# ISTITUTO NAZIONALE DI FISICA NUCLEARE

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## A VERY EFFICIENT FEL FOR THE MICROWAVE SOURCE FELTRON

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**ABSTRACT**
The paper examines the conditions for obtaining a 70% efficiency in an FEL. A couple of formulae useful to designers are given. The enhancement of the FEL efficiency at that extent is used to simplify the design of a multibeam FEL.

## 1 - INTRODUCTION

FELTRON is a 200 MW - 20 GHz pulsed μ-wave FEL driven by an Electrostatic Accelerator for high gradient LINAC[1], see Fig.1. One of

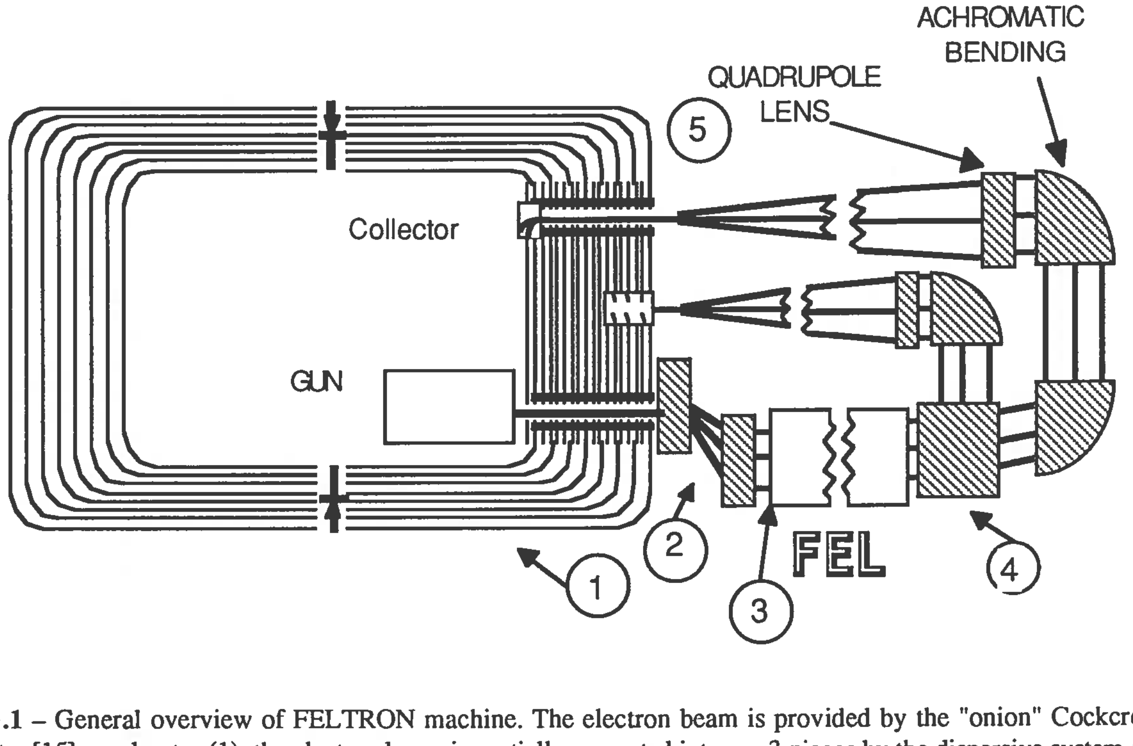


**FIG.1** – General overview of FELTRON machine. The electron beam is provided by the "onion" Cockcroft-Walton[15] accelerator (1); the electron beam is spatially separated into n = 3 pieces by the dispersive system (2); the FEL (3) provides the three 400 MW RF beams; the dispersive system (4) separates the slow and the fast electron beams; the quadupole lens (5) conveys the three fast beams into the decelerating column for recovery.

the main feature of the source is that it has ten parallel outputs. In fact, the high gradient structure needs to be fed at each meter and the power source must be able to supply a section of ten meters. That feature is accomplished generating a set of ten parallel electron beams (eb) and so in turn a set of ten parallel FELs. This complex scheme comes from the fact that the efficiency of the FEL has been assumed 40%, as it was proven feasible at the Livermore experiment (2). The aim of this paper is to show that the FEL efficiency can reach a value of 70%. Hence, the number of channels can be halved, reducing so the design complexity. The very high FEL efficiency is important, not only because it determines the machine design, but also because it increases the total efficiency.

The efficiency of an FEL operating in the high gain+tapered regime is roughly given by

$$\eta \approx b \frac{\gamma_i - \langle\gamma_f\rangle}{\gamma_i - 1} \tag{1}$$

where b is the bunching coefficient, $\gamma_i$ is the initial value of the Lorentz factor and $<\gamma_f>$ is final value of the bunched part of the electron beam.

