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A THEORETICAL ANALYSIS OF THE RESPONSE OF A DOUBLE BALANCED MIXER FOR LINEARITY IMPROVEMENT OF PHASE DETECTION

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ABSTRACT:

The Double Balanced Mixer (DBM) is a 3-ports device producing an output signal as result of the interaction between two input waveforms. A theoretical analysis of this interaction and its dependence from the levels of the input signals is the topic of this paper, to demonstrate the linearity improvement which can be achieved for phase detection, using the DBM with comparable input amplitudes.

1. Introduction

The circuit model of a DBM is reported in Fig. 1, where the 3 ports are shortly indicated as LO (Local Oscillator), RF (Radio Frequency) and IF (Intermediate Frequency).

Because of the high symmetry of the circuit, it exhibits a very good insulation between the 3 ports. This means that a signal applied to a single port produces, at the first order, no voltage at the other two. Practically the isolation falls with frequency down to a typical value of 30 dB at the highest operative frequency of the device.

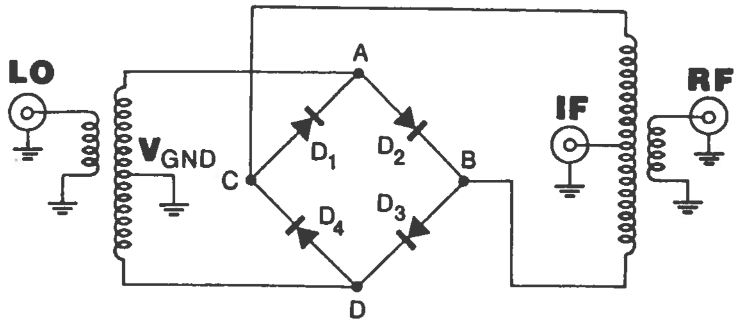

Fig. 1: DBM circuit

The network of Fig. 1 is a non-linear one, therefore the device behavior can not be described using the superposition theorem; the simultaneous application of voltages at two different gates produces a degradation of the electrical symmetry inside the circuit and then a voltage appears at the remaining gate. So that, the output signal is determined by the levels and frequencies of the input waveforms and depends on the choice of the input gates. Particularly, when the device is used in homodyne systems, the gates LO and RF are chosen as input ports; the processing waveform is usually connected to the port RF, the LO one being driven by a reference signal.

2. Driving RF port with a low-level signal (compared with the LO signal)

2.1 General Equation

Let us consider an RF signal much smaller than that applied at LO port, the latter being high enough to fully forward drive the diodes of the ring. Hence the conduction status of the diodes is just controlled by the instantaneous voltage $V_{LO}(t)$: when its value is greater than the diodes threshold V_γ the $D_2 - D_3$ pair is switched on, while, for negative values, when $V_{LO}(t) < -V_\gamma$, the pair $D_1 - D_4$ is forward biased. As a consequence, in the former

case the B node is a virtual ground and the output voltage $V_{IF}(t)$ is given by the equation $V_{IF}(t) = -V_{RF}(t)$; in the latter case the C node is a virtual ground and the output voltage becomes $V_{IF}(t) = V_{RF}(t)$.
Conversely, when the module of $V_{LO}(t)$ is lower than V_γ no voltage appears at the IF output port, the 4 nodes of the ring becoming "floating points". The overall behavior of the device is therefore described by the following equation:

$$V_{IF}(t) = -s(t) \cdot V_{RF}(t) \tag{1}$$

where:

$$s(t) = \begin{cases} 0 & if \;\; |V_{LO}(t)| < V_\gamma \\ sgn[V_{LO}(t)] & if \;\; |V_{LO}(t)| > V_\gamma \end{cases} \tag{2}$$

