ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Genova

INFN/TC-96/18
17 Ottobre 1996

G. Gemme, R. Parodi, G. Bienvenu, T. Garvey:

A PROPOSAL FOR MEASUREMENTS OF THE LIMITING ELECTRIC AND MAGNETIC FIELDS OF SUPERCONDUCTING NIOBIUM CAVITIES AND OF NIOBIUM BASED COMPOUNDS

SIS–Pubblicazioni
dei Laboratori Nazionali di Frascati

INFN – Istituto Nazionale di Fisica Nucelare
Sezione di Genova

INFN/TC–96/18
17 Ottobre 1996

A PROPOSAL FOR MEASUREMENTS OF THE LIMITING ELECTRIC AND MAGNETIC FIELDS OF SUPERCONDUCTING NIOBIUM CAVITIES AND OF NIOBIUM BASED COMPOUNDS.

G. Gemme and R. Parodi,
INFN – Sezione di Genova, Via Dodecaneso 33, I–16146 Genova (Italy)

G. Bienvenu and T. Garvey,
LAL–Université de Paris–Sud, Batiment 200 F–91405 Orsay Cedex (France)

INTRODUCTION

For precision studies of the top quark and to search for the Higgs boson physicists require e^+e^- collider energies higher than those available today. The superconducting linear collider study, TESLA, offers the possibility of achieving centre of mass energies in the 0.5 – 1 TeV range.

A challenging feature of TESLA is the problem of routinely reaching accelerating fields of 25 – 30 MV/m in superconducting cavities with low cryogenic losses. Even this field level which, according to our present knowledge of the behaviour of RF superconductors, is well below the maximum theoretical limit of 50 – 60 MV/m for pure Niobium, is still roughly two to three times higher than the maximum field routinely achieved in superconducting structures operating in electron accelerators [1].

The major limitation in bulk niobium cavities is the *Non Resonant Electron Loading* (NREL) due to field emitted electrons from high field regions close to the irises of the structures. To investigate this limitation, and the effect of processing the cavity surfaces to reduce the NREL, we propose to use a pulsed RF technique similar to the one developed at SLAC in the early 1980's [2] to push the surface electric fields to the 50–100 MV/m range in order to study the field emission mechanism. The surface conditions before and after the application of the field will be carefully investigated by XPS analysis. Our investigation is mainly aimed at establishing the limiting value for the accelerating field for cavities by trying to push the maximum surface magnetic field as close as possible to the critical field of niobium.

As a second challenging, but appealing, aim we plan to look for correlations between the chemical state of the clean niobium surface before exposure to strong RF fields and the modifications to the surface found at the emitting sites after the application of the high RF power.

To perform this research we propose to use the NEPAL facility at LAL–Orsay so taking advantage of the available high pulsed peak power (up to 35 MW) in the normal mode of operation and up to 150 MW in the MPC mode [3]. The proposed test scheme (using a high power RF pulse in the 1– 4.5 μs range and a repetition–rate of 100 Hz) will allow us to push the surface field to the desired levels even under very severe NREL [1]. The above mentioned choice of pulsed RF also provides us with a comfortable margin on the requirements of the cryogenic system needed to perform the measurements.

The 3 GHz operating frequency of the NEPAL RF system is only a little more than twice the operating frequency foreseen for the TESLA collider, giving us some confidence that the results of our investigations can provide useful information for the development of the project.

The process of cold electron emission from a metallic surface is governed by the well–known Fowler–Nordheim (FN) law [4] in the case of DC fields and the *modified FN* law [5] in the case of RF fields. Both mechanisms are known to be frequency independent at least in the RF frequency range up to 10 GHz.

1. PULSED MEASUREMENTS ON SUPERCONDUCTING RF CAVITIES: A REVIEW

The method of measuring the RF properties of superconducting cavities by using a very short radio frequency pulse of very high power was pioneered in the early eighties by I. Campisi and Z.D. Farkas [2] at SLAC.

The method was shown to be very useful for investigations of the limiting fields in SC cavities but was not exploited in depth due to the then growing interest in very high fields in accelerating structures (80–100 MV/m) far beyond the maximum theoretical field of 50–60 MV/m theoretically foreseen for a niobium based accelerator. At the beginning of the eighties the quest for accelerating fields higher than 80–100 MV/m excluded the use of superconducting cavities due to the intrinsic limit on the maximum surface magnetic field set by the critic field H_C of the superconductor. In practical superconductors the highest critical field is roughly 200 mT (for niobium), setting a maximum theoretical accelerating field of 50 MV/m for an accelerating structure.