The bunching, given an electron beam of reasonably good qualities (these, indeed, can be very good from an electrostatic accelerator), is around 80%, $\gamma_f$ can be around 2 in efficient devices, as shown below.

The final value of $\gamma$ is very near to $\gamma_z$, the longitudinal Lorentz factor.

In fact, the physics of the FEL, because of the synchronism condition, imposes the equality between the velocity of the ponderomotive potential and the z-velocity of the electron beam (eb) and it is constant along the wiggler, i.e.

$$v_{pm} = c\,\beta_z = \text{cost} \tag{2}$$

The longitudinal $\gamma_z$ is reduced from the total $\gamma$ by the perpendicular velocity, $\beta_x$, $\beta^2 = \beta_z^2 + \beta_x^2$,

$$\gamma_z = \left(1 - \beta_z^2\right)^{-1/2} \cong \frac{\gamma}{\left(1 + \gamma^2 \beta_x^2\right)^{1/2}} = \frac{\gamma}{\left(1 + a_0^2\right)^{1/2}} \tag{3}$$

In the tapered wiggler section, the energy is extracted from the electrons while the adimensional wiggler vector potential $a_o$

$$a_0 = \frac{b_0}{k_0} = \frac{e\,B_0}{\sqrt{2}\,m\,c^2 k_0} = 0.66\,B_0\,(T)\lambda_0\,(cm) \tag{4}$$

is decreased up to a value near to zero by reducing the magnetic field. The transverse velocity

$$\beta_x \cong \frac{a_0}{\gamma} \tag{5}$$

goes to zero together with $a_o$. The energy associated with perpendicular part of the motion is extracted from the eb and transformed into em energy. The tapering is a mean to transfer transverse energy of the eb (energy associated to the transverse motion) into the longitudinal energy of the eb, so to keep constant $\gamma_z$ while the eb is decelerating. This is necessary because the conversion of the eb kinetic energy leads to a decrease of $\gamma$ and in turn of $\gamma_z$ as indicated by eq. (3). The energy extraction is at expenses of the longitudinal part, because the perpendicular velocity does not decrease as far as $a_0$ is constant, as shown by eq. (5). Thus, the decrease of $a_0$ is the mean to reduce the transverse energy while leaving constant the longitudinal one.

From the above arguments, we figure out that, in order to get high efficiency, we must design a system that has a large fraction of the eb energy in the transverse part (large transverse velocity $\beta_x$) and a relatively small fraction in the longitudinal one (small velocity $\beta_z$). From eq. (3), given a certain $\gamma$, in order to have a small $\gamma_z$, $a_0$ must be large.

Therefore, either the field $B_0$ or the wiggler period $\lambda_0$ or both must be large. Owing to the wiggler technology, the field can be high when the period is long. Combining eq (4) with the synchronism condition (2), which is conveniently expressed in terms of the wiggler parameters

$$\lambda_s = \frac{\lambda_0}{2}\left(\frac{1 + a_0^2}{\gamma^2}\right) \qquad (6)$$

we get that $B_0$ and $\lambda_0$ can be large when the radiation wavelength is long enough. In fact, assuming $\gamma_z = 2$, as said before, we obtain $\lambda_0 = 8\lambda_s$.

Being in our problem $a_0 \gg 1$, eq.(3) can be approximated

$$\gamma_z^i \cong \frac{\gamma_i}{a_0} \qquad (7)$$

Willing $\gamma_z = 2$ we get the relation between the initial wiggler parameters and the initial energy

$$a_0^i = \frac{\gamma_i}{2} \qquad (8)$$

The assumption of the final value of the Lorentz factor equal or less than 2, $\gamma_f \le 2$, needs some other considerations. That final value depends on the ratio between the ponderomotive force and the space charge force when the eb has lost most of its energy. That ratio depends on the current and the beam emittance. However, with reasonable beams and few hundreds amps, we guess that the electrons can be decelerated till to an energy equal-less than 0.5 MeV. In fact, Raman FELs worked with an energy of 1.5 MeV and got high efficiency [3,4].

We note, from relation (1), that the conversion efficiency depends on the initial energy as on the final one. If the final energy of the electrons is as small as quoted, and the initial $\gamma_i$ is enough high, $\gamma_i > 10$, the eq (1) can be approximated

$$\eta \approx b \qquad (9)$$

that is the efficiency is proportional to the bunching.

In this result it is given for granted that it is possible to taper the wiggler as long as required and, further, that the electrons do not spill out of the potential well. Here we notice that the longer is wavelength the easier is to satisfy the requirement $a_0 = \gamma_i/2$, because the wiggler period becomes longer. The relation of the peak magnetic field and the wiggler gap g and $\lambda_0$, of a hybrid wiggler, is [5]:

$$B_0 = 3.33\, B_r\, e^{-\frac{g}{\lambda_0}\left(5.47 - 1.8\frac{g}{\lambda_0}\right)} \qquad (10)$$

where $B_r$ is the permanent field. The equation is represented in Fig. 2 assuming a $B_r$=1T.

