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A WARM FLUID MODEL FOR COASTING BEAM LONGITUDINAL INSTABILITIES

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Abstract

A simple warm fluid model for the longitudinal motion of a coasting beam is proposed. It foresees accurately the rise time of the instability and the frequency shift when the working point of the machine is far outside the stability region. The model takes into account the effects of the beam momentum spread through the fluid kinetic pressure. The closure of the fluid model is obtained by imposing that the "heat flux" is zero in the equation for the energy conservation. The proposed model provides accurately the growth times and the frequency shift for the longitudinal instabilities obtained by using the kinetic model, the SCOP-RZ/PATRIC PIC code as well as for that observed at the ESR in 1997.

1. Introduction

The study of coherent instabilities of particle beams in circular accelerators and storage rings, caused by the interaction with self-induced electromagnetic fields, has received recently new attention in connection with the research on high-current machines operating below transition energy [1].

The mechanism of the longitudinal instability in a coasting beam is intrinsically of fluid nature, see, for instance, the comprehensive review in [2]. By making use of the cold fluid model it is possible to describe the initial phase of the instability growth. In particular, it is possible to give simple analytical expressions for the rise time and the frequency shift of the unstable slow

wave as functions of the beam parameters as well as of the total impedance, [2]. Indeed, these formulae are reliable only far outside the stability region corresponding to the initial beam velocity distribution.

This limit of validity is due to the fact that in the cold fluid model the “slowing” effect of the Landau damping is absent (*e.g.*, [2], [3]). The mechanism of beam stabilisation, known as Landau damping, gives rise to a finite stability region in the impedance plane, and can be in no case predicted by a model where the wave-particle interaction is not taken into account. In other words, we must never expect that a fluid model can give as a result the existence of a stable beam even when the resistive part of the impedance is different from zero.

Anyway even quite far outside the stability boundary the cold fluid model still causes an underestimation of the rise time and of the absolute value of the frequency shift.

In this paper we propose a warm fluid model in which the effects of the beam momentum spread are taken into account through the “kinetic pressure” of the beam. The closure of the fluid model is obtained by imposing that the “heat flux” is zero in the equation for the energy conservation. The pressure effect on the beam dynamics plays the same role as the beam space charge impedance. In particular, we show that the effect of the initial pressure cannot be neglected when the Keil-Schnell impedance (*e.g.*, [2]) is of the same order of magnitude as the total impedance. Finally we apply these results to the ESR storage ring and we find a good agreement between the results obtained with our model, those obtained with kinetic model, the PATRIC PIC code as well as that observed at the ESR in 1997.

2. Review of the kinetic model

Let us consider a coasting beam moving in a circular machine, (*i.e.*, a storage ring or a circular accelerator), with nominal longitudinal velocity U_0 and nominal circular “equilibrium” orbits with radius r_0 ; $C_0 = 2\pi r_0$ is the circumference length. In the following we will always use the symbols θ and r for the azimuthal and radial co-ordinates, respectively. Furthermore, with ω and ε we will indicate the angular frequency, $\omega = \dot{\theta}$, and the total energy of the particle; $\omega_0 = U_0 / r_0$ is the nominal angular frequency. The radius of the equilibrium orbit of the particle depends on the angular frequency, $r = r(\omega)$, according to the momentum compaction and slip factors, and the revolution frequency of the generic particle depends on its energy ε , according to the frequency dispersion of the ring $\omega = \omega(\varepsilon)$, (*e.g.*, [2]).

We start describing the beam by means of the distribution function $g(\theta, w; t)$ in the phase space (θ, w) where w is defined as

$$w(\varepsilon) \triangleq 2\pi \int_{\varepsilon_0}^{\varepsilon} \frac{d\varepsilon}{\omega(\varepsilon)}. \quad (1)$$

Let us suppose that the longitudinal component $E = E(\theta, r; t)$ of the electric field acting on the particles (due to the space charge and to the interaction of the beam with the surroundings) depends on the radial co-ordinate r as $1/r$,

$$E(r, \theta; t) = -\frac{\phi(\theta; t)}{2\pi r}, \quad (2)$$

where the “potential” function $\phi(\theta; t)$ is independent to the particle radius. Under this assumption

θ and w are conjugate variables. An immediate consequence of this property is that the distribution function $g(\theta, w; t)$ is solution of the kinetic equation

$$\frac{\partial g}{\partial t} + \omega(w) \frac{\partial g}{\partial \theta} - q\phi(\theta; t) \frac{\partial g}{\partial w} = 0. \quad (3)$$

where q is the electric charge of the particle. We have neglected collision phenomena because they do not play a considerable role on the time scale of the instability we are interested in (this assumption may be always checked “a posteriori”). In this model the electric field E is the averaged field over small phase space volumes so as to be cleaned out of the fluctuations due to the microscopic discrete nature of the particle beam.

