $W$ -boson and charged Higgs particle exchanges. To determine the magnitude of these couplings, let us introduce the new charged fields  $G^+, H_1^+, H_2^+$  through the decomposition

$$\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \phi_3^+ \end{pmatrix} = O \begin{pmatrix} G^+ \\ H_1^+ \\ H_2^+ \end{pmatrix}, \quad (36)$$

where the matrix  $O$  is given by Eq. (20) and  $G^+$  corresponds to the pseudo-Goldstone boson. Going to the physical basis for the charged Higgs fields, the couplings to the  $d_R$  and  $u_R$  quarks are given by

$$\begin{aligned} L_Y^+ = & -\sqrt{2} \bar{u}_L M_d d_R G^+ - \bar{u}_L A_1^d d_R H_1^+ - \bar{u}_L A_2^d d_R H_2^+ \\ & -\sqrt{2} \bar{d}_L M_u u_R G^- - \bar{d}_L A_1^u u_R H_1^- - \bar{d}_L A_2^u u_R H_2^- + \text{h.c.}, \end{aligned} \quad (37)$$

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<sup>2</sup>According to the polar decomposition theorem, the mass matrices  $M_{u,d}$  can always be written as a product of a hermitian matrix and a unitary matrix. The latter can be rotated away by a redefinition of the right quark fields. Notice however that the form of the coupling matrices  $g_2^d$  and  $g_3^u$  given in Eq. (32) is in general not invariant under such a transformation. Here we shall assume that the quark mass matrices are hermitian in the basis where the couplings have the special form (32). Our analysis can be easily extended to a more general case.

where we have introduced the coupling matrices

$$A_i^u = e^{-i\alpha_2} (g_1^u O_{1(i+1)} + g_3^u e^{-i\alpha_3} O_{3(i+1)}) , \quad (38)$$

$$A_i^d = e^{i\alpha_3} (g_1^d O_{1(i+1)} + g_2^d e^{i\alpha_2} O_{2(i+1)}) , \quad i = 1, 2 . \quad (39)$$

After performing a rotation to the quark mass eigenbasis we obtain

$$A_i^u = V^\dagger e^{-i\alpha_2} \left[ \frac{\sqrt{2}}{v_1} D_u O_{1(i+1)} + G_3^u e^{-i\alpha_3} \left( O_{3(i+1)} - \frac{v_3}{v_1} O_{1(i+1)} \right) \right] , \quad (40)$$

$$A_i^d = V e^{i\alpha_3} \left[ \frac{\sqrt{2}}{v_1} D_d O_{1(i+1)} + G_2^d e^{i\alpha_2} \left( O_{2(i+1)} - \frac{v_2}{v_1} O_{1(i+1)} \right) \right] , \quad (41)$$

with  $G_2^d, G_3^u$  defined in Eqs. (25) and (31), respectively.

It is clear that in order to analyze the charged Higgs contributions to the relevant flavour-changing processes we need to know the structure of the unitary matrices  $U_{uL}, U_{uR}$ . To be able to predict the size of such contributions, we shall assume that the up quark mass matrix  $M_u$  is approximately given by the texture zero structure [14],

$$M_u = \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & d \end{pmatrix} . \quad (42)$$

In this case

$$U_{uL} \sim U_{uR} \sim \begin{pmatrix} 1 & \sqrt{m_u/m_c} & \sqrt{\epsilon m_u m_c^2/m_t^3} \\ -\sqrt{m_u/m_c} & 1 & \sqrt{\epsilon m_c/m_t} \\ \sqrt{\epsilon m_u/m_t} & -\sqrt{\epsilon m_c/m_t} & 1 \end{pmatrix} , \quad (43)$$

where

$$\epsilon \equiv \frac{d - m_t}{m_c} . \quad (44)$$

Such a choice is of course in agreement with our previous assumption of the matrices  $U_{uL}$  and  $U_{uR}$  being close to the identity matrix.

Under the above conditions, the coupling matrix  $G_3^u$  defined in Eq. (34) takes the simple form

$$G_3^u = g_t \begin{pmatrix} \epsilon m_u/m_t & -\epsilon \sqrt{m_u m_c}/m_t & \sqrt{\epsilon m_u/m_t} \\ -\epsilon \sqrt{m_u m_c}/m_t & \epsilon m_c/m_t & -\sqrt{\epsilon m_c/m_t} \\ \sqrt{\epsilon m_u/m_t} & -\sqrt{\epsilon m_c/m_t} & 1 \end{pmatrix} . \quad (45)$$

In the context of our model, where the hierarchy  $v \ll \sigma_b \ll \sigma_t$  is expected among the VEV's, Eqs. (20), (40) and (41), together with (45) and (35), yield

$$A_1^u \simeq \frac{\sqrt{2}}{v} V^\dagger e^{i\varphi_b} \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_c(1 - \epsilon e^{-\varphi_t}) & -\sqrt{\epsilon m_c m_t} e^{-i\varphi_t} \\ 0 & -\sqrt{\epsilon m_c m_t} e^{-i\varphi_t} & m_t(1 - e^{-i\varphi_t}) \end{pmatrix}, \quad (46)$$

$$(A_1^d)_{ij} \simeq \frac{\sqrt{2}}{v} e^{i\varphi_t} \sum_{k=1}^3 V_{ik} [(D_d)_{kj} - m_b V_{3k}^* V_{3j} e^{-i\varphi_b}], \quad (47)$$

$$A_2^u \ll A_1^u, \quad A_2^d \ll A_1^d, \quad (48)$$

after the corresponding identification  $v_1 = v, v_2 = \sigma_b, v_3 = \sigma_t$  and  $\alpha_2 = -\varphi_b, \alpha_3 = \varphi_t$ . In particular, this implies that the contributions to flavour-changing processes coming from the charged Higgs  $H_2^+$  will be strongly suppressed, provided that the Higgs mass  $m_{H_2^+} \simeq m_{H_1^+}$ . In what follows we assume that the latter condition is satisfied. Moreover, we shall discuss two limiting cases:  $\epsilon \simeq m_u/m_c \simeq 0$ , which corresponds to  $b \simeq m_c$  in Eq. (42), and  $\epsilon \simeq 1$ , i.e.  $b \simeq m_u \simeq 0$ .

## 4 New physics and $\varepsilon_K, \Delta m_{B_d}, \Delta m_{B_s}, \Delta m_D$

Within the SM, the CKM matrix is constrained by unitarity and experimental data. These constraints are usually expressed in terms of the Wolfenstein parameters  $A, \rho$  and  $\eta$  [15], and presented as a unitarity triangle in the complex plane  $(\bar{\rho}, \bar{\eta})$  (see Fig. 1 below) [1, 16]. They can be summarized as follows [17, 18]:

From semileptonic  $K$  and  $B$  decays we have<sup>3</sup>

$$\begin{aligned} |V_{us}| &= \lambda = 0.2205 \pm 0.0018, & |V_{cb}| &= 0.040 \pm 0.002, \\ |V_{ub}| &= (3.56 \pm 0.56) \times 10^{-3}, & A &= \frac{|V_{cb}|}{\lambda^2} = 0.826 \pm 0.041, \end{aligned} \quad (49)$$

which implies

$$R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.39 \pm 0.07, \quad (50)$$

with  $\bar{\rho} = \rho(1 - \lambda^2/2)$ ,  $\bar{\eta} = \eta(1 - \lambda^2/2)$ . The above results are extracted from tree level decays with large branching ratios and therefore their determination is essentially independent of physics beyond the SM.

