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**PRIMARY IONIZATION AND ENERGY LOSS CALCULATION FOR HELIUM,
NEON, ARGON AND KRYPTON**

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Abstract

We performed a calculation of the energy loss dE/dx and the primary ionization density dN/dx for a charged heavy particle crossing a gas volume as a function of the particle β . We start from the Maxwell equation and, using the experimental values of the photo-absorption cross section, we take into account the different material properties. Our calculation of dE/dx is in excellent agreement with the Bethe-Bloch function and in the case of dN/dx we found a relativistic rise and a good agreement with the primary ionization experimental data we compare. We consider the possibility to use primary ionization measure to make the particle identification in the large tracking chambers.

1 Introduction

An estimation of the primary ionization induced by a charged heavy particle inside a medium and its dependence from the velocity is very interesting for several applications. The main one is the possibility to count primary ionization acts to measure particle velocity, if this number is a function of beta. The primary ionization follows poissonian statistics and its distribution width is very smaller compared to the total charge distribution (Landau), allowing for a better Particle Identification method. In the first two sections of the paper we calculate the mean energy loss and the mean number of primary electrons. In the third section we study the possibility to use primary ionization counting for particle identification in an ideal detector and draw some final remarks.

2 dE/dx calculation

In order to calculate the energy loss and the number of primary ionization acts of a charged heavy particle inside a medium, we follow the calculation made by Allison et al.. [1]. Starting from Maxwell equation, we calculate the average energy loss by electromagnetic interaction in the continuum

$$\frac{dE}{dx} = \frac{e\vec{E}(\vec{\beta}ct, t) \cdot \vec{\beta}}{\beta} \quad (1)$$

where $E(\vec{r}, t)$ is the electrical field resulting from the electromagnetic interaction of the medium with the crossing particle and βc is the particle velocity.

We take into account material properties, introducing complex dielectric constant and we are able to evaluate its real and imaginary parts by the photo-absorption cross section values and the Kramer-Krönig relation (PAI model).

$$\begin{aligned} \sigma_{\gamma}(E) &= \frac{ZE}{N\hbar c} \frac{\epsilon_2}{\sqrt{\epsilon_1}} \simeq \frac{ZE}{N\hbar c} \epsilon_2 \\ \epsilon_1(E) &= 1 + \frac{2}{\pi} P \int_0^{\infty} \frac{\sigma_{\gamma}(x) dx}{x^2 - E^2} \end{aligned} \quad (2)$$

where P indicates that the principal value of the integral is to be taken, N is the electron density of the medium and Z the atomic number. We made the low density assumption so that $\epsilon_1 \simeq 1$ and $\epsilon_2 \ll 1$.

In order to calculate the energy loss, we start from formula (1) and the Maxwell equation in a medium, then we express the electrical field in terms of the scalar and vector potentials Fourier's transforms. After integration in Fourier momentum, the energy loss is then given by the formula

$$\begin{aligned} \frac{dE}{dx} &= \int_0^{\infty} \frac{2e^2}{hc\beta^2} \left\{ \frac{Nc}{Z} \sigma_{\gamma}(E) \log \left[(1 - \beta^2 \epsilon_1)^2 + \beta^4 \epsilon_2^2 \right]^{-1/2} \right. \\ &\quad \left. + \frac{Nc}{Z} \sigma_{\gamma}(E) \log \left(\frac{2m_e c^2 \beta^2}{E} \right) + E \left(\beta^2 - \frac{\epsilon_1}{|\epsilon_2|^2} \right) \Theta + \frac{1}{EZ} \int_0^E \sigma_{\gamma}(E') dE' \right\} \\ \Theta &= \arg(1 - \beta^2 \epsilon_1 + i\beta^2 \epsilon_2) \end{aligned} \quad (3)$$

If we interpret the process as a sum of discrete collisions, we can write the energy loss by ionization as

$$\frac{dE}{dx} = - \int_{I_{ion}}^{E_{max}} NE \frac{d\sigma}{dE} dE \quad (4)$$

giving us the following expression for the differential cross section

$$\begin{aligned} \frac{d\sigma}{dE} = & \frac{\pi\alpha}{\beta^2} \left\{ \frac{\sigma_\gamma(E)}{EZ} \log \left[(1 - \beta^2 \epsilon_1)^2 + \beta^4 \epsilon_2^2 \right]^{-1/2} \right. \\ & \left. + \frac{\sigma_\gamma(E)}{EZ} \log \left(\frac{2m_e c^2 \beta^2}{E} \right) + \frac{\Theta}{Nhc} \left(\beta^2 - \frac{\epsilon_1}{|\epsilon_2|^2} \right) + \frac{1}{E^2} \int_{I_{ion}}^{E_{max}} \sigma_\gamma(E') dE' \right\} \end{aligned} \quad (5)$$

E_{max} is the maximum energy transfer in a single collision, that, for relativistic kinematical reasons, is given by

$$E_{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2} \quad (6)$$

and I_{ion} is the atom ionization energy. In the previous cross section formula, the third and fourth terms are the low and high energy collisions contributes, the first term is the relativistic correction to them and the second term is responsible of the Cerenkov radiation.