To derive the cold fluid model we need the kinetic description of the beam in the velocity phase space. Let us introduce the two independent variables s and u

$$\begin{aligned} s &= r_0 \theta, \\ u &= r_0 v. \end{aligned} \quad (4)$$

and let be $f(s, u; t)$ the distribution function of the beam in the phase space (s, u) (the spatial coordinate s is defined on the interval $(0, C_0)$). The distribution function $f(s, u; t)$ is related to the distribution function $g(\theta, w; t)$ through the relation

$$f(s, u, t) = \frac{1}{r_0^2} \left| \frac{dw}{d\omega} \right| g(\theta, w, t). \quad (5)$$

Assuming a small relative energy spread of the beam particles, we may replace ω by its nominal mean value ω_0 in the integral (1) and we may linearize the dispersion relation that links the angular frequency to the particle energy,

$$\omega(\Delta\varepsilon) = \omega_0 + \kappa_0 \Delta\varepsilon, \quad (6)$$

where $\Delta\varepsilon \hat{=} \varepsilon - \varepsilon_0$, $\kappa_0 \hat{=} -\eta\omega_0 / (\beta_0^2 \varepsilon_0)$, $\beta_0 = U_0 / c$, $\varepsilon_0 = m_0 c^2 \gamma_0$, η is the frequency slip factor, m_0 is the rest mass of the particles and $\gamma_0 = (1 - \beta_0^2)^{-1/2}$. By using these approximations we obtain for the leading term

$$w(\varepsilon) \cong 2\pi(\varepsilon - \varepsilon_0) / \omega_0, \quad (7)$$

and

$$f(s, u; t) \cong \frac{2\pi}{r_0^2 |\kappa_0| \omega_0} g(\theta, w; t). \quad (8)$$

Therefore the distribution function $f(s, u; t)$ is solution of the Vlasov equation

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial s} - \frac{q}{m^*} \frac{\phi(s; t)}{2\pi r_0} \frac{\partial f}{\partial u} = 0. \quad (9)$$

To derive the equation (9) we have used the relation

$$\frac{\omega_0 r_0}{\beta_0^2 \epsilon_0} = \frac{U_0}{\beta_0^2 \epsilon_0} = \frac{1}{p_0}, \quad (10)$$

where p_0 is the nominal momentum of the beam particle. Furthermore, we have introduced the “effective” particle mass

$$m^* = -\frac{p_0}{U_0 \eta}. \quad (11)$$

Note that below transition m^* is positive.

Finally we have to specify the form of the driving term ϕ present in the Vlasov equation. In general $\phi(t)$ is made up of two different contributions: an external voltage acting on the beam, which can represent either a bunching field oscillating in the RF cavity or a residual field detuned with respect to the beam revolution frequency, and a self-voltage coming from the interaction of the beam with the beam itself and with the surrounding environment.

It is customary to represent the latter on a given harmonic as the product of the beam current component at that harmonic times the coupling longitudinal impedance that synthetically describes all the mentioned interactions of the beam. Thus the electric “potential” $\phi(s, t)$ is linked to the beam electric current intensity

$$I(s, t) = q \int_{-\infty}^{+\infty} u f(s, u, t) du, \quad (12)$$

through the relation

$$\phi(s, t) = \sum_m \dot{Z}(m\omega_0) I_m(t) e^{-imk_0 s} + \phi_{ext}(s, t), \quad (13)$$

where $k_0 = (\omega_0 / u_0) = 1 / r_0$,

$$I_m(t) = \frac{1}{C_0} \int_0^{C_0} I(s, t) e^{imk_0 s} ds, \quad (14)$$

$\dot{Z} = \dot{Z}(\omega)$ is the total longitudinal impedance of the machine and $\phi_{ext}(s, t)$ is an external voltage *i.e.*, it could take into account the action of an RF cavity. Using the value of the impedance estimated exactly at $\omega = m\omega_0$ is in fact an approximation, since the time dependence of the beam current signal makes it not purely oscillating at the multiples of the revolution frequency but at shifted values.

The potential $\phi(t)$ depends only on the total impedance of the machine \dot{Z} , on the applied RF voltage and on the beam current intensity, therefore it is completely determined by fluid variables.

In this paper we analyse the situation where no external voltage is acting on the beam and the only electromagnetic force that determines the beam evolution is the one associated to the beam-beam and the beam-environment interaction.