Next, for the  $CP$  violating parameter  $\varepsilon_K$  (and assuming  $\varepsilon_K \gg \varepsilon'$ ),

$$\varepsilon_K \simeq \frac{e^{i\pi/4} \text{Im}(M_{12}^K)}{\sqrt{2} \Delta m_K}, \quad M_{12}^K = \frac{\langle K^0 | \mathcal{H}_{\text{eff}}(\Delta S = 2) | \bar{K}^0 \rangle}{2m_K}, \quad (51)$$

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<sup>3</sup>All our input parameters are taken from [1, 17].

the calculation of the box diagrams describing the  $K^0 - \bar{K}^0$  mixing in SM gives

$$\varepsilon_K^{\text{SM}} = e^{i\pi/4} C_\varepsilon \hat{B}_K \text{Im}(\lambda_t) [\text{Re}(\lambda_c^*) (\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)) - \text{Re}(\lambda_t^*) \eta_2 S_0(x_t)] ,$$

$$C_\varepsilon = \frac{G_F^2 m_W^2 f_K^2 m_K}{6\sqrt{2}\Delta m_K} = 3.84 \times 10^4 , \quad \lambda_i = (V_{is}^* V_{id}) , \quad x_i = \frac{m_i^2}{m_W^2} . \quad (52)$$

Comparing this result with the experimental value  $|\varepsilon_K| = (2.280 \pm 0.013) \times 10^{-3}$ , one obtains a constraint in the form of the hyperbola

$$\bar{\eta} [(1 - \bar{\rho}) A^2 \eta_2 S_0(x_t) + P_c(\varepsilon)] A^2 \hat{B}_K = 0.226 . \quad (53)$$

In the above formulas,  $P_c(\varepsilon) = 0.31 \pm 0.05$  summarizes the charm-charm and charm-top contributions in the SM,  $\hat{B}_K = 0.80 \pm 0.15$  is a nonperturbative parameter, the correction factors  $\eta_1 = 1.38 \pm 0.20$ ,  $\eta_2 = 0.57 \pm 0.01$ ,  $\eta_3 = 0.47 \pm 0.04$  describe the short-distance QCD effects,  $f_K = 160$  MeV is the kaon decay constant,  $m_K = 497.672 \pm 0.031$  MeV is the kaon mass and  $\Delta m_K = (3.489 \pm 0.008) \times 10^{-12}$  MeV is the mass difference in the  $K$  system. The gauge independent functions  $S_0$  which govern the FCNC processes are approximately given by

$$S_0(x_t) = 2.46 \left( \frac{m_t}{170 \text{ GeV}} \right)^{1.52} , \quad S_0(x_c) = x_c ,$$

$$S_0(x_c, x_t) = x_c \left[ \ln \frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln x_t}{4(1-x_t)^2} \right] , \quad (54)$$

where  $m_c = 1.30 \pm 0.05$  GeV and  $m_t = 165 \pm 5$  GeV correspond to the running quark masses defined as  $m_q \equiv m_q(m_q^{\text{pole}})$ .

Substituting the numerical values for the parameters in Eq.(53) the  $\varepsilon_K$  constraint reads

$$\bar{\eta} [(1 - \bar{\rho})(0.91 \pm_{0.14}^{0.16} + (0.31 \pm 0.05))] = 0.41 \pm_{0.10}^{0.15} . \quad (55)$$

Next, the amplitude for the  $\Delta B = 2$  transition in the  $B_{d,s}^0 - \bar{B}_{d,s}^0$  systems is given in the SM by

$$M_{12}^{\text{SM}}(B_q) = \frac{\langle B_q^0 | \mathcal{H}_{\text{eff}}(\Delta B = 2) | \bar{B}_q^0 \rangle}{2m_{B_q}} = \kappa_q (V_{tb} V_{tq}^*)^2 , \quad q = d, s , \quad (56)$$

$$\kappa_q = \frac{G_F^2}{12\pi^2} m_W^2 \eta_B m_{B_q} f_{B_q}^2 \hat{B}_{B_q} S_0(x_t) ,$$

so that the mass differences are

$$\Delta m_{B_q} = 2 |M_{12}^{\text{SM}}(B_q)| = 2 |\kappa_q (V_{tb} V_{tq}^*)^2| . \quad (57)$$

Here  $\eta_B = 0.55 \pm 0.01$  is a QCD correction coefficient,  $m_{B_d} = 5.28$  GeV and  $m_{B_s} = 5.37$  GeV are the  $B$ -meson masses and the factor  $f_{B_q} \hat{B}_{B_q}^{1/2}$  measures the hadronic uncertainties. Recent lattice QCD estimates give  $f_{B_d} \hat{B}_{B_d}^{1/2} = 200 \pm 40$  MeV and  $\xi_s \equiv (f_{B_s} \hat{B}_{B_s}^{1/2}) / (f_{B_d} \hat{B}_{B_d}^{1/2}) = 1.14 \pm 0.08$ .

Combining the experimental value  $\Delta m_{B_d} = 0.471 \pm 0.016 \text{ ps}^{-1}$  with Eq. (57) we can determine the parameter

$$R_t \equiv \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right| = \left[ \frac{|V_{td}|}{8.8 \times 10^{-3}} \right] \left[ \frac{0.040}{|V_{cb}|} \right] = 0.98 \pm_{-0.22}^{+0.37}. \quad (58)$$

On the other hand, the measurement  $\Delta m_{B_s} > 12.4 \text{ ps}^{-1}$  allows us to determine  $R_t$  in a different way, namely,

$$R_t = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{ts}} \right| = \frac{\xi_s}{\lambda} \sqrt{\frac{m_{B_s}}{m_{B_d}}} \sqrt{\frac{\Delta m_{B_d}}{\Delta m_{B_s}}} < 1.03 \pm_{-0.14}^{+0.15}. \quad (59)$$

Fig. 1 summarizes the constraints given by Eqs. (50), (55), (58) and (59) in the plane  $(\bar{\rho}, \bar{\eta})$  within the SM. The dot-filled area corresponds to the presently allowed region if no new physics beyond the SM is invoked.

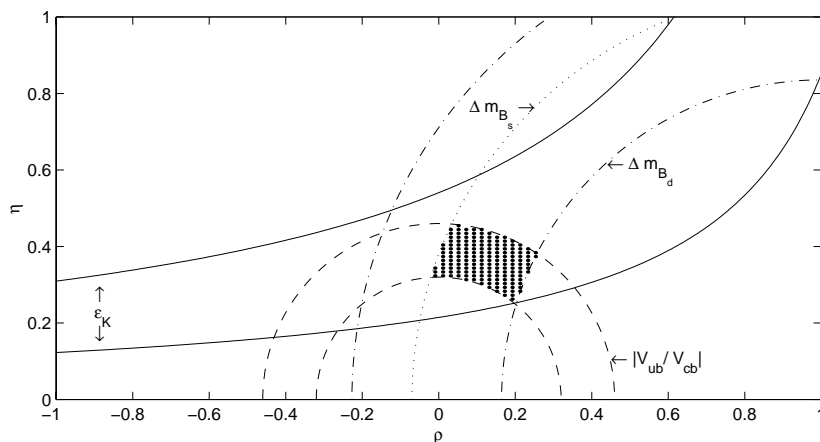


Figure 1: Constraints on the plane  $(\rho, \eta)$  coming from the measurements of  $|V_{ub}/V_{cb}|$  (dashed),  $\varepsilon_K$  (solid),  $\Delta m_{B_d}$  (dot-dashed) and  $\Delta m_{B_s}$  (dotted) within the Standard Model. The dot-filled area corresponds to the presently allowed region.

