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**COUPLING IMPEDANCE MEASUREMENTS:
AN IMPROVED WIRE METHOD**

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A possible scheme to perform bench measurements of coupling impedances [1, 2, 3, 4] is to insert in the accelerator component under test a wire on which a current pulse of the same shape of the bunch is riding. For a discussion of the method, we may refer to a paper by Sands e Rees [5].

This method needs a complex numerical manipulation, by means of FFT and IFFT, because of the unavoidable presence of the multiple reflections in the measuring apparatus. We want to discuss an improved method of measurement which does not need this manipulation. The discussion of the bench measurements will be given in a quite general way, and we will recover the results well known in literature as particular cases. The validity limits will be given as well.

1 Analysis of the coaxial line method for impedance measurements

The insertion of the wire behaves together with the component as a coaxial line. We describe how the characteristics of a coaxial line are influenced by the presence of an accelerator component.

We use the telegraphist equations in the time domain and we synthesize the time domain response by means of an adequate Fourier Transform.

The method is quite general, so that we will describe it regardless the characteristics of the component to be measured (Device Under Test = DUT) and of the reference line (Reference = REF).

In order to extract the coupling impedance from the measurements we transform the coaxial component in an equivalent line with distributed parameters.

The equivalent line of the REF has a resistance R_0, an inductance L_0, and a capacitance C_0 per unit length: the Device Under Test introduces a further complex term $\zeta = Z_{||}/l = R + jX$, which represents the DUT impedance exceeding the REF one (fig. 1)[3], where l is the length of the DUT and $Z_{||}$ is its longitudinal impedance.

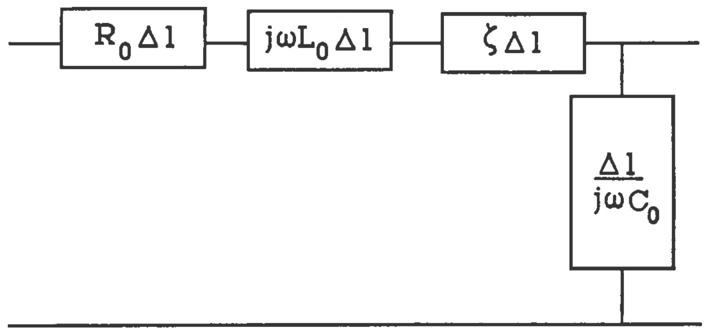

Figure 1: *The line equivalent cell for DUT*

The DUT propagation constant is given by the following expression:

$$k_D = \omega \sqrt{\left(1 - j\frac{R_0 + \zeta}{\omega L_0}\right) C_0 L_0} \tag{1}$$

while the REF one is

$$k_R = \omega \sqrt{\left(1 - j\frac{R_0}{\omega L_0}\right) C_0 L_0} \tag{2}$$

Furthermore the characteristic impedance of the REF is:

$$Z_0 = \sqrt{\frac{L_0 - jR_0/\omega}{C_0}} \tag{3}$$

In principle it is possible to get the quantity $Z_{||}$ from k_D, k_R e Z_0 by means of the formula

$$Z_{||} = \zeta l = jZ_0 \frac{(k_D^2 - k_R^2)l^2}{k_R l} \tag{4}$$

Now we show that from transmission scattering coefficients, S_{12}, of the equivalent lines of DUT and REF it is possible to get k_D, k_R, and then $Z_{||}$.

The scattering matrix of a bipolar circuit is defined as

$$\boldsymbol{S} = (\boldsymbol{Z} - Z_c\boldsymbol{I})(\boldsymbol{Z} + Z_c\boldsymbol{I})^{-1} \tag{5}$$

where

Z_c is a reference impedance;

$\boldsymbol{Z}$ is the impedance matrix of the circuit defined as follows:

$$Z = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \tag{6}$$

The matrix $\boldsymbol{Z}$ relates the voltage vector and the current vector by means of the following matrix equation:

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \boldsymbol{Z} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \tag{7}$$

For reciprocal circuits the condition $Z_{12} = Z_{21}$ is always satisfied, and all passive circuits satisfy it. Furthermore, for a symmetrical circuit we have $Z_{11} = Z_{22}$: in this case we may define a characteristic impedance as $Z_0 = \sqrt{|\boldsymbol{Z}|}$.