The impedance $\dot{Z}(\omega)$ consists of the space charge impedance $\dot{Z}_m^{(sc)}$ of the beam itself and the impedance of the surroundings $\dot{Z}_m^{(sur)}$:

$$\dot{Z}_m = \dot{Z}_m^{(sc)} + \dot{Z}_m^{(sur)}, \quad (15)$$

where [2]

$$\dot{Z}_m^{(sc)} = -imX_{sc}. \quad (16)$$

The reactance X_{sc} in the relation (16) is the space charge reactance of the beam, which does not depend on the wave number m if it is not too high (for ESR must be $m \ll 10^3$).

3. The equations of the fluid model with temperature

In order to deduce from the kinetic model the equations for the dynamics of the macroscopic fluid quantities characterising the beam, such as the distribution of its line density, the mean velocity and the current intensity along the ring, we have to calculate the momenta of Vlasov equation (9) by multiplying it with powers of the velocity u and integrating over the velocity space. With this regard, we firstly remind how the numeric line density $n(s,t)$ and the mean velocity $U(s,t)$ of the beam are related to the distribution function f ,

$$\begin{aligned} n(s,t) &\triangleq \int_{-\infty}^{\infty} f(s,u,t)du, \\ U(s,t) &\triangleq \frac{\int_{-\infty}^{\infty} uf(s,u,t)du}{n(s,t)}. \end{aligned} \quad (17)$$

The beam current intensity $I(s,t)$ is given by

$$I(s,t) = qn(s,t)U(s,t) \cong qU_0n(s,t). \quad (18)$$

In equation (18) we have approximated the actual averaged velocity of the beam with the nominal mean velocity U_0 because in our case the shifting of the actual mean velocity from the nominal one is very small compared with U_0 .

Let us start with the zero-order moment obtained by multiplying by $u^0 = 1$ and integrating on the velocity space. The first term of equation (9) becomes

$$\left\langle \frac{\partial f}{\partial t} \right\rangle = \frac{\partial n}{\partial t}, \quad (19)$$

the second

$$\left\langle u \frac{\partial f}{\partial s} \right\rangle = \frac{\partial}{\partial s}(nU), \quad (20)$$

and the third

$$\frac{q}{m^*} \frac{\phi(s,t)}{2\pi r_0} \left\langle \frac{\partial f}{\partial u} \right\rangle = \frac{1}{2\pi r_0} \frac{q}{m^*} \phi(s,t) [f(s,u,t)]_{u=-\infty}^{u=\infty} = 0, \quad (21)$$

where the symbol $\langle f \rangle$ indicates $\int_{-\infty}^{+\infty} f du$, namely the averaging over the velocity space. Since no particles can have infinite velocity, f falls very rapidly as $u \rightarrow \pm\infty$. By using the expressions (19)-(21), from equation (9) we have the continuity equation in the configuration space s

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial s}(nU) = 0. \quad (22)$$

If we multiply now equation (9) by u and then integrate it on the velocity space the first term becomes:

$$\left\langle u \frac{\partial f}{\partial t} \right\rangle = \frac{\partial}{\partial t} \langle nU \rangle, \quad (23)$$

the second

$$\langle u^2 \frac{\partial f}{\partial s} \rangle = \frac{\partial}{\partial s} (nU^2) + \frac{\partial}{\partial s} \langle (u - U)^2 f \rangle, \quad (24)$$

and the third

$$-\frac{q}{m^*} \frac{\phi}{2\pi r_0} \langle u \frac{\partial f}{\partial u} \rangle = \frac{q}{m^*} \frac{n\phi}{2\pi r_0}. \quad (25)$$

In this way we get the equation of the “momentum” conservation

$$\frac{\partial}{\partial t} (m^* nU) + \frac{\partial}{\partial s} [(m^* nU)U + \Pi] = -\frac{1}{2\pi r_0} qn\phi, \quad (26)$$

where the new “macroscopic” quantity $\Pi(s, t)$ given by

$$\Pi(s, t) = \langle m^* (u - U)^2 f \rangle \quad (27)$$

takes into account the effects due to the spread in the longitudinal velocity of the particle beam. The quantities $P(s, t) = m^* n(s, t)U(s, t)$ and $\Pi(s, t)$ may be considered, respectively, as the longitudinal line momentum density and the longitudinal “kinetic pressure” of the beam. The pressure contribution in (26) may be interpreted as the additional momentum flux due to the particle motion relative to the mean one.