The model of interest to us and presented in Section 2 contains new physical fields when compared to the SM. As the masses of such fields are expected to be much larger than  $m_W$ , their contributions to charged current tree level decays should be negligible. They can however significantly contribute to quantities such as  $\varepsilon_K$  and  $\Delta m_{B_{d,s}}$ , thus playing an important role in the determination of the unitarity triangle. To establish their impact, first we shall compute the new contributions to  $\varepsilon_K$ ,  $\Delta m_{B_d}$  and  $\Delta m_{B_s}$  coming from the FCNC processes induced by the heavy neutral Higgs field. Then we shall compare these contributions with the ones induced by the new heavy charged Higgs fields. As we shall see, if the mass scale for the heavy charged Higgs ( $m_{H^+}$ ) is of the same order than the scale for the heavy neutral Higgs ( $m_{H^0}$ ), new physics contributions are always dominated by tree-level FCNC effects. Finally, new contributions to the  $\Delta m_D$  mass difference are also expected and they are discussed at the end of this section.

## 4.1 FCNC contributions

Since the couplings of down quarks to the neutral scalar fields  $R$  and  $I$  are flavour-violating (cf. Eqs. (23) and (24)), they will induce a tree-level FCNC contribution to  $K^0 - \bar{K}^0$  mixing. In the framework of our model such couplings are determined by Eqs. (24), (35) and given by

$$\Gamma_{ij}^d = -\frac{\sqrt{v^2 + \sigma_b^2}}{v} e^{i\varphi_b} (G_2^d)_{ij} \simeq -\frac{\sqrt{2} m_b}{v} e^{i\varphi_b} V_{3i}^* V_{3j}, \quad (60)$$

where we have used the fact that  $v \ll \sigma_b$ ,  $m_b \simeq g_b \sigma_b / \sqrt{2}$ . To estimate the hadronic matrix elements it is customary to use the so-called vacuum insertion approximation [16]. In this approximation, the new physics contribution to the matrix element  $M_{12}$  of the transition  $K^0 - \bar{K}^0$  will be given by

$$M_{12}^{\text{new}}(K) = \frac{\langle K^0 | \mathcal{H}_{\text{eff}}^{\text{new}} | \bar{K}^0 \rangle}{2m_K} = -e^{2i\varphi_b} \frac{f_K^2 m_K \hat{B}_K m_b^2}{4v^2 m_{H^0}^2} (V_{td}^* V_{ts})^2 \left[ \frac{1}{6} + \left( \frac{m_K}{m_d + m_s} \right)^2 \right], \quad (61)$$

where  $\mathcal{H}_{\text{eff}}^{\text{new}}$  is the effective  $\Delta S = 2$  Hamiltonian induced by the neutral Higgs exchange. It is now straightforward to compute the FCNC contribution to the  $CP$ -violating parameter  $\varepsilon_K$  defined in Eq. (51). We obtain

$$\varepsilon_K^{H^0} = -C_\varepsilon^{(0)} |V_{us}|^2 |V_{cb}|^4 \{ [(1 - \rho)^2 - \eta^2] \sin 2\varphi_b + 2\eta(1 - \rho) \cos 2\varphi_b \} \frac{m_b^2}{m_{H^0}^2}, \quad (62)$$

where

$$C_\varepsilon^{(0)} = \frac{m_K f_K^2 \hat{B}_K}{4\sqrt{2} \Delta m_K v^2} \left[ \frac{1}{6} + \left( \frac{m_K}{m_d + m_s} \right)^2 \right] \simeq 6.8 \times 10^{12} \left[ \frac{\text{GeV}}{v} \right]^2, \quad (63)$$

with  $m_s(m_c) = 130$  MeV,  $m_d(m_c) = 8$  MeV. Substituting the central values  $|V_{us}| = 0.2205$ ,  $|V_{cb}| = 0.040$ ,  $m_b = 4.25$  GeV in Eq. (62) and assuming  $\varphi_b \simeq \pi/2$ , we find

$$\varepsilon_K^{H^0} \simeq 30.6 \eta (1 - \rho) \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^0}} \right]^2. \quad (64)$$

A lower bound on the scale of the heavy neutral Higgs can be then obtained by requiring the new physics contribution (64) to be smaller than the SM contribution, i.e.  $|\varepsilon_K^{H^0}| < |\varepsilon_K^{\text{SM}}|$ . Since for the central values of the parameters, the contribution to  $\varepsilon_K$  in the SM (cf. Eq. (52)) is approximately given by

$$|\varepsilon_K^{\text{SM}}| \simeq 5.2 \times 10^{-3} \eta (1.34 - \rho), \quad (65)$$

we find for  $0 \lesssim \rho \lesssim 0.3$ ,

$$m_{H^0} \gtrsim 65 \text{ TeV} \left[ \frac{\text{GeV}}{v} \right]. \quad (66)$$

In particular, for  $v = \sqrt{2} m_c \simeq 1.84 \text{ GeV}$  we obtain

$$m_{H^0} \gtrsim 35 \text{ TeV} . \quad (67)$$

New FCNC contributions induced by the heavy neutral Higgs field  $H^0$  in the mass differences  $\Delta m_{B_q}$  ( $q = d, s$ ) are easily obtained from the previous results on  $\varepsilon_K$ . For the matrix elements  $M_{12}^{\text{new}}(B_q)$  we have

$$M_{12}^{\text{new}}(B_q) = -e^{2i\varphi_b} \kappa_q^{(0)} (V_{tq}^* V_{tb})^2 \frac{m_b^2}{m_{H^0}^2} , \quad (68)$$

$$\kappa_q^{(0)} = \frac{f_{B_q}^2 \hat{B}_{B_q} m_{B_q}}{4v^2} \left[ \frac{1}{6} + \left( \frac{m_{B_q}}{m_q + m_b} \right)^2 \right] ,$$

with

$$\kappa_d^{(0)} \simeq 0.09 \text{ GeV} \left[ \frac{\text{GeV}}{v} \right]^2 , \quad \kappa_s^{(0)} \simeq 0.12 \text{ GeV} \left[ \frac{\text{GeV}}{v} \right]^2 . \quad (69)$$

This implies

$$\begin{aligned} \Delta m_{B_d}^{H^0} &= 2|M_{12}^{\text{new}}(B_d)| = 2\kappa_d^{(0)} |V_{us}|^2 |V_{cb}|^2 [(1-\rho)^2 + \eta^2] \frac{m_b^2}{m_{H^0}^2} \\ &\simeq 3.84 \times 10^2 \text{ ps}^{-1} [(1-\rho)^2 + \eta^2] \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^0}} \right]^2 , \end{aligned} \quad (70)$$

$$\Delta m_{B_s}^{H^0} = 2|M_{12}^{\text{new}}(B_s)| = 2\kappa_s^{(0)} |V_{cb}|^2 \frac{m_b^2}{m_{H^0}^2} \simeq 1.02 \times 10^4 \text{ ps}^{-1} \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^0}} \right]^2 . \quad (71)$$

These FCNC contributions to  $\Delta m_{B_q}$  are to be compared with the SM contributions given by Eq. (57) and which can be approximately written as

$$\Delta m_{B_d}^{\text{SM}} \simeq 0.48 \text{ ps}^{-1} [(1-\rho)^2 + \eta^2] , \quad \Delta m_{B_s}^{\text{SM}} \simeq 13.04 \text{ ps}^{-1} . \quad (72)$$

We have then

$$w_q \equiv \frac{\Delta m_{B_q}^{H^0}}{\Delta m_{B_q}^{\text{SM}}} = \frac{\kappa_q^{(0)} m_b^2}{\kappa_q m_{H^0}^2} \simeq 0.19 \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{65 \text{ TeV}}{m_{H^0}} \right]^2 \lesssim 0.19 , \quad (73)$$

if the lower bound given in Eq. (66) for  $m_{H^0}$  is satisfied. We see that the contributions to  $\Delta m_{B_q}$  coming from the neutral Higgs are much smaller than the SM ones. In Fig. 2 we illustrate our results for  $m_{H^0} = 35 \text{ TeV}$  and  $v = \sqrt{2} m_c$ . We notice that while  $\varepsilon_K$  is quite sensitive to new physics, the constraints coming from  $B^0 - \bar{B}^0$  mixing practically do not change when compared to the SM results.

