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TOPGUN: A NEW WAY TO INCREASE THE BEAM BRIGHTNESS OF RF GUNS

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ABSTRACT

In this paper a new method is presented to neutralize the RF induced emittance blow up generated inside RF electron Guns. The method is based on a multi-mode operation of the RF Gun cavity, which must be able to support both the accelerating mode ($TM_{010-\pi}$) and a higher harmonic mode. The analytical study of the beam dynamics, which has been found in good agreement with the numerical simulations, shows that the growth of the normalized rms emittance, produced by the time dependent RF forces during the acceleration in the Gun, can be cancelled up to fourth order terms. This is of great relevance for the improvement of RF Gun performances, since the RF field contribution to the emittance blow up becomes negligible and no more dependent on the bunch size. As shown in this paper, with such a new Gun, which is called "TOPGUN", the way to attain high brightness beams becomes straightforward.

INTRODUCTION

In the first Section of this paper we present a general discussion on the beam characteristics produced by standard RF Guns, concerning in particular the quality of transverse and longitudinal phase space distributions observed at the Gun exit. It is shown that the presence of non linear terms in the longitudinal phase space distribution, essentially due to the non linear dependence of the energy gain versus injection phase, is the key issue which must be considered to correct and neutralize the transverse emittance blow up.

Section 2 is dedicated to an analytical study of the beam dynamics in a multi-mode RF Gun: following a previous analysis [1], we develop the calculations taking into account the presence of an odd harmonic resonant mode superimposed to the main accelerating mode, and we search for a condition of vanishing non linear terms in the energy-phase relationship. It is shown that, once satisfied this condition, which will be called "straight-topping", all the terms up to the fourth order in the rms normalized transverse emittance are cancelled out. At the same time the contribution to the longitudinal emittance produced by the RF field is vanishing too, so that the output beam can be magnetically compressed at the Gun exit, to get a very high peak current.

A preliminary and not optimized geometry for a possible RF structure able to support both the accelerating $TM_{010\text{-}\pi}$ mode and a third harmonic $TM_{012\text{-}\pi}$ mode is studied in Section 3. This geometry is still of the Brookhaven type[10] and the fundamental frequency has been kept close to the BNL value. A scheme of principle for the independent coupling of the two modes, based on two wave guides each coupled to just one mode, is also presented.

The results of some numerical simulations, performed on the proposed RF structure using the self-consistent e.m. PIC code ITACA, are shown in Section 4. The agreement with the analytical predictions is really quite good. In fact the contributions coming from the non linear transverse RF components, not taken into account in the analytical study, are not minimized in the chosen geometry, and they are mainly responsible for the left contribution to the RF induced emittance observed in the computation. However, the possibility to achieve a transverse emittance of 2 mm·mrad with 1 nC bunches at 4 MeV is demonstrated.

Finally, the great advantages of the TOP-GUN are discussed in the concluding Section, especially in terms of better performances for the emittances and beam quality. The new criteria for the operation of RF Guns, based on new scaling laws for the optimization of the beam quality, are also addressed.

1 - THE BEAM AT THE EXIT OF A STANDARD RF GUN

It is well known that the beam brightness produced by RF Guns, defined as: $B_n=2I/(4\pi\varepsilon_n)^2$ [2] (where I is the peak current of the bunch and ε_n the rms normalized transverse emittance), is mainly limited by two effects:

- the emittance growth due to space charge forces (caused by the non linear transverse components of the space charge field and by its phase dependence, i.e. its variation versus the longitudinal position inside the bunch)
- the time (or phase) dependence of the RF transverse forces, which produces a transverse momentum at the Gun exit which is strongly correlated to the longitudinal position inside the bunch (i.e. the phase). This is responsible for the so called fan-like shape of the transverse phase space [3].

It must be noticed that the non linear transverse components of the RF field give also a contribution to the emittance blow up, introducing a distortion of the phase space distribution (the fan outline becomes curved). However, because a particular shaping of the iris profile[4] allows to minimize the non linear transverse components of the RF field, making negligible their contribution to the emittance, in the following discussion we neglect, as usual[1], this effect.

From the analytical study made by K.J.Kim[1] it can be shown that the total rms emittance growth, $\Delta\varepsilon_{tot}$ [m rad], at the Gun exit can be written as:

$$\Delta\varepsilon_{tot}^2 = \left(a_{RF}^2 E_0 \sigma_r^2 \sigma_z^2\right)^2 + \left(a_{sc} \frac{Q}{E_0 (3\sigma_r + 5\sigma_z)}\right)^2 + 2\,J\,a_{RF}\,a_{sc} \frac{Q(\sigma_r\sigma_z)^2}{3\sigma_r + 5\sigma_z} \qquad 1)$$

where: E_0 is the peak electric field on the cathode surface [MV/m], σ_r and σ_z are the widths of the gaussian bunch current distribution [m], Q the bunch charge [nC], while a_{RF} is defined as a_{RF}=.83k ($k=\omega_{RF}/c$ [m^{-1}]) and a_{SC} is given by $a_{SC}=5.7\cdot 10^{-6}/\sin\phi_0$, ϕ_0 being the injection phase.

The first term in the sum gives the contribution to the emittance coming from the time dependence of the linear RF field with resonant frequency ω_{RF}, while the second one represents the effect of the space charge forces. The third term gives a contribution coming from the correlation between RF and space charge effects. The correlation factor J is given by:

$$J(\theta) = \frac{3\theta+5}{\sqrt{2}} \int_0^\infty (2+x)^{-2} (2+\theta x)^{-1.5} dx \quad , \quad \theta = \frac{\sigma_r}{\sigma_z}$$

The optimum injection phase ϕ_0, i.e. the phase at which the centre of the laser pulse must strike the cathode surface in order to minimize the RF contribution to the emittance, is specified by the equation

$$\frac{\pi}{2} = \phi_0 + \frac{1}{2\alpha\cdot\sin\phi_0} \qquad 2) \qquad \text{where } \alpha = \frac{eE_0}{2kmc^2}$$

Expression 1) tell us that, once chosen the frequency and field of the RF Gun, for a fixed bunch charge some optimum values for σ_r and σ_z can be found which minimizes the emittance growth. An increase of the bunch length causes actually a decrease of the space charge contribution to the emittance but increases at the same time the emittance growth due to RF field. The same holds for the bunch radius σ_r.

Just as an example we show in Fig.1 the behaviour of $\Delta\varepsilon_{tot}$ when Q=1 nC, ν_{RF}=2856 MHz and E_0=100 MV/m are selected, i.e. the typical operating values of the Brookhaven ATF RF Gun.

The equi-level plot for the emittance blow up as a function of σ_r and σ_z is drawn in Fig.1: the numbers on the lines give the amount of emittance blow up on that line (in mm·mrad). Two minimum regions are shown in the plot ($\Delta\varepsilon_{tot} \approx$ 5 mm·mrad): one for disk-like bunches (large σ_r and short σ_z, like for the anticipated operating point of the ATF Gun[4]) and the other one for cigar-like bunches (small σ_r and long σ_z). The peak current is of course decreasing as σ_z^{-1}, hence cigar-like bunches have a lower peak current if the bunch charge is kept constant. However, as discussed elsewhere[5] - and as shown below in Section 2 and 4 - their current can be increased via a magnetic compression as long as their longitudinal phase space distribution does not exhibit a significant non linear distortion.

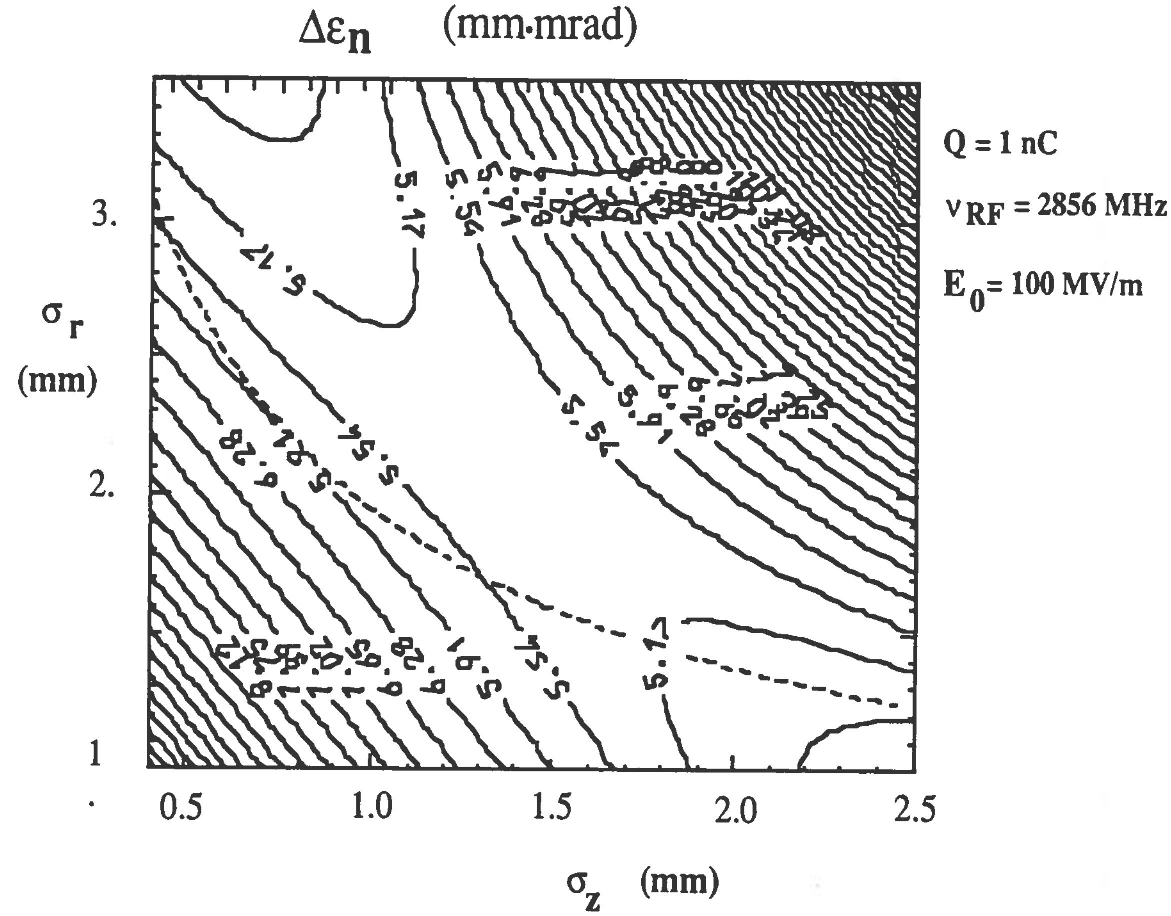

Fig. 1 - Equi-level lines for the quantity $\Delta\varepsilon_{tot}$ (equation 1) as a function of σ_r and σ_z, for fixed Q, ν_{RF} and E_0.

Moreover, the dashed line splits the plot into two regions: the region below the line is not permitted because needs at the cathode surface a current density larger than 500 A/cm^2, which is a typical value for present photo-cathode technology (cigar-like bunches are in this sense more difficult to generate). Finally, the upper right corner of the plot is the RF-dominated region for $\Delta\varepsilon_{tot}$ (RF induced blow up is dominant), while the left lower corner is the space-charge dominated region.

We want to stress at this point that in a standard RF Gun it is not possible to reduce the emittance simply by decreasing the bunch charge density, i.e. using longer and larger bunches in order to keep low the space charge field, since the RF induced emittance blow up becomes the dominant effect. The possibility to cancel this RF contribution would allow automatically to damp down also the space charge emittance simply by using larger sizes for the bunch (for a given bunch charge) or by increasing the peak field on the cathode surface.

A different point of view [6], in the subject of beam quality optimization in operating RF Guns, looks at the optimum field, at a fixed frequency, to achieve a certain peak current and emittance: the optimum field is defined by equating in 1) the the RF field contribution (first term in the r.h.s.) to the space charge one (second term in the r.h.s.). This is of relevance when one wants to generate a high peak current I (>100 A) just at the gun exit, without any further compression.

Alternate strategies have been recently proposed[7] to attain high brightness using magnetic compression. These require a low longitudinal emittance, i.e. a small curvature in the longitudinal phase space distribution, in order to get a high efficiency in the magnetic compression.

It is well known that the basic mechanism of the RF induced emittance blow-up consists in the correlation between the exit transverse momentum and the injection phase, as given by the formula:

$$p_r = \alpha k r \cdot \left(\sin\langle\phi\rangle + \Delta\phi \cdot \cos\langle\phi\rangle - \frac{\Delta\phi^2}{2} \sin\langle\phi\rangle \right) \qquad 3)$$

which gives the so called fan-like shape of the transverse phase space distribution at the gun exit. In this expression r is the radial position (assumed to be constant during the acceleration) of a generic electron of the bunch, whose exit phase ϕ (defined as $\phi = \omega T_f - kL + \phi_0$, L being the Gun length, ϕ_0 the injection phase at the cathode and T_f the exit time) is supposed to be slightly distributed around an average exit phase of the bunch $\langle\phi\rangle$, such that $\phi = \langle\phi\rangle + \Delta\phi$. It can be noted that the analytical estimation for the exit phase ϕ is actually given by the r.h.s. of eq. 2).

Moreover, since the fields (both RF and space charge) are assumed to be axi-symmetrical, the phase spaces (x,p_x) and (y,p_y) must be identical, hence x and p_x could be substituted in 3) by r and p_r respectively. It must be noted that expression 3) for the exit momentum has been obtained taking into account only the linear transverse components of the RF field, assumed to be a pure first (spatial) harmonic of a $TM_{010\text{-}\pi}$ resonant mode: $E_z(z,t) = E_0\cos(kz)\sin(\omega t+\phi_0)$ (z=0 is the cathode position).

Using the standard definition for the rms normalized emittance[8] :

$$\varepsilon_x = \sqrt{\langle x^2\rangle\langle p_x^2\rangle - \langle x p_x\rangle^2} \qquad 4)$$

where $\langle\rangle$ means an average over the phase space distribution, the emittance at the Gun exit is given by a quadratic sum of the induced emittance blow up plus the temperature emittance at the cathode. In the following we will take care only of the emittance blow up, assuming a zero cathode temperature.

Substituting for p_x its expression given by eq. 3), and assuming that the phase distribution is symmetric with respect to $\langle\phi\rangle$ (i.e. $\langle\Delta\phi\rangle = \langle(\Delta\phi)^3\rangle = ... = 0$), the emittance turns out to be:

$$\varepsilon_x^{RF} = \varepsilon_{min} + \alpha k\langle x^2\rangle\sqrt{\langle(\Delta\phi)^2\rangle}\,\left|\cos\langle\phi\rangle\right| \qquad 5)$$

where

$$\varepsilon_{min} = \frac{\alpha k\langle x^2\rangle}{2}\sqrt{\langle(\Delta\phi)^4\rangle - \langle(\Delta\phi)^2\rangle^2}$$

For a gaussian bunch, recalling that $\langle x^2\rangle = \sigma_r$, $\langle(\Delta\phi)^4\rangle = 3k^4\sigma_z^4$ and $\langle(\Delta\phi)^2\rangle = k^2\sigma_z^2$, it is easy to verify that ε_{min} gives just the square root of the first term on the r.h.s. of eq. 1), corresponding to the RF induced emittance blow up.