Of course we need now one more equation in order to express the dynamics of the kinetic pressure. In the cold fluid approximation we would have simply closed the system with the assumption $\Pi \rightarrow 0$ working as a constitutive relation. But if our aim is to try and set up a more precise fluid model that takes into account the presence of a finite beam energy spread, then we have to proceed with at least one further step in the hierarchy of the momenta equations and introduce only later an external assumption in order to close our resulting model. Therefore, we multiply now equation (9) by u^2 and then average it on the velocity space; the first term becomes:

$$\left\langle m^* u^2 \frac{\partial f}{\partial t} \right\rangle = \frac{\partial}{\partial t} (m^* nU^2) + \frac{\partial \Pi}{\partial t}, \quad (28)$$

the second

$$\left\langle m^* u^3 \frac{\partial f}{\partial s} \right\rangle = \frac{\partial}{\partial s} [(m^* nU^2 + 3\Pi)U + \Theta], \quad (29)$$

and the third

$$\frac{q}{m^*} E(s, t) \left\langle u^2 \frac{\partial f}{\partial u} \right\rangle = -2nU \frac{q}{m^*} E(s, t). \quad (30)$$

After summing up these last three terms and by making use of the first two equations of the fluid model, that is (22) and (26), so as to express by means of spatial partial derivatives the time partial derivatives $\partial n/\partial t$, $\partial U/\partial t$ and $\partial U^2/\partial t$, we finally obtain the equation of “conservation of energy” in the simple form:

$$\frac{\partial}{\partial t} \left(\frac{\Pi}{n^3} \right) + U \frac{\partial}{\partial s} \left(\frac{\Pi}{n^3} \right) + \frac{1}{n^3} \frac{\partial \Theta}{\partial s} = 0 \quad (31)$$

Here $\Theta(s, t)$, defined as

$$\Theta(s, t) = \left\langle m^* (u - U)^3 f \right\rangle, \quad (32)$$

takes into consideration the flux of “disordered” kinetic energy due to the disordered motion, namely that associated to the disordered part of the particles longitudinal motion. Then, the quantity $\Theta(s, t)$ may be considered as a “heat flux”. Now we assume that during the beam motion there's no heat flux, that is $\Theta \rightarrow 0$. Under this assumption we get the closed system of partial differential equations in the unknowns $n(s, t)$, $U(s, t)$ and $\Pi(s, t)$:

$$\begin{cases} \frac{\partial n}{\partial t} + \frac{\partial}{\partial s} (nU) = 0, \\ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial s} + \frac{1}{n} \frac{\partial P}{\partial s} = -\frac{q}{2\pi r_0 m^*} \phi(s, t), \\ \frac{\partial}{\partial t} \left(\frac{P}{n^3} \right) + U \frac{\partial}{\partial s} \left(\frac{P}{n^3} \right) = 0. \end{cases} \quad (33)$$

It is useful to reformulate the fluid equations (33) in such a way as to hide the “fast” component of the dynamics, that is the one due to the beam revolution around the ring - its characteristic time being $2\pi/\omega_0$ - and provide the only “slow” components of the beam evolution. For this purpose we perform the following linear transformation of variables:

$$\begin{cases} U = U_0 + V, \\ s = U_0 t + x. \end{cases} \quad (34)$$

By applying this transformation to the system (33) we obtain

$$\begin{cases} \frac{\partial \Lambda}{\partial t} + \frac{\partial}{\partial x} (\Lambda V) = 0 \\ \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \frac{q}{\Lambda} \frac{\partial}{\partial x} (R \Lambda^3) = -\frac{1}{2\pi r_0} \frac{q}{m^*} \psi(x, t) \\ \frac{\partial R}{\partial t} + V \frac{\partial R}{\partial x} = 0 \end{cases} \quad (35)$$

where $\Lambda(x, t)$, $V(x, t)$, $R(x, t)$ are given by

$$\Lambda(x, t) = qn(x + U_0 t, t), \quad V(x, t) = U(x + V_0 t, t) - U_0, \quad R(x, t) = \Pi(x + U_0 t, t) / \Lambda^3(x, t), \quad (36)$$

$\psi(x, t)$ is

$$\psi(x, t) = U_0 \sum_m \dot{Z}(m v_0) \Lambda_m(t) e^{-i m k_0 x}, \quad (37)$$

and

$$\Lambda_m(t) = \frac{1}{C_0} \int_0^{C_0} \Lambda(x, t) e^{i m k_0 x} dx. \quad (38)$$

The quantity $\Lambda(x, t)$ represents the linear charge density. The equations (35) have to be solved with the boundary conditions

$$\begin{cases} \Lambda(x = 0, t) = \Lambda(x = C_0, t) \\ V(x = 0, t) = V(x = C_0, t), \\ R(x = 0, t) = R(x = C_0, t) \end{cases} \quad (39)$$

coming from the periodicity of the structure, and given initial conditions.