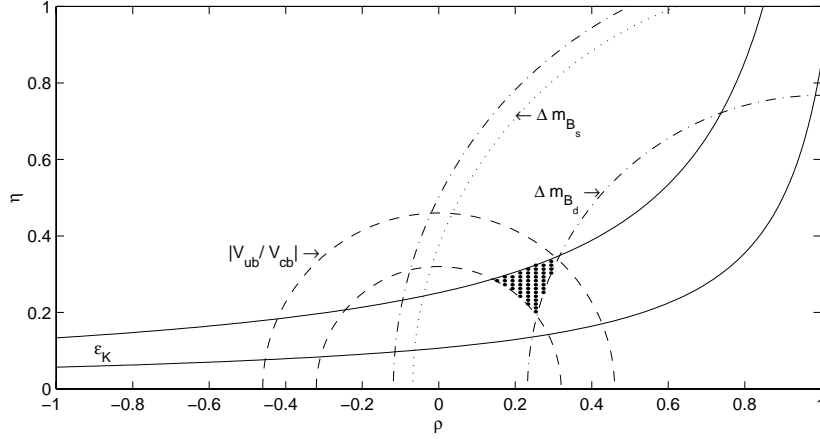


Figure 2: Constraints on the plane  $(\rho, \eta)$  after including the new FCNC contributions induced by the heavy neutral Higgs  $H^0$ . The curves are given for  $m_{H^0} = 35$  TeV,  $v = \sqrt{2} m_c$  and we assume  $m_{H_1^+} \simeq m_{H_2^+} \gg m_{H^0}$ . The dot-filled area corresponds to the region allowed by the present experimental bounds.

## 4.2 Charged-current contributions

The main contributions to flavour-changing processes induced by the charged Higgs field  $H_1^+$  are described by box diagrams where  $H_1^+$  and the  $W$  gauge boson are circulating inside the box<sup>4</sup>. Their computation is also straightforward.

For the new physics contribution to the amplitude  $M_{12}$  in the  $K^0 - \bar{K}^0$  system we find

$$\begin{aligned}
 M_{12}^{\text{new}}(K) &= \frac{\sqrt{2} G_F m_W^2 m_K f_K^2 \hat{B}_K}{24\pi^2 v^2} [\lambda_t^* \Gamma_{tt} f_t + \lambda_c^* \Gamma_{cc} f_c + (\lambda_{ct}^* \Gamma_{ct} + \lambda_{tc}^* \Gamma_{tc}) f_{ct}] \frac{m_t^2}{m_{H^+}^2}, \\
 f_t &= -\frac{x_t}{2} \left[ 1 + \ln x_t - \frac{3}{x_t - 1} + \frac{3 \ln x_t}{(x_t - 1)^2} \right], \quad f_c = -2x_c (1 + \ln x_c), \\
 f_{ct} &= \sqrt{x_c x_t} \left[ \frac{2x_c \ln x_c}{x_t} + \frac{3}{2} \frac{\ln x_t}{x_t - 1} + \frac{1}{2} \ln \frac{x_{H^+}}{x_t} \right], \\
 \Gamma_{ij} &= g_{id} g_{js}^*, \quad \lambda_{ij} = V_{is}^* V_{jd}, \quad i = c, t,
 \end{aligned} \tag{74}$$

with  $\lambda_i$  and  $x_i$  defined in Eq. (52). Moreover, according to Eq. (46),

$$\begin{aligned}
 g_{t\alpha} &= -V_{c\alpha}^* e^{-i\varphi_t} \sqrt{\frac{\epsilon m_c}{m_t}} + V_{t\alpha}^* (1 - e^{-i\varphi_t}), \\
 g_{c\alpha} &= V_{c\alpha}^* (1 - \epsilon e^{-i\varphi_t}) \frac{m_c}{m_t} - V_{t\alpha}^* e^{-i\varphi_t} \sqrt{\frac{\epsilon m_c}{m_t}}, \quad \alpha = d, s, b.
 \end{aligned} \tag{75}$$

<sup>4</sup>As discussed at the end of Section 3, the contributions coming from the charged Higgs field  $H_2^+$  can be neglected if  $m_{H_2^+} \simeq m_{H_1^+}$  (cf. Eq. (48)).



Therefore, the charged Higgs contribution to  $\varepsilon_K$  reads

$$\varepsilon_K^{H^+} = C_\varepsilon^{(+)} [A_t f_t + A_c f_c + A_{ct} f_{ct}] \frac{m_t^2}{m_{H^+}^2}, \quad (76)$$

where

$$\begin{aligned} A_t &= \text{Im}(\lambda_t^* \Gamma_{tt}), \quad A_c = \text{Im}(\lambda_c^* \Gamma_{cc}), \quad A_{ct} = \text{Im}(\lambda_{tc}^* \Gamma_{tc} + \lambda_{ct}^* \Gamma_{ct}), \\ C_\varepsilon^{(+)} &= \frac{G_F m_W^2 m_K f_K^2 \hat{B}_K}{24\pi^2 v^2 \Delta m_K} \simeq 0.93 \times 10^9 \left[ \frac{\text{GeV}}{v} \right]^2. \end{aligned} \quad (77)$$

When the texture parameter  $\epsilon = 0$ , the coefficients  $A_i$  in Eq. (76) are approximately given by

$$\begin{aligned} A_t &\simeq 4(1 - \cos \varphi_t) |V_{cb}|^2 (1 - \rho) J, \quad A_c \simeq 0, \\ A_{ct} &\simeq 2 \frac{m_c}{m_t} (1 - \cos \varphi_t) J, \quad J = |V_{us}|^2 |V_{cb}|^2 \eta, \end{aligned} \quad (78)$$

while for  $\epsilon = 1$  they are

$$A_t \simeq A_c \simeq J \frac{m_c}{m_t}, \quad A_{ct} \simeq 2J(1 - \rho) \frac{m_c}{m_t}. \quad (79)$$

We can now give a numerical estimate of the above contributions. Using the fact that  $\cos \varphi_t \simeq 1 - m_b^2/(2m_t^2)$  we obtain

$$\varepsilon_K^{H^+} \Big|_{\epsilon=0} \simeq -0.016 \eta (1 - \rho) \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^+}} \right]^2, \quad (80)$$

$$\varepsilon_K^{H^+} \Big|_{\epsilon=1} \simeq -2.55 \eta (23.6 + \rho) \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^+}} \right]^2. \quad (81)$$

If we require that  $|\varepsilon_K^{H^+}| < |\varepsilon_K^{\text{SM}}|$ , where  $\varepsilon_K^{\text{SM}}$  is given in Eq. (65), then we can find the following lower bounds on the charged Higgs mass for  $0 \lesssim \rho \lesssim 0.3$ ,

$$m_{H^+} \gtrsim 1.5 \text{ TeV} \left[ \frac{\text{GeV}}{v} \right] \quad \text{for } \epsilon = 0, \quad (82)$$

$$m_{H^+} \gtrsim 101 \text{ TeV} \left[ \frac{\text{GeV}}{v} \right] \quad \text{for } \epsilon = 1. \quad (83)$$

This in particular implies for  $v \simeq \sqrt{2} m_c$ ,

$$m_{H^+} \gtrsim 800 \text{ GeV} \quad \text{if } \epsilon = 0, \quad m_{H^+} \gtrsim 55 \text{ TeV} \quad \text{if } \epsilon = 1. \quad (84)$$