In the early eighties the research for future e^+e^- colliders was mainly driven by the results of the experiments on the generation of bursts of microwave power at the GW level obtained by A. Sessler in the FEL experiments at Lawrence Livermore National Laboratories. Assuming that GW pulsed RF sources would be available in the short term the linac community started to design and develop linacs for operation at very high accelerating fields (80–100 MV/m) operating at very high frequencies (17–30 GHz).

In the same period (1985–6) all the available resources at SLAC (money and manpower) were diverted to the task of completing the SLC project and only a small effort in the then highly promising field of high gradient experiments was supported. For this reason the interesting investigation on the limiting fields in pulsed superconducting cavities was stopped.

Despite the low number of tests, (a total of five different tests are discussed in the final report on this experiment [2]) the potential usefulness of the method is already well evident, suggesting a very powerful tool for the investigation of the maximum achievable fields in SC cavities.

The focal point of the method (already demonstrated in the tests performed at SLAC) is the strong coupling between the cavity under test and the RF system. The use of high peak power give us the possibility (see section 2) of reaching high fields in a SC cavity with a very high unloaded quality factor and a corresponding long (compared to the length of the RF pulse) rise time, τ_0, of the stored fields in comparison with the pulse length, T.

Let us consider as an example,

$$\tau_0 > \mathbf{100\ T} \tag{1}$$

Recall that the filling time τ_0 of an unloaded cavity is related to the quality factor $\mathbf{Q_0}$ and the resonant frequency **f** by the equation

$$\tau_0 = \mathbf{Q_o/2\pi f} \tag{2}$$

Combining the equations 1 and 2 and taking a pulse length of 4μsec and the operating frequency of 3 GHz of the NEPAL klystron it follows that the lower limit on the quality factor of a cavity is $7.5\text{x}10^{6}$.

This value is roughly three to ten times lower than the mean Q_0 value routinely measured in superconducting cavities at 3GHz and 4.2 Kelvin.

Under certain conditions (see appendix) the energy U stored in the cavity at the end of the pulse of width T is simply,

$$\mathbf{U = 0.81\ PT}$$

Pulsed high power experiments provide us with a very comfortable experimental way to obtain useful information on the surface fields without forcing us to push the experimental set up to reach the very high Q_0 mandatory in low power continuous wave tests to achieve very high fields. In fact the experiment should be performed at very low duty cycle, allowing the cavity under test to forget the thermal effect of the surface dissipation from one pulse to the next. As an example, this method allows us to reach high fields operating the cavity under test in a liquid helium bath at 4.2 K. At this temperature and 3 GHz the quality factor of a cavity is $\approx 10^8$; well beyond the value of $7.5\text{x}10^6$ set by equations (1) and (2).

Under similar experimental conditions in the SLAC experiments Campisi [2] reached electric surface fields higher than 60 MV/m. No benefit was found in running the cavity at lower temperature and gaining a factor of about 50 on the quality factor. To achieve the same field intensity using a low power continuous wave set up an input power of 15 Watts would be needed for a quality factor of $5\text{x}10^9$ and cavity operation at 1.8 K. This power corresponds to a heat flux higher than the critical heat flux leading to the transition of the niobium in the high power dissipation region of the cavity.

Also in the case of cw tests the power dissipation in the helium bath will easily overload the refrigeration system, producing an increase of the temperature of the cooling bath and a thermal runaway of the system.

For the reasons discussed, above a quality factor of 10^{10} is mandatory (in low power continuous wave tests) to reach high fields, reducing the power dissipation to few Watts, with a large gain in thermal stability of the cavity.

In the GHz frequency range this value of Q_0 is obtained by operating the cavity at 1.5 – 1.8 Kelvin in superfluid helium. To obtain that temperature we need a huge pumping system to lower the bath temperature and the superfluid helium adds complications to the design and the operation of the test set up.

The possibility of obtaining high fields even with moderately high Q_0 does not relax the requirement of careful preparation of the cavity surface. The cavity must be capable of providing high Q_0 values, without being field limited by any surface contamination. The argument for reaching high surface fields using a cavity with a 'not so high' quality factor is even more important for the part of our proposal concerning the demountable cavity . We foresee the use of this cavity for the surface analysis of the electron emission region before and after the high field tests.

As shown in section 3, to do that part of our investigation, a demountable end wall is foreseen, using a well–designed choke flange to keep the power dissipation on the indium or lead seal in the joint region to a minimum.

The experimental difficulty of obtaining a very low loss joint and the subsequent limitation of the quality factor to 3 to 5×10^9 was the reason in the past for the unreliable behaviour of this type of cavity which was unable to reach the design fields needed for the experiments. This limitation hindered experiments aimed at understanding the influence of the surface state on the Fowler–Nordheim emission of electrons by "clean" metallic surfaces exposed to high RF electric fields [5].