In our case the circuit is a coaxial line for which the impedance matrix behaves as

$$\boldsymbol{Z} = \frac{-jZ_0}{\sin kl} \begin{pmatrix} \cos kl & 1 \\ 1 & \cos kl \end{pmatrix} \tag{8}$$

Resorting to the definition of scattering matrix as given by equation (5) we obtain

$$\boldsymbol{S} = \frac{1}{|\boldsymbol{Z}| + Z_c^2 + Z_c(Z_{11} + Z_{22})} \begin{pmatrix} |\boldsymbol{Z}| - Z_c^2 & 2Z_c Z_{12} \\ 2Z_c Z_{12} & |\boldsymbol{Z}| - Z_c^2 \end{pmatrix} \tag{9}$$

which for a symmetrical line ($Z_{11} = Z_{22}$) becomes

$$\boldsymbol{S} = \frac{\begin{pmatrix} (Z_0^2 - Z_c^2)\sin kl & -2jZ_c Z_0 \\ -2jZ_c Z_0 & (Z_0^2 - Z_c^2)\sin kl \end{pmatrix}}{(Z_0^2 + Z_c^2)\sin kl - 2jZ_c Z_0 \cos kl} \tag{10}$$

We see that if, and only if, the reference impedance Z_c is equal to th characteristic impedance of the line Z_0, namely if the line is matched, the scattering matrix, denoted now with the index 0, is zero on the diagonal and the only scattering parameter becomes

$$S_{12}^0 = exp(-jkl) \tag{11}$$

Consequently, according to eq. (4) with the condition $Z_c = Z_0$, the longitudinal impedance can be written as

$$Z_{||} = Z_0 \ln \frac{S_{12}^{0R}}{S_{12}^{0D}} \left[1 + \frac{\ln S_{12}^{0D}}{\ln S_{12}^{0R}}\right] \tag{12}$$

which gives $Z_{||}$ as a function of the scattering parameters S_{12}^{0D} and S_{12}^{0R} of DUT and REF respectively.

Allowing for the transverse impedance, this quantity can be calculated in a similar way, measuring the transmission coefficients at the ends of two wires supporting opposite currents which are equivalent to a current dipole exciting transverse modes. One should point out that the measurement gives the longitudinal impedance of a dipole mode.

Resorting to the definition of the transverse impedance [1] and to Panowski–Wentzel theorem we get the expression

$$Z_{\perp} = \frac{cZ_0}{\omega\Delta^2} \ln \frac{S_{12}^{0R}}{S_{12}^{0D}} \left[1 + \frac{\ln S_{12}^{0D}}{\ln S_{12}^{0R}}\right] \tag{13}$$

where Δ is the distance between the wires.

The expressions in the literature [5, 2, 4] are slightly different from eq. (12) and from eq. (13) and are only a first approximation of them.

Indeed, if $\ln S_{12}^{0R}$ is quite close to $\ln S_{12}^{0D}$, their ratio in eqs. (12) and (13) can be approximated by 1, while expanding $\ln(S_{12}^{0R}/(S_{12}^{0D})$ in series we get the standard formulas

$$Z_{\|} = 2Z_0 \frac{S_{12}^{0R} - S_{12}^{0D}}{S_{12}^{0R}} \tag{14}$$

$$Z_{\perp} = 2\frac{cZ_0}{\omega\Delta^2} \frac{S_{12}^{0R} - S_{12}^{0D}}{S_{12}^{0R}} \tag{15}$$

which however should only be used in the case of matched line.

It is worth to emphasize that most of measuring devices give the scattering matrix referred to an impedance $Z_c = 50\Omega$ which in general differs from the line impedance Z_0.

Our aim is then to find a suitable algorithm and to envisage a correct set of measurements in order to get the DUT and REF matched scattering matrices from the mismatched scattering matrices, to be inserted in eqs. (12) and (13).