Let us now consider the charged Higgs contributions to  $B_{d,s}^0 - \bar{B}_{d,s}^0$  mixing. Their computation is analogous to the one in the  $K^0 - \bar{K}^0$  system. We obtain

$$M_{12}^{\text{new}}(B_q) = \kappa_q^{(+)} \left[ A_{tt}^{(q)} f_t + A_{cc}^{(q)} f_c + (A_{ct}^{(q)} + A_{tc}^{(q)}) f_{ct} \right] \frac{m_t^2}{m_{H^+}^2}, \quad (85)$$

$$\kappa_q^{(+)} = \frac{\sqrt{2} G_F m_W^2 m_{B_q} f_{B_q}^2 \hat{B}_{B_q}}{24\pi^2 v^2}, \quad A_{ij}^{(q)} = V_{ib} V_{jq}^* g_{iq} g_{jb}^*, \quad q = d, s,$$

with

$$\kappa_d^{(+)} \simeq 0.95 \times 10^{-4} \text{ GeV} \left[ \frac{\text{GeV}}{v} \right]^2, \quad \kappa_s^{(+)} \simeq 1.26 \times 10^{-4} \text{ GeV} \left[ \frac{\text{GeV}}{v} \right]^2, \quad (86)$$

$f_i$  and  $g_{ij}$  defined in Eqs. (74) and (75), respectively.

For  $\epsilon = 0$  the amplitude (85) is dominated by the top quark contributions described by the coefficient  $A_{tt}^{(q)}$ . We have

$$A_{tt}^{(q)} \simeq 2(V_{tb}V_{tq}^*)^2 (1 - \cos \varphi_t) \simeq (V_{tq}^*)^2 \frac{m_b^2}{m_t^2}. \quad (87)$$

The top-charm terms proportional to  $A_{tc}^{(q)}$  will however give an important contribution to the amplitude in the case of  $\epsilon = 1$ . In the latter case we find for the relevant coefficients:

$$\begin{aligned} A_{tt}^{(q)} &\simeq V_{tq}^* V_{cq}^* V_{tb}^2 (1 - e^{-i\varphi_t}) \sqrt{\frac{m_c}{m_t}} \simeq i V_{tq}^* V_{cq}^* \frac{m_b}{m_t} \sqrt{\frac{m_c}{m_t}}, \\ A_{tc}^{(q)} &\simeq (V_{cq}^*)^2 V_{tb}^2 \frac{m_c}{m_t} \simeq (V_{cq}^*)^2 \frac{m_c}{m_t}. \end{aligned} \quad (88)$$

Therefore, the new contributions to the mass differences  $\Delta m_{B_q}$  will be given by

$$\begin{aligned} \Delta m_{B_d}^{H^+} \Big|_{\epsilon=0} &= 2\kappa_d^{(+)} |V_{us}|^2 |V_{cb}|^2 [(1 - \rho)^2 + \eta^2] |f_t| \frac{m_b^2}{m_{H^+}^2} \\ &\simeq 1.65 \text{ ps}^{-1} [(1 - \rho)^2 + \eta^2] \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^+}} \right]^2, \end{aligned} \quad (89)$$

$$\Delta m_{B_s}^{H^+} \Big|_{\epsilon=0} = 2\kappa_s^{(+)} |V_{cb}|^2 |f_t| \frac{m_b^2}{m_{H^+}^2} \simeq 22.4 \text{ ps}^{-1} \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^+}} \right]^2. \quad (90)$$

Comparing these values with the SM result given in Eqs. (72) we arrive at

$$\frac{\Delta m_{B_q}^{H^+}}{\Delta m_{B_q}^{\text{SM}}} \simeq 1.55 \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{1.5 \text{ TeV}}{m_{H^+}} \right]^2. \quad (91)$$

We see that for  $m_{H^+}$  close to the lower bound (82) the charged Higgs contributions to  $B_{d,s}^0 - \bar{B}_{d,s}^0$  mixing are of the same order of magnitude than the SM ones. A slightly higher value of  $m_{H^+}$  is required if we impose  $\Delta m_{B_q}^{H^+} < \Delta m_{B_q}^{\text{SM}}$ . From Eq. (91) it follows then

$$m_{H^+} \gtrsim 1 \text{ TeV} \quad \text{for} \quad \epsilon = 0, \quad (92)$$

with  $v \simeq \sqrt{2} m_c$ .

In the case of  $\epsilon = 1$  we have

$$\begin{aligned} \Delta m_{B_d}^{H^+} \Big|_{\epsilon=1} &= 2\kappa_d^{(+)} |V_{us}|^2 \left| -iV_{cb}(1 - \rho + i\eta) \sqrt{\frac{m_c}{m_t}} f_t + \frac{m_c}{m_b} f_{ct} \right| \frac{m_b m_t}{m_{H^+}^2} \\ &\simeq 140 \text{ ps}^{-1} \left| 1.8 - \eta + 0.7 \ln \left( \frac{m_{H^+}}{\text{TeV}} \right) + i(1 - \rho) \right| \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^+}} \right]^2, \end{aligned} \quad (93)$$

$$\begin{aligned} \Delta m_{B_s}^{H^+} \Big|_{\epsilon=1} &= 2\kappa_s^{(+)} \left| -iV_{cb} \sqrt{\frac{m_c}{m_t}} f_t + \frac{m_c}{m_b} f_{ct} \right| \frac{m_b m_t}{m_{H^+}^2} \\ &\simeq 3.8 \times 10^3 \text{ ps}^{-1} \left| 1.8 + 0.7 \ln \left( \frac{m_{H^+}}{\text{TeV}} \right) + i \right| \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^+}} \right]^2. \end{aligned} \quad (94)$$

Comparing these contributions with the SM result (72) we find:

$$\frac{\Delta m_{B_q}^{H^+}}{\Delta m_{B_q}^{\text{SM}}} \lesssim 0.15, \quad (95)$$

for  $m_{H^+} \gtrsim 55 \text{ TeV}$  and  $v \simeq \sqrt{2} m_c$  as given by the bound (83).

In Fig. 3 we illustrate the constraints on the  $(\rho, \eta)$ -plane for  $m_{H^+} = 2 \text{ TeV}$ ,  $v = \sqrt{2} m_c$  and the parameter  $\epsilon = 0$ . We assume  $m_{H^0} \gg m_{H^+}$  and thus, only the contribution coming from the flavour-changing charged current is taken into account. The dot-filled area corresponds to the allowed region. A similar plot is given in Fig. 4 for the case  $\epsilon = 1$  and  $m_{H^+} = 80 \text{ TeV}$ .

### 4.3 FCNC and $\Delta m_D$

In the up quark sector,  $D^0 - \bar{D}^0$  mixing is perhaps one of the most interesting processes given that this process is highly suppressed in the SM:  $\Delta m_D^{\text{SM}} < 10^{-15} \text{ GeV}$ . To estimate the size of FCNC contributions to the mass difference  $\Delta m_D$  we use the expression (29) and the approximate form (43) for the matrices  $U_{uL}, U_{uR}$ . The relevant couplings are then given by

$$\Gamma_{ij}^u = -\frac{\sigma_b \sigma_t}{v^2} e^{-i\varphi_t} (G_3^u)_{ij} \simeq -\frac{2m_b m_t}{v^2} e^{-i\varphi_t} (G_3^u)_{ij}, \quad (96)$$

where  $G_3^u$  is given in Eq. (45) and we have used  $m_b \simeq g_b \sigma_b / \sqrt{2}$ ,  $m_t \simeq g_t \sigma_t / \sqrt{2}$ ,  $g_{b,t} \simeq 1$ , to approximate the right hand side of Eq. (96).