The use of RF pulsed power as proposed, will ease the requirements on the quality factor as previously explained above. Again, the only requirement on the quality factor is the one given above, i.e. a lower limit of about 10^7.

This requirement can be, and was in effect, easily overcome [5] in the past at Cornell, where a similar cavity equipped with an indium joint reached a quality factor more than 5 times 10^9. We are confident that a similar result can be achieved in the cavity foreseen for our investigation of the chemical composition of the niobium surface on the electron field emission.

In this case also the cavity surface must be cleaned to give us information about the effects of surface contamination in the emitting region.

The use of very short high power pulses allows us to circumvent the technical limitation of the losses in the demountable joint that in the past produced erratic behaviour and spoiled the results of similar investigations.

2. BASICS OF HIGH PEAK POWER PULSED MEASUREMENTS

The NEPAL Test Facility at LAL is very well equipped for investigating the effects of high power on the maximum field obtainable in SC accelerators and for studies of surface treatments on emitted current. NEPAL is equipped with a high power klystron rated up to 35 MW with 4.5 μs pulses at a rep–rate of 25, 50 or 100 Hz. The klystron also has a SLED type system (MPC), that boosts the peak power up to 200 MW at the cost of a reduced pulse length of 0.8 μs.

Despite the reduced length of the RF power pulse the system is well suited for driving a superconducting cavity up to the breakdown field and far beyond.

For a suitable choice of coupling factor (see appendix) the stored energy in the cavity at the end of the 35 MW, 4.5μs pulse will be 128 Joules, which is more than enough to reach the limiting field in a cavity having an electric field coefficient $K_E = E_s/\sqrt{\omega U}$ ranging in the few hundreds as is usual SC accelerating cavities, as the one shown in Figure 1.

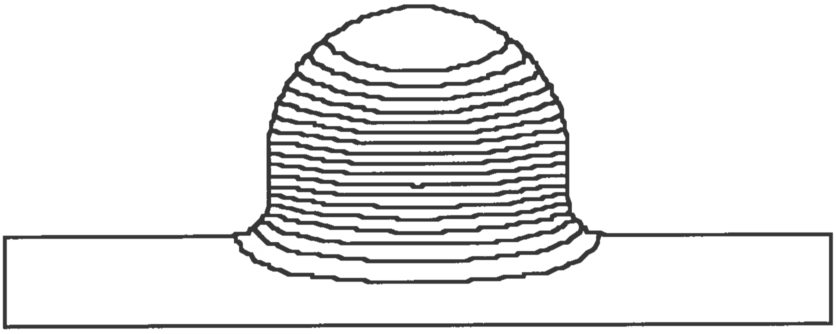

FIG. 1 – Geometry of a 3 GHz superconducting cavity with superimposed E–Field lines.

The relevant RF roperties of the cavity, computed by our Oscar2D code [6], are reported in table I.

Table I

frequency	3 GHz
Γ	230 Ω
R/Q	133 Ω
Ep/Ea	1.9
$K_E = E_p/\sqrt{(\omega U)}$	520 [V]/[Watt]$^{0.5}$
$K_H = H_p/\sqrt{(\omega U)}$	9x10^{-4} [mT]/[Watt]$^{0.5}$
Ep/Hp	0.58[MV/m/mT]

The $\mathbf{K_E}$ and $\mathbf{K_H}$ parameters give the surface electric and magnetic field of the cavity given the RF energy stored in the cavity. The coefficient Γ relates the surface resistance $\mathbf{R_S}$ and the $\mathbf{Q_0}$ of the cavity through the equation

$$Q_0 = \frac{\Gamma}{R_S}$$

From the data of Table I it is straightforward to see that we need 3.8 Joules of stored energy in the cavity to reach the critical superheating critical field of 240 mT.

We recall that in the SERA2 accelerating structure (one meter long) tested at the NEPAL Facility the stored energy is 10 Joules for an the accelerating field of 30 MV/m.

It is easy to convince ourselves that equally in the case of a superconducting cavity with a very long un–loaded filling time (say one second) driven by a short RF power pulse of few microseconds the stored energy in the cavity can reach very high values allowing for the build up of extremely high accelerating and surface fields easily exceeding the critical fields of the niobium.

By definition
$$Q_0 = \frac{2\pi f U}{P_d}$$

The stored energy, as a function of time is given by (see appendix),

$$U = \eta\frac{PQ_L}{2\pi f}\left[1\text{-}\exp(-\frac{t}{2\tau})\right]^2$$

Where η is a constant depending only upon the coupling coefficient of the cavity to the input port. The value of η is 2 for critical coupling of the cavity to the input network and 4 in the case of a heavily overcoupled cavity as in the SLAC experiment.