It is clear from eqs. 5) that this contribution has a sharp minimum at $\langle\phi\rangle = \pi/2$, where the first order term in the (p_r,ϕ) correlation is vanishing: this produces the minimum aperture of the fan in the transverse phase space, as shown in Fig.2a and 2b, where two typical transverse phase space distributions (r,p_r) at the Gun exit are plotted for the two values $\langle\phi\rangle = 90°$ and $\langle\phi\rangle = 110°$ respectively. In particular, it can be seen that the first order term, not present in Fig.2a, makes wider the fan in Fig.2b. Moreover we note that the minimum condition is actually a critical operating condition: a little shift of $\langle\phi\rangle$ (namely $\delta\langle\phi\rangle$) away from the optimum value sets up indeed the second term in the r.h.s of the first of eqs. 5), which scales like $\delta\langle\phi\rangle\cdot\sigma_z$ and gives a large contribution.

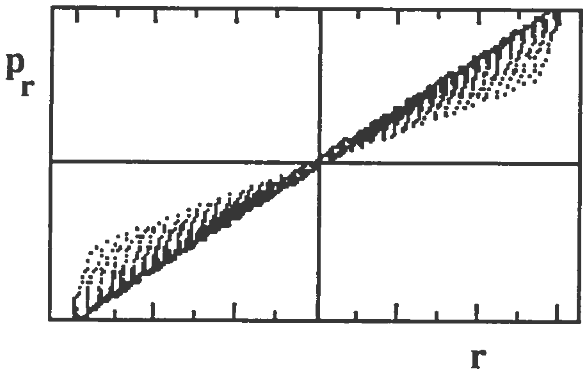

Fig. 2a - Transverse phase space (r,p_r) distribution, at the Gun exit, for the case of minimum emittance (i.e. $\langle\phi\rangle = 90°$).

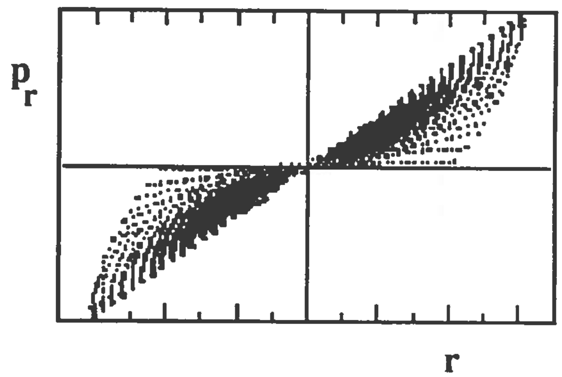

Fig.2b - Transverse phase space (r,p_r) distribution, at the Gun exit, obtained with a higher exit phase $\langle\phi\rangle = 110°$.

In other words, the minimum emittance blow up is a difficult condition to guarantee as a stable one in the real operation of a RF Gun: the real emittance blow up due to the RF field is therefore underestimated by the first term in the r.h.s. of eq. 1).

2 - BEAM DYNAMICS IN THE MULTI-MODE RF GUN

2.1 - The effect of the "Straight Topping"

In order to improve the performances of a standard RF Gun we started noting that the final energy of an electron at the Gun exit can be written as (see Appendix 1):

$$\gamma_f = 1 + \alpha \cdot [\pi(N+1/2)\sin\phi + \cos\phi] \qquad 6)$$

where ϕ is still the electron exit phase and N+1/2 is the number of cells of the Gun cavity. Taking into account a small deviation in the phase $\Delta\phi$, we get the linear and second order terms in the energy distribution acording to the equation:

$$\gamma_f = 1 + \alpha \cdot \{ [\pi (N+1/2) \sin\langle\phi\rangle + \cos\langle\phi\rangle] \cdot (1 - 0.5\,\Delta\phi^2) + + [\pi (N+1/2) \cos\langle\phi\rangle - \sin\langle\phi\rangle] \cdot \Delta\phi \} \qquad 6')$$

The second order term in $\Delta\phi^2$ is clearly a source of longitudinal rms emittance blow up. Indeed, the exit longitudinal momentum $p_z = mc\beta_z\gamma$ can be considered simply proportional to γ_f (being $\beta_z \approx 1$ at the gun exit) and the rms longitudinal emittance, given by:

$$\varepsilon_z = k^2 \sqrt{\langle(\Delta\phi)^2\rangle\langle(\Delta p_z)^2\rangle - \langle\Delta\phi\Delta p_z\rangle^2}$$

can be cancelled out if the second order term in 6') is vanishing. This would require $\cot\langle\phi\rangle = -\pi(N+1/2)$, i.e. a value for $\langle\phi\rangle$ far from the minimum transverse emittance condition $\langle\phi\rangle = \pi/2$. For such a value of $\langle\phi\rangle$ the exit momentum is $p_z = 1 + \alpha[(N+1/2)\pi - \Delta\phi - .75\pi\Delta\phi^2]$ while the longitudinal emittance is $\varepsilon_z = 0.5\sqrt{3}(N+1/2)\pi\alpha k^2\sigma_z^3$. This emittance blow up is substantially due to the curvature of the longitudinal phase space, given by the second order term $\Delta\phi^2$ in equation 6'), as shown in Fig. 3a and 3b, where two typical longitudinal phase space distributions are plotted for $\langle\phi\rangle = 90°$ and $\langle\phi\rangle = 110°$.

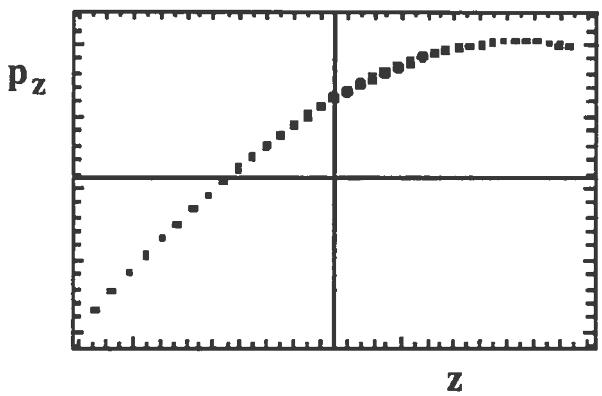

Fig. 3a - Longitudinal phase space (z,p_z) distribution, at the Gun exit, for the case of minimum emittance (i.e. $\langle\phi\rangle$=90°)

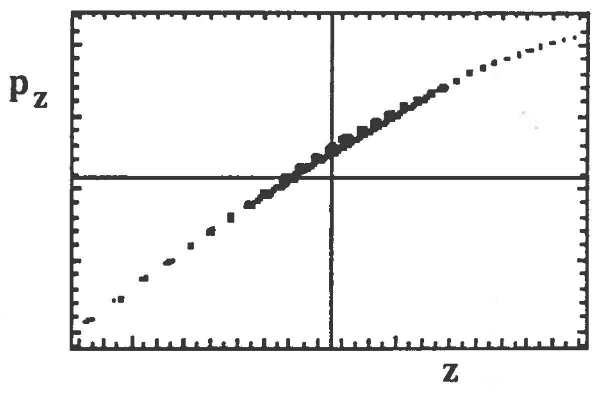

Fig.3b - Longitudinal phase space (z,p_z) distribution, at the Gun exit, obtained with a higher phase $\langle\phi\rangle$=110° .

It is evident that the condition of minimum transverse emittance does not match with that one of the longitudinal emittance: indeed, higher injection phases give less curvature in the longitudinal phase space distribution, i.e. a smaller longitudinal emittance, but higher transverse emittance values.

Hence we look for a condition of zero curvature in the longitudinal phase space distribution: this will be similar to a flat-topping condition, except that in a flat-top operation usually the first order term in the energy phase relationship is also vanishing. For this reason we call the condition under study a "straight-topping" condition, because the first order term will be left free to give a constant slope in the energy-phase curve.

As for the flat-topping we try to add to the main accelerating field of the RF Gun, given by a TM_{010}-π mode, a n^{th} harmonic field (n integer) as given in the following:

$$E_z(z,t) = E_0(z)\cos(kz)\sin(\omega t+\phi_0) + E_n(z)\cos(nkz)\sin(n\omega t+n\phi_0) \qquad 7)$$

where $E_0(z)$ and $E_n(z)$ are step-like functions of the type $E_0(z) = \theta[(N+1/2)\frac{\lambda}{2}-z]E_0$ and $E_n(z) = \theta[(N+1/2)\frac{\lambda}{2}-z]E_n$ (E_0 and E_n being the peak fields at the cathode surface) which account for the discontinuity of the RF field at the cavity exit, while ω is the frequency of the TM_{010}-π mode and θ is the unitary step function. Following Kim's method[1] to compute the final energy γ_f and phase ϕ, by simply summing the separate contribution of the n^{th} harmonic field we get:

$$\gamma_f = 1 + \alpha\left[\pi(N+\tfrac{1}{2})\sin\phi + \cos\phi\right] + \alpha_n\left[n\pi(N+\tfrac{1}{2})\sin(n\phi) + \begin{cases}\cos(n\phi), & n \text{ odd}\\ 0, & n \text{ even}\end{cases}\right] \qquad 8)$$

and

$$\phi = \frac{1}{2\alpha\sin\phi_0 + 2n\alpha_n\sin(n\phi_0)} + \phi_0 \qquad 8')$$

where we have defined α_n as: $$\alpha_n = \frac{eE_n}{2nkmc^2}$$

In the following we will study odd harmonics: as shown below, only such harmonics can satisfy a straight-top condition up to fourth order. Assuming now a symmetric phase distribution over the bunch length, with $\phi = \langle\phi\rangle + \Delta\phi$, we compute up to 3^{rd} order the contributions to the longitudinal normalized momentum at the Gun exit ($p_z \approx \gamma_f$), for the case $\langle\phi\rangle = \pi/2$:

$$\begin{aligned} p_z = & \left[1 + \pi(N+1/2)(\alpha + (-1)^{(n-1)/2} n\alpha_n)\right] + \\ & + \left[-\alpha - (-1)^{(n-1)/2} n\alpha_n\right]\Delta\phi + \\ & + \left[(\pi/2)(N+1/2)(-\alpha - (-1)^{(n-1)/2} n^3\alpha_n)\right](\Delta\phi)^2 + \\ & + \left[(1/6)(\alpha + (-1)^{(n-1)/2} n^3\alpha_n)\right](\Delta\phi)^3 + \ldots \end{aligned} \qquad 9)$$

It is easy to verify that the condition

$$\alpha_n = (-1)^{(n-1/2)} \frac{\alpha}{n^3} \qquad 10)$$

which is equivalent to

$$E_n = (-1)^{(n-1/2)} \frac{E_0}{n^2} \qquad 10')$$

assures a fourth-order straight-topping, i.e. a vanishing second and third derivative in the energy-phase relationship at the phase $<\phi> = \pi/2$, which guarantees a minimum transverse emittance.

It is interesting to compute the final momentum as given by expression 9) for the case of n=3 and n=5 (the lowest odd harmonics which can be added to the main accelerating field) given respectively by proper tuned $TM_{012-\pi}$ and $TM_{014-\pi}$ resonant modes of a 1+1/2 cell structure.

3rd harmonic n=3 $\quad E_3 = \frac{E_0}{9} \quad p_z = [1+1.5\pi\alpha(1-\frac{1}{9})] - \alpha(1-\frac{1}{9})\Delta\phi$

5th harmonic n=5 $\quad E_5 = \frac{E_0}{25} \quad p_z = [1+1.5\pi\alpha(1+\frac{1}{25})] - \alpha(1+\frac{1}{25})\Delta\phi$

The third harmonic causes a slight (-11%) decrease both of the final average energy (the zero order term in the expression for p_z) and of the energy spread (linear term in $\Delta\phi$), while the fifth harmonic increases both terms by a smaller amount (+4%).

In Fig.4a and 4b the effect of a 3rd harmonic straight-toppping, is shown for the case of standard BNL values (ν_{RF}=2856 MHz, E_0=100 MV/m and α=1.64): the exit energy γ_f, plotted versus the exit phase ϕ, displays a linear behaviour in the region around $\phi=\pi/2$ when the straight-topping is applied, while the curvature of the dashed line (no straight topping) is clearly visible.

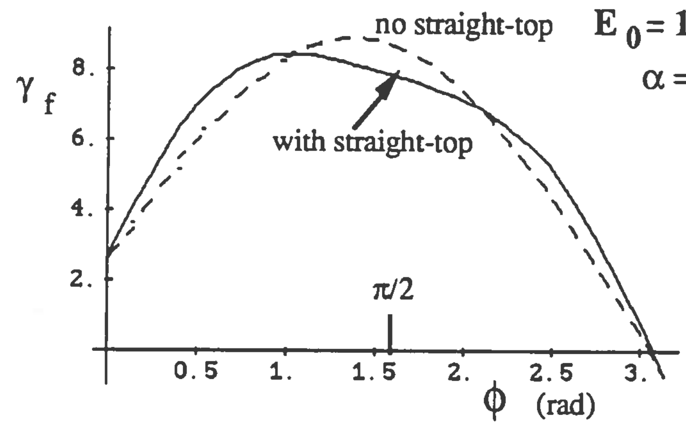

Fig.4a - Exit energy γ_f as a function of the exit phase ϕ for a standard RF Gun (dashed line) and for a RF Gun with straight-topping (solid line).

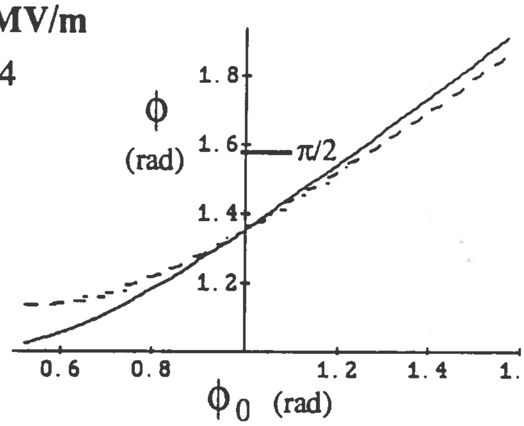

Fig.4b - Exit phase ϕ as a function of the injection phase ϕ_0 for a standard RF Gun (dashed line) and for a RF Gun with straight-topping (solid line).

The the exit phase shift produced by the straight-toppping (i.e. by the superposition of a third harmonic) is quite negligible, as can be seen in Fig.4b: indeed, to keep $\phi=\pi/2$ at the Gun exit, one has to decrease by a negligible amount the injection phase ϕ_0, from 71° down to 70°. Since the injection phase determines the emittance blow up due to space charge, we can conclude that the straight-toppping operation does not have, from this point of view, any negative effect on the space charge induced emittance.

2.2 - Analytical estimation of the rms transverse emittance

To compute the left contributions to the transverse emittance once the straight-top condition has been satisfied, we start with the evaluation of the transverse momentum at the Gun exit.