Starting from photo-ionization cross section data [2], we are now able to calculate the energy loss and primary ionization acts number. In figure 1 we show the photo-absorption cross section of the Argon gas. It is evident the internal shells structure, that we point out with shell names.

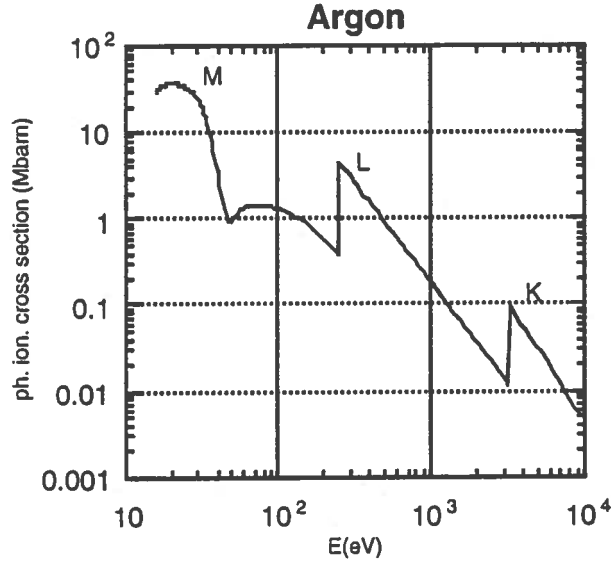


Figure 1: Argon photo-absorption cross section. The different peaks correspond to the opening of more internal shells. Data come from reference.

The Thomas-Reiche-Khun sum rule relates the integrated cross section to some material properties

$$\int_0^\infty \sigma_\gamma(E) dE = \frac{he^2 Z}{4\epsilon_0 m_e c} \quad (7)$$

In table 1 we compare the equation values to the integral of the quoted σ_γ for the four gases.

Table 1: Theoretical values and our calculation of Thomas-Reiche-Khun sum rule relation.

Gas	Theoretical	Our calculation
Helium	219.5	173.2
Neon	1098	1102
Argon	1975	1807
Krypton	3951	3380

Except for Neon, all calculated values are smaller than theoretical ones and we observe consistent discrepancies for Helium and Krypton. In the Krypton case it could be explained with deficiencies in experimental data at high energy values. In fact, in the Neon and Argon cases we have data up to 60 keV photons but for Helium and Krypton data stop at 8 keV and for Krypton the more internal shells are completely absent.

In order to calculate dE/dx and dN/dx we integrate by parts the high energy collision term, to avoid double integrals and reduce it to a sum of yet calculated terms. In the figure 2 we compare values given by formula (4) for dE/dx to the Bethe-Bloch curve for Argon including the Sternheimer correction term [3].

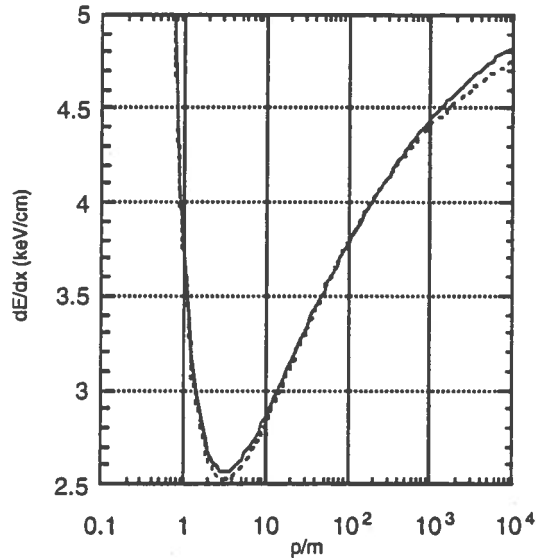


Figure 2: Comparison of our calculation of dE/dx with the Bethe-Bloch formula.

For the mean ionization potential I , used in the Bethe-Bloch curve and defined by

$$\log I = \frac{4\epsilon_0 m_e c}{h e^2 Z} \int_0^\infty \sigma_\gamma(E) \log E dE \quad (8)$$

we took the value calculated by Lapique et al. [4] equal to 180 eV, in contrast with the experimental value around 200 eV usually given in the literature. The value that we found is 140 eV and the difference can be explained by the fact that they include in the I calculation terms keeping into account the atom excitation. On the other hand, when we calculate I , we integrate formula (8) from I_0 to E_{max} , drastically reducing interactions with atom excitation. This argument can as well explain the discrepancies in the Thomas-Reiche-Khun relation values. The agreement between our calculation of dE/dx and the standard Bethe-Bloch curve with relativistic corrections is completely satisfactory.