As
$$Q_L = 2\pi f\tau$$

For
$$t/\tau << 1\ ,$$

(as for a critically coupled SC cavity and a RF pulse length of 4.5µs)
using the first order series expansion for the exponential function in the expression for the stored energy we obtain

$$U = P\tau(\frac{t}{\tau})^2\frac{\eta}{4}$$

In the case
$$\frac{t}{\tau} >> 1,$$

(as for the case of a copper structure or, as in our case, a highly overcoupled superconducting cavity) we obtain for the maximum stored energy

$$U = \eta P\tau$$

The previous arguments tell us that in a superconducting cavity the experimental conditions can be chosen in such a way that the stored energy in the resonator under test is largely independent of the unloaded cavity quality factor and is determined only by the peak RF power, RF pulse length and the coupling coefficient of the cavity to the RF system.

In both of the above cases the cavity reaches surface fields corresponding to the energy stored in the cavity in a way independent from the Q_0 variations induced by the RF power. The value of the maximum stored energy in the cavity for the NEPAL RF system (35MW and 4.5 µsec) would be 128 Joules for a heavily overcoupled SC cavity, having a loaded Q of 34000 and a filling time 1.8 µsecond (see appendix).

This value exceeds, by a factor ≈ 40, the maximum energy needed to drive a cavity to the superheating field of niobium.

The conclusion of this discussion is that the NEPAL Facility is well equipped to drive superconducting cavities to surface fields exceeding the maximum field that can be theoretically sustained by a SC cavity.

3. SURFACE ANALYSIS

To understand the effect on field emission of the chemical state of the surface and to assess the modifications of the surface due to the electron discharge we propose to perform XPS analysis of the emitting region using a Φ–5600Ci Esca Multitechnique System operating in Genoa. To perform this investigation we plan to take a chemical image of the cavity surface before and after the exposure to high RF fields to make correlations regarding the physical state of the surface, the field emission current and the physical and chemical state of the surface after the electron discharge.

We plan to use a special cavity with a demountable wall for these measurements. This cavity, similar to the one used by Moffat at Cornell for investigation of high fields in cavities [5], is shown in Figure 2. The RF properties of the cavity are given in Table II. The end wall is joined to the cavity body by a choke flange to minimise the RF current in the indium O–Ring used to seal the cavity.

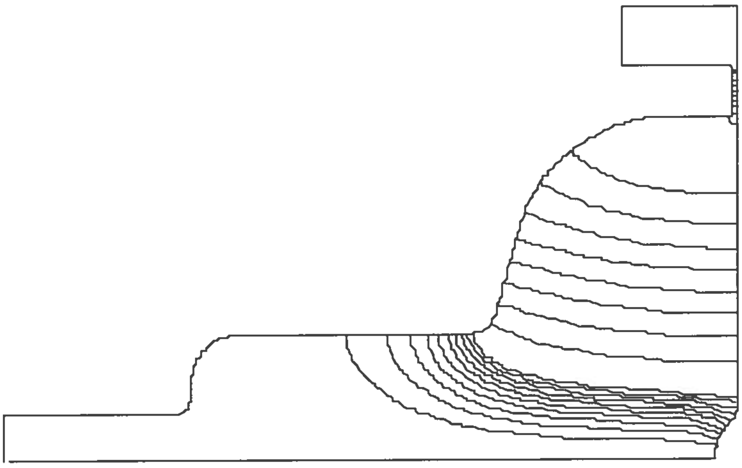

FIG. 2 – Demountable cavity for XPS Analysis of the emitting surfaces.

Table II

frequency	3 GHz
Γ	199 Ω
$K_E=E_p/(\omega U)^{0.5}$	1080 [V/m]/[Watt]$^{0.5}$
$K_H=H_p/(\omega U)^{0.5}$	1×10^{-3} mT/[Watt]$^{0.5}$
E_p/H_p	1 [MV/m/mT]

In our case this indium seal is not so critical as it is in cw measurements because for pulsed measurements the only requirement to be fulfilled is to reach the maximum stored energy in the cavity, expressed by the relation between the filling time τ of the cavity and the RF pulse length T as shown in the appendix.

The proposed cavity shape gives us the further benefit of allowing us to double the maximum surface field on the niobium end wall by simply dimpling the wall at the center with a radius of 8 mm by pressing the niobium sheet against a die. In that way we are confident we can succeed in pushing the electric surface fields in that region well beyond the maximum field foreseen for the operation of cavities in the e^+-e^- colliders. The shaping of the end wall reduces the high field region of the cavity to a circle of 3 mm radius, as shown in Figure 3. This value matches the large field analysis area of the Esca system to be used for the surface analysis of the state of the surface before and after exposure to high fields.