Keeping the dominant term we obtain

$$\begin{aligned} M_{12}^{\text{new}}(D) &= -\epsilon^2 e^{-2i\varphi_t} \kappa_u^{(0)} \frac{m_b^2}{m_{H^0}^2}, \\ \kappa_u^{(0)} &= \frac{4m_u m_c f_D^2 \hat{B}_D m_D}{v^4} \left[ \frac{1}{6} + \left( \frac{m_D}{m_u + m_c} \right)^2 \right], \\ &\simeq 5.4 \times 10^{-3} \text{ GeV} \left[ \frac{\text{GeV}}{v} \right]^4, \end{aligned} \quad (97)$$

where  $f_D \sqrt{\hat{B}_D} \simeq 225$  MeV [19],  $m_D = 1.86$  GeV and we take  $m_u \simeq 5$  MeV. Therefore,

$$\Delta m_D = 2|M_{12}(D)| = 2\epsilon^2 \kappa_u^{(0)} \frac{m_b^2}{m_{H^0}^2} \simeq 4.6 \epsilon^2 \times 10^{-11} \text{ GeV} \left[ \frac{\text{GeV}}{v} \right]^4 \left[ \frac{65 \text{ TeV}}{m_{H^0}} \right]^2 . \quad (98)$$

Using the bound on  $m_{H^0}$  coming from  $K$  physics (cf. Eq.(67)), one obtains the upper limit

$$\Delta m_D \lesssim 1.35 \epsilon^2 \times 10^{-11} \text{ GeV} . \quad (99)$$

We notice that except when  $\epsilon \ll 1$ , our model predicts a value for  $\Delta m_D$  much larger than in the SM. This is a clear signature of the model.

Comparing the above value with the experimental limit

$$(\Delta m_D)_{\text{exp}} < 5 \times 10^{-14} \text{ GeV} , \quad (100)$$

we find an upper limit on the parameter  $\epsilon$ ,

$$\epsilon \lesssim 0.06 , \quad (101)$$

which in turn translates into constraints on the texture form (42) assumed here for the up quark mass matrix  $M_u$ .

At this point it is worth recalling that flavour-changing contributions induced by new physics crucially depend on the specific patterns of the fermion mass matrices. In our analysis we have assumed a simple but quite generic *Ansatz* for the up-quark mass matrix, expressed in terms of quark mass ratios and a free parameter  $\epsilon$ . This parameter is therefore expected to be constrained in order to avoid dangerous large contributions to low-energy observable effects. It is clear that more specific mass matrix textures (e.g. triangular-type textures in the up-quark sector) could lead to a natural suppression of these contributions and the above constraints thus be avoided.

## 5 Electric dipole moment of the neutron

The electric dipole moment (EDM) of a fermion  $\Psi$  is defined as

$$iF_D(k^2) \bar{\Psi}(p_1) \gamma_5 \sigma^{\mu\nu} \Psi(p_2) F_{\mu\nu} ,$$

where  $F_{\mu\nu}$  is the electromagnetic tensor and  $k_\mu = p_{2\mu} - p_{1\mu}$ . In the present model, we can expect a large contribution to the EDM of the neutron induced by the spontaneous  $CP$  violation if the phases  $\varphi_{t,b} \neq 0, \pi$ . In the latter case, the exchange of the heavy neutral and the heavy charged Higgs fields are expected to contribute to the EDM of the neutron. For the down quark contribution with exchange of a heavy neutral Higgs the dominant term is given by ( $f = d, u$ )

$$\left| \frac{F_D(0)}{e} \right|_f \simeq \frac{|Q_f|}{16\pi^2} \text{Im}(\Gamma_{31}^f \Gamma_{13}^f) \frac{\sqrt{x_f} (x_f^2 - 1 - 2x_f \ln x_f)}{(x_f - 1)^3 m_{H^0}} , \quad (102)$$

where  $x_{d,u} \equiv m_{b,t}^2/m_{H^0}^2$ ,  $Q_u = 2/3$ ,  $Q_d = -1/3$ ,  $\Gamma_{ij}^d$  and  $\Gamma_{ij}^u$  are defined in Eqs. (60) and (96), respectively. Using the fact that  $m_b \simeq g_b \sigma_b/\sqrt{2}$ ,  $m_t \simeq g_t \sigma_t/\sqrt{2}$  and assuming as before  $g_{b,t} \simeq 1$ , we have

$$\Gamma_{31}^d \simeq -\frac{\sqrt{2}m_b}{v} e^{i\varphi_b} V_{td}, \quad \Gamma_{13}^d \simeq -\frac{\sqrt{2}m_b}{v} e^{i\varphi_b} V_{td}^*, \quad (103)$$

$$\Gamma_{31}^u = \Gamma_{13}^u \simeq -\frac{2m_b\sqrt{\epsilon m_u m_t}}{v^2} e^{-i\varphi_t}. \quad (104)$$

Since  $x_f \ll 1$  and  $\sin 2\varphi_{b,t} \simeq 2m_b/m_t$ , Eqs. (102) are approximately given by

$$\left| \frac{F_D(0)}{e} \right|_d \simeq \frac{1}{12\pi^2} |V_{td}|^2 \frac{m_b^4}{v^2 m_t m_{H^0}^2} \simeq 8 \times 10^{-30} \text{ cm} \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{65 \text{ TeV}}{m_{H^0}} \right]^2, \quad (105)$$

$$\left| \frac{F_D(0)}{e} \right|_u \simeq \frac{\epsilon}{3\pi^2} \frac{m_u m_b^3 m_t}{v^4 m_{H^0}^2} \simeq \epsilon \times 10^{-23} \text{ cm} \left[ \frac{\text{GeV}}{v} \right]^4 \left[ \frac{65 \text{ TeV}}{m_{H^0}} \right]^2, \quad (106)$$

after substituting the values  $m_u \simeq 5 \text{ MeV}$ ,  $m_b \simeq 4.25 \text{ GeV}$ ,  $m_t \simeq 165 \text{ GeV}$  and  $|V_{td}| \simeq 0.01$ .

Thus, using the lower bound on  $m_{H^0}$  given by Eq. (67), we obtain the following upper bounds on the electric dipole moments of the quarks:

$$\left| \frac{F_D(0)}{e} \right|_d \lesssim 8 \times 10^{-30} \text{ cm}, \quad \left| \frac{F_D(0)}{e} \right|_u \lesssim 3 \epsilon \times 10^{-24} \text{ cm}. \quad (107)$$

Taking into account that  $m_u/m_c \simeq 10^{-3} \lesssim \epsilon \leq 1$  and assuming

$$\left| \frac{F_D(0)}{e} \right|_n \simeq \left| \frac{F_D(0)}{e} \right|_d + \left| \frac{F_D(0)}{e} \right|_u, \quad (108)$$

we conclude from Eqs. (107) that the EDM will be always dominated by the up-quark contribution. Moreover,

$$10^{-27} \text{ cm} \lesssim \left| \frac{F_D(0)}{e} \right|_n \lesssim 10^{-24} \text{ cm} \quad (109)$$

in the allowed range of the parameter  $\epsilon$ . Of course, the above conclusions hold for our specific choice of the up-quark mass matrix texture (42). If such is the case, then the predicted EDM is expected to be very close to the present experimental limit [1]

$$\left| \frac{F_D(0)}{e} \right|_{\text{exp}} < 10^{-26} \text{ cm}. \quad (110)$$

The comparison of Eqs. (107) with the experimental bound (110) allows us to further constrain the parameter  $\epsilon$ . We find

$$10^{-3} \lesssim \epsilon \lesssim 3 \times 10^{-3}, \quad (111)$$

which is more restrictive than the upper bound previously found from  $D^0 - \bar{D}^0$  mixing (see Eq. (101)). Notice however that the constraint (111) was obtained assuming the lower bound on the Higgs mass given by Eq. (67) and implied by  $\varepsilon_K$ . We could of course relax this constraint by pushing the heavy neutral Higgs mass to a higher scale, but then the “raison d’être” of our model would be lost and its predictions would be close to the SM ones. From a phenomenological point of view we find more plausible to fix the Higgs scale from the constraints coming from  $K$  and  $B$  physics and, in particular, from the  $CP$ -violating parameter  $\varepsilon_K$ . Other constraints, such as the EDM of the neutron, will then give us a hint on what kind of quark mass matrix textures are favoured in the theory.