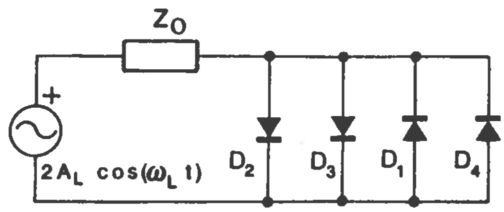

Fig. 2: DBM equivalent circuit for small RF signals

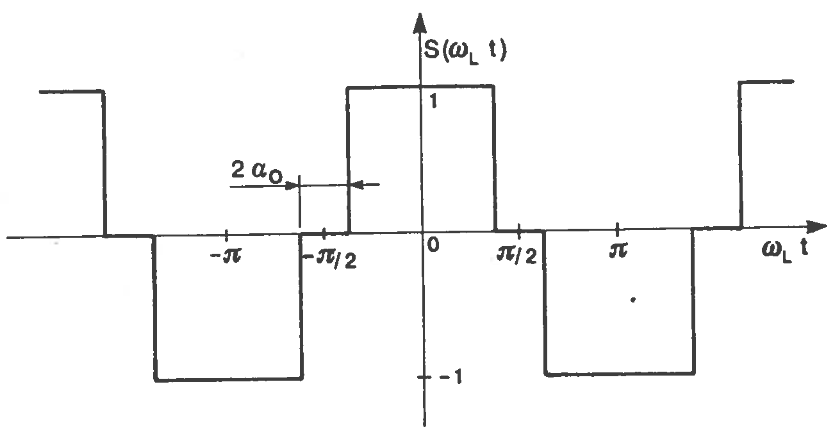

Fig. 3: "switching function" s(t)

Equations 1) and 2) point out that the DBM, driven with a small RF signal, performs an analog multiplication between the RF waveform and a three-levels square wave, whose shape depends on the LO voltage istantaneous sign and amplitude (see Fig. 3). For this reason the DBM is employed for frequency translation, phase detection and coherent demodulation.
Let us consider a sinusoidal source, with frequency ω_L, feeding the LO port. Representing this source as a voltage generator having an amplitude $2A_L$, twice the nominal value, with a series resistance Z_o, equal to the transmission line characteristic impedance (usually 50 Ohm), the load voltage equals the nominal value, if the source is properly matched. Since the central tap LO transformer is equivalent to an ideal 2:1 transformer, the primary voltage and current have to be respectively one half and twice, with respect to those at the secondary. The resulting equivalent circuit for the primary is presented in Fig. 2.
The function s(t), given by Eq. 2, has the plot reported in Fig. 3, i.e. a square wave having unit amplitude, ω_L frequency and a cutoff angle $2\alpha_o$, corresponding to the time interval in which the LO signal is included in between the two threshold values $\pm V_\gamma$.
As it's easy to demonstrate that:

$$\sin\alpha_o = V_\gamma/2A_L \tag{3}$$

calculating the Fourier series expansion of $s(t)$, we obtain:

$$s(t) = \sum_{n=-\infty}^{+\infty} s_n e^{jn\omega_L t} = \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi/2}\cos(n\alpha_o)e^{jn\omega_L t} \tag{4}$$

where:

$$s_n = 1/T \cdot \int_{-T/2}^{+T/2} s(t)e^{-jn\omega_L t} = \frac{\sin(n\pi/2)}{n\pi/2}\cos(n\alpha_o) \tag{5}$$

Let us consider a periodical input signal at RF port, with frequency ω_R:

$$V_{RF}(t) = V_{RF}(\omega_R t + \Phi) \tag{6}$$

whose Fourier series expansion is given by:

$$V_{RF}(t) = \sum_{m=-\infty}^{+\infty} v_{Rm} e^{jm(\omega_L t+\Phi)} \tag{7}$$

Then Eq. 1 may be rewritten in the following form:

$$V_{IF}(t) = -\sum_{m,n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi/2} \cos(n\alpha_o) v_{Rm} e^{j[n\omega_L t + m(\omega_L t + \Phi)]} \tag{8}$$

This last expression, sometime reported in equivalent form, is called "the general mixer equation".

The resulting signal is a superposition of spectral lines, containing all the frequencies obtained by a linear combination (with integer coefficients) of the two fundamental frequencies ω_R and ω_L, except the even coefficients of ω_L, which correspond to null values of s_n (see Eq. 5).