In this way the analysis can be performed in a single step avoiding the search of the vacuum discharge site on a broad area around the irises. In this way we also plan to quantitatively investigate the effect on NREL of the exposure of a clean niobium surface to the

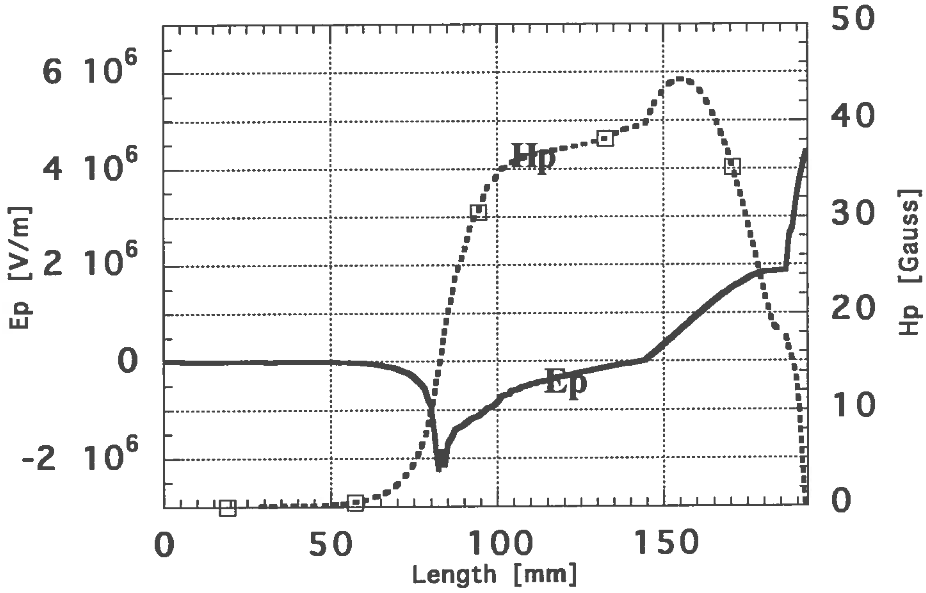

FIG. 3 – Peak surface fields along the cavity walls; the high field region is reduced to a circular spot of 3 mm radius around the cavity axis

4. THE MEASURING SYSTEM

The layout of the measurement set up is shown schematically in Figure 4, and is very similar to the one used by Campisi and Farkas [2]. The cavity is decoupled from the klystron through a 6db hybrid coupler, to protect the klystron against an excess of reflected power. The transmitted and reflected powers are sampled using two waveguide couplers; insulators are used to terminate the lines at the power meter sensors and a diode is used for transmitted RF power detection.

The incident and reflected powers are detected using a dual peak power meter calibrated to give both peak and average power. The cavity is coupled to a waveguide through an iris and the waveguide act also as a vacuum manifold to continuously pump the cavity by using a 15 $^l/_s$ Starcell™ Ion Pump. A calibrated diode (or a spectrum analyser) is used as a video detector to monitor the power transmitted through the cavity and collected by the transmission probe.