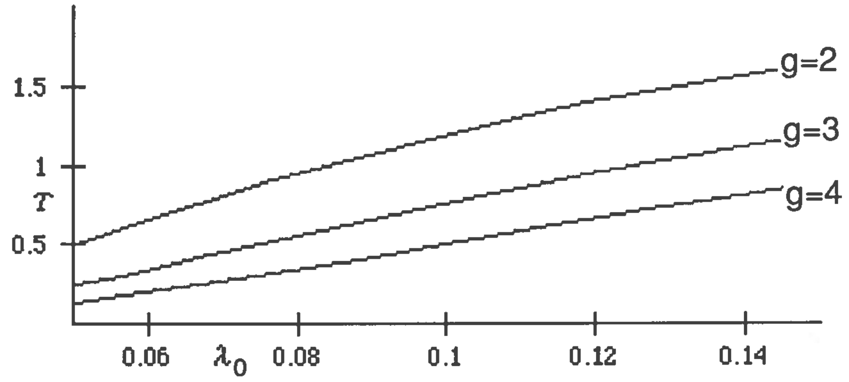


**FIG. 2** – Diagrams of the wiggler peak in Tesla versus the wiggler period $\lambda_0$ (m) for the gap value 2,3,4 cm respectively of a hybrid wiggler.

A μ-wave FEL meets all the requirements: beam qualities, long wiggler period and spatial definition of the radiation beam.

The high value of the efficiency makes the μ-wave FEL, in the high gain+tapered regime, a very important RF tube.

We have used for the above discussion the relations of free-space FEL. This is not correct for a waveguide FEL (as is that of FELTRON project). It has been done for easy arguments. The synchronism condition of the waveguide

$$\lambda_s = \frac{\lambda_0 \left(1 + a_0^2\right)}{\gamma^2} \left[ \frac{1}{\beta_z} \quad \frac{1}{1 \pm \beta_z \sqrt{1 - \left( \frac{\lambda_0 \sqrt{1 + a_0^2}}{2\, h\, \gamma\, \beta_z} \right)^2}} \right] \tag{11}$$

which comes from the combination of the dispersion relation and the FEL synchronism relation in the case of the waveguide

$$\left(\frac{\omega}{c}\right)^2 = k_g^2 + k_{co}^2 \tag{12}$$

$$\frac{\omega}{c} = \beta_z \left(k_g + k_0\right) \tag{13}$$

would have made the relations and arguments more cumbersome without changing the basic conclusions.

In eqs (11), (12) and (13) $\beta_z$ is

$$\beta_z = \sqrt{1 - \frac{1 + a_0^2}{\gamma^2}} \tag{14}$$

with h the height of the waveguide, the cut-off wavenumber holds $k_{co} = \pi / h$ , $k_g$ is waveguide wavenumber,and $k_s^2 = k_g^2 + k_{co}^2$. In the waveguide two frequencies are possible at each $\gamma$, as shown in the diagram of Fig. 3. It is quite obvious that the higher frequency must be chosen, because it is the nearer to the free space case, then it requires a high value of $a_0$.