2 The improved method of measurement

In the previous section we have shown that it is possible to get the coupling impedance from the measurements of the scattering coefficients S_{12}^0 of the device under test and of the reference line. It is worthwhile to remark that these coefficients should be measured (or computed) with reference to the impedance $Z_c = Z_0$. Generally speaking, this is not possible by means of a direct measurement, since the scattering coefficients given by the measuring device are always referred to a value of $Z_c = 50\Omega$.

In addition to this, we point out that, for mechanical and electrical reasons, in some cases both DUT and REF cannot be directly connected with the ports of the measuring instrument: indeed we need some electrical and mechanical adaptors, like cones and lumped Γ-circuits. As a consequence, the scattering matrices which are measured are those of the whole configurations represented in Fig. 2.

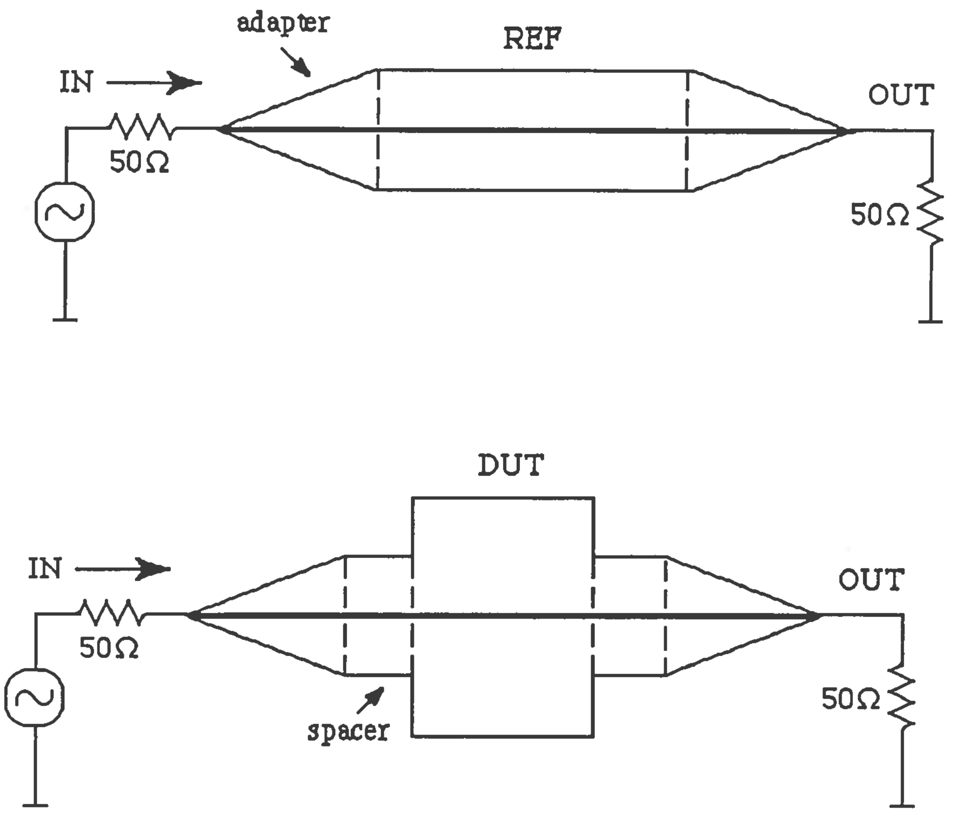

Figure 2: *Experimental configuration: a) Reference line, b) Device Under Test*

We must face the twofold difficulty to get, from the measurements, the scattering matrix of the device under test (or of the reference line) **alone**, and to find this matrix referred to an impedance Z_c equal to the characteristic impedance of the line.