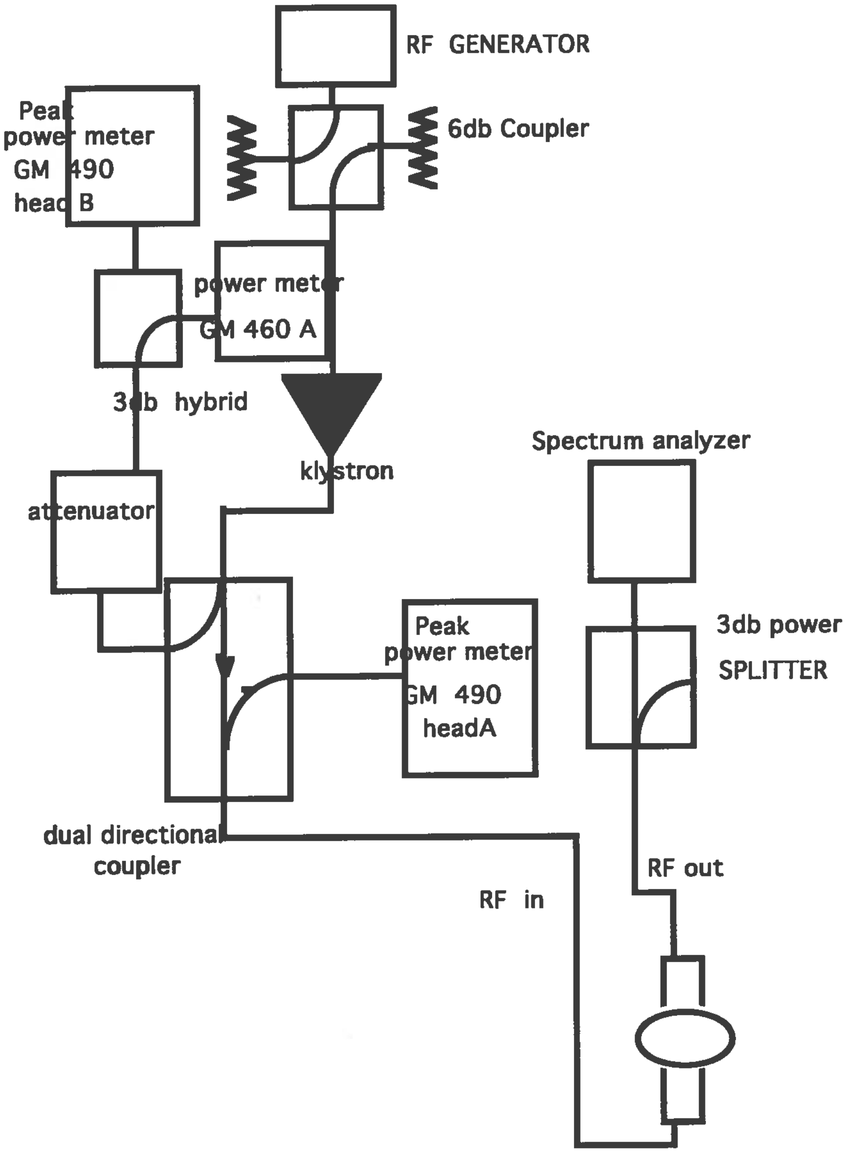

FIG. 4 – Experimental set–up for pulsed measurements.

The transmitted power is also used to cross check the measurement of the field level in the cavity remembering that the transmitted power **P_t** is proportional to the energy stored in the cavity, **U_s**.
The measurement of the fields in a cavity is straightforward when the stored energy in the cavity is known. In fact widespread computer codes for the simulation of RF cavities give the coefficient relating the stored energy to the field value.

For steady state measurements it is easy to obtain the value of the stored energy by a careful measurement of the quality factor Q_0 and of the RF power P_d dissipated in the cavity.

For transient measurements the exact evaluation of the field value is less certain, and so a coefficient relating the stored energy and the field value and some assumptions are needed.

Recalling that $$Q_o = \frac{\omega U}{P_d} \text{ and that } Qe = \frac{\omega U}{Pe}$$

$$\frac{Pe}{P_d} = \beta = \frac{Q_o}{Q_e} \text{ it follows that}$$

$$P_d Q_0 = P_e Q_e$$

giving us the possibility of computing the field by only measuring the coupling coefficient of the port and the RF power emitted by the port used for the measurement.

In the case of a heavily overcoupled port (even if the Q_0 changes), under the condition $Q_0 >> Q_e$ the measurement of the decay time of the emitted power gives us the Q_e value and the peak power value of at the beginning of the decay of the emitted power.

The value of the surface fields can easily be obtained by measuring, via a peak power meter, the peak value and the decay time of the emitted power.

For this reason we carefully design the coupling port to have the cavity under test close to critical coupling when normal conducting at 10 K.

This means for a niobium cavity operating at 3 GHz we need an external quality factor of less than 50,000, the Q_0 value at 4.2 K being of the order of 10^8.
To avoid overheating at the cavity waveguide coupler we plan to use iris coupling between the input waveguide and the cavity under test. The opening of the iris, giving the desired coupling coefficient will be determined using the HFSS™ code, from Hewlett–Packard, currently used at Genoa for the design of cavities and couplers. A first sketch of the coupling system is shown in Figure 5.