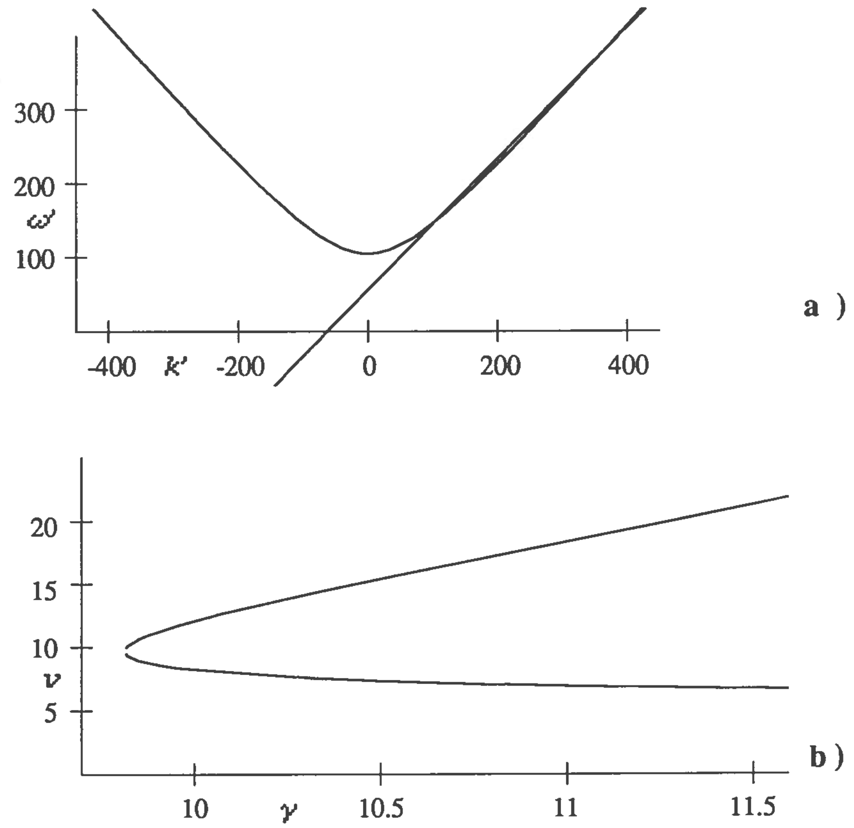


**FIG. 3** – a) electron beam synchronism with the electromagnetic wave in the waveguide; b) synchronism relationship on the plane (ν,γ). The parameters are those of Table I.

About the overall efficiency, since the electrostatic accelerator has an estimated efficiency of 80%, the total efficiency of the source would be $\eta=\eta_{acc}*\eta_{FEL}=50\%$. This is a satisfactory number. Anyway, it can be further increased by recovering the spent electron beam. The part of the electron beam which does not participate to the interaction within the tapered FEL section, will be called fast beam section. The strongly decelerated part will be called slow beam section.

In an electrostatic accelerator , as at the UCSB experiment (6), the recovery is made using a decelerating column+collector (possibly a multistage collector when the eb has a large energy spread, ref. 7). The recovery requires that the electrons have enough energy to reach the high voltage terminal. In addition, they must hit the collector at enough low energy to have a handleable cooling problem. The collector is set at the high voltage terminal, and, there, it is difficult to have an efficient cooling system. At the UCSB experiment the recovery of a 2 Amps electron beam has been successfully done. However, that FEL pertains to the so-called Madey class FEL. The eb remains substantially cold because $\Delta\gamma/\gamma\approx 1/N$ and N, the number of undulator periods, is 150. In our case the energy spread of the fast eb is around 5%. With an eb of 5 MeV, this leads to an energy spread of 250 KeV. A multistage collector of this energy acceptance is a difficult item. An electrostatic accelerator of the Cockcroft-Walton type, as we have proposed for FELTRON, with its capacitors column dividing the total voltage, is suitable to recover charges with different energy. These charges will end up to the proper capacitor.

The beam qualities in electrostatic accelerators can be very good: the normalized emittance can be estimated less than $10^{-4}$ m rad and the energy spread $\Delta\gamma/\gamma < 1\%$.

A possible sketch for the recovery of the fast 20% beam is presented.

In this article we will refer to the set of parameters relative to the FELTRON project, even if results have general validity.

## 2 – 70% EFFICIENCY FEL DESIGN

The analytical and numerical work is based on the single particle equations of motion derived by Bonifacio, Pellegrini, Narducci and rewritten for a waveguide by the Livermore group (8,9). We use also their notations. The particle equations describe the motion of electrons in energy ($\gamma$) and phase ($\theta$) in the ponderomotive potential formed by the wiggler and laser fields. The arguments for neglecting the space charge force are discussed below. The field equation relates the radiation amplitude to the driving eb current. The field amplitude growth is expressed in terms of $\gamma$ and $\theta$ of the electrons.

For the simulations and calculations, we refer to the FELTRON parameters listed in Table I which has been set to comply with the high efficiency requirements

**TABLE I – FEL PARAMETERS.**

| | | | |
|---|---|---|---|
| Energy (MeV) ($\gamma$=11.2) | 5.3 | $\mu$-wave wavelength (cm) | $\lambda_s = 1.5$ |
| Current (A) | 100 | Wiggler period (cm) | $\lambda_0 = 10$ |
| Energy Spread | $\leq 1.5\%$ | Peak Field (kG) | $B_0 = 7.5$ |
| Normalized emittance (m rad) | $\varepsilon_n < 10^{-3}$ | Wiggler parameter | $a_0 = 5$ |
| Electron Beam radius (mm) | 4 | Gap cm | $g = 3$ |
| Pierce parameter | $\rho = 3\ \%$ | Waveguide ($cm^2$) | $a{\cdot}h = 3*3$ |
| $\gamma_{\parallel}$ | 2.2 | Small Signal Gain | $G_0 = 11$ |
| Undulator length (m) | 6 | mode $TE_{01}$ | |
| Undulator type Hybrid with canted poles | | Frequency GHz | 20 |
| Polarization | planar | | |

This set of parameters comes from the tradeoff between the field amplitude, wiggler period and waveguide height compatible with the highest possible wiggler parameter $a_0$. We would have needed $a_0$= 5.5, as indicated by eq.(7), but the value of $B_0$ would have exceeded the value indicated by eq. (11).

In what follows, z is the axis of the FEL and of the electron beam propagation. The electromagnetic field has a wavelength $\lambda_s$ and a complex electric field amplitude $E_s = |E_s| e^{i\phi}$.