From the axial field on axis (equation 7) we get, via a linear expansion off-axis, the transverse component of the force:

$$F_r = e(E_r - \beta c B_\theta) = - e\frac{r}{2}\left(\frac{\partial}{\partial z} E_z + \frac{\beta}{c}\frac{\partial}{\partial t} E_z\right)$$

According to the "impulsive approximation" discussed in Appendix 1, we assume that the electrons - after undergoing a step-like transition nearby the cathode surface from the actual injection phase ϕ_0 up to the exit (asymptotic) phase ϕ - cross the RF Gun cavity at $\beta = 1$.

From the point of view of transverse kicks experienced by electrons, we can say that the global effect of the multi-cell Gun cavity can be computed taking into account only the last half cell: indeed, each second half cell (included the cathode half cell) gives a transverse kick contribution equal in amplitude but with opposite sign as the one given by the first half cell of the next cell. In other words, the total transverse momentum produced by the Gun cavity up to the last half cell is actually zero. This can be proven as in the following.

The transverse force becomes (at $\beta \approx 1$):

$$F_r = -\frac{er}{2c}\frac{dE_z}{dt}$$

hence the transverse momentum change from the centre of a cell (which is reached by an electron at time T_c) up to the centre of the next one (which is reached at time $T_{nc} = T_c + \pi/\omega_{RF}$) is given by:

$$\Delta p_r = \frac{-er}{2c}\int_{T_c}^{T_{nc}} \frac{dE_z}{dt}dt = \frac{-er}{2c}\left[E_z(T_{nc}) - E_z(T_c)\right] = \frac{-er}{2c}\left[E_0\sin(\phi+2\pi) - E_0\sin\phi\right] = 0$$

since the phase shift is exactly 2π for a synchronous $\beta = 1$ electron (ϕ is the electron phase - with respect to the RF wave-field - which has been assumed as a constant from the cathode up to the exit).

Taking into account that at the Gun exit $E_z(T_{exit})$ is vanishing, the transverse momentum of the last half cell becomes:

$$p_r = \frac{-er}{2mc^2} \int_{T_{lc}}^{T_{exit}} \frac{dE_z}{dt} dt = \frac{-er}{2mc^2} E_0 \sin\phi = \alpha k r \sin\phi$$

which is actually the same result obtained by the Kim's calculation.

Considering now the presence of an odd n^{th} harmonic, under the straight-topping condition, $E_n = E_0/n^2$, we obtain the exit transverse momentum just summing the n^{th} harmonic contribution given by:

$$p_r = \alpha k r \cdot \sin\phi + \frac{\alpha k r}{n^2} \sin(n\phi) \qquad 11)$$

Again we compute the blow up of the rms normalized transverse emittance via the expression

$$\varepsilon_x^{RF\text{-}top} = \alpha k <x^2> \sqrt{<\left(\sin\phi + \frac{\sin(n\phi)}{n^2}\right)^2> - <\sin\phi + \frac{\sin(n\phi)}{n^2}>^2} \qquad 12)$$

which gives the following result, taking into account only terms up to $(\Delta\phi)^3$ in the expansion of $\sin\phi$ and $\sin(n\phi)$:

$$\varepsilon_x^2 = [\tfrac{1}{4}(<(\Delta\phi)^4> - <(\Delta\phi)^2>^2)][\sin<\phi> + \sin(n<\phi>)]^2$$
$$[<(\Delta\phi)^2> - \tfrac{1}{3}<(\Delta\phi)^4>]\cos^2<\phi>$$
$$[\tfrac{1}{n^2}<(\Delta\phi)^2> - \tfrac{1}{3}<(\Delta\phi)^4>]\cos^2(n<\phi>) \qquad 13)$$
$$\tfrac{2}{n}[<(\Delta\phi)^2> - \tfrac{1}{6}<(\Delta\phi)^4>(1+n^2)]\cos<\phi>\cos(n<\phi>)$$

If n = 3,7,11,.the previous expression is vanishing at $<\phi>=\pi/2$, meaning that the minimum emittance contains only terms of the order of $<(\Delta\phi)^4>$.

Away from the minimum we have:

$$\varepsilon_x^{RF} = O(<(\Delta\phi)^4>) + \alpha k <x^2> \sqrt{<(\Delta\phi)^2>} \left| \cos<\phi> + \frac{\cos(n<\phi>)}{n} \right| \approx$$
$$\approx O(<(\Delta\phi)^4>) + \alpha k <x^2> \sqrt{<(\Delta\phi)^2>} \left(\frac{n^2-1}{6}\right)\left|\delta^3\right| \qquad \text{if } <\phi> = \pi/2+\delta \;,\; \delta \ll 1 \qquad 14)$$

Taking as example the case of n=3, we show in Fig.5 the behaviour of the blow up of the rms normalized transverse emittance as a function of the exit phase ϕ, for two representative bunch sizes, keeping again usual BNL values for α and E_0.

The first one is a disk-like bunch with σ_ϕ = .036 rad and $<x^2> = \sigma_r$ = 3 mm. The minimum emittance without straight-topping (i.e. the quantity ε_{min} of expression 2) at $<\phi> = \pi/2$) is about 1 mm·mrad, while in presence of the 3rd harmonic field becomes actually zero. The lower dashed line gives the behaviour of the emittance without straight-topping, while the lower solid line corresponds to the straight-topping: it is evident the effect of straight-topping, not only in zeroing the minimum value, but also in strongly decreasing the variation around the minimum for exit phases different from $\pi/2$.

The effect of straight-topping is even more impressive for the case of a cigar-like bunch (σ_ϕ = 15 rad and $<x^2> = \sigma_r$ = 2 mm) having a minimum emittance of about 6 mm·mrad, which increases up to 25 mm·mrad at $<\phi>$ = 1.9 rad = 109°. Applying the 3rd harmonic straight-topping the emittance can be kept less than 1 mm·mrad all over a 30° RF interval centered around $<\phi> = \pi/2$. The space charge contribution is not taken into account here, but it must be stressed that for the cigar-like bunch one should get a reduction by a factor at least 4 in the space charge induced emittance, due to the density decrease, with respect to the disk-like bunch. As shown elsewhere [1,3] and in Section 4, this space charge contribution can be kept below 1 mm·mrad.

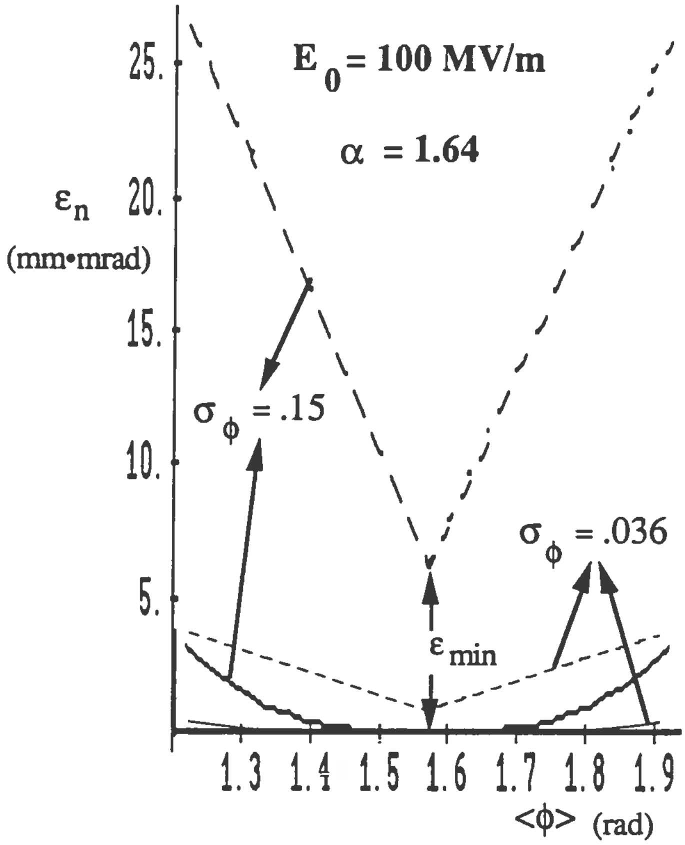

Fig.5 - Transverse emittance blow up induced by RF field as a function of the average exit phase for a disk-like bunch (σ_ϕ=.036 rad) and for a cigar-like one (σ_ϕ=.15 rad). Dashed lines give the behaviour of a standard RF Gun, while solid lines the results from a RF Gun with 3rd harmonic straight-topping.

The $<\phi>$ dependence of the rms emittance has been strongly reduced by the superposition of the straight-topping 3rd harmonic field. Nevertheless, we have to study the perturbations produced by a slight off-set of the straight-topping condition, caused by a small variation of the n^{th} harmonic amplitude E_0, or phase $n\phi_0$, or frequency $n\omega_{RF}$, with respect to the nominal values required by straight-topping.

We start taking into account a slight deviation δ of the harmonic amplitude, as given by:

$$E_{0n} = \frac{E_0\,(1+\delta)}{n}$$

Repeating the calculations for the transverse emittance as in expressions 11) to 13), we obtain:

$$\varepsilon_x = \alpha k <x^2> \frac{|\delta|}{2} \sqrt{<(\Delta\phi)^4> - <(\Delta\phi)^2>^2} \;=\; \frac{|\delta|}{\sqrt{2}}\,\varepsilon_{min} \qquad 15)$$

which tells us that the emittance change is small, being a fraction δ of the minimum emittance ε_{min} (as given by eq. 2). A 1% accuracy in the control of the n^{th} harmonic amplitude is therefore enough to get an actual emittance negligible with respect to the value one would get without straight-topping.

We now let the phase of the n^{th} harmonic free to have a slight shift $\delta\phi$ (caused by some error in the phase control of the n^{th} harmonic) with respect to the nominal value, such that the term $\sin(n\phi)$ in eq.12 becomes $\sin(n\phi+\delta\phi)$. The exit emittance at $<\phi> = \pi/2$ comes out to be:

$$\varepsilon_x = \alpha k <x^2> \sqrt{<(\Delta\phi)^2>}\;\frac{|\delta\phi|}{n}$$

In order to compare such a value with the minimum emittance ε_{min} of eq. 2) (no straight-topping at $<\phi> = \pi/2$), we take a gaussian bunch of length σ_ϕ: in this case the emittance given by the above equation can be rewritten as:

$$\varepsilon_x = \frac{\sqrt{2}\,|\delta\phi|}{n\cdot\sigma_\phi}\,\varepsilon_{min} \qquad 16)$$

This result gives an upper limit to the max deviation of the n^{th} harmonic phase for a given reduction of the minimum emittance in absence of straight-topping. If one want to reduce ε_{min} by one order of magnitude the maximum deviation $\delta\phi$ must obey the condition $|\delta\phi| < \sigma_\phi/5$ when the 3rd harmonic field is used to achieve straight-topping. Since the interest of straight-topping lies in the domain of long (cigar-like) bunches, usually σ_ϕ is in the range of 10° RF: a control of the 3rd harmonic phase within 2° RF (in unit of the n^{th} harmonic frequency) is therefore required.

Regarding possibble errors in the n^{th} harmonic frequency, at this stage of the analysis we can simply say that a slight deviation from the harmonic frequency rule, such that $\nu_{RF}(n^{th}$ harmonic) = $(n+\delta n)\nu_{RF}$ (first harmonic), can be regarded (for single-bunch dynamics) as a phase shift, equal to that observed at the end of the RF Gun cavity between the first and the n^{th} harmonic field. Since the time spent by an electron inside the RF Gun cavity is given by: $\omega T = \phi - \phi_0+(N+1/2)\pi$ (ϕ being the exit phase, as usual), the phase shift is simply given by: $\delta n \cdot \omega T$. For a 1+1/2 cell Gun (ωT is about 2π) the phase error can be considered equal to $2\pi\delta n$. So that, on the basis of equation 16) we can say that the frequency error δn should be contained within a few per mil in order to assure an emittance reduction of one order of magnitude.

2.3 - Analytical estimation of the rms longitudinal emittance

As anticipated at the beginning of this Section, the vanishing of the second and third order terms in the energy-phase relationship strongly damps the longitudinal emittance blow up. The rms longitudinal emittance is indeed given by:

$$\varepsilon_z = \sqrt{<(\delta z)^2><(\delta p_z)^2> - <\delta z \delta p_z>^2} \qquad 17)$$

where: <> is an average all over the bunch density distribution, $\delta z = z - <z>$ and $\delta p_z = p_z - <p_z>$.

Taking p_z at the exit of the RF Gun, as given by equation 9) for the case of $<\phi> = \pi/2$ (recalling that under the straight-topping condition terms like $(\Delta\phi)^2$ and $(\Delta\phi)^3$ are vanishing), we can assume that the exit momentum can be written as $p_z = a \cdot \delta z + b \cdot (\delta z)^4$ (a and b being coefficients which depend on α, n and N). In this case, keeping $\Delta\phi = k\delta z$, equation 17) gives the result:

$$\varepsilon_z = b\sqrt{<(\delta z)^2><(\delta z)^8>} = O(\sigma_\phi^5)$$

which confirms that the rms longitudinal emittance blow up contains only terms of the order of $\sigma_\phi{}^5$. This fact greatly increases the efficiency of a magnetic compressor, added downstream the RF Gun, as shown later on in Section 4 by the results of some numerical simulations.

3 - A PRELIMINARY TOPGUN RF STRUCTURE

One of the main problems to fulfil the requirements of the straight-topping condition is to find a geometry for the RF Gun cavity able to support a n^{th} ($n = 3, 7, 11,...$) harmonic resonant mode whose frequency is just n times the resonant frequency of the $TM_{010\text{-}\pi}$ accelerating mode. Hereafter we will focus our attention on the 3^{rd} harmonic, which is surely the simplest harmonic to add. So that, $n = 3$ is assumed in the following two Sections.

It is well known that in an ideal multi-cell structure the $TM_{010\text{-}\pi}$ mode can be represented by a pure spatial first harmonic, as the first term in the r.h.s. of eq. 7). In order to produce the second term (for $n = 3$) one needs a $TM_{012\text{-}\pi}$ mode, for which each cell is $\frac{3\lambda_3}{2}$ long (where λ_3 is the wavelength of the 3^{rd} harmonic), versus a $\lambda/2$ length of each cell for the $TM_{010\text{-}\pi}$ mode ($\lambda = 3\lambda_3$ is the wavelength of the 1^{st} harmonic $TM_{010\text{-}\pi}$ mode).

Assuming that each cell of the ideal multi-cell structure can be represented by a pill-box of length $\lambda/2$, the natural ratio between the wavelength of the TM_{010} and the TM_{012} resonant modes is easy to be obtained.

In fact, if a is the radius of the pill-box cell, the resonant wavelength for the TM_{010} mode is given by $\lambda = 2.61a$ ($2.61 = 2\pi/p_{01}$, p_{01} being the first root of the zero-order Bessel function J_0), while the synchronism condition fix the cell length, d, to one half of the wavelength, hence $d = 1.305a$. The resonant wavelength of the TM_{012} mode is given by:

$$\lambda_3 = \sqrt{\frac{1}{\frac{1}{d^2} + \frac{1}{(2.61a)^2}}}$$

hence λ_3 comes out to be $\lambda_3 = \lambda/\sqrt{5} = 0.45\lambda$. The natural ratio is therefore far from the required value of 1/3.