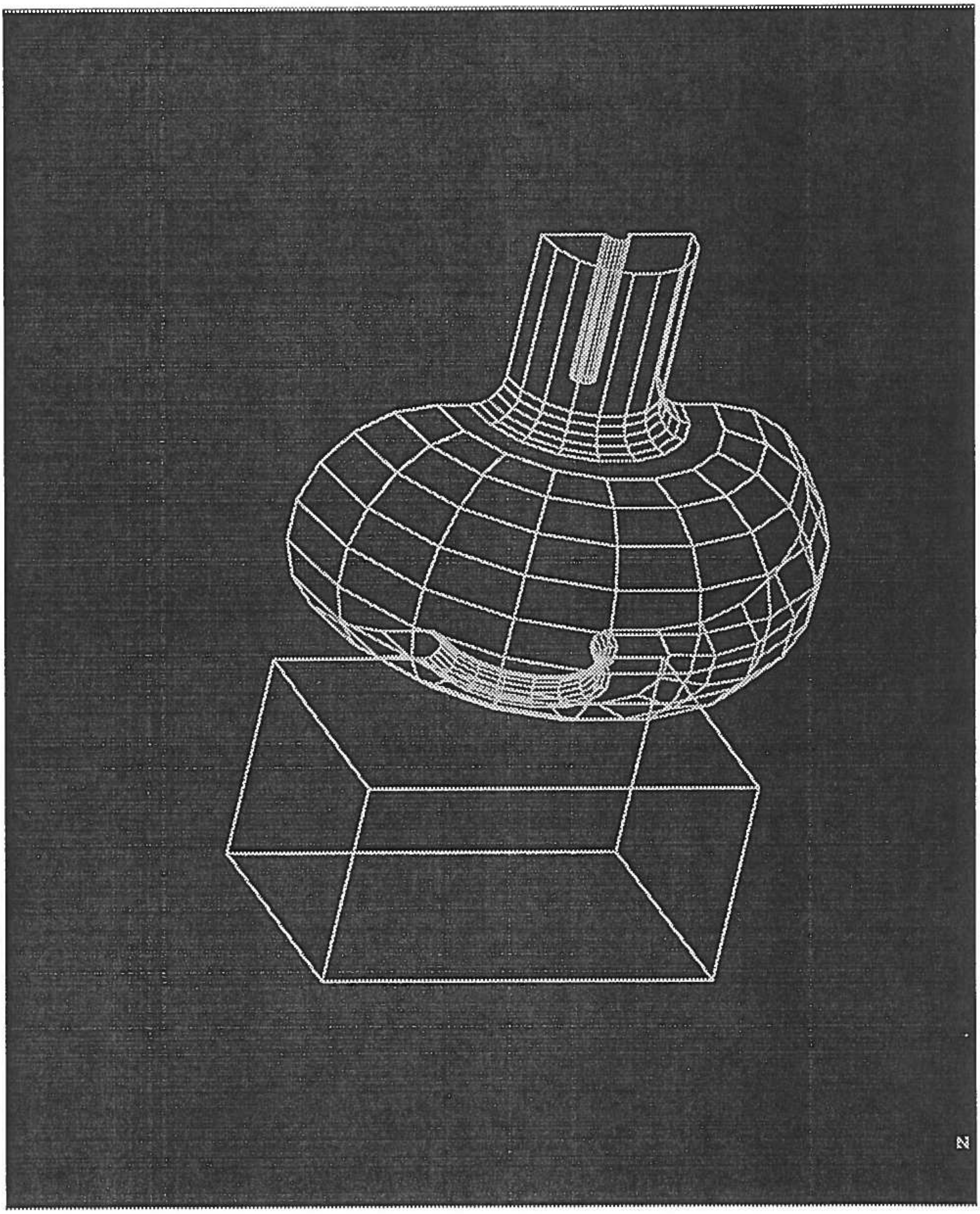

FIG. 5 – Sketch of the Cavity–Waveguide coupler used to taylor to the right dimension the Iris of the cavity using HFSS Copling factor $\beta \approx 1000$.

References

[1] H.T. Edwards et al. "TESLA Parameters Update–A Progress Report on the TESLA Collider Design' Proceedings of "The 6th Workshop on RF Superconductivity" October 4–8, 1993, (CEBAF), page 361, R.Sundelin Editor.

[2] I.E. Campisi, Z.D.Farkas "HIGH GRADIENT, PULSED OPERATION OF SUPERCONDUCTING NIOBIUM CAVITIES", SLAC/AP–16 February 1984.

[3] P.Brunet, " Nouvelles Experiences de Physique des Accelerateurs au LAL", Sera 94–267, NEPAL note 94–5.

[4] U Klein " An investigation on limit fields and surface resistance of superconducting resonators", PhD Thesis Gesamthochschule Wuppertal WUB DI 81–2.

[5] D.L. Moffat et al, "Superconducting RF cavities Designed to Attain High Surface Electric Fields" Proceedings of "The 4th Workshop on RF Superconductivity" August 14–18,1989 Tsukuba, Japan, Editor Y Kojima KEK Report 89–1, January 1990.

[6] P.Fernandes, R.Parodi, "Oscar2d, A Computer Code For The Design Of Rf Cavities And Structures", Proceedings of the LINAC 86 Conference SLAC San Francisco, USA, June 1986. Also SLAC report 303 page 330.