3 dN/dx Calculation

To calculate the primary ionization acts, we have to count only the ionization contribute to σ_γ and we have to integrate from I_{ion} to E_{max} to exclude the most part of atom excitation. As the equation (4), we have

$$\frac{dN}{dx} = \int_{I_{ion}}^{E_{max}} N \frac{d\sigma}{dE} dE \quad (9)$$

In figure 3 we plot the energy loss curves (dashed lines), expressed in keV/cm and the primary ionization acts for centimetre (continuum line) versus particle momentum in GeV.

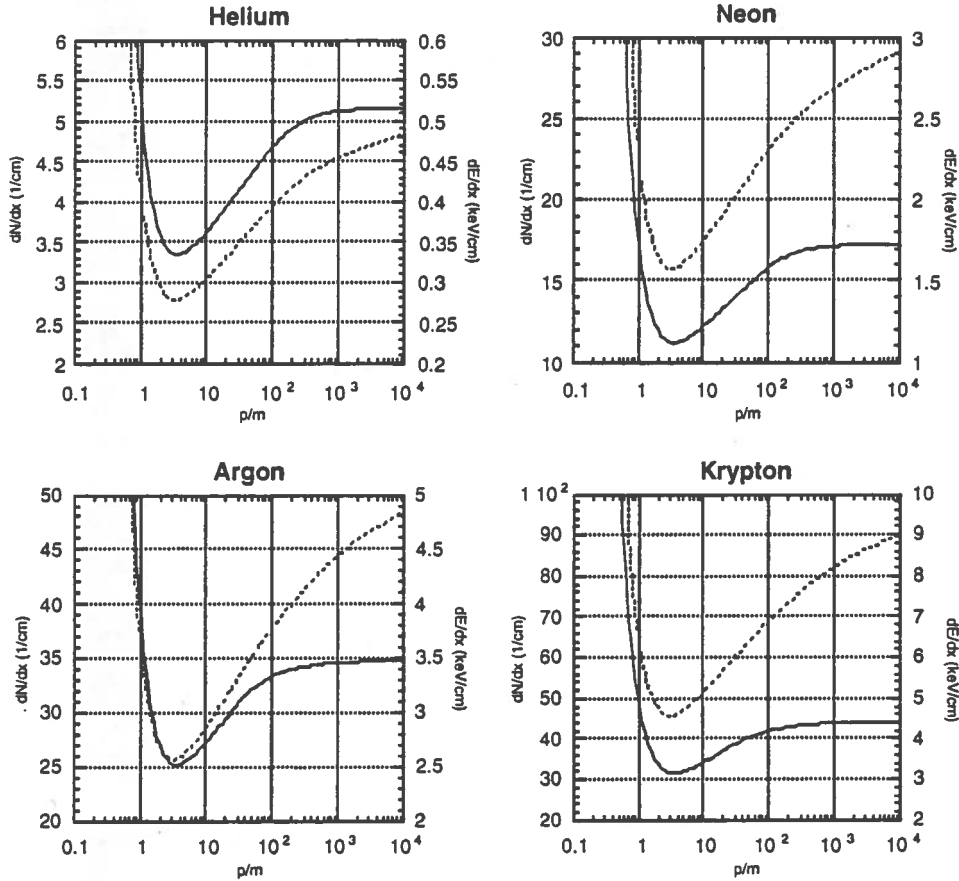


Figure 3: dE/dx (dashed) and dN/dx (solid) from our calculation for the four noble gases considered.

First of all we note that also the primary electrons number has the relativistic rise but, depending on the specific gas, it reaches a flat plateau for a p/m value of around 100, before the energy loss curve. It is possible to show that we have the relativistic rise up to a threshold value proportional to the mean energy exchanged in the collision. In this situation the collision are predominant in the more external shells, where the transferred energy is the lowest. Then the relative weight of this shell is much larger in the primary ionization value than in the total one. In table 2 we summarise the several properties of dN/dx curves for the four gases. In particular, we report the amount of total relativistic rise (percent respect to the minimum value), the maximum relativistic rise slope (as the number of ionization for decade) and the cut value up to which we have the relativistic rise (conventionally we take the value at which it becomes half). For the primary electrons number, the agreement with the measures that we found in the literature [6, 5] is fully successful. As we expect, in the Helium case, we obtain a very low primaries number and a consequent low absolute relativistic rise. On the other hand it shows the largest momentum range of relativistic rise and it is due to the fact mentioned above and the higher ionization potential of Helium respect to the other gases.