4. Linear analysis of the adiabatic fluid model

Let us perform now a linearization of the equations of the system (35) in order to find out which new terms are introduced by our considering the particle beam as a fluid with its own momentum spread and not purely cold. We expect this to modify the way the rise time of an instability must be evaluated; what is interesting, then, is to see how, and to which extent, the formulae for the characterization of *e-folding* time and frequency shift of an instability change after taking into account the term due the pressure gradient in the momentum conservation equation and treating it by means of the third equation of our system. Therefore we write

$$\Lambda(x, t) = \Lambda_0 + \delta\Lambda \quad V = \delta V \quad \Pi = \Pi_0 + \delta\Pi, \quad (40)$$

and then substitute these perturbed quantities in the equations of motion, keeping only the terms where the perturbations appear with a unitary power (linear terms):

$$\begin{cases} \frac{\partial \delta\Lambda}{\partial t} + \Lambda_0 \frac{\partial \delta V}{\partial x} = 0 \\ \frac{\partial \delta V}{\partial t} = -\frac{q}{\Lambda_0} \frac{\partial \delta\Pi}{\partial x} - \frac{1}{2\pi r_0} \frac{q}{m^*} \psi(x, t) \\ \frac{\partial}{\partial t} \left(\frac{\delta\Pi}{\Lambda_0^3} - \frac{3\Pi_0}{\Lambda_0^3} \frac{\delta\Lambda}{\Lambda_0} \right) = 0 \end{cases} \quad (41)$$

Observing that the third equation easily provides

$$\delta\Pi = 3\Pi_0 \frac{\delta\Lambda}{\Lambda_0} + \text{const.} , \quad (42)$$

we finally find the system for the perturbations in velocity and density:

$$\begin{cases} \frac{\partial\delta\Lambda}{\partial t} + \Lambda_0 \frac{\partial\delta V}{\partial x} = 0, \\ \frac{\partial\delta V}{\partial t} = -\frac{q}{m^*} \frac{U_0}{2\pi} \left(\frac{6\pi m^* \Pi_0}{U_0 \Lambda_0^2} + X_{sc} \right) \frac{\partial\Lambda}{\partial x} - \frac{qU_0}{2\pi r_0 m^*} \sum_m \dot{Z}_m^{(surr)}(m v_0) \lambda_m(t) e^{-imk_0 s}. \end{cases} \quad (43)$$

From the last equation it becomes clear that the influence of the pressure term acts in the linear regime exactly as a further contribution to the space charge impedance seen by the beam. In other words, one could take into account the fact that our beam has got a finite spread in the velocities by considering in the classical fluid equations a modified space charge impedance, which is the sum of the one relative to the cold fluid model and a second term depending on the initial beam pressure and, consequently, on the initial beam momentum spread. Thus the second equation of system (43) suggests the definition of a sort of “kinetic” reactance according to:

$$X_{kin} = \frac{6\pi m^* \Pi_0}{\Lambda_0 I_0} \quad (44)$$

If we now introduce the beam longitudinal temperature

$$T_0 = \frac{q\Pi_0}{\Lambda_0 k} , \quad (45)$$

we can then rewrite the kinetic reactance as

$$X_{kin} = 6\pi \left(\frac{kT_0}{qI_0} \right) , \quad (46)$$

and the total modified space charge impedance will simply become:

$$X_{eq} = X_{sc} + X_{kin} = X_{sc} + 6\pi \left(\frac{kT_0}{qI_0} \right). \quad (47)$$

From the definition of Π , Eq. (29), it is straightforward to find out that the relation between the value of Π_0 , or equivalently of T_0 , and the beam initial velocity spread is the following:

$$\Pi_0 = \frac{\Lambda_0 \Delta v_{HWHM}^2}{2 \ln(2)}. \quad (48)$$

This last equality allows us to evaluate the quantity Π_0 from the observables Δv_{HWHM} and Λ_0 .