Coming back to a real structure, we consider now the standard SLAC geometry for iris-coupled cells, like the one in use for the BNL RF Gun cavity (iris curvature radius = 10 mm , iris diameter = 20 mm , cell length = 52.3 mm , cell radius = 41.6 mm). For this geometry the resonating frequency of the $TM_{012\text{-}\pi}$ mode (as computed with Superfish) is 9635 MHz, versus a $TM_{010\text{-}\pi}$ frequency of 2854 MHz. The capacitive coupling between adjacent cells produces therefore a ratio $\lambda_3/\lambda = 0.3$.

In order to study how to increase this ratio up to the required value (0.33) we followed a procedure already in use and described elsewhere [9], based on the Slater theorem on frequency shifts caused by cavity boundary perturbations. For axi-symmetric fields the theorem can be summarized in the formula:

$$\Delta\nu\,(r,z) = \frac{\pi\nu}{2U}\, r \cdot [\mu H^2(r,z) - \varepsilon E^2(r,z)] \qquad 19)$$

which gives the frequency shifts $\Delta\nu(r,z)$ produced by a removal of 1 cm^2 area from the cavity section in the (r,z) plane at the point (r,z), where a peak magnetic field amplitude H and a peak electric field amplitude E are present (U is the total energy stored in the cavity and ν is the actual resonating frequency).

A special version of Superfish[9] has been used, able to produce equi-level plots for the quantity $\Delta\nu$ given by equation 19) all over the cavity section: this is an useful tool to understand at a glance what is the frequency shift sensitivity against boundary perturbations of different regions in the cavity. .

The application of such a procedure to our problem is shown in Fig.6, where two half cells of the BNL geometry are shown with equi-level plots of $\Delta\nu$ [MHz/cm^2]. The plot concerning the $TM_{010\text{-}\pi}$ mode is drawn in the right half cell, while in the left cell the lines $\Delta\nu$=const. are plotted for the $TM_{012\text{-}\pi}$ mode. The number marked on the lines give the corresponding value of $\Delta\nu$: it can be clearly seen what regions of the cell must be changed in order to decrease the $TM_{012\text{-}\pi}$ frequency leaving unchanged the frequency of the $TM_{010\text{-}\pi}$ mode. In fact, a reduction of the iris curvature radius can give a substantial reduction of the $TM_{012\text{-}\pi}$ frequency (the sign of the shift is in fact positive in that region, meaning that a cavity area removal would increase the frequency). On the other side, the reduction of iris radius should leave, at first order, unchanged the $TM_{010\text{-}\pi}$ frequency because of the counter-balancing positive and negative contributions produced by a section increase in that cavity region.

The new iris profile, sketched in Fig.6 by a dashed line, should give the required result in term of the needed frequency values. Indeed, after a few iterating tries, we found that an iris curvature radius decreased down to 7.5 mm is able to support a $TM_{012\text{-}\pi}$ mode which is actually a 3rd harmonic of the $TM_{010\text{-}\pi}$ mode.

The preliminary geometry for the TOPGUN cavity is sketched in Fig.7: the boundary of the 1+1/2 cell cavity is shown together with the $r \cdot H_\phi$ = const. line pattern, both for the $TM_{010\text{-}\pi}$ mode (whose frequency has been slightly shifted down to 2817 MHz) and for the $TM_{012\text{-}\pi}$ mode (which has become a 3rd harmonic of the main accelerating $TM_{010\text{-}\pi}$ mode) with a resonant frequency of 8464 MHz (i.e. within 1 part per mil). The cell radius R_c and the iris aperture R_i have been kept unchanged with respect to the BNL geometry (i.e. 41.6 mm and 10 mm respectively).

The shunt impedance r_s for the $TM_{010\text{-}\pi}$ mode is unaffected by the geometry modification - the shunt impedance is still r_s = 42 MΩ/m, corresponding to a 5.6 MW power consuption for 4.3 MeV energy gain, 100 MV/m being the peak field at the cathode surface - while the same quantity for the 3rd harmonic $TM_{012\text{-}\pi}$ mode is r_{s3} = 2.3 MΩ/m. Using a 3rd harmonic field amplitude E_3 given by the straight-topping rule (E_3=E_0/9), the power dissipated by the $TM_{012\text{-}p}$ mode onto the cavity surface is about 22% of the $TM_{010\text{-}\pi}$ power consumption (recalling that the power consuption scales like the square of the field divided by the shunt impedance).

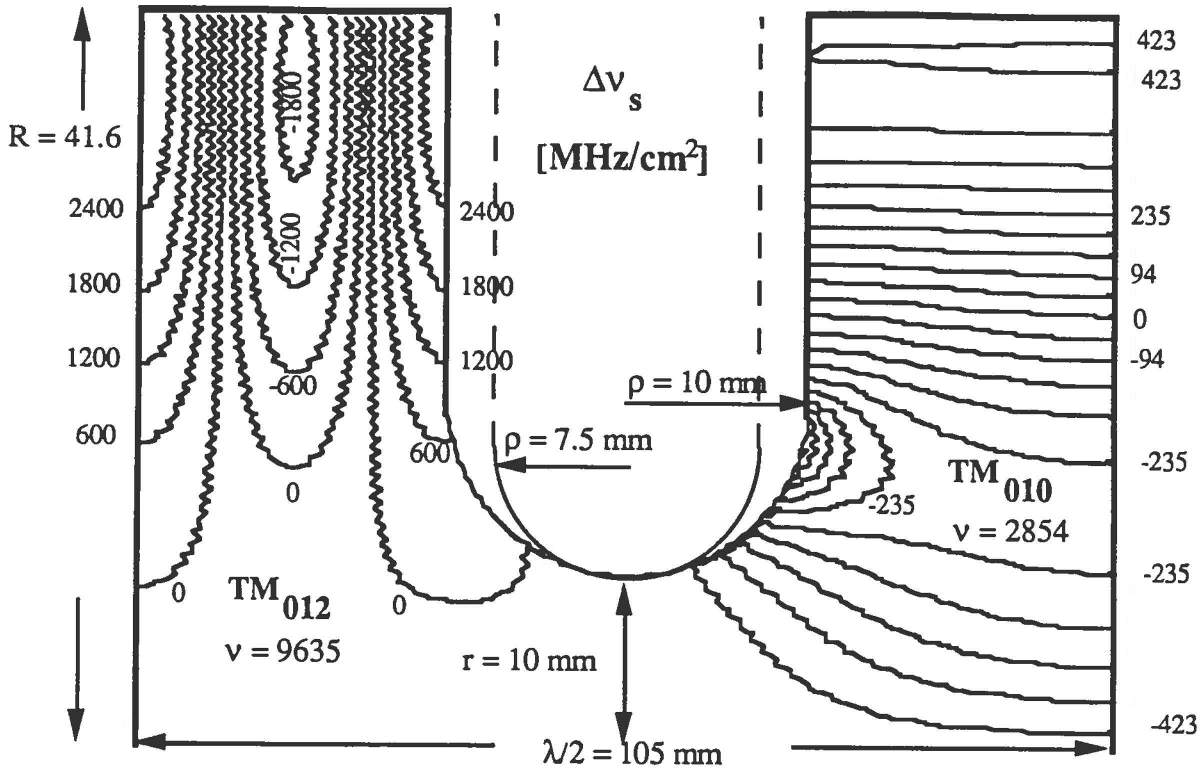

Fig. 6 - Equi-level plots for the frequency shift sensitivity Δν [MHz/cm^2] against cavity boundary perturbations for the two resonant $TM_{010\text{-}\pi}$ (on the right) and $TM_{012\text{-}\pi}$ (on the left) modes in the cell of the BNL type geometry.

The drawing plotted in Fig.7 must be regarded as corresponding to the (r,z) section of an axi-symmetric structure up to the radius R_c, the maximum radius of the Gun cavity, while the waveguide sections plotted above and below the cavity must be regarded as the (x,y) sections of two rectangular waveguides tangentially connected to the RF Gun cavity itself. The propagation z-axis of the two waveguides is therefore perpendicular to the drawing plane: hence, the longitudinal H_z component of the magnetic field of a TE_{n0} mode, propagating inside the waveguide, can couple efficiently to the azimuthal H_ϕ component of the axi-symmetric field of a TM_{0np} resonant mode in the cavity, through the indicated slots on the cavity to waveguide connections. Such a coupling procedure has been tested and successfully used at the BNL RF Gun[10].

We propose here a two waveguide scheme allowing to couple separately the two resonant modes in the Gun cavity. It is based on a second waveguide propagating a TE_{40} mode, whose H_z field component has the right distribution along the x-axis in order to couple, via five slots, with the $TM_{012\text{-}\pi}$ mode in the cavity. The two slots of the TE_{10} waveguide must be positioned around the nodes of the $TM_{012\text{-}\pi}$ mode, in order to minimize the coupling to this mode, and the five slots of the TE_{40} waveguide must be dimensioned in order to give a net zero power flux onto the $TM_{010\text{-}\pi}$ mode. To assure this condition the central one of the three slots in the second cell must be double-sized with respect to the side slots. In fact, the H_ϕ field of the $TM_{010\text{-}\pi}$ mode is actually constant versus z on the outer surface of the cavity (at $r=R_c$) so that the opposite fluxes from each one of the two slots in the first cell are exactly counter-balancing, while the sum of the fluxes from the two side slots of the second cell are just opposite with respect to the flux through the central slot.

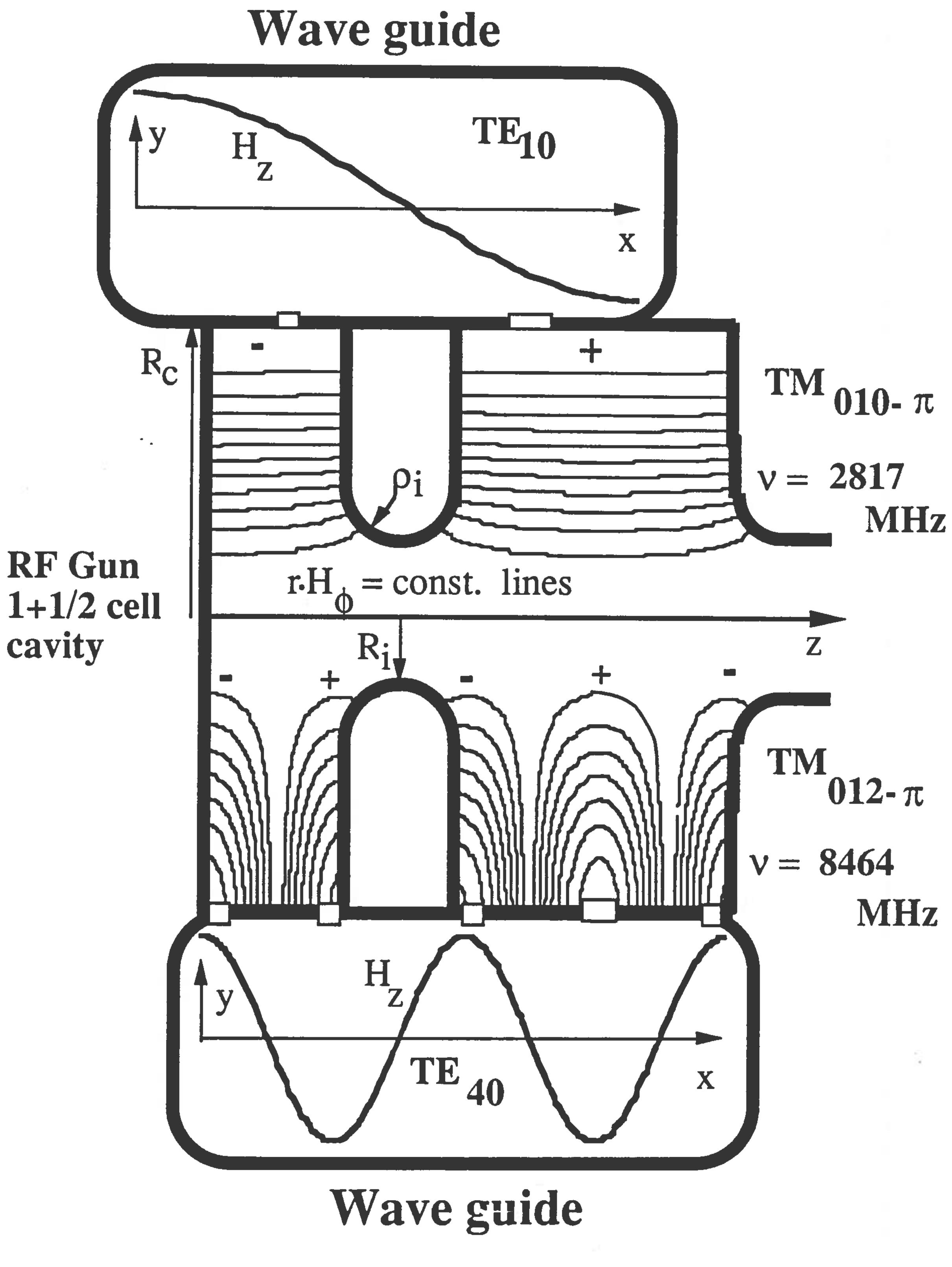

Fig.7 - TOP-Gun structure scheme. The field pattern of the main accelerating $TM_{010\text{-}\pi}$ mode is plotted together with the field lines of the straight-topping 3rd harmonic $TM_{012\text{-}\pi}$ mode. A scheme of principle for the coupling of the two resonant modes via two rectangular waveguides is also drawn.

A real operating solution should be found via a proper positioning of the two waveguides and by an optimization of the two vertical y-size of the waveguides, which are actually free parameters (the TE_{10} and TE_{40} cutoff frequencies are indeed determined only by the horizontal x-size).

Since the cut-off wavelength for the TE_{40} mode is equal to a/2 (being a the horizontal x-size of the waveguide) one can choose the size a just equal to the cavity length, which is 9/4 of the $TM_{012\text{-}\pi}$ wavelength: in such a way the peaks of the TE_{40} H_z distribution along x match the corresponding maxima of the H_ϕ distribution along z in the cavity. Therefore the wavelength of the TE_{40} comes out to be 4/9 of the horizontal size a, hence just below the cut-off wavelength.

The electric field distribution on axis resulting from the superposition of the two modes is shown in Fig.8, where solid and dotted lines give the field distribution of the two single modes for peak values at z=0 (the cathode surface) of 100 MV/m for the first harmonic $TM_{010\text{-}\pi}$ mode and 11.1 MV/m for the 3rd harmonic $TM_{012\text{-}\pi}$ mode, respectively. The ratio between the two peak fields is just the one required by the theoretical straight-topping condition of equation 10').