Table 2: Several properties of the primary ionization curve for the four considered gases: ionization at the minimum, ionization at the plateau, amount of total relativistic rise (%), maximum slope and momentum up to which we have the relativistic rise.

gas	minimum	plateau	$\Delta N/N(\%)$	max slope	$(p/m)_{max}$
Helium	3.35	5.15	54.1	1.16	210
Neon	11.2	17.2	13.5	3.78	190
Argon	25.2	34.8	38.0	7.72	75
Krypton	31.6	44.2	39.7	9.68	75

4 Application to an ideal detector

We study now the possibility to use primary ionization counting to help the particle identification inside the tracking chamber. Unfortunately we aren't able to make the same calculations for the quencher gases usually added to noble gases for chamber mixtures. Then we have no way to simulate precisely the real detector gas mixture, especially for the Helium based mixture, where the ionization is so low that the quencher gas has a crucial rule ¹. For the quencher gas we could consider Methene (CH₄), that has a behaviour more similar to the noble gas.

To study the particle identification power of a such detector, we define the D parameter as

$$D_{(i,j)}(p, x) = \frac{N_i(p, x) - N_j(p, x)}{\sqrt{\min(N_i, N_j)}} \quad (i, j) = (\mu, \pi), (\pi, K), (K, p) \quad (10)$$

Essentially $D(p, x)$ is the difference between primary electrons created by two different particles, measured in units of sigma. $D(p, x)$ is a function of the particle momentum and the track length of the particle inside the medium.

We show in figure 4 the D parameter versus particles momentum p , for the three couples of particle above indicated and for one meter of pure Helium and pure Argon. As we expected, the Helium lower ionization allows a worst particle identification respect to the Argon. Anyway we have to note that the momentum region where, for example, $D_{(\pi, K)}$ is greater than two is almost the same of the Argon case (2-40) GeV. The reason of this is that the Helium dN/dx curve has a longer relativistic rise (see also the table 2).

If we use a (80-20)% Helium-Methene gas mixture, we can assume 16 primary electrons for centimetre for a minimum particle [6], increasing the primaries number at least for a factor of three. Then we expect an increase of our discrimination power for a factor of $\sqrt{3}$. In this way we should have, for example, a separation between pions and kaons greater than three sigma from 2 GeV up to 40 GeV, very important region for the B mesons physics experiments.

We think that this method is very promising and we performed an experimental test to support and demonstrate our previous considerations. The analysis of this test is now finished and is going to be submitted for publication.

Acknowledgement

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¹If we want to resolve primary electrons from primary ones, we have to take in account their diffusion in the gas. In effect, in the more optimistic case, when the impact parameter of the track respect to the wire is zero, we must have a mean distance between two near clusters greater than diffusion length on the total path. For a generic track, we have to consider the geometrical reduction of the distance between two cluster. It's easy to see that this fact force to work with low Z gases mixture

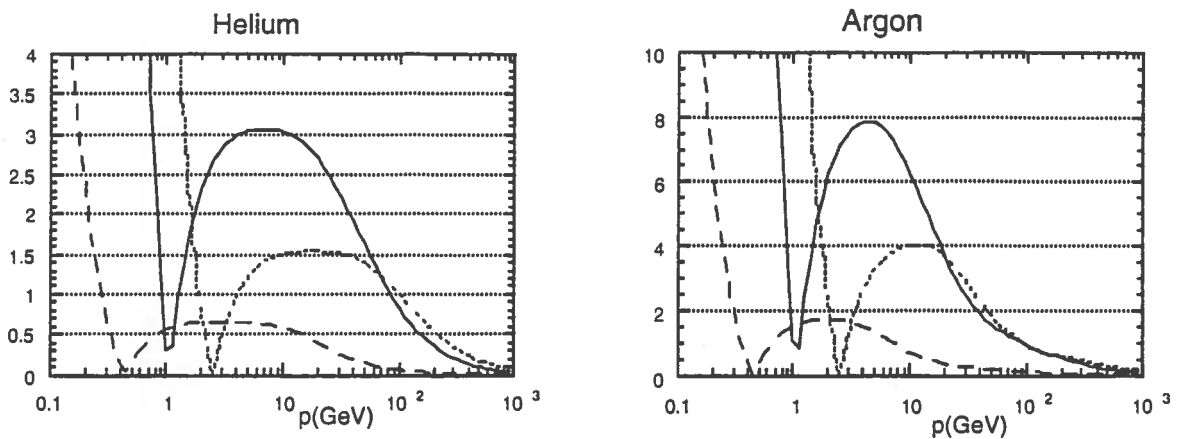


Figure 4: Particle identification parameter $D(p, x)$ for pure Helium and Argon, versus particle momentum in GeV, for π/K (solid), K/P (point) and μ/π (dashed).

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