Now assuming that $\delta\Lambda$ and δV have a space-time dependence of the kind

$$\begin{aligned}\delta\Lambda(x, t) &= A_n e^{i(\Delta\omega t - nk_0 x)} + c. c., \\ \delta V(x, t) &= B_n e^{i(\Delta\omega t - nk_0 x)} + c. c.,\end{aligned}\quad (49)$$

we obtain for the complex frequency $\Delta\omega = \Delta\omega_r + i\Delta\omega_i$

$$\begin{aligned}\Delta\omega_r &= \pm\omega_0 \left[\frac{1}{2R_0} \left(\sqrt{Z_r^2 + Z_i^2} - Z_i \right) \right]^{1/2}, \\ \Delta\omega_i &= \pm\omega_0 \left[\frac{1}{2R_0} \left(\sqrt{Z_r^2 + Z_i^2} + Z_i \right) \right]^{1/2},\end{aligned}\quad (50)$$

where

$$Z_r = \text{Re}\{\dot{Z}^{(sur)}(n\omega_0)\}, \quad (51)$$

$$Z_i = \text{Im}\{\dot{Z}^{(sur)}(n\omega_0)\} - mX_{eq}, \quad (52)$$

and the characteristic resistance R_0 is given by

$$R_0 = 4\pi \frac{1/2 |m^*| U_0^2}{qI_0}. \quad (53)$$

A useful way of writing the equivalent space charge impedance may be as in the following:

$$X_{eq} \cong X_{sc} + 3.1 |Z_{KS}|, \quad (54)$$

where

$$Z_{KS} = 0.7 \frac{2\pi p_0 \beta c |\eta|}{qI_0} \left(\frac{\delta p}{p_0} \right)_{HWHM}^2 \quad (55)$$

is simply the Keil-Schnell impedance (e.g., [2]) relative to the case under consideration (form factor around the unity - initial velocity beam distribution assumed Gaussian). In conclusion the pressure effect on the beam dynamics plays the same role as the beam space charge impedance. This effect cannot be neglected when the Keil-Schnell impedance is of the same order of magnitude as the total impedance.

5. Application to the ESR beam and conclusions

In the case of the ESR the total impedance acting on the beam has a real part, which is mainly the resistive part of the cavity impedance, and an imaginary part, which is given by the sum of the imaginary part of the cavity impedance and the space charge reactance from the beam itself. Thus the impedance of the surrounding may be represented as

$$\dot{Z}_m^{(cav)} = \frac{R_s}{1 + iQ \left(\frac{m\omega_0}{\omega_c} - \frac{\omega_c}{m\omega_0} \right)}, \quad (56)$$

where R_s , Q and ω_c are, respectively, the shunt resistance, the quality factor and the fundamental

eigenfrequency of the RF cavity. In Table I the beam and machine parameters relative to the instability measurements that were carried out at the ESR in February 1997 are summarized [4].

Table I

E_{kin}	340 MeV/U
γ	1.36
$(\Delta p / p)_{FWHM}$	$1.1 \cdot 10^{-5}$
Z_{KS}	220 Ω
$\nu_0/2\pi$	1.886633 MHz
r_0	17.22 m
η	-0.367
I_0	0.366 mA
X_{sc}	670 Ω
R_s	1300 Ω
Q	50

Here we refer only to the rise times and the frequency shifts obtained by the linear kinetic theory because they agree very well with those obtained by using the PATRIC PIC code, as well as with that observed at the ESR in 1997, [4] and [5]. The curve of the instability rise times versus cavity frequency offset $\Delta f \equiv (\omega_0 - \omega_c)/2\pi$ (\square in fig. 1) was calculated from the intersections of the rise time trajectories with the cavity detuning curve plotted in the impedance plane; similarly, the curve of the expected unstable wave frequency shifts versus frequency offset (\square in fig. 2) has been evaluated from the intersections of the lines $\Delta\omega_r = \text{const.}$ with the cavity detuning curve plotted in the impedance plane. These two curves, along with the points that come from the beam simulations with the PATRIC code, are given in [5].

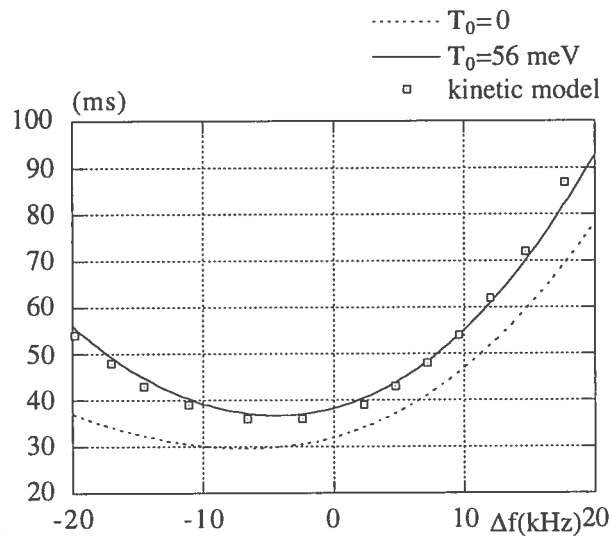


Fig. 1 – Comparison of the growth times obtained by means of the warm fluid model with those obtained by means of kinetic and cold fluid model.