Let us now consider the charged Higgs contributions. In this case the dominant contribution is coming from the diagrams with a top quark circulating inside the loop. The photon line can be attached to the Higgs line or to the quark line. Therefore we have

$$\left| \frac{F_D(0)}{e} \right|_{H^+} \simeq \frac{1}{16\pi^2} \text{Im} [(A_1^d)_{31} (A_1^u)_{13}] \left\{ Q_{H^-} \frac{\sqrt{x_u} (x_u^2 - 1 - 2x_u \ln x_u)}{(x_u - 1)^3} + Q_u \frac{\sqrt{x_u} (x_u^2 - 4x_u + 3 + 2 \ln x_u)}{(x_u - 1)^3} \right\} \frac{1}{m_{H^+}}, \quad (112)$$

with  $x_u$  now defined as  $x_u \equiv m_t^2/m_{H^+}^2$ ,  $Q_{H^-} = -1$ ;  $A_1^u$  and  $A_1^d$  given by Eqs. (46) and (47), respectively. Since

$$(A_1^u)_{13} \simeq \frac{\sqrt{2}}{v} e^{i\varphi_b} V_{td}^* m_t (1 - e^{i\varphi_t}) \simeq -\frac{i\sqrt{2}}{v} e^{i\varphi_b} V_{td}^* m_b, \quad (113)$$

$$(A_1^d)_{31} \simeq -\frac{\sqrt{2}}{v} e^{i(\varphi_t - \varphi_b)} V_{td} m_b, \quad (114)$$

we obtain

$$\left| \frac{F_D(0)}{e} \right|_{H^+} \simeq \frac{3}{8\pi^2} \frac{m_b^2 m_t}{v^2 m_{H^+}^2} |V_{td}|^2 \simeq 2 \times 10^{-22} \text{ cm} \left[ \frac{\text{GeV}}{v} \right]^2 \left[ \frac{\text{TeV}}{m_{H^+}} \right]^2. \quad (115)$$

Imposing the experimental constraint given in Eq. (110), we get a stronger constraint on  $m_{H^+}$  than the lower bound (82), namely,

$$m_{H^+} \gtrsim 140 \text{ TeV} \left[ \frac{\text{GeV}}{v} \right]. \quad (116)$$

In particular, for  $v \simeq \sqrt{2} m_c$  we obtain

$$m_{H^+} \gtrsim 75 \text{ TeV}. \quad (117)$$

We also remark that in leading order this bound is independent of the texture parameter  $\epsilon$ .

## 6 FCNC in top decays: the example $t \rightarrow q \gamma$

The FCNC decays  $t \rightarrow q \gamma$  and  $t \rightarrow q Z$  are strongly suppressed in the SM at the level of  $10^{-12}$ . Observation of any of these events would be an indication of physics beyond the SM. It is of particular interest to study the order of magnitude that our model predicts for such processes. The amplitude of  $t \rightarrow q \gamma$  can be parametrized as

$$M_\gamma \equiv \bar{q}(p_1) \left[ i(A + B\gamma_5)\sigma^{\mu\nu} \frac{q_\nu}{m_t} \right] t(p_2)A_\mu , \quad (118)$$

where  $A_\mu$  is the photon field and  $q_\mu \equiv p_{2\mu} - p_{1\mu}$ . The decay width of this process is given by

$$\Gamma(t \rightarrow q \gamma) = \frac{m_t}{8\pi} (|A|^2 + |B|^2) , \quad (119)$$

and is dominated by the process  $t \rightarrow b W$ ,

$$\Gamma(t \rightarrow b W) = \frac{G_F}{8\sqrt{2}\pi} |V_{tb}|^2 m_t^3 \left( 1 - \frac{m_W^2}{m_t^2} \right) \left( 1 + \frac{m_W^2}{m_t^2} - 2\frac{m_W^4}{m_t^4} \right) . \quad (120)$$

Once the coefficients  $A$  and  $B$  are known, it is straightforward to compute the branching ratio corresponding to the process  $t \rightarrow q \gamma$ . Recently, an experimental limit on this process has been reported [20]

$$B(t \rightarrow q \gamma) < 0.032 , \quad (121)$$

which can be translated into a limit on the parameters  $A$  and  $B$ ,

$$|A|^2 + |B|^2 < 6.5 \times 10^{-3} . \quad (122)$$

This limit should improve with the future LHC, which is expected to decrease the above bound by two orders of magnitude [21]

$$B(t \rightarrow q \gamma) < 10^{-4} , \quad (123)$$

i.e.

$$|A|^2 + |B|^2 < 2 \times 10^{-5} . \quad (124)$$

In our model one expects that a such process will be induced by one loop diagrams as is the case for the EDM. Moreover, both contributions, with a heavy neutral Higgs exchange and with a charged Higgs exchange, should be taken into account. One expects then using the bounds given by Eqs. (67) and (117),

$$A \approx B \simeq \frac{m_b^2}{m_{H^0}^2} \lesssim 10^{-8} \quad (125)$$

for the neutral Higgs exchange, while

$$A \approx B \simeq \frac{m_t^2}{m_{H^+}^2} \lesssim 5 \times 10^{-6} \quad (126)$$

for the charged Higgs exchange.

Since the above bounds are further multiplied by additional suppression factors coming from the CKM matrix, we can conclude that the prediction of the present model for the process  $t \rightarrow q \gamma$  is out of the reach of the next future colliders.

## 7 Implications on $CP$ asymmetries

As discussed in Section 4, if  $m_{H^+} \simeq m_{H^0}$  then the flavour-changing charged Higgs contributions to  $\Delta m_{B_{d,s}}$  are negligible. In order to study the implications of the model on  $CP$  asymmetries, we shall assume that new physics appears only through tree-level FCNC effects.

An easy way to parametrize the effects of new physics on  $B_q^0 - \bar{B}_q^0$  mixing is by introducing the parameters  $r_q^2$  and the phases  $2\theta_q$  through the relation

$$M_{12}(B_q) = M_{12}^{\text{SM}} + M_{12}^{\text{new}} \equiv r_q^2 e^{2i\theta_q} M_{12}^{\text{SM}}(B_q) . \quad (127)$$

On the other hand, from Eqs. (56) and (68) we have

$$M_{12}(B_q) = (1 - w_q e^{2i\varphi_b}) M_{12}^{\text{SM}}(B_q) , \quad (128)$$

with  $w_q$  defined in Eq. (73). Comparing Eqs. (127) and (128) we find the relations

$$r_q^2 = \sqrt{1 - 2w_q \cos 2\varphi_b + w_q^2} , \quad \tan 2\theta_q = \frac{-w_q \sin 2\varphi_b}{1 - w_q \cos 2\varphi_b} . \quad (129)$$

Within the SM, the  $CP$  asymmetry  $a_{\psi K_s}$  in  $B_d^0 (\bar{B}_d^0) \rightarrow \psi K_S$  decays is related to the angle  $\beta$  of the unitarity triangle,

$$a_{\psi K_s} = \sin 2\beta , \quad \beta = \arg \left[ -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right] . \quad (130)$$

While global analyses of the CKM unitarity triangle yield the values

$$(\sin 2\beta)_{\text{SM}} = \begin{cases} 0.75 \pm 0.06 & [22] , \\ 0.73 \pm 0.20 & [23] , \\ 0.63 \pm 0.12 & [24] , \end{cases} \quad (131)$$

the recent experimental measurements of the above time dependent  $CP$  asymmetry give

$$(\sin 2\beta)_{\psi K_S} = \begin{cases} 0.12 \pm 0.37 \pm 0.09 & (\text{BABAR}) [25] , \\ 0.45 \pm 0.44 \pm 0.08 & (\text{BELLE}) [26] , \\ 0.79 \pm 0.42 & (\text{CDF}) [27] . \end{cases} \quad (132)$$



The above experimental values imply the average

$$(\sin 2\beta)_{\psi K_S} = 0.42 \pm 0.24 . \quad (133)$$

Although the SM estimates are consistent with the present experimental results, the small values of  $\sin 2\beta$  found by BABAR and BELLE collaborations might indicate the presence of new physics contributions.