We will now show that, by means of the measurements of the matrix $\boldsymbol{S}^M$ of the whole configuration as shown in Fig. 2 (where M stays for the DUT or for the REF), and of the matrix $\boldsymbol{S}^C$ of the configurations of the two contiguous adaptors as shown in Fig. 3, it is possible to get the quantity S^0_{12} satisfying to both the above requirements without any restriction as to the impedance Z_c respect to which we measure the matrices $\boldsymbol{S}^M$ and $\boldsymbol{S}^C$.

This means that we may get the quantity of interest by means of the measurements of the mismatched matrices, as they are given by the measuring instruments, namely with $Z_c = 50\ \Omega$.

In order to analyze the electric characteristics of the configurations shown in fig. 2 and fig. 3 it is convenient to use the Hybrid matrix $\boldsymbol{H}$ treatment, instead of characterize them by means of the usual $\boldsymbol{S}$ matrices.

It is well known that input and output currents and voltages are related by

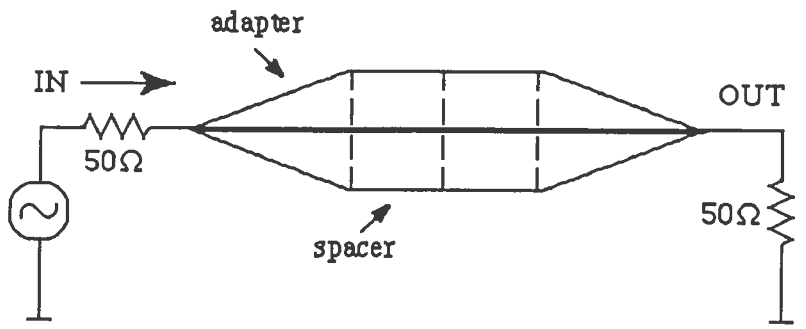

Figure 3: *Configuration of the contiguous adaptors*

the matrix $\boldsymbol{H}$ as follows

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} \equiv \boldsymbol{H} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} \tag{16}$$

We have chosen to use hybrid parameters because the representation by means of $\boldsymbol{H}$ matrices is very helpful to treat cascade circuits as those shown in fig. 2 and fig. 3 are.

In Appendix A some useful relations can be found, which involve matrices $\boldsymbol{H}$, $\boldsymbol{Z}$ and $\boldsymbol{S}$.

As a reminder, hereafter a few properties of hybrid matrices are given. First of all, in a symmetric circuit

$$|\boldsymbol{H}| \equiv h_{11}^2 - h_{12}h_{21} = 1 \tag{17}$$

is always satisfied.

Furthermore, the relationship between the scattering matrix and the hybrid parameters, in the simplest case of a symmetric line is

$$\boldsymbol{S} = \frac{\begin{pmatrix} Y_c h_{12} - Z_c h_{21} & 2 \\ 2 & Y_c h_{12} - Z_c h_{21} \end{pmatrix}}{Y_c h_{12} + Z_c h_{21} + 2h_{11}} \tag{18}$$

where the admittance Y_c is the inverse of Z_c.

Allowing for a matched circuit, the matched matrix $\boldsymbol{S}^0$ must be zero on the diagonal, as already said in sec. 1, which implies that

$$Z_0^2 = \frac{h_{12}}{h_{21}} \tag{19}$$

and, therefore, eq. (18), with the condition (17), becomes (as shown in Appendix A):

$$\boldsymbol{S}^0 = \begin{pmatrix} 0 & h_{11} - j\sqrt{1 - h_{11}^2} \\ h_{11} - j\sqrt{1 - h_{11}^2} & 0 \end{pmatrix} \tag{20}$$

If we interpret the parameter h_{11} as

$$h_{11} = \cos\Theta$$

where Θ in a complex angle, then the scattering parameter S_{12} can be written as

$$S^0_{12} = exp(-j\Theta)$$

which is in agreement with eq. (11).

The result of eq. (20) is very important, because it shows that the knowledge of the only parameter h_{11} is enough to get the quantity S^0_{12} we are interested in [6].