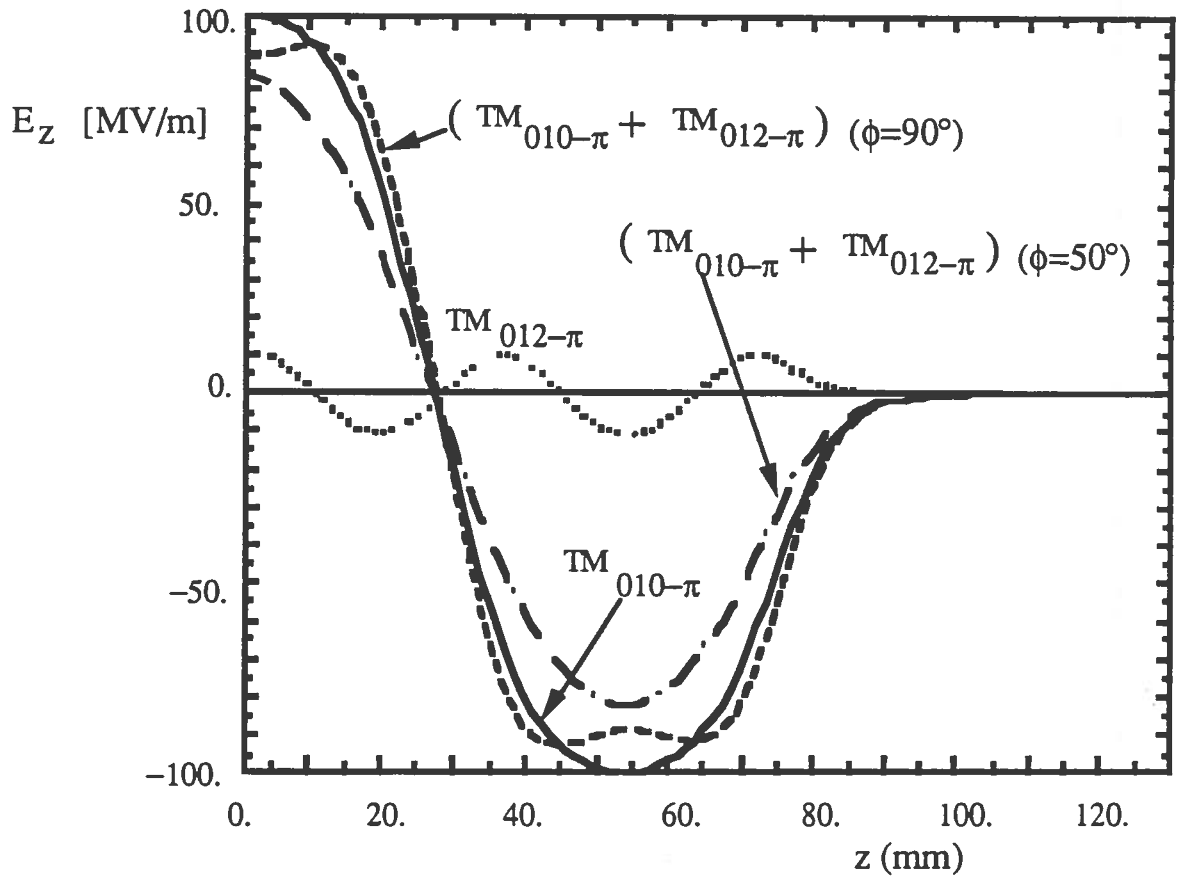

Fig.8 - Electric field distribution on axis for the case of: standard RF Gun (solid line), TOP-GUN with 3rd harmonic at phase $\phi=90°$ (dashed line), TOP-GUN with 3rd harmonic at phase $\phi=50°$ (dotted-dashed line). The 3rd harmonic field distribution on axis is shown by the dotted line.

The distribution of the total field given by the superposition of the two modes is plotted at two representative phases: the dotted-dashed line corresponds to a phase ϕ=50° RF, which is a typical value around the injection phase of cigar-like (long) bunches, while the dashed line corresponds to a phase ϕ=90° RF, which gives the peak field distribution on axis.

It must be noted that the reduction of the iris curvature radius will unavoidably increase the non linear terms in the transverse RF field components. The proposed geometry for the TOPGUN must therefore be regarded as a first tentative and preliminary geometry needed just to verify via the numerical simulations that the straight-topping condition really works in damping the RF emittance blow up, as anticipated by the analytical study.

Further optimizations are surely needed to minimize the non-linear RF components: an iris profiling according to the well known criterium[4], able at the same time to preserve the condition that the $TM_{012\text{-}\pi}$ mode must be the 3rd harmonic of the $TM_{010\text{-}\pi}$ accelerating mode, will be the subject of a further study.

4 - PRELIMINARY NUMERICAL SIMULATIONS

In order to verify the validity of the analytical study on a real structure, we used the PIC electromagnetic code ITACA to study numerically the beam dynamics in the TOPGUN structure with a straight-topping 3rd harmonic mode.

The code, developed at Milan[11], is presently in use at BNL-ATF to compare the numerical calculations to the experimental results: in the preliminary comparisons quite good agreements have been found between measured and computed data [12].

Since the interest of the TOP-GUN scheme is that to achieve the maximum benefit from the damping of the RF induced emittance blow up, we chose bunch parameters typical of cigar-like bunches - emittance blow up dominated by RF field - in order to proof the damping mechanism. A bunch rms length of 18° RF has been selected, corresponding to σ_ϕ = .15 rad (σ_z = 2.5 mm), with a bunch width σ_r = 2 mm. The typical ATF value for the electric peak field at the cathode has been taken, namely E_0 = 100 MV/m. From the analytical straight-topping condition, given by eq. 10'), the choice E_3 = 11.11 MV/m follows.

First of all, we computed the emittance blow up in absence of 3rd harmonic field (i.e. E_3 = 0.): the solid line plotted in fig.9 gives the rms normalized emittance at the TOP-GUN exit as a function of the injection phase ϕ_0. Since the energy of the emerging photo-electrons at the cathode surface has been taken zero the emittance at the cathode surface is negligible, hence the exit emittance gives just the emittance blow up due to RF field if the space-charge field contribution is not taken into account. The code has a special option to switch off the space charge field, allowing to study separately the effect of the RF field on the beam dynamics.

The injection phase ϕ_0.must be considered here as the time (in unit of RF degrees) when the laser pulse head strikes the cathode surface.

Since the time gaussian laser pulse distribution is described in the numerical simulations between $-3\sigma_\phi$ and $+3\sigma_\phi$ around the peak of the laser pulse, one must add just $3\sigma_\phi$.(i.e. 27° RF) to ϕ_0 in order to find the average injection phase $<\phi_0>$ which is referred to in the analytical study (equation 8' and fig.4b). In the present case the minimum emittance has been found at ϕ_0=38° RF, which corresponds to $<\phi_0>$ = 65° RF. The corresponding exit average phase $<\phi>$ comes out to be 103°, to be compared to 90° analytically calculated. It is however well known[1] that the analytically predicted value slightly underestimates the exit phase as a function of the injection phase.

Applying a 3rd harmonic field of different amplitudes around the prescribed level $E_3 = E_0/9$, a strong damping of the emittance blow up is obtained, as shown by the three curves plotted in fig.9 (curve "a" E_3 = 11.1 MV/m , curve "b" E_3 = 13 MV/m , curve "c" E_3 = 14 MV/m). The injection phase ϕ_3 of the 3rd harmonic field has been varied, according to the analytical rule: $\phi_3 = 3\phi_0$.

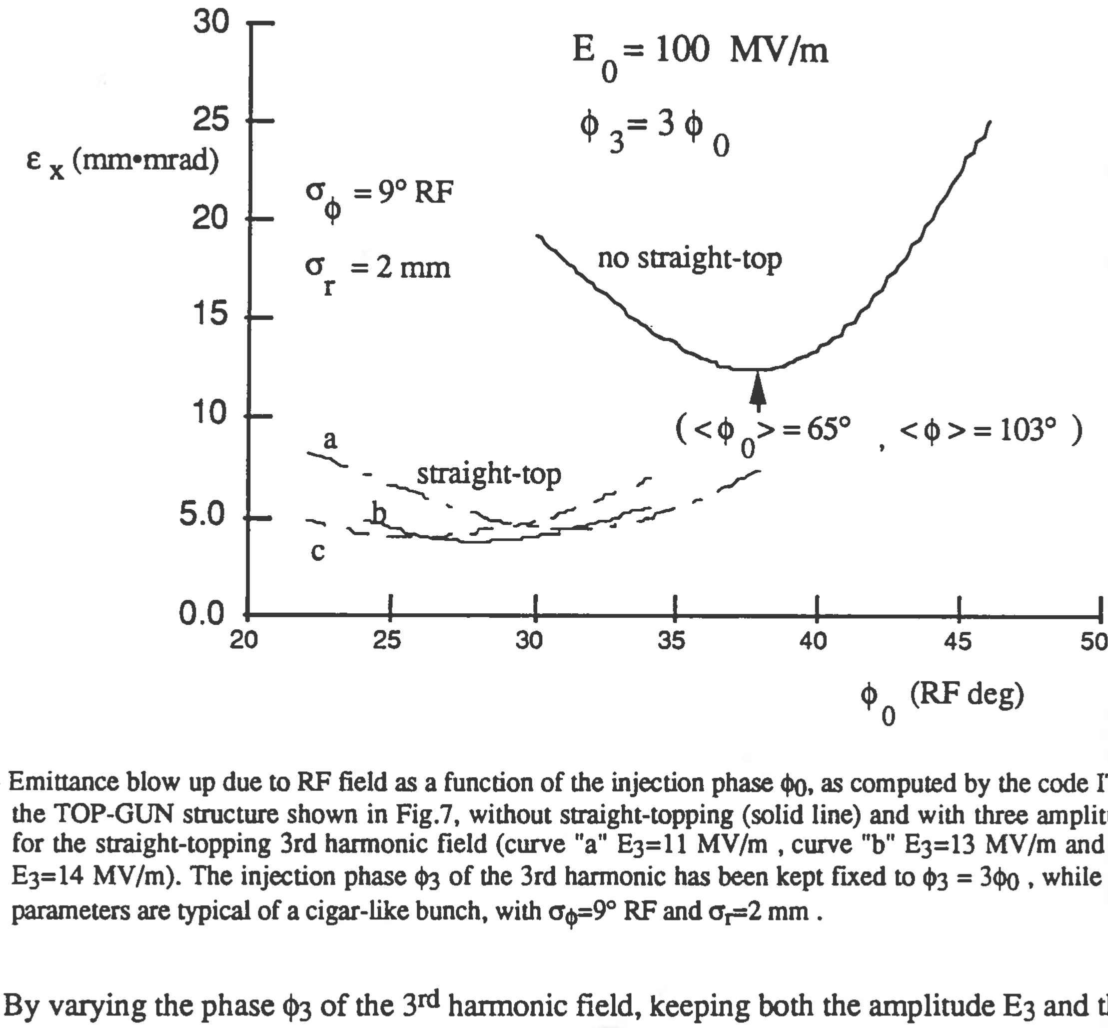

Fig.9 - Emittance blow up due to RF field as a function of the injection phase ϕ_0, as computed by the code ITACA for the TOP-GUN structure shown in Fig.7, without straight-topping (solid line) and with three amplitude levels for the straight-topping 3rd harmonic field (curve "a" E_3=11 MV/m , curve "b" E_3=13 MV/m and curve "c" E_3=14 MV/m). The injection phase ϕ_3 of the 3rd harmonic has been kept fixed to $\phi_3 = 3\phi_0$, while the bunch parameters are typical of a cigar-like bunch, with σ_ϕ=9° RF and σ_r=2 mm .

By varying the phase ϕ_3 of the 3rd harmonic field, keeping both the amplitude E_3 and the phase ϕ_0 of the first harmonic field fixed at values close to the minima of the curve "b" and "c" of Fig.9, we obtain the curves plotted in Fig.10, where the behaviour of the rms normalized emittance versus the 3rd harmonic field ϕ_3 is reported. Still lower values for the emittance are found by properly adjusting ϕ_3, as indicated by the square dots on the lines, which correspond to the theoretical rule $\phi_3 = 3\phi_0$.

Such preliminary results confirm the possibility to damp down the emittance blow up due to RF field by nearly one order of magnitude: an absolute minimum at 2 mm·mrad, against 13 mm·mrad of the case without straight-topping, is indeed achieved. Moreover, the emittance variation is less sensitive against the phase ϕ_0 of the main accelerating field when the straight-topping scheme is in use, as anticipated by the analytical study. The same holds for the amplitude E_3 of the 3rd harmonic field, whose variations does not produce significant changes on the exit emittance.

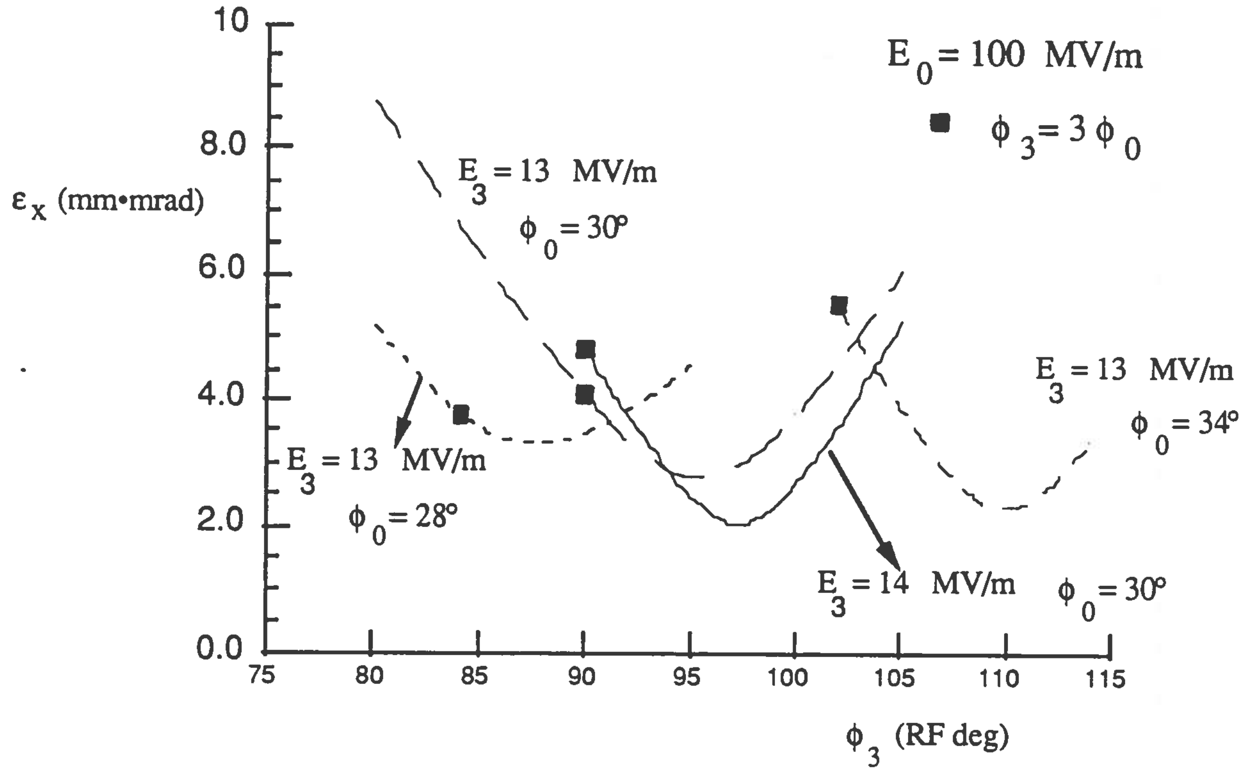

Fig.10 - Emittance blow up due to RF field as a function of the 3rd harmonic injection phase ϕ_3 with the same input bunch parameters as for Fig.9. Two different levels of the 3rd harmonic field amplitude (13 and 14 MV/m) have been taken, with the indicated injection phase ϕ_0 values for the $TM_{010\text{-}\pi}$ field. The square dots mark the values of ϕ_3 which satisfy the $\phi_3 = 3\phi_0$ rule.