APPENDIX

Service d'Etudes et Réalisations d'Accélérateurs

SERA 96 T. Garvey 30/3/96

MEMO: Superconducting Activities on NEPAL

The time dependence of the stored energy while filling an RF cavity is given by,

$$\frac{dU}{dt} = 2\sqrt{\frac{U\omega P_f}{Q_e}} - \frac{\omega U}{Q_L}$$

(to see why, refer to the paper "Response of Superconducting Cavities to High Peak Power", Hays and Padamsee, US PAC 1995)
where P_f is the forward power, ω is the radian frequency of the cavity, U is the instantaneous stored energy, Q_e is the external quality factor ($Q_e = Q_0/\beta$) and Q_L is the loaded quality factor ($Q_L = Q_0/(1+\beta)$).
From this expression it is easy to show that the RF field, E, increases as

$$E = E_0\,(1\text{-}\exp(-\frac{t}{2\tau}))$$

where E_0 is the steady state field and $\tau = Q_L/\omega$.

As $U = kE^2$ where k is a geometry dependent factor we see that

$$U(t) = U_0\left[1\text{-}\exp(-\frac{t}{2\tau})\right]^2 \qquad (1)$$

where U_0 is the steady state energy which is given by $U_0 = \frac{4\tau^2\omega P_f}{Q_e}$.
(One can substitute back and check that eqn.(1) is a valid solution to the differential equation above).
Equation (1) can then be written as, $U(t) = \frac{4\beta}{1+\beta} P_f \tau\left[1\text{-}\exp(-\frac{t}{2\tau})\right]^2$ (2)

For $t/\tau << 1$ we can approximate eqn.(2) as $U(t) = \frac{\beta}{1+\beta} P_f t\,(\frac{t}{\tau})$.

Thus *the energy initially increases quadratically with time*. Therefore if $t/\tau <<1$ we cannot hope to have large energies in the cavity, nor high gradients.

Example: Assume critical coupling β=1, $Q_e = Q_0 = 2\ Q_L$. For 3 GHz cavities at 4.2K we have Q_0 of the order of 10^8 (see below) therefore τ = 2.65 ms. Therefore, even with P_f = 7 MW and a pulse width T = 4.5 μs we have U(T) = 27 mJ !

If we look again at the expression for U(T), the energy stored at the end of a pulse of width T we see that we can write it as

$$U(T) = \frac{2\beta}{1+\beta} P_f T \frac{[1-\exp(-\alpha)]^2}{(\alpha)}$$

where $\alpha = T/2\tau$. Obviously there will be a value for α which optimises the transfer of energy to the cavity (just like the efficiency of a travelling wave structure). This value is 1.26 and means that **for a given pulse–width there is a given value of the loaded Q for optimum energy transfer,** which will be 81% of the pulse energy (note that P_fT is just the energy in the pulse). For a pulse of 4.5 μs long we need a Q_L of 33,750. Shorter pulses will require correspodingly smaller values of Q_L. Such Q's imply that one needs to be strongly coupled to the cavity and the ratio of $\beta/(1+\beta) = 1$.

A more complicated derivation of the maximum efficiency can be found in the note by Z.D. Farkas "Superconducting Cavities and Modulated RF", US PAC 1981.

In conclusion, to do interesting experiments in NEPAL strong coupling is required.

The Q_0 of a superconducting cavity is given by $Q_0 = \frac{G}{R_{BCS} + R_{res}}$, where G is a geometric factor, R_{BCS} is the the surface impedance given by BCS theory and R_{res} is the residual surface impedance due to defects in the material. The value of R_{BCS} is given by

$$R_{BCS} = A\frac{\omega^2}{T}\exp(-\frac{\Delta}{kT})$$

Where A is a constant of the order of 10^{-24} $\Omega K s^2$, T is the temperature of the material and $2\Delta = 3.52kT_c$ where T_c is the critical temperature of the material (9.2K for niobium). For niobium at 4.2K and 3 GHz we have R_{BCS} = 1.8 μΩ. This greatly exceeds the residual surface impedance which can then be neglected. For a pillbox of length l and radius b we have $G = 453\Omega/(1+b/l)$. If we say that $l = \lambda/2$ (π mode cavity) and $2\pi b/\lambda = 2.405$ (TM_{01} mode) we have b/l = 0.77 and G = 256Ω. The value quoted by Parodi, which I presume is computer calculated for his cell geometry, is 230 Ω, close enough. Finally, using the formula above we have $Q_0 = 1.3\text{x}10^8$, which justifies the figure used above.