2.2 Coherent input signals

Let us consider coherent input signals in the same hypotesis introduced above, i.e. $\omega_L = \omega_R \equiv \omega_o$. Then Eq. 8 becomes:

$$V_{IF}(t) = -\sum_{m,n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi/2} \cos(n\alpha_o) v_{Rm} e^{j[(n+m)\omega_o t + m\Phi]} \tag{9}$$

and the dc level V_o of this waveform, which can be obtained by the use of a low-pass filter (LPF), is given by:

$$\begin{aligned} V_o \equiv \overline{V_{IF}(t)} &= -\sum_{k=-\infty}^{+\infty} \frac{\sin(k\pi/2)}{k\pi/2} \cos(k\alpha_o) v_{Rk} e^{jk\Phi} = \\ &= -\sum_{k=0}^{+\infty} 2\frac{\sin(k\pi/2)}{k\pi/2} \cos(k\alpha_o) |v_{Rk}| \cdot \cos[k\Phi + Arg(v_{Rk})] \end{aligned} \tag{10}$$

which gives, for a general RF waveform, the dependence of the dc output level V_o from the phase offset Φ between the two input signals. Particularly, if the RF signal is a monochromatic one, i.e. :

$$v_{R\pm1} = A_R/2 \qquad v_{Rk} = 0 \quad \forall k \neq \pm 1 \tag{11}$$

Equation 10 becomes:

$$V_o = -\frac{2}{\pi} \cos(\alpha_o) A_R \cos(\Phi) \tag{12}$$

This simple result points out that, in the above specified conditions, the dc output level of a DBM depends on the RF signal amplitude and phase (referred to the LO signal); thus the device is suitable for homodyne systems and phase detection.
Looking at Eq. 12, we can note that the term $\cos(\alpha_o)$ is produced by the diode threshold V_γ. In fact from Eq. 3 we have:

$$\cos(\alpha_o) = \sqrt{1 - (V_\gamma/2A_L)^2} \qquad (13)$$

which, if $V_\gamma/A_L << 1$, becomes:

$$\cos(\alpha_o) \simeq 1 - 1/8(V_\gamma/A_L)^2 \qquad (14)$$

Equation 14 shows that the term $\cos(\alpha_o)$ gives a second order contribution to Eq. 12; this is because, if $V_\gamma/A_L << 1$, the LO signal crosses the cutoff region when its derivative has a maximum and the cross time is very short, compared to the signal period. Thus s(t) approaches a square wave, given by the istantaneous sign of the LO signal, i.e. :

$$s(t) \simeq sgn[\cos(\omega_L t)] \qquad (15)$$

This result is very important and it will be used in the following, in which the DBM will be supposed to perform like with ideal diodes ($V_\gamma = 0$).

3. Driving RF port with a high-level signal (of the order of the LO signal)

3.1 General approach

Up to this point we have considered a LO signal high enough to forward drive the diodes, together with a RF signal substantially smaller. When this last condition fails, the theoretical approach to the device behavior has to be reconsidered and, because of the previous results, we do that neglecting the diode threshold voltage.
If LO and RF signals are of the same order of magnitude, the diode conduction is determined by an istantaneous comparison between the two inputs. Moreover, it's easy to demonstrate that the diodes conduction status is controlled by the input having the istantaneous higher module.
When the RF signal prevails, the $D_1 - D_2$ and $D_3 - D_4$ pairs are alternatively switched on,

grounding nodes A and D for, respectively, negative and positive voltage values.
Conversely, when the LO signal prevails the $D_1 - D_4$ and $D_2 - D_3$ pairs are alternatively switched on, grounding nodes C and B for, respectively, negative and positive voltage values.
As a synthesis, the device behavior can be now shortly described by the following equation:

$$V_{IF}(t) = \begin{cases} -\,sgn[V_{LO}(t)] \cdot V_{RF}(t) & if \quad \left|\dfrac{V_{LO}(t)}{V_{RF}(t)}\right| > 1 \\ \\ -\,sgn[V_{RF}(t)] \cdot V_{LO}(t) & if \quad \left|\dfrac{V_{LO}(t)}{V_{RF}(t)}\right| < 1 \end{cases} \qquad (16)$$