If we had assumed a cold fluid model for our beam and we had applied the formulae (50) with an impedance that was purely the sum of the cavity impedance (function of the cavity frequency offset Δf) and the space charge impedance not corrected, we would have found curves (dotted lines in figures 1 and 2) τ_r vs Δf and $\Delta\omega_r$ vs Δf lying below the ones estimated with the kinetic model; $\tau_r = 1/|\Delta\omega_i|$ is the growth time of the instability. The error in the estimation of the instability rise times and frequency shifts reaches down to 30-40% in the central part of the curves, where we are far enough from the stability boundary, and yet it is quite high to be tolerated. If we apply the correction due to the pressure we found out that the resulting τ_r vs Δf and $\Delta\omega_r$ vs Δf curves (full lines in figures 1 and 2), are in very good agreement with the points estimated with the kinetic model. This means that the correction introduced in the space charge reactance, which is to be replaced by the sum of the old one plus the kinetic reactance (46), allows us to predict the right instability parameters in case the working point lies far outside the stability region.

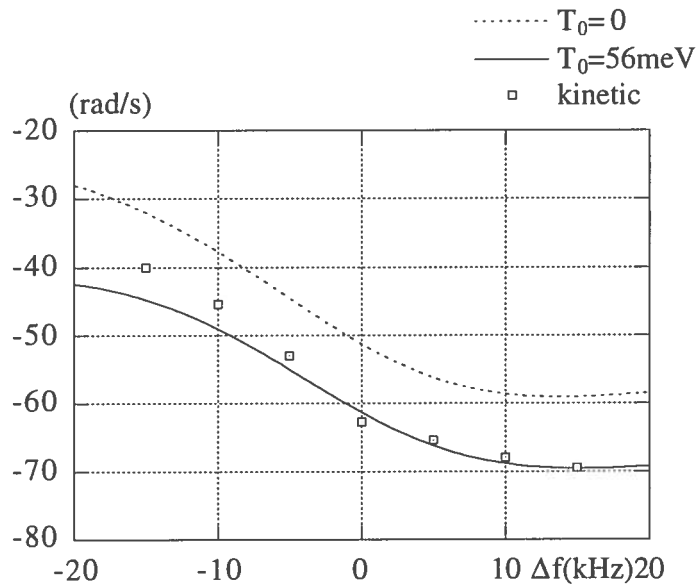


Fig. 2 – Comparison of the frequency shifts obtained by means of the warm fluid model with those obtained by means of kinetic and cold fluid model.

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$$Z_{KS} = 0.7 \frac{2\pi p_0 \beta c |\eta|}{q I_0} \left(\frac{\delta p}{p_0} \right)_{FWHM}^2 \quad (55)$$

is simply the Keil-Schnell impedance (*e.g.*, [2]) relative to the case under consideration (form factor around the unity - initial velocity beam distribution assumed Gaussian). In conclusion the pressure effect on the beam dynamics plays the same role as the beam space charge impedance. This effect cannot be neglected when the Keil-Schnell impedance is of the same order of magnitude as the total impedance.

4. Application to the ESR beam and conclusions

In the case of the ESR the total impedance acting on the beam has a real part, which is mainly the resistive part of the cavity impedance, and an imaginary part, which is given by the sum of the imaginary part of the cavity impedance and the space charge reactance from the beam itself. Thus the impedance of the surrounding may be represented as

$$\dot{Z}_m^{(cav)} = \frac{R_s}{1 + iQ \left(\frac{m\omega_0}{\omega_c} - \frac{\omega_c}{m\omega_0} \right)}, \quad (56)$$

where R_s , Q and ω_c are, respectively, the shunt resistance, the quality factor and the fundamental eigenfrequency of the RF cavity. In Table I the beam and machine parameters relative to the instability measurements that were carried out at the ESR in February 1997 are summarized [4].

Table I

E_{kin}	340 MeV/U
γ	1.36
$(\Delta p / p)_{FWHM}$	$1.1 \cdot 10^{-5}$
Z_{KS}	220 Ω
$\nu_0/2\pi$	1.886633 MHz
r_0	17.22 m
η	-0.367
I_0	0.366 mA
X_{sc}	670 Ω
R_s	1300 Ω
Q	50

Here we refer only to the rise times and the frequency shifts obtained by the linear kinetic theory because they agree very well with those obtained by using the PATRIC PIC code, as well as with that observed at the ESR in 1997, [4] and [5]. The curve of the instability rise times versus cavity frequency offset $\Delta f \equiv (\omega_0 - \omega_c)/2\pi$ (□ in fig.1) was calculated from the intersections of the rise time trajectories with the cavity detuning curve plotted in the impedance plane; similarly, the curve of the expected unstable wave frequency shifts versus frequency offset (□ in fig. 2) has been evaluated from the intersections of the lines $\Delta\omega_r = const.$ with the cavity detuning curve plotted in the impedance plane. These two curves, along with the points that come from the beam simulations with the PATRIC code, are given in [5].