We use the dimensionless vector potentials of the field.

$$a_0 = \frac{b_0}{k_0} = \frac{e B_0}{\sqrt{2}\, m c^2 k_0} = 0.66\, B_0\,(T) \lambda_0\,(cm) \qquad (15)$$

$$a_s = \frac{e_s}{k_s} = \frac{e E_s}{\sqrt{2}\, m c^2 k_s} = 0.22 \times 10^{-6}\, E_s\,(V/m)\, \lambda_s\,(m) \qquad (16)$$

Here $E_s$ is the field amplitude, which in a waveguide of a·h transverse dimensions holds

$$E_s \cong \sqrt{754\ P_{RF}/(ah/2)} \tag{17}$$

$P_{RF}$ is the RF power travelling in the waveguide.

The motion equations of the j-th electron and the field in a waveguide are (in the paraxial approximation)

$$\frac{d\,\gamma_j}{d\,z} = \left[ i\,\frac{a_0\,f_B}{\gamma_j\,\beta_z}\,e_s \quad + e_z \right] \times\ e^{i\,\vartheta_j} \tag{18}$$

$$\frac{d\,\vartheta_j}{d\,z} = (\ k_0 + \delta k - \frac{\omega_s\,/\,c}{\beta_z\,(1+\beta_z)}\,\frac{1+a_0^2}{\gamma_j^2}\ ) +\ 2\left[\frac{a_0\,f_B}{\beta_z\,(1+\beta_z)\,\gamma_j^2}\,e_s\,e^{i\,\vartheta_j}\right] \tag{19}$$

$$\frac{d\,e_s}{d\,z} = i\,\frac{1}{2}\,\frac{(\omega_p^*)^2}{k_g\,c^2}\,a_0\,f_B\,\frac{1}{N}\sum_1^N{}_j\,\frac{e^{-\,i\,\vartheta_j}}{\gamma_j}\cos\left(\frac{\pi\,y_j}{b}\right) \tag{20}$$

where the relative phase of the electron in a waveguide is

$$\vartheta_j = (\ k_0 + k_g)\ z\ -\ \omega_s\ \ t \tag{21}$$

the space charge adimensional field is

$$e_z = \frac{e\,E_z}{m\,c^2} \ = \ -i\,\frac{2\,\omega_p^2}{(k_0 + k_g)\,c^2} \tag{22}$$

the starred plasma frequency is:

$$(\omega_p^*)^2 = \frac{e^2 n^*}{\varepsilon_0 m} = \frac{e}{\varepsilon_0 m c}\,\frac{I}{(ah/2)} \tag{23}$$

the current density is calculated on the radiation mode area, and the plasma frequency is:

$$\omega_p^2 = \frac{e^2 n_b}{\varepsilon_0 m} \tag{24}$$

with the bunched charge density

$$n_b = \left(\frac{I\,/\,c}{A_{sc}}\right)\frac{1}{N}\sum^N{}_j\ e^{-i\,\vartheta_j} = \frac{J_{eb}}{c}\,b \tag{25}$$

where $f_B$ is the standard difference of the Bessel functions (10), I is the beam current, N the number of electrons per period of the ponderomotive potential and $e_s$ has to be considered the complex amplitude of the $TE_{01}$ mode in the waveguide; $A_{sc}$ in eq. (25) is the electron beam area whose value is enhanced (because of the waveguide wall effect) of a factor three with respect to the actual value. We did not exploit the usual approximation $\gamma^2 >> 1$ and then $[(1/\beta_z) - 1] \cong (1/\gamma_z^2)$, we have used instead the exact expression because, within the tapered section $\gamma$ can become small. Anyway, the above approximation is well satisfied till to $\gamma$ near three, that is for all but the very last part of the electrons trip inside the wiggler.

In view of performing the calculation of the bunching b and the final energy $\gamma_f$, we do not need to consider the three dimensional effects, emittance, energy spread and field distribution in the transverse plane. In fact, these effects can only produce non-significant variations to those two quantities. In addition, in our project the emittance can be small enough to neglect the relative term [11,12] in eq. (18).

Within the tapered wiggler section, following Kroll, Morton and Rosenblat [13] the FEL equations are changed introducing the total phase $\psi = \vartheta + \Phi$ and imposing it to be constant through the tapered section by a linear variation of the magnetic field.

The bunching b is determined by the FEL amplifier within the first normal wiggler section, while $\gamma_f$ is determined by the dynamics within the second tapered section. We assume that after saturation the bunched electrons are trapped into the ponderomotive well, the bucket, built by the interference of the wiggler wave, $a_0 \sin k_0 z$, and the radiation wave $a_s \sin(kz-\omega t)$ and do not spill out during the bucket deceleration.