3.2 Coherent input signals

Let us now consider two coherent sinusoidal input signals, having a phase offset Φ and comparable levels. i.e. :

$$V_{LO}(t) = A_L \cos(\omega_o t) \qquad (17)$$

$$V_{RF}(t) = A_R \cos(\omega_o t + \Phi) = kA_L \cos(\omega_o t + \Phi) \qquad (18)$$

where the dimensionless parameter k express the amplitude ratio between the RF on LO signals. While for $k \to 0$ the low-level approach is quite adequate, whenever the value of k increases, Eq. 16) becomes:

$$V_{IF}(\omega_o t, \Phi) = \begin{cases} -\,sgn[\cos(\omega_o t)]kA_L \cos(\omega_o t + \Phi) & if \quad \left|\dfrac{\cos(\omega_o t)}{\cos(\omega_o t + \Phi)}\right| > k \\ \\ -\,sgn[\cos(\omega_o t + \Phi)]A_L \cos(\omega_o t) & if \quad \left|\dfrac{\cos(\omega_o t)}{\cos(\omega_o t + \Phi)}\right| < k \end{cases} \qquad (19)$$

and the dc level $V_{o_k}(\Phi)$ of this waveform is given, as it is reported in the Appendix, by:

$$V_{o_k}(\Phi) \equiv \overline{V_{IF}(\omega_o t, \Phi)} = 1/T \cdot \int_{-T/2}^{+T/2} V_{IF}(\omega_o t, \Phi)dt = \frac{2}{\pi} A_R \cdot g_k(\Phi) \qquad (20)$$

where:

$$g_k(\Phi) \equiv -\frac{\sqrt{1 + k^2 + 2k\cos(\Phi)} - \sqrt{1 + k^2 - 2k\cos(\Phi)}}{2k} \qquad (21)$$

A typical IF waveform is presented in Fig. 4.
The $g_k(\Phi)$ function express the normalized response with respect to the RF level and therefore it gives the dependence of the shape of detection characteristic from the value of the k parameter.

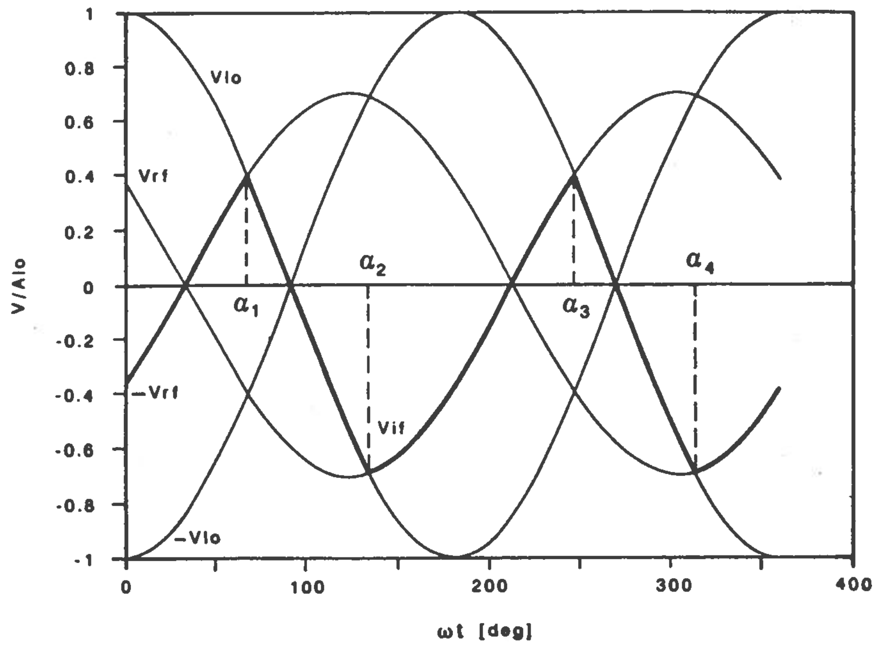

Fig. 4: IF output waveform for large RF signals

The function plot is presented in Fig. 5, for two different values of k (0 and 1 respectively); since the function $g_k(\Phi)$ is an even one, i.e. $V_{o_k}(-\Phi) = V_{o_k}(\Phi)$, the plot is reported only for the $[0, \pi]$ Φ range.
For any value of k, inside the 0 to 1 range, the $g_k(\Phi)$ function intercepts the points $(0, -1)$, $(\pi/2, 0)$ and $(\pi, 1)$, the k parameter being just the controller of the fitting plot shape.
When RF signal amplitude is substantially smaller than the LO one, we are in the $k \rightarrow 0$ case and then Eq. 21) becomes:

$$g_{k \rightarrow 0} = -\cos(\Phi) \qquad (22)$$

which is equal to Eq. 12 when $\alpha_o \simeq 0$.
Conversely, increasing the value of the k parameter, the device characteristic curve becomes

more linear, progressively approaching a triangle waveform. This behavior is quite agreeable because it performs a linear region broadening.
As a matter of fact, computing the characteristic derivative, we obtain:

$$\frac{dg_k}{d\Phi} = \frac{\sin(\Phi)}{2}\left(\frac{1}{\sqrt{1+k^2+2k\cos\Phi}} + \frac{1}{\sqrt{1+k^2-2k\cos\Phi}}\right) \qquad (23)$$

which, independently from k, and has a maximum for $\Phi = \pi/2$ given by:

$$\frac{dg_k}{d\Phi}(\pi/2) = \frac{1}{\sqrt{1+k^2}} \qquad (24)$$

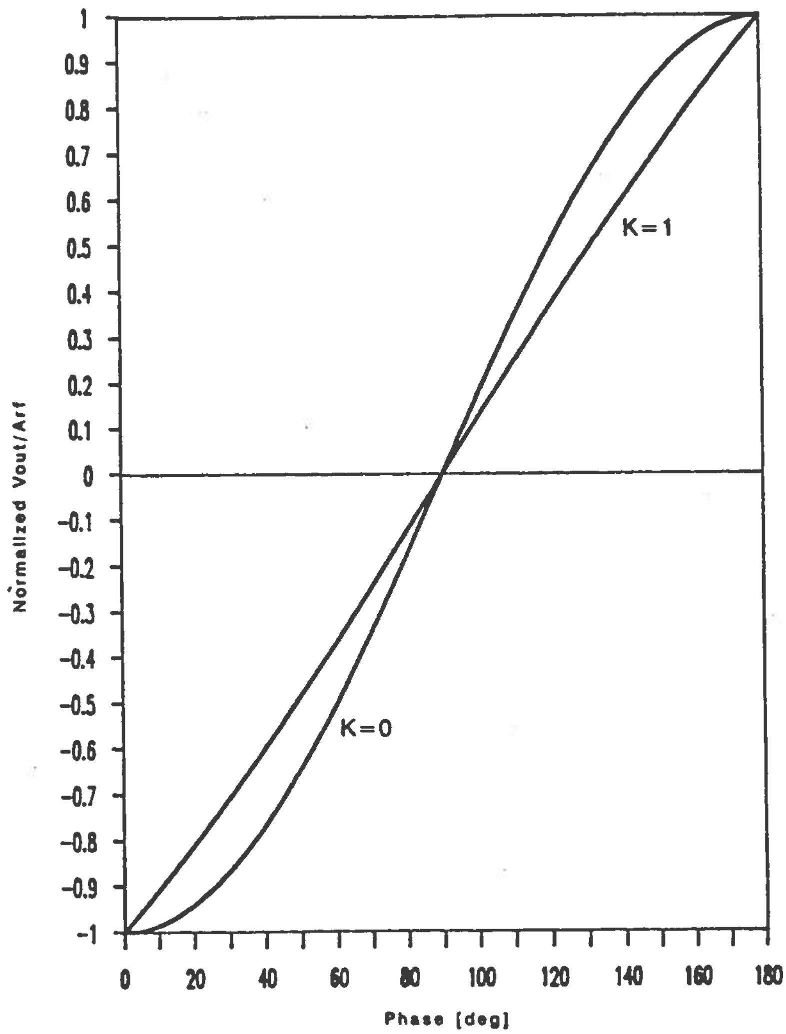

Fig. 5: $g_k(\Phi)$ plot

Finally, if we fix for phase detection a maximum allowable linearity error ϵ_l (defined as the ratio between the derivative deviation from maximum and the maximum value itself), the instantaneous working point of the DBM has to be hold inside an interval (centered on the $\Phi = \pi/2$) having a width $\Delta\Phi_k(\epsilon_l)$, which depends on ϵ_l.

It's easy to demonstrate that the linear region width depends on k. Particularly for k=0 and k=1 is given respectively by:

$$\Delta\Phi_0 = 2\sqrt{2\epsilon_l} \quad [rad] \tag{25}$$

$$\Delta\Phi_1 = 4\sqrt{2\epsilon_l} \quad [rad] \tag{26}$$