The residual rms normalized emittance (2 mm·mrad) is partially due to the contribution coming from the non linear transverse components in the RF field, as can be inferred from Fig.11, where the behaviour of the rms normalized emittance ε_x versus the average <z> position of the bunch is plotted (dashed line) together with the rms normalized emittance ε_{cs} associated to a central slice of the bunch (centered around <z>), carrying 10% of the bunch charge. The emittance ε_{cs} is therefore much less sensitive to the linear RF field contribution, as can be seen in Fig.11: the strong modulations displaied by the ε_x curve (caused by the alternate opening and closing of the fan-like distribution in the transverse phase space) are in fact much less pronounced in the ε_{cs} curve. The quantity ε_{cs} gives therefore at the exit an estimation of the contribution coming from the non linear terms in the RF field transverse components: in the upper diagram, where no straight-topping has been applied, the difference between ε_x and ε_{cs} is much larger than in the lower diagram, with the straight-topping in use, indicating that the straight-topping operation really damps down the emittance blow up due to the linear RF field.

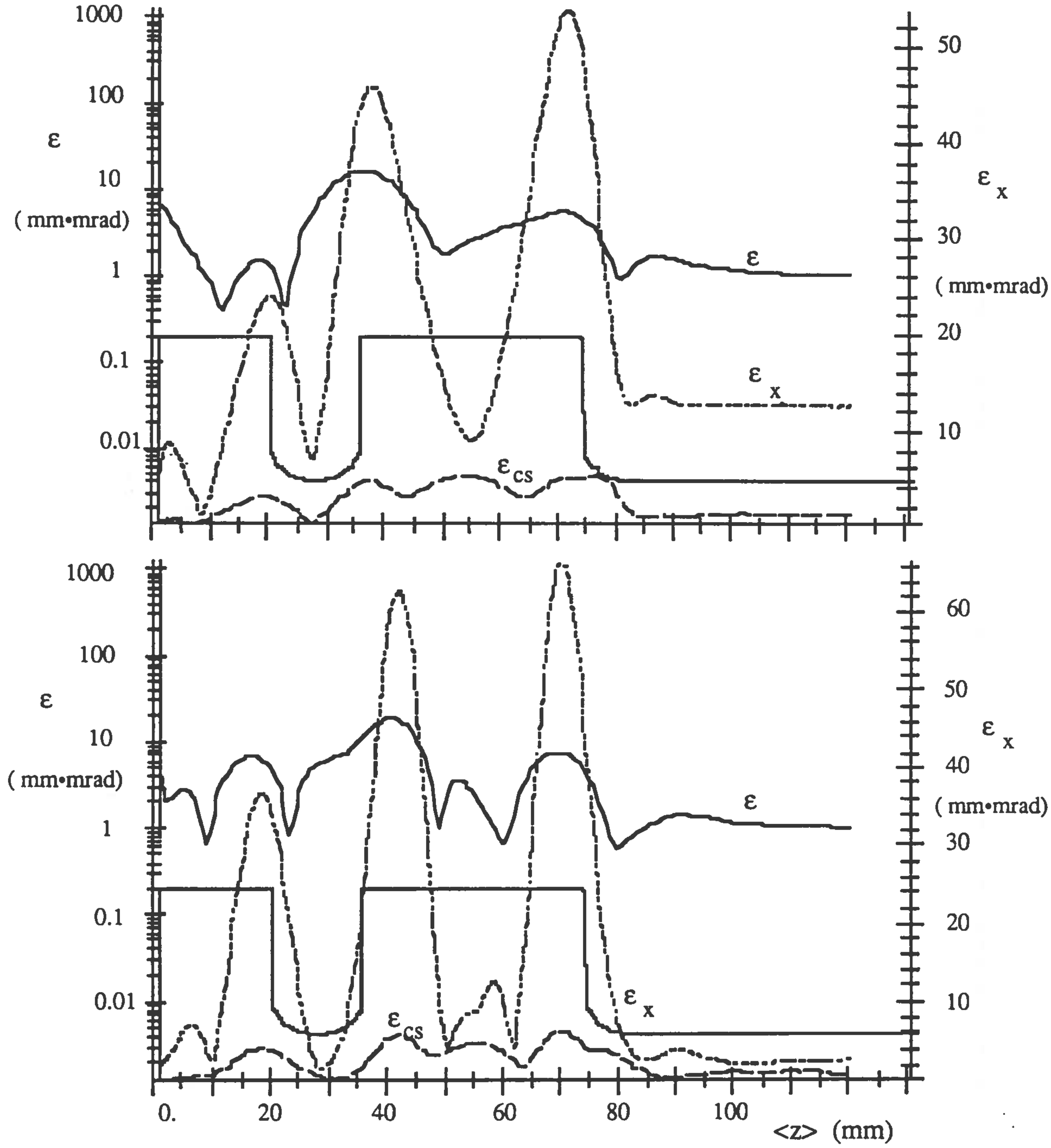

Fig.11 - Transverse emittance as a function of the bunch average position <z> plotted along the acceleration into the preliminary TOPGUN structure: without the 3rd harmonic field (upper diagram) and with a straight-topping 3rd harmonic (lower diagram) of amplitude $E_3 = 14$ MV/m and phase $\phi_3 = 95°$ (the 1st harmonic phase is $\phi_0=30°$, while in the upper diagram $\phi_0 = 38°$). The dashed line gives the normalized rms emittance ε_x, the solid line gives the transverse emittance ε, while the long-dashed line at the bottom of each diagram gives the normalized emittance ε_{cs} associated to a thin slice of the bunch centered around <z> and carrying.about 10% of the total bunch charge.

In order to give a better estimation of the contribution coming from the non linear RF field components we calculated the emittance at the TOPGUN exit without straight-topping for a fixed ϕ_0, namely ϕ_0=30°, as a function of the laser pulse length. This is plotted in Fig.12 versus the bunch rms length σ_ϕ: in absence of non linear contributions this emittance should scale like σ_ϕ (see equation 5).

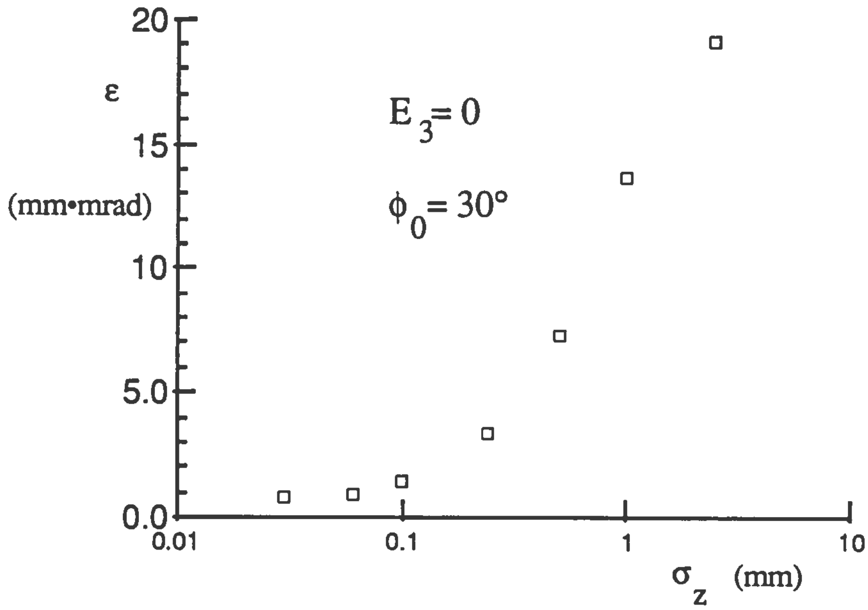

Fig.12 - Normalized transverse emittance ε as a function of σ_z for the case of a TOPGUN operated without the 3rd harmonic straight-topping, with an injection phase ϕ_0 = 30° and peak field E_0 = 100 MV/m (σ_r=2 mm).

The saturation of the emittance shown in Fig. 12 at the level of about 0.9 mm·mrad for very short bunches ($\sigma_z < 0.1$ mm , i.e. $\sigma_\phi < 0.3°$ RF) can be ascribed at the left contribution coming from the non linear RF field transverse components, once the linear contribution (scaling like σ_ϕ) has become negligible.

This suggests that, using an optimized geometry for the TOPGUN structure, one should achieve a residual rms normalized transverse emittance below 1 mm·mrad also for long cigar-like bunches (without space charge contribution, whose effect is shown at the end of the Section).

The transverse phase space distributions, plotted in Fig.13a and 13b, display quite well the mechanism of the emittance blow up neutralization. In fact, it can be easily seen that the fan aperture of the (r,p_r) distribution shown in the upper diagram of Fig.13b - corresponding to a straight-topping operation with E_3 = 14 MV/m, ϕ_3 = 95° and ϕ_0 = 30° - is strongly reduced with respect to the one shown in the upper diagram of Fig.13a - representing the (r,p_r) distribution at the TOPGUN exit without 3rd harmonic field in use.

To better enlighten the difference between the two phase space distributions, two transformations have been applied which give rise to the distributions plotted in the diagrams at the bottom of both figures.

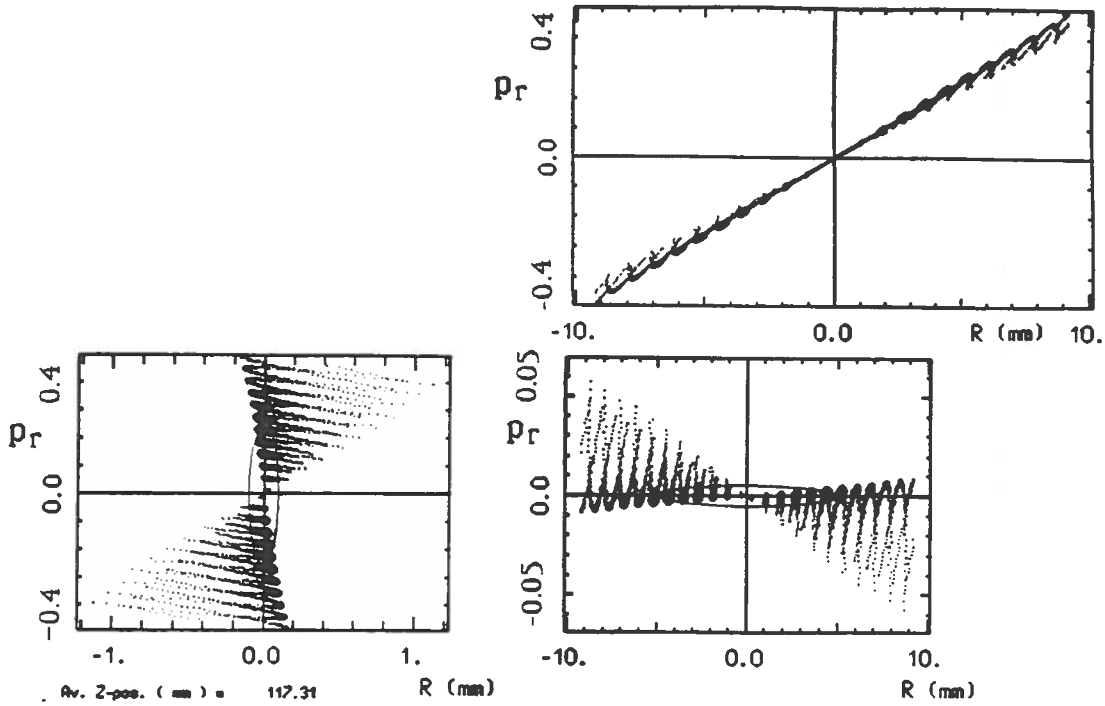

Fig. 13a - Transverse phase space distribution (r,p_r) (p_r in unit of mc), at the TOP-GUN exit, obtained without straight-topping (ϕ_0=38°). Bunch parameters are σ_ϕ=.15 rad and σ_r=2mm .

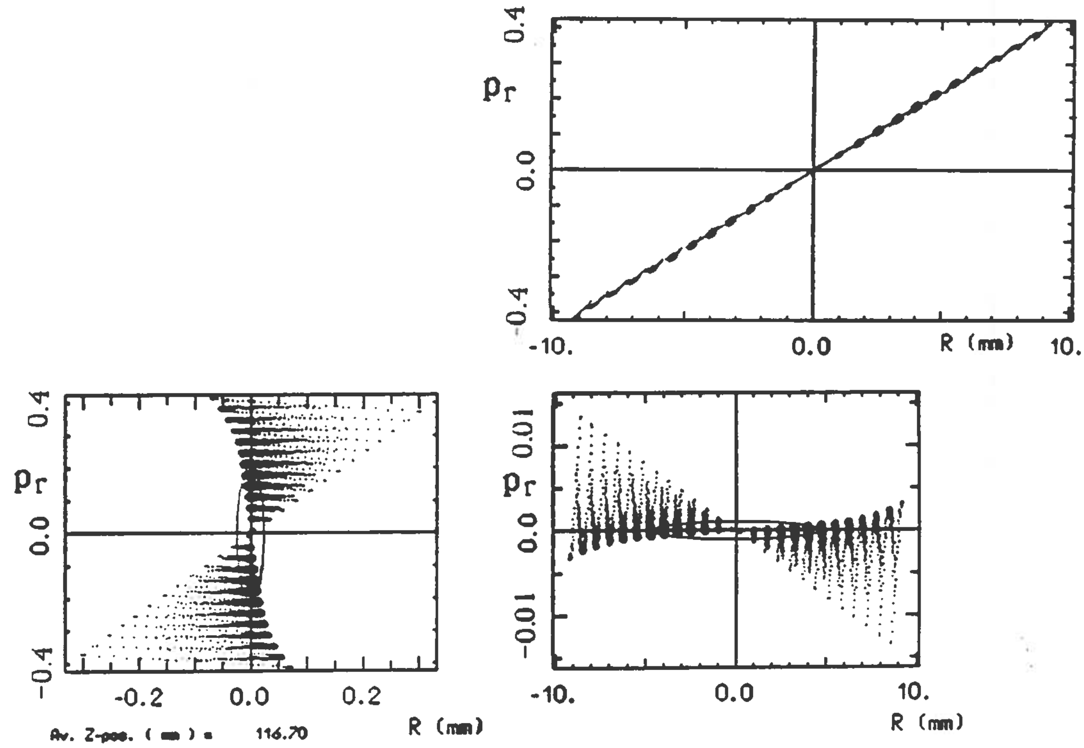

Fig. 13b - Transverse phase space distribution (r,p_r) (p_r in unit of mc), at the TOP-GUN exit, obtained with a 3rd harmonic field applied (E_3=14 MV/m ϕ_3=95° and ϕ_0=30°).