Let us turn now to the object of our investigation. We have already said, without demonstrating it, that from the measurement of the (mismatched) matrices $\boldsymbol{S}^C$ and $\boldsymbol{S}^M$ we are able to get the transmission parameter we are interested in. We start from the relevant hybrid matrices $\boldsymbol{H}^C$ and $\boldsymbol{H}^M$. Let us label the circuits preliminarily: A are the mechanical (cones, spacers) and electrical (possible Γ circuits) adaptors; D and R are the device under test and the reference line; A^{-1} is the reversed adaptor. The configuration in fig. 2a is then labelled ARA^{-1}, whilst the one in fig. 2b will be called ADA^{-1}; the configuration in fig. 3 will be instead AA^{-1}.

Let us start with the analysis of the hybrid matrix $\boldsymbol{H}^C$ relevant to the configuration AA^{-1}. Naming the matrix of the adaptor $\boldsymbol{H}_A$ and defining the matrix $\boldsymbol{L}$ as

$$\boldsymbol{L} \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{21}$$

the hybrid matrix $\boldsymbol{H}^C$ becomes

$$\boldsymbol{H}^C = \boldsymbol{H}_A(\boldsymbol{L}\boldsymbol{H}_A^{-1}\boldsymbol{L}) = \begin{pmatrix} a & b \\ (a^2-1)/b & a \end{pmatrix} \tag{22}$$

Bear in mind that the hybrid matrix of the reversed adaptor is the inverse of $\boldsymbol{L}\boldsymbol{H}_A\boldsymbol{L}$. In the equation above the quantities a and b are measured quantities, or can be obtained from the measurements of the $\boldsymbol{S}^C$ as shown in Appendix A.

Allowing for the matrix

$$\boldsymbol{H}^C\boldsymbol{L} = \boldsymbol{H}_A\boldsymbol{L}\boldsymbol{H}_A^{-1} \tag{23}$$

we note that the matrix $\boldsymbol{H}_A$ can diagonalize the matrix $\boldsymbol{H}^C\boldsymbol{L}$ of eq. (23) and that the eigenvalues of $\boldsymbol{H}^C\boldsymbol{L}$ are -1 and 1.

We point out that from the matrix $\boldsymbol{H}^C$ it is not possible to get the hybrid matrix $\boldsymbol{H}_A$ of the adaptor alone, since there is an infinite number of unit matrices which diagonalize $\boldsymbol{H}^C\boldsymbol{L}$. Let $\boldsymbol{U}$ be an arbitrary unit matrix which diagonalize

$\boldsymbol{H}^C\boldsymbol{L}$, the hybrid matrix $\boldsymbol{H}_A$ of the adaptor alone can be obtained from $\boldsymbol{U}$, to within an arbitrary constant, according to the equation

$$\boldsymbol{H}_A = \boldsymbol{U}\begin{pmatrix} t & 0 \\ 0 & t^{-1} \end{pmatrix} \equiv \boldsymbol{UT} \tag{24}$$

where t is an arbitrary complex number. Indeed, plugging eq. (24) in eq. (23) we get

$$\boldsymbol{H}_A\boldsymbol{L}\boldsymbol{H}_A^{-1} = \boldsymbol{UTLT}^{-1}\boldsymbol{U}^{-1} \equiv \boldsymbol{ULU}^{-1} \tag{25}$$

We chose for the matrix $\boldsymbol{U}$ the expression

$$\boldsymbol{U} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & b \\ (a-1)/b & a+1 \end{pmatrix} \tag{26}$$

therefore the matrix $\boldsymbol{H}_A$ of the adaptor alone is

$$\boldsymbol{H}_A = \boldsymbol{UT} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & b \\ (a-1)/b & a+1 \end{pmatrix}\begin{pmatrix} t & 0 \\ 0 & t^{-1} \end{pmatrix} \tag{27}$$

Let us turn now our attention to the hybrid matrix $\boldsymbol{H}^M$ relevant to the configuration ADA^{-1} or ARA^{-1}. Letting $\boldsymbol{H}^X$ the unknown matrix of the DUT or of the REF, we obtain the expression

$$\boldsymbol{H}^M = (\boldsymbol{H}_A)\boldsymbol{H}^X(\boldsymbol{L}\boldsymbol{H}_A^{-1}\boldsymbol{L}) \equiv \begin{pmatrix} h_{11}^M & h_{12}^M \\ h_{21}^M & h_{22}^M \end{pmatrix} \tag{28}$$

In this equation too, the matrix $\boldsymbol{H}^M$ is a measured quantity or can be obtained by measuring the relevant scattering matrix as shown in Appendix A.