Till to saturation (within the 1.5 m of normal wiggler) an instability makes the bunching b to grow exponentially up to a value around 80% and, in the meanwhile, an RF wave grows exponentially to a power $P_{RF} \approx (1.3\ \rho)\, P_{eb}$. Here, ρ is the so-called Pierce parameter and holds

$$\rho = 0.171\ \frac{B_0^{2/3}\,\lambda_0^{4/3}}{\gamma_r}\left(\frac{I}{ah/2}\right)^{1/3} f_B^{2/3} \qquad (26)$$

The value of b comes out to depend on the beam qualities only: the energy spread introduces a detuning between electrons and the ponderomotive force (which in turn affects the phase of the electrons) and the emittance determines the eb spot size and so in turn the space charge force and the amplitude of the betatron motion. However, owing to the instability mechanism, the bunching is weakly dependent on the electron beam qualities. Once the energy spread is less than the parameter ρ and the normalized emittance is $\varepsilon_n \leq 2\pi/\lambda$ the bunching b is always around 75%. This has been shown by simulations and confirmed by the Livermore experiment [2,10,12].

With the set of motion eqs. (18), (19) and (20) and their variations within the tapered section, we have calculated the bunching b, the output power, the final beam distribution and the eb evolution within the phase space as function of z by a 1-D computer program. The results are shown in Figs. 4, 5, 6 and 7. For the bunching and power we have obtained respectively 80% and ~330 MW. The FEL efficiency results in η ~70%. This result is in good agreement with the analytical one.

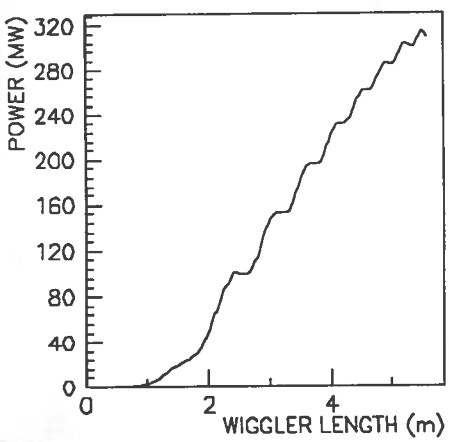


**FIG. 4** – Power versus wiggler length computed with the field tapering shown in Fig. 5.

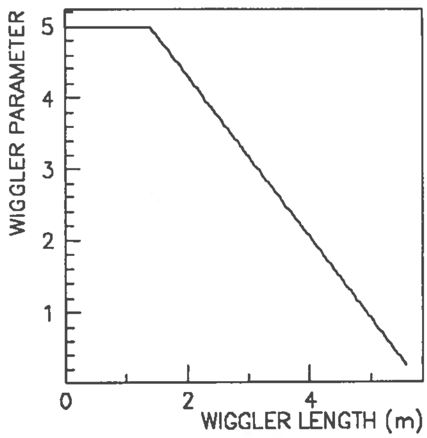


**FIG. 5** – Diagram of the tapering of the wiggler field along the z axis.

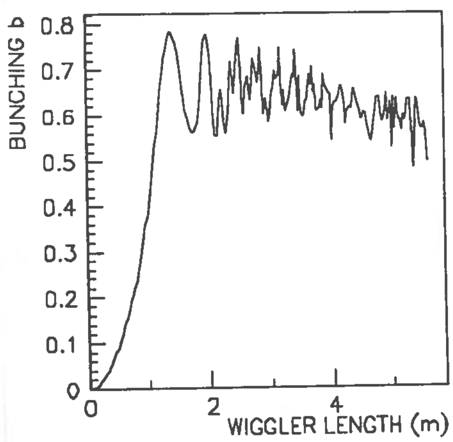


**FIG. 6** – Bunching coefficient b versus wiggler length.

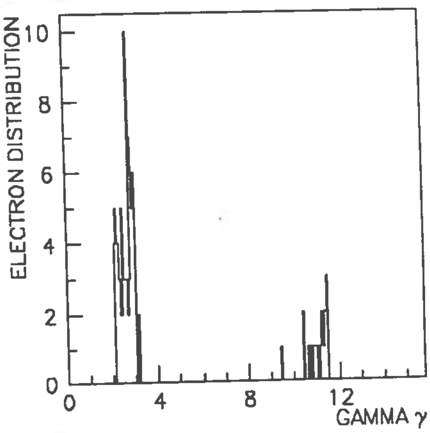


**FIG. 7** – Final energy distribution versus $\gamma$; there are two peaks: the one at high $\gamma$ is due to untrapped electrons, the one, centered around $\gamma = 2.5$, is due to electrons trapped inside the bucket.