The distribution on the left side has been obtained transforming the "r" phase space variable into the new "r_f", as given by

$$r_f = r - \frac{p_r \cdot \langle r p_r \rangle}{\langle p_r^2 \rangle}$$

which transforms the correlated ($<rp_r> \neq 0$) distribution into an uncorrelated one ($<r_f p_r> = 0$), minimizing the rms beam transverse size $\sqrt{<r_f^2>}$. This is now given by:

$$\sqrt{<r_f^2>} = \varepsilon/\sqrt{<p_r^2>} \, .$$

Such a virtual transformation should give the phase space distribution at the beam focus (located around the average z-position $<z_f> = <z> - <\beta_z><rp_r>/<p_r^2/\gamma>$) in absence of energy spread (the real transformation would be $r_f = r - \frac{p_r \cdot <rp_r>}{\gamma <p_r^2/\gamma>}$) and is useful to better visualize the fan aperture by removing the large correlation term $<rp_r>$ from the original distribution.

The distribution on the right side has been obtained transforming the "p_r" phase space variable into the new "p_{rf}", as given by

$$p_{rf} = p_r - \frac{r \cdot <rp_r>}{<r^2>}$$

which transforms the correlated ($<rp_r> \neq 0$) distribution into an uncorrelated one ($<rp_{rf}> = 0$), minimizing the rms beam transverse momentum $\sqrt{<p_{rf}^2>}$. This is now given by:

$$\sqrt{<p_{rf}^2>} = \varepsilon/\sqrt{<r^2>} \, .$$

Such a transformation gives also a phase space distribution which better displays the momentum-phase correlation effect, in terms of the fan aperture: the rms beam transverse size remains indeed unchanged (@ R_{rms} = 4.9 mm for Fig.13a and @ R_{rms} = 4.66 mm for Fig.13b) but the rms transverse momentum (expressed in unit of mc) decreases down to p_{rrms} = .005 for Fig.13a and p_{rrms} = .001 for Fig.13b.

The difference is actually due to the neutralization of the momentum-phase correlation produced by the straight-topping operation: the fan in Fig.13b is so closed that the filamentation effect caused by non linear RF components gives a visible curvature in the distribution (not visible in the lower right-side diagram of Fig.13a) which becomes an important source of emittance, as shown by the plotted rms ellipse contour *.

This confirms again that the non linear RF components do not allow to reduce further the emittance blow up and an optimized geometry must be used to reduce their contributions if one wants to achieve the maximum benefit from the straight-topping operation of the TOPGUN.

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*Since the code uses actually variable charged particles for the simulation, the gaussian charge density distribution of the bunch is produced with uniformly distributed particles (in the (r,z) bunch section) with a particle charge distribution which is gaussian in the longitudinal z-position inside the bunch. In the figure, the charge of each particle is graphically represented by assigning a variable size dot at each particle - the size of the dot scaling like the charge of the particle. It can be seen that the particle lying in the core of the bunch have actually the largest dots: the emittance is largely due to the fan aperture associated to such particles, which is rather small compared to the curvature of the distribution. The residual fan aperture associated to the particles lying in the head or in the tail of the bunch (which are represented by small-size dots) has a negligible effect on the emittance since its contribution to the average quantities $<r^2>$, $<rp_r>$, etc is very low.

The effect of the straight-topping operation onto the longitudinal emittance is also largely positive: a strong damping of the emittance blow up is here also achieved, as shown in Fig.14, where the longitudinal emittance ε_z is plotted (solid line) as a function of the average bunch position $<z>$ along the acceleration through the TOPGUN. The upper diagram corresponds to an acceleration without straight-topping, while the lower diagram with the straight-topping in use (same bunch and field parameters of Fig.11, i.e. $\sigma_r = 2$ mm, $\sigma_\phi = .15$ rad, $E_3 = 14$ MV/m, $\phi_0 = 30°$, $\phi_3 = 97°$).

The longitudinal emittance at the exit is reduced by a factor 5.

Such a large reduction of ε_z is largely due to the minimisation of the non linear terms in the energy gain versus phase relationship. This is clearly shown by the longitudinal phase space distributions, plotted in Fig.15 for the two cases: without straight-topping, upper diagram, and with the straight-topping 3rd harmonic field applied, lower diagram. The bunch and fields parameters are again the same ones of Figs. 13a and 13b. It is really evident how the two phase space distributions follow strictly the two energy-phase relationships plotted in Fig. 4a (except for a mirror-like reflection due to the definition of the phase $\phi = \omega t - kz + \phi_0$, which has just an opposite sign with respect to the phase space variable z). It must be noted the slight difference between the exit average energy achieved without straight topping (4.37 MeV for $\phi_0 = 38°$) and the same quantity in presence of 3rd harmonic field (3.96 MeV for $\phi_0 = 30°$ and $\phi_3 = 95°$).

The main effect of the energy gain curve linearisation is a much larger efficiency of a possible magnetic compression applied onto the beam downstream the Gun output.

Applying to the distributions of Fig.15 the transformation:

$$\delta z_c = \delta z - \frac{\delta p_z <\delta_z \delta p_z>}{<\delta p_z^2>}$$

where $\delta z = z - <z>$ and $\delta p_z = p_z - <p_z>$ are the position and momentum deviations in the longitudinal phase space, we obtain the distributions plotted in Fig.16. Such a transformation produces an uncorrelated ($<\delta z \delta p_z> = 0$) distribution in the longitudinal phase space which minimizes the rms bunch length $<\delta z^2>$ and can be regarded as the first order approximation of a magnetic compression applied onto a relativistic ($<\beta_z> \approx 1$) beam.

While the two distributions in Fig.15 have fairly the same peak current (40 A), since the rms bunch length is nearly the same, after compression the distribution in the lower diagram (with straight-topping) exhibits a 436 A peak current against a 73 A peak current of the upper diagram (without straight-topping). These currents must be regarded as the maxima peak current values achievable via a magnetic compression.

Since the normalized beam brightness scales like the peak current divided by the square of the normalized rms transverse emittance, and taking into account that the emittance of the beam in the upper diagram is 13 mm·mrad while the one of the lower diagram is 2 mm·mrad, the beam brightness is enlarged, via the exploitation of the straight-topping 3rd harmonic, by a factor 250!

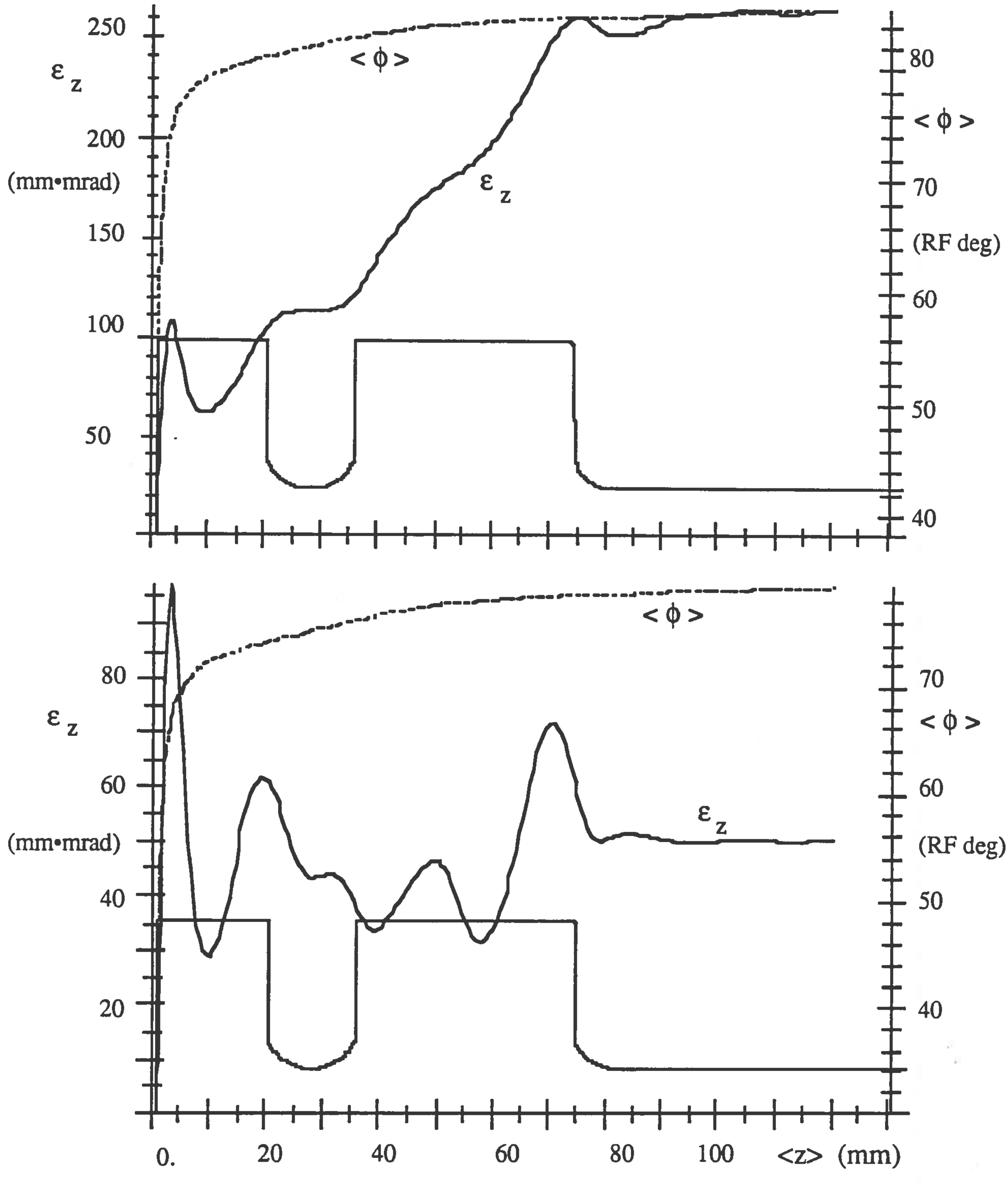

Fig.14 - Longitudinal rms emittance a a function of the average <z> position of the bunch plotted along the bunch acceleration through the TOP-GUN structure (solid line) without the 3rd harmonic field (upper diagram) and with the 3rd harmonic field (lower diagram), for the same bunch and field parameters as for Figs. 13a and 13b. The dashed line gives the average bunch phase $<\phi>$. For the upper diagram the injection phase is 38° while the exit phase 85°; for the lower diagram the injection phase is 30° while the exit phase is 79° .

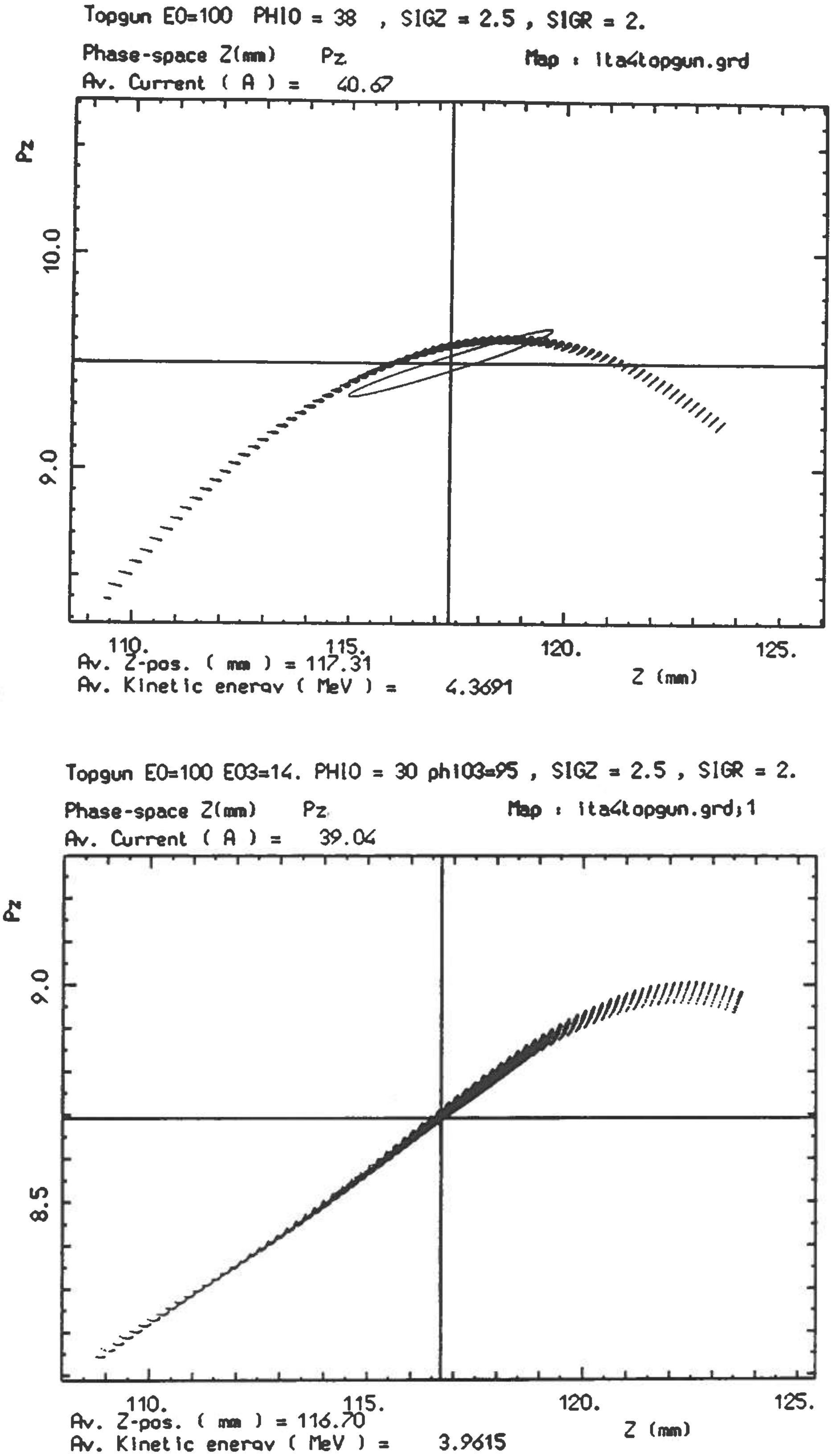

Fig.15 - Longitudinal phase space distributions (z,p_z) (p_z in unit of mc) at the TOPGUN exit for the two cases: without straight-topping (upper diagram) and with the 3rd harmonic field applied (lower diagram). The plotted ellipses correspond to the rms ellipses of the distributions (in the lower diagram the ellipse contour is scarcely visible). Bunch and field parameters are those of Figs. 13a and 13b.