From eq. (28) we get the following formula for the hybrid matrix $\boldsymbol{H}^X$:

$$\boldsymbol{H}^X = \boldsymbol{H}_A^{-1}\boldsymbol{H}^M\boldsymbol{L}\boldsymbol{H}_A\boldsymbol{L} \tag{29}$$

Introducing in eq. (29) the matrix $\boldsymbol{H}_A$ found with the first measurement and expressed by eq. (27), to within an arbitrary constant, we obtain

$$\boldsymbol{H}^X = \boldsymbol{T}^{-1}\boldsymbol{U}^{-1}\boldsymbol{H}^M\boldsymbol{LULT} = \begin{pmatrix} h_{11}^X & h_{12}^X \\ h_{21}^X & h_{11}^X \end{pmatrix} \tag{30}$$

We remark that in eq. (30) all the matrices are known (because they are measured or can be obtained from measured quantities) except the matrix $\boldsymbol{T}$ which, by definition, is diagonal and unitary,

It is easy to demonstrate that, by premultiplicating and postmultiplicating an arbitrary matrix by the matrices $\boldsymbol{T}^{-1}$ and $\boldsymbol{T}$, this operation does not modify the values of the diagonal elements. Therefore h_{11}^X equals the element 11 of the matrix before the operation, i.e.

$$h_{11}^X = [\boldsymbol{U}^{-1}\boldsymbol{H}^M\boldsymbol{LUL}]_{11} \tag{31}$$

The quantity defined by the equation above is known because the matrices $\boldsymbol{U}$ and $\boldsymbol{H}^M$ are known, since they can be deducted from the measured scattering matrices relative to the configurations of fig. 2 and of fig. 3, as it has already been pointed out.

The expression of h_{11}^X (see Appendix A) is

$$h_{11}^X = \frac{1}{2}\left[2ah_{11}^M - bh_{21}^M - \frac{a^2-1}{b}h_{12}^M\right] \tag{32}$$

from which, according to eq. (20) and to the algebra developed in Appendix A we get the result we are looking for:

$$\begin{aligned} S_{12}^{0X} &= \frac{1}{2S_{12}^C S_{12}^M}\Big\{{S_{12}^C}^2 + {S_{12}^M}^2 - \left(S_{11}^C - S_{11}^M\right)^2 + \\ &-\sqrt{\left[(S_{12}^C - S_{12}^M)^2 - (S_{11}^C - S_{11}^M)^2\right]\left[(S_{12}^C + S_{12}^M)^2 - (S_{11}^C - S_{11}^M)^2\right]}\Big\} \end{aligned} \tag{33}$$

We point out that the measured matrices, $\boldsymbol{S}^M$ and $\boldsymbol{S}^C$, are not necessarily the adapted ones, and then they can be measured by means of standard network analyzer.

This expression can be written for $X = R$ and $X = D$, namely when M stands for ARA^{-1}, and when M stands for ADA^{-1}, respectively. Therefore, we obtain the transmission parameters S_{12}^{0R} and S_{12}^{0D}, which, plugged in eq. (12) yield the longitudinal impedance for the case of one wire measurement, while, plugged in eq. (13), yield the transverse impedance in the case of two-wire measurement.

As a conclusion, in order to derive exactly the longitudinal and transverse impedances we may measure the mismatched scattering $\boldsymbol{S}^C$, $\boldsymbol{S}^D$ e $\boldsymbol{S}^R$, relative to circuital configurations described above.