In fact, the efficiency, by definition, is

$$\eta = \frac{P_{RF}}{P^i_{eb}} = \frac{\Delta P_{eb}}{P^i_{eb}} = \frac{\sum_j \left(\gamma^i_j - \gamma^f_j\right)}{\sum_j \left(\gamma^i_j - 1\right)}$$

$$= \frac{N_{ub}\left(\gamma^i - \langle\gamma^f_{ub}\rangle\right)}{N\left(\gamma^i - 1\right)} + \frac{N_b\left(\gamma^i - \langle\gamma^f_b\rangle\right)}{N\left(\gamma^i - 1\right)} = b\,\frac{\Delta\gamma_b}{\left(\gamma^i - 1\right)} + (1 - b)\frac{\Delta\gamma_{ub}}{\left(\gamma^i - 1\right)} \tag{27}$$

where N is the total particles number, $N_b$ and $N_{ub}$ are the bunched and unbunched particles numbers respectively, the same for γ.

When b is nearby one, $\langle \gamma^f_{ub} \rangle << \langle \gamma^f_b \rangle$ and $\gamma \gg 1$, we get, dropping the indices,

$$\eta \approx b \frac{\Delta \gamma}{\gamma}$$

The energy distribution in diagram of Fig. 7 shows the shift of the average energy distribution and the calculated final value of γ. We see that all the trapped electrons remain inside the bucket till to the end. The energy distribution of these electrons around the average value coincides with the height of the bucket, which in turn is about the Pierce parameter ρ.

The Livermore group have shown, with the more sophisticated code FRED, that some electrons spill out from the bucket. Anyway, this fact does not change the gross result.

We have compared simulations with and without the space charge. The differences are within few percentages. This is agreement with the results of ref. [11].

The importance of the longitudinal space-charge force is weighted by its ratio with the pondero-motive force

$$\frac{F_{sc}}{F_{pm}} = \frac{|e_z|}{a_0 f_B |e_s| / \gamma} \qquad (28)$$

This ratio results much less than 1 in both regimes, the saturation and the tapered one. In the former, γ remains substantially unchanged, whilst in the latter it lowers up to a value around 2. This value is anyway high for space charge effect.

The gain, phase shift, bunching and power up to saturation could be calculated through a single root of a cubic equation with real coefficient, obtained by linearizing the FEL set of equations (see the procedure of Bonifacio-Pellegrini-Narducci in ref. [9] and refs. [10,12]).

## 3 – GENERAL CONSIDERATIONS

The quasi-null effect of the space charge indicates that no collective interaction (Raman regime) can establish. This is expected within the untapered section because the Pierce parameter ρ is enough small (less than 0.1). Within the tapered wiggler section, the following physical considerations explain why the collective instability does not appear: a) γ is still too high, b) the radiation wave is so strong as to be able to drive any electron beam dynamics; c) the average velocity of the eb decreases too fast to allow the growth of a plasma wave: the dispersion relation for the wave [14]

$$\omega = k_z v \pm \frac{\omega_p}{\gamma} \qquad (29)$$

is not stable because the eb velocity v continuously changing; d) the eb syncrotron motion inside the bucket opposes the ordered collective motion of the plasma wave; e) the electron beam is too warm for the dispersive growth rate of a Raman FEL: the required condition

$$\frac{\Delta \gamma}{\gamma} < \frac{\omega_p / \gamma}{\omega_0 \equiv k_0 c} \qquad (30)$$

does not hold because the eb uniformly fills the buckets, whose dimension is

$$\Delta \gamma / \gamma \approx 2 \sqrt{a_0 a_s} \tag{31}$$

It is assumed that the fast eb does not participate to anything being out of resonance.

In the case of $\gamma_z$ enough small, the deceleration process stops to operate, ultimately, at $a_0$ value such that the space-charge force becomes comparable with the ponderomotive force and the energy spread is comparable with the average one. We observe that we do not deal with diffraction because the radiation is guided by the μ-wave cavity.

In the case of $\gamma_z$ high enough , the deceleration process stops because the ponderomotive force ultimately vanishes(with $a_0$).

## 4 – A TWO SECTIONS TAPERED WIGGLER

A wiggler able to comply with the requirement of eq. (7) is, in many cases, technically impossible. There is a small space to obtain a further increase of the efficiency by making the tapering through the reduction of both the period and the field of the wiggler. This can be accomplished simply by dividing the tapered wiggler into two sections having different periods: the period of the second section $\lambda_2$ shorter than the period of the first section $\lambda_1$. The tapering is made in both sections by decreasing the magnetic field. For simplicity we have renamed the two wigglers parameters with indices 1 and 2: $\lambda_1, \lambda_2, B_1, B_2, a_1, a_2$.