If we had assumed a cold fluid model for our beam and we had applied the formulae (50) with an impedance that was purely the sum of the cavity impedance (function of the cavity frequency offset Δf) and the space charge impedance not corrected, we would have found curves (dotted lines in figures 1 and 2) τ_r vs Δf and $\Delta\omega_r$ vs Δf lying below the ones estimated with the kinetic model; $\tau_r = 1/|\Delta\omega_i|$ is the growth time of the instability. The error in the estimation of the instability rise times and frequency shifts reaches down to 30-40% in the central part of the curves, where we are far enough from the stability boundary, and yet it is quite high to be tolerated. If we apply the correction due to the pressure we found out that the resulting τ_r vs Δf and $\Delta\omega_r$ vs Δf curves (full lines in figures 1 and 2), are in very good agreement with the points estimated with the kinetic model. This means that the correction introduced in the space charge reactance, which is to be replaced by the sum of the old one plus the kinetic reactance (46), allows us to predict the right instability parameters in case the working point lies far outside the stability region.

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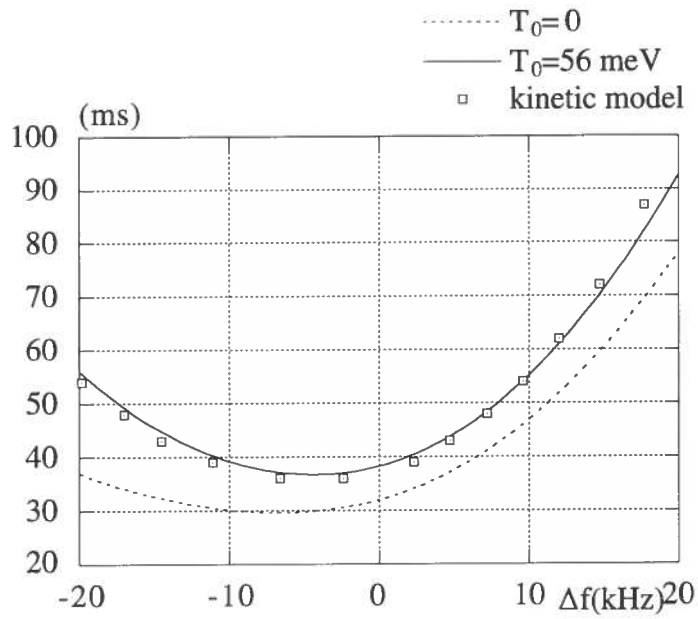


Fig. 1. Comparison of the growth times obtained by means of the warm fluid model with those obtained by means of kinetic and cold fluid model.

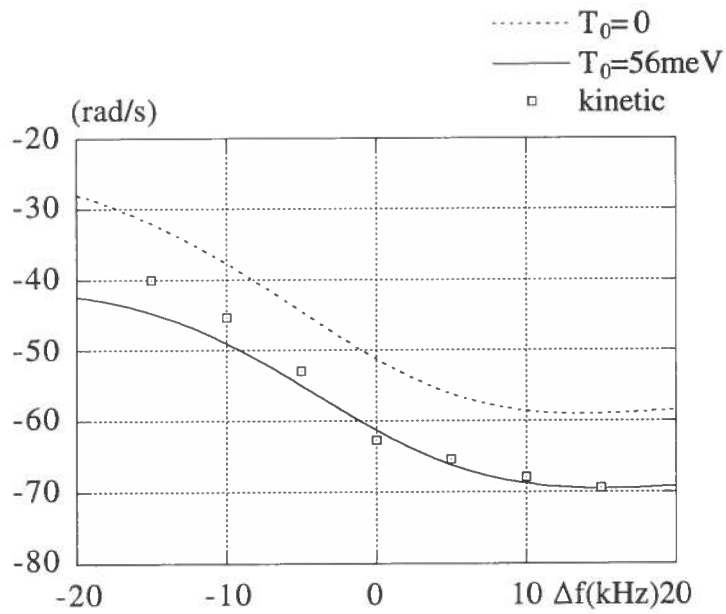


Fig. 2. Comparison of the frequency shifts obtained by means of the warm fluid model with those obtained by means of kinetic and cold fluid model.