A

The general relationship between the matrix $\boldsymbol{S}$ and hybrid parameters is:

$$\boldsymbol{S} = \frac{\begin{pmatrix} Y_c h_{12} - Z_c h_{21} + (h_{11} - h_{22}) & 2 \\ 2 & Y_c h_{12} - Z_c h_{21} + (h_{22} - h_{11}) \end{pmatrix}}{Y_c h_{12} + Z_c h_{21} + h_{11} + h_{22}} \tag{34}$$

For a symmetrical circuit ($h_{11} = h_{22}$) we may define the characteristic impedance Z_0. If the circuit is matched ($Z_c = Z_0$) the diagonal is zero, which implies

$$Z_0 = \sqrt{\frac{h_{12}}{h_{21}}} \tag{35}$$

In this case eq. (34) takes the form

$$\boldsymbol{S}^0 = \frac{1}{\sqrt{h_{12}h_{21}} + h_{11}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = [h_{11} - j\sqrt{h_{11}^2 - 1}] \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{36}$$

where we took in account that

$$|\boldsymbol{H}| \equiv h_{11}^2 - h_{12}h_{21} = 1$$

The inverse relation of eq. (34) is

$$\boldsymbol{H} = \frac{1}{2S_{12}} \begin{pmatrix} 1 - |\boldsymbol{S}| + S_{11} - S_{22} & Z_c(1 + |\boldsymbol{S}| + S_{11} + S_{22}) \\ Y_c(1 + |\boldsymbol{S}| - S_{11} - S_{22}) & 1 - |\boldsymbol{S}| + S_{22} - S_{11} \end{pmatrix} \tag{37}$$

In order to derive eq. (33) from eq. (32) we resort to the expression of the hybrid matrix $\boldsymbol{H}^C$ as a function of the measurable matrix $\boldsymbol{S}^C$ given by eq. (37); the result is

$$a = \frac{1}{2S_{12}^C}(1 - |\boldsymbol{S}^C|) \tag{38}$$

$$b = \frac{Z_c}{2S_{12}^C}(1 + |\boldsymbol{S}^C| + 2S_{11}) \tag{39}$$

where, because of the symmetry, we took $S_{11}^C = S_{22}^C$.

Likewise, for the quantities h_{11}^M, h_{12}^M h_{21}^M we get:

$$h_{11}^M = \frac{1}{2S_{12}^M}(1 - |\boldsymbol{S}^M|)$$

$$h_{12}^M = \frac{Z_c}{2S_{12}^M}(1 + |\boldsymbol{S}^M| + 2S_{11}^M)$$

$$h_{21}^M = \frac{Y_c}{2S_{12}^M}(1 + |\boldsymbol{S}^M| - 2S_{11}^M)$$

These expressions inserted in eq. (32) and eventually in eq. (20) yield the eq. (33).

References

[1] G. Nassibian, F. Sacherer: "*Methods for Measuring Transverse Coupling Impedances in Circular Accelerators*", in Nuclear Instruments and Methods 159 (1979) pp.21-27.

[2] H. Hahn, F. Pedersen: "*On a Coaxial Wire Measurements of the Longitudinal Coupling Impedance*", BNL 50870, in Particle Accelerators and High-Voltage Machines TID-4500, April 1978.

[3] L.S.Walling et al.: "*Transmission Line Impedance Measurements for an Advanced Hadron Facility*", in Nuclear Instruments and Methods of Physics Research, A 281 (1989), pp.433-447.

[4] F. Caspers: "*Beam Impedance Measurements Using the Coaxial Wire Method*", in Proc. of the Workshop on Impedance and Current Limitations, ESRF-Grenoble, France, October 1988, CERN PS/88-59.

[5] M. Sands, J. Rees: "*A Bench Measurement of the Energy Loss of a Stored Beam to a Cavity*", SLAC-Report PEP-95, August 1974.

[6] M. Cardito, G. Di Massa, F. Galluccio, R. Losito, M.R. Masullo, V.G. Vaccaro: "*Transverse Beam Impedance Measurement. A New Coaxial Method.*", INFN sez. di Napoli, May 1993, in Particle Accelerators Conference, Washington, USA, May 1993 (PAC 93).