A final simulation for the TOP-GUN has been carried out via a fully self-consistent electro-magnetic calculation, able to take into account also the electro-magnetic self-field produced by the bunch current and charge density distribution.

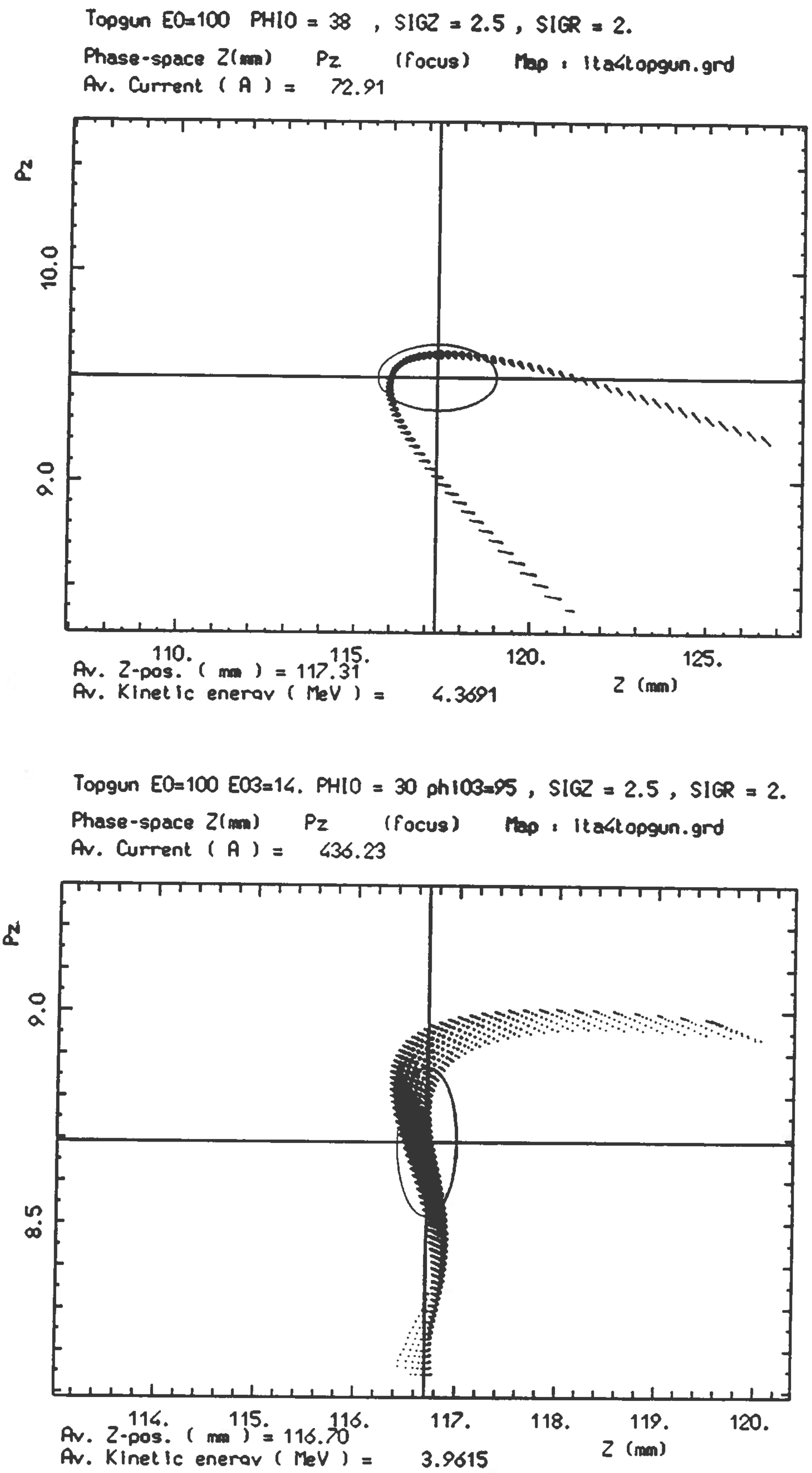

Fig.16 - Longitudinal phase space distributions (z,p_z) (p_z in unit of mc) after a magnetic compression has been applied downstream the TOPGUN output. Upper diagram: without straight-topping. Lower diagram: with straight-topping. Bunch and field parameters as for Figs. 13a and 13b. Rms ellipses are plotted for both distributions.

The bunch and field parameters are again the same as for previous calculations, but we left free to vary the phase ϕ_3 of the 3rd harmonic field: the resulting rms normalized emittance is plotted in Fig.17 as a function of ϕ_3.

The space charge contribution rises the minimum emittance up to 3.5 mm·mrad and shifts slightly the minimum phase, from 97° to 101°. This shift should be due to the correlation effect between space charge field and RF field. Because of the long cpu-time required by e.m. calculations no systematic search has been performed on the three free parameters, namely E_3, ϕ_3 and ϕ_0. Further study on this topic, both analytically and numerically performed, will be the subject of a next paper.

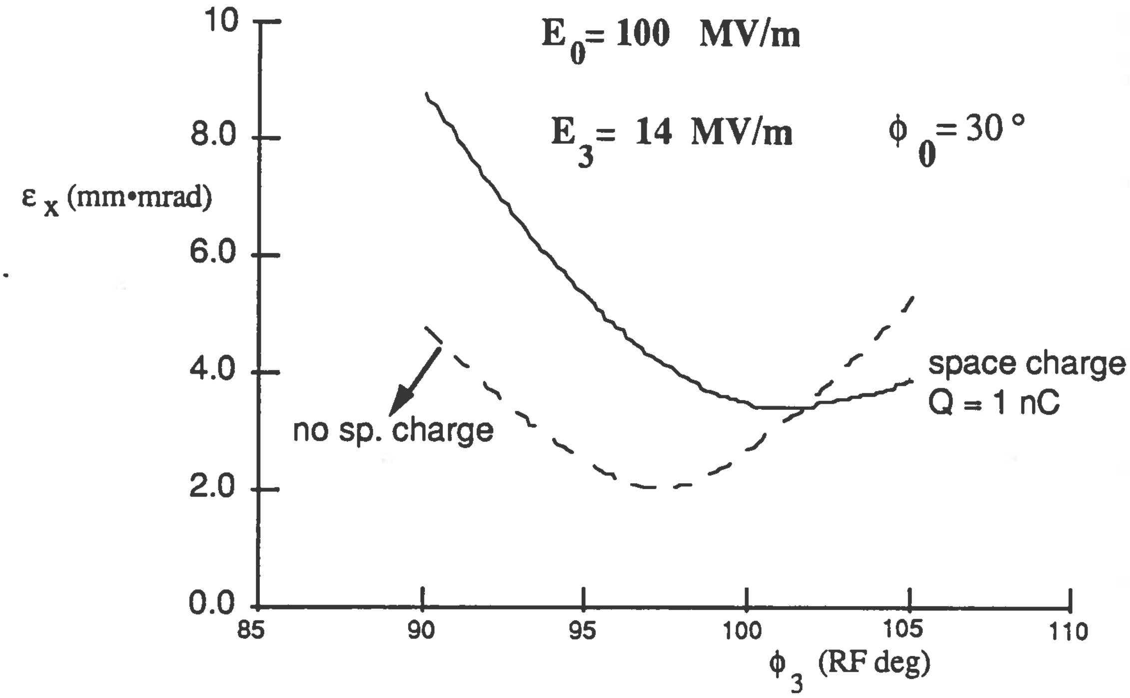

Fig.17 - Normalized rms emittance at the exit of the TOPGUN structure for a 1 nC bunch as a function of the 3rd harmonic phase ϕ_3 (solid line), compared to the emittance in absence of space-charge field (dashed line).

The starting energy of the photo-electrons has been taken .05 eV, as produced by a Copper cathode illuminated by a quadrupled Nd-Yag laser: the solid curve in Fig.17 takes therefore into account also the contribution of the uncorrelated "temperature" emittance at the cathode surface (not considered for the RF calculation reported by the dashed curve).

The final output beam characteristics are summarized in Table I, where they are compared to the corresponding quantities referred to the computed results for a typical cigar-like bunch produced by the BNL-ATF GUN.

It must be stressed that the longitudinal emittance does not suffer any significant degradation caused by the space charge forces, because of the low charge density in the long cigar-like bunch. The maximum peak current after compression is therefore unaltered.

The reported beam brightness values are computed considering a 200 A peak current after compression for both cases. Higher currents would indeed cause a large emittance degradation in the compression process performed at 4 MeV: a higher beam energy is required to compress further without a significant emittance degradation[13].

TABLE I

		TOPGUN Cigar-like bunch	BNL-ATF GUN Disk-like bunch
Peak cathode field E_0	[MV/m]	100	100
av. injection phase $<\phi_0>$	[RF deg]	57	60
3rd harm. field E_3	[MV/m]	14	-
3rd harm. phase ϕ_3	[RF deg]	100	-
Bunch charge	[nC]	1	1
Laser pulse length σ_t	[ps]	8.3	2
Laser width σ_r	[mm]	2	3
Output energy	[MeV]	3.93	4.17
Rms Energy spread	[%]	±1.8	±0.5
Output charge	[nC]	1	0.89
Output rms radius σ_r	[mm]	4.1	4.1
Rms bunch length σ_z	[mm]	2.5	0.6
Cathode norm. emittance	[m·rad]	$4.3 \cdot 10^{-7}$	$6.5 \cdot 10^{-7}$
Exit rms norm. emitt. ε_x	[m·rad]	$3.5 \cdot 10^{-6}$	$6. \cdot 10^{-6}$
Rms longit. emitt. ε_z	[m·rad]	$3.9 \cdot 10^{-5}$	$4.4 \cdot 10^{-6}$
Peak current	[A]	45	170
Max. Compressed peak curr.	[A]	460	850
Norm. brightness* B_n	[A/m²·rad²]	$2.1 \cdot 10^{11}$	$7. \cdot 10^{10}$

* @ I = 200 A ($B_n = 2I/(4\pi\varepsilon_x)^2$)

Finally, we would like to stress that the benefit of the straight-topping operation could be in principle exploited also in the domain of disk-like bunches using higher cathode peak-fields in order to damp down the emittance blow up due to space charge fields, without increasing RF field emittance.

5 - CONCLUSIONS

The new proposed operation of a RF Gun with a multi-mode structure looks very promising in improving the output beam quality.

A preliminary structure for a TOPGUN shows the possibility to improve by large factors the beam brightness.

Further studies are needed to investigate a possible damping mechanism, based on a straight-topping operation, for the linear correlated space charge contribution to the emittance blow up .

Finally a possible unique feature of the TOPGUN for the generation of a bunch train with micro-repetition rate at very high frequency is sketched in Appendix 2..

ACKNOWLEDGMENTS

We thank Todd I. Smith for clarifying discussions and really enlightening suggestions on emittance effects of higher harmonic modes in RF cavities.

APPENDIX 1

We discuss here a very simple way to compute the exit phase ϕ and final energy γ_f of an electron accelerated inside a RF Gun: we call it "impulsive approximation". It is just equivalent to Kim's approximation, but a different approach is followed.

Assuming a strong electric field amplitude E_0 applied at the cathode surface, the photo-electrons emerging from the cathode will be quickly accelerated, reaching in a short time (and within a small distance just nearby the cathode surface) $\beta \approx 1$. One can approximate the RF resonant field, over this time-interval, with a constant electric field of amplitude $E = E_0 \sin\phi_0$.

It is well known that a particle being accelerated from rest by a uniform field (z-oriented) reaches asymptotically a motion described by:

$$z \approx ct - \Delta \qquad \text{with} \quad \Delta = \frac{mc^2}{eE_0 \sin\phi_0}$$

being Δ the distance travelled by the particle to get a kinetic energy just equal to the rest-mass energy mc^2 (i.e. $\gamma = 2$ at $z = \Delta$).

With the typical values of E_0 and ϕ_0 in use for RF Guns, Δ turns out to be a small fraction of the RF wavelength λ (with BNL parameters, $\Delta \approx \lambda/18$), hence the non relativistic motion of the electrons is really confined nearby the cathode surface.

Since Δ is just the spatial delay between the electron and the forward travelling wave component of the standing resonant RF field (propagating along z at $v = c$), the asymptotic phase shift of the electron with respect to the RF field is given by:

$$(\phi - \phi_0) = k\Delta \qquad k = 2\pi/\lambda$$

where ϕ represents the asymptotic exit phase. Subsituting for Δ and recalling the definition of the dimensionless parameter α ($\alpha = eE_0/2mc^2k$), one gets the asymptotic exit phase as given by the r.h.s. term of equation 2) (Kim's result)

$$\phi = \frac{1}{2\alpha \sin\phi_0} + \phi_0$$

In order to compute the energy gain we assume now that the electrons are emitted from the cathode surface at $\beta \approx 1$ at a phase ϕ: this is equivalent to look at the non relativistic part in the electron motion just as a re-definition of the starting conditions at the cathode surface.

The kinetic energy gain ΔT will be given by:

$$\Delta T = eE_0 \int_0^{(N+1/2)\lambda/2} \cos(kz)\sin(\omega t + \phi)dz = \frac{eE_0}{2k}\left[\pi(N+\tfrac{1}{2})\sin\phi + \cos\phi\right]$$

which gives actually the same result of equation 6) if one substitute for α the previous definition (and recalling that $\gamma_f = 1+\Delta T/mc^2$).

APPENDIX 2

We present here a possible unique feature of the TOPGUN for the generation of train of bunches with micro-repetition rate at very high frequency.

The exit longitudinal phase space distributions (see Fig.15) show that it is possible to guarantee a linear energy-phase correlation within about 60 RF deg (1 rad) of the Gun RF period.

Setting $\Delta\phi = 1$ in equation 9), under the straight-topping condition of eq. 10'), it is immediate to check that the maximum energy spread ΔT at the TOP-GUN exit does not depend on the number of cell (i.e. on the final energy) and it is given by:

$$\Delta T\ [\mathrm{keV}] = \pm\, 70 \cdot E_0\ [\mathrm{MV/m}] \cdot \lambda_{RF}\ [\mathrm{m}]$$

Assuming an unform distribution of N_b short bunches all over the linear part of the energy gain curve (see Fig.4a), ΔT will be the difference in the final energy between the central bunch and the head (or tail) one. If the bunches must be spaced every one period of a frequency ν_a (possibly the resonant frequency of a high gradient structure) the TOPGUN frequency must be ν_G as given by:

$$\nu_G = \frac{\nu_a}{2\pi N_b}$$

which gives, taking into account that $\lambda_{RF} = c/\nu_G$,

$$\Delta T\ [\mathrm{keV}] = \pm\, 440\ N_b\ E_0[\mathrm{MV/m}]\ \lambda_a[\mathrm{m}]$$

with $\lambda_a = c/\nu_a$.

Taking parameter values typical for a TeV collider - i.e. $N_b = 10$, $\nu_a = 30$ GHz - the TOPGUN frequency is fixed around 500 MHz. Therefore, using a field $E_0 = 30$ MV/m one gets an energy spread in the bunch train below 1.3 MeV, which can be easily recovered applying a linear energy ramp to the train.

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