The point where to join the two wigglers is determined both by the enhancement of the FEL parameter $a_2$ referred to $a_1$ that can be achieved and by the fact that $a_1$ must be enough high to have still the bucket dimension at its normal width. The bucket dimension $a_0(t)a_s(t)$ is constant as far as $a_0^2 \gg 1$. In fact, differentiating the synchronism condition (6)

$$\frac{\Delta\gamma}{\gamma} \cong \frac{\Delta B}{B_0} = \frac{\Delta a_0}{a_0} \tag{32}$$

and combining with the fact that the eb energy is completely transferred into the radiation beam

$$\Delta P_{eb} = m c^2 \Delta\gamma = \Delta P_{RF} \tag{33}$$

lead to a constant bucket dimension. We notice that eqs (32 and 33) hold only within the previous limit because, being it a kinematic relation, it does not deal with the dynamics of the electron beam, described by the equations set -18,19,20-. The space charge, the detrapping due to non adiabatic process and, furthermore, the saturation of the $\gamma$ towards $\gamma_z$ are not accounted for. The linear relation (33) holds anyway as long as $a_0^2 \gg 1$, that is as long as we are far from the saturation of the synchronism condition ( when $a_0$ becomes about 1).

The output RF power, as far as relation (32) holds, can be calculated by

$$P_{RF} = b \times P_{eb} \times \frac{\Delta a_0}{a_0} + \rho\ (1\text{-}b)\ P_{eb} \tag{34}$$

This formula reproduces very well the simulations results up to an $a_1$ value around 2, that is when $B_1$= 3 kG. There, the output power results in

$$P_{RF} = 245 \text{ MW} \qquad \text{at} \qquad a_0 = 2$$

and the efficiency is already 50%.

Joining at that point the previous two wigglers, we would have for the wiggler 2

$\lambda_2 = 7.1$ cm
$B_2 = 5$ kG
$a_2 = 2.3$

These numbers come from the equality

$$\lambda_1 \left(1 + a_1^2\right) = \lambda_2 \left(1 + a_2^2\right) \quad (35)$$

and from the fact that $B_2$ must comply with the curves of fig. 1. The ratio between the new and old transverse energy obtained with that change is (2.3/2), which is really small. Therefore, it could be more advantageous to change wiggler 1 at lower $a_1$ , although this is against the previous considerations. In fact, choosing $a_1$=1 ( $B_1$=1.5 kG) we could have $\lambda_2 = 6$ cm, $B_2$ = 3.8 kG and $a_2 = 1.5$, that is a more favorable ratio $a_2 / a_1$.

Anyway, it is clear from these numbers, that this scheme is a refinement compared with that with one wiggler only.

## 5- FELTRON AFTER 70% FEL EFFICIENCY

The efficiency of 70% changes dramatically the parameters tradeoff of a machine. Considering our FELTRON project, it becomes possible to pass from ten channels (ten outputs) to five channels. In fact, the increase of the FEL efficiency of a factor 1.75 (from 40% to 70%) almost doubles the output RF power given a certain current. Hence, it is natural to increase the current of only 20% (passing from 100 to 120 A in FELTRON electrostatic accelerator) so to double the output RF power per electron beam compared with the previous case. This will lead to an RF power of 400 MW instead of the required 200 MW. Each RF beam can, then, be divided into two parts, each one feeding one meter of LINAC. The scheme of the machine is shown in Fig. 7.

THE RECOVERY. There is no doubt that the recovery of a set of parallel electron beams, and moreover with a large energy spread, is a great challenge. Willing anyway to do it, the number of electron beams ought to be reduced up to three, as shown in Fig. 7.

We must separate the fast and slow electrons by a dispersive system. The three fast beams of 20 A each must then be focused in the horizontal plane (by means of a quadrupole lens for example) into the decelerating column. Inside the decelerating column the beam is guided by the focusing system of the column [1,16]. The three slow beams should be treated in the same way as refer to the focusing in a single beam, but then this beam must be recovered in a short decelerating column. The Cockcroft-Walton, with its series of capacitors which divides the total voltage, is capable to recover charge at an energy lower than the initial one. In fact, that charge can be recovered at the proper capacitor. We remark that the overall efficiency can reach a value higher than 90%!

## 6 – CONCLUSIONS

An FEL operating in the high gain+tapered regime with an efficiency higher than 70% is possible in the high frequency micro-wave band, centimeter region. The condition is that the initial energy is higher than 10 MeV, $\gamma_i > 10$ and the wiggler magnetic field is such high that the longitudinal Lorentz factor inside the wiggler is nearly 2, $\gamma_{||} \approx 2$. This leads to the following relation between the radiation wavelength and the wiggler period: $\lambda_0 = 8\lambda_s$. That high energy extraction is based on two facts: one is that the FEL process can extract all the energy set in the transverse component of the motion, the other is that the bunching of the electron beam can reach a value of 80%. The transverse energy extraction is given by the basic FEL equation

$$\frac{d\gamma}{dt} = \frac{e}{mc^2} v_\perp E_\perp$$

which tells that the transverse velocity is progressively reduced to zero, being the tapering able to keep constant the electron phase with respect to the electric field. The electrons are bunched at the beginning of the interaction by an instability process which squeezes the electrons contained in one wavelength at a such extent to obtain a bunching of 80%.

In simulations, the space charge effect does not show any important effect up to the end of the interaction.

With the recovery, an pverall efficiency around 90% can be reached. We stress that the recovery systemis a difficult item.

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