If the new physics modifies the phase of the mixing amplitude, then the asymmetry will also get a contribution from the  $\theta_d$  phase,

$$a_{\psi K_S} = \sin 2(\beta + \theta_d) . \quad (134)$$

Moreover, if we assume that the  $\theta$  term in Lagrangian (6) is the only source of  $CP$  violation in our model, then  $\varphi_b \simeq -\pi/2 + m_b/m_t$ . In this case Eqs. (129) imply

$$r_d \simeq \sqrt{1 + w_d} , \quad \tan 2\theta_d \simeq \frac{2w_d}{1 + w_d} \frac{m_b}{m_t} . \quad (135)$$

We see that the new phase  $\theta_d$  is suppressed by the ratio  $m_b/m_t$ . Using the upper bound  $w_d \lesssim 0.19$  given by Eq. (73) we find

$$r_d \lesssim 1.1 , \quad r_s \simeq r_d , \quad \tan 2\theta_d \lesssim 0.008 . \quad (136)$$

Although these predictions are consistent with the global average (133), we notice that the deviations from the SM predictions are very small in this case and, consequently, it is not possible to achieve consistency [18] with the small values reported by the BABAR collaboration [25].

In order to illustrate the dependence of our result on the strong  $CP$  phase  $\theta$ , let us assume that  $CP$  violation in the CKM matrix is independent of the value of  $\theta$ . In other words, let us assume that besides the angle  $\theta$ , there exist other sources of  $CP$  violation and we consider  $\theta$  as an arbitrary parameter. Of course, when  $\theta$  is different from  $\pi/2$ , the isospin symmetry between the top and bottom quarks has to be explicitly broken in the effective Lagrangian (3). From the minimization of the potential (cf. Eqs. (10)) and taking  $m_b \ll m_t$ , one easily finds

$$\varphi_b \simeq -\theta + \frac{m_b}{m_t} , \quad \varphi_t \simeq \frac{m_b}{m_t} . \quad (137)$$

At first order in  $m_b/m_t$  we obtain in this case:

$$r_d = \sqrt{1 - 2w_d \cos 2\theta + w_d^2} , \quad \tan 2\theta_d = \frac{w_d \sin 2\theta}{1 - w_d \cos 2\theta} . \quad (138)$$

It is interesting to study how the angle  $\theta_d$  varies as a function of  $\theta$ . The extrema values for  $\theta_d$  are obtained when  $\cos 2\theta = w_d$  and this implies

$$r_d = \sqrt{1 - w_d^2} , \quad \tan 2\theta_d = \pm \frac{w_d}{\sqrt{1 - w_d^2}} . \quad (139)$$

With  $w_d \lesssim 0.19$  as given by Eq. (73) we have then

$$r_d \gtrsim 0.98, \quad |\tan 2\theta_d| \lesssim 0.19. \quad (140)$$

Thus, if the strong  $CP$  phase  $\theta$  is assumed to be a free parameter of the model, the constraint (136) on the new physics contribution to the  $CP$  asymmetries in  $B$  decays is relaxed and we can reach a rather sizeable phase  $\theta_d \simeq 6^\circ$ . This in turn would allow to accommodate [18] the present experimental measurements, including the small values obtained by BABAR and BELLE collaborations.

## 8 Conclusion

In this paper we have studied the phenomenological constraints on a model where  $CP$  violation is dynamically induced by a strong  $CP$  phase  $\theta$  [11]. The most promising tests for the model are given by the new experimental prospects to measure  $\Delta M_D$  or to improve the experimental limit on the electric dipole moment of the neutron.

Contrary to naive expectations, the fact that the new force responsible for the electroweak symmetry breaking and for the spontaneous  $CP$  violation is only sensitive to the third generation of quarks does not imply that the most stringent constraints come from processes involving the heavy flavours ( $t$  and  $b$ ). We have shown that the stringent constraints on the scale of new physics come from  $K$  physics and from the electric dipole moment of the neutron. This means that even if FCNC processes are naturally suppressed in the model by the CKM matrix elements, this suppression is not strong enough to allow for a mass scale of the heavy Higgs to be of the order of few TeV.

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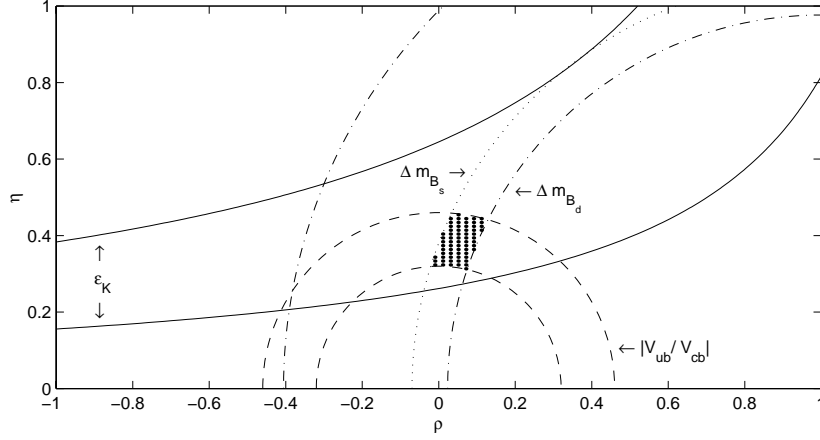


Figure 3: Constraints on the plane  $(\rho, \eta)$  after including the new flavour-changing charged contributions induced by the heavy charged Higgs  $H_1^+$ . The curves are given for  $m_{H_1^+} = 2$  TeV,  $v = \sqrt{2}m_c$  and the parameter  $\epsilon = 0$ . We assume  $m_{H_1^+} \simeq m_{H_2^+} \ll m_{H^0}$ .

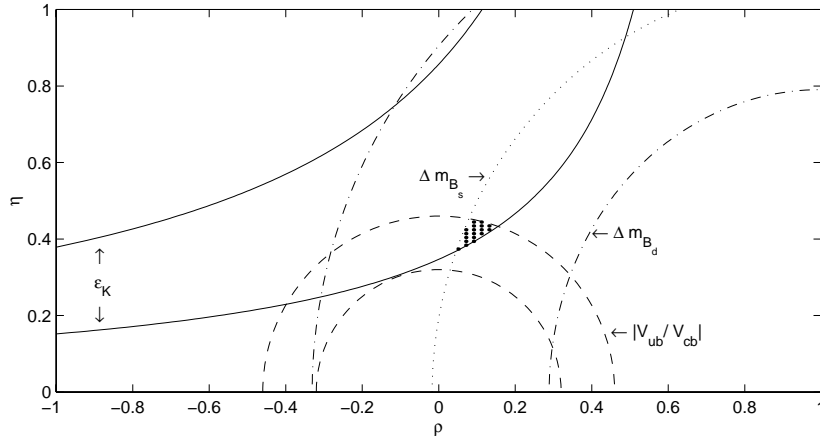


Figure 4: As in Fig. 3, but taking the parameter  $\epsilon = 1$  and  $m_{H_1^+} = 80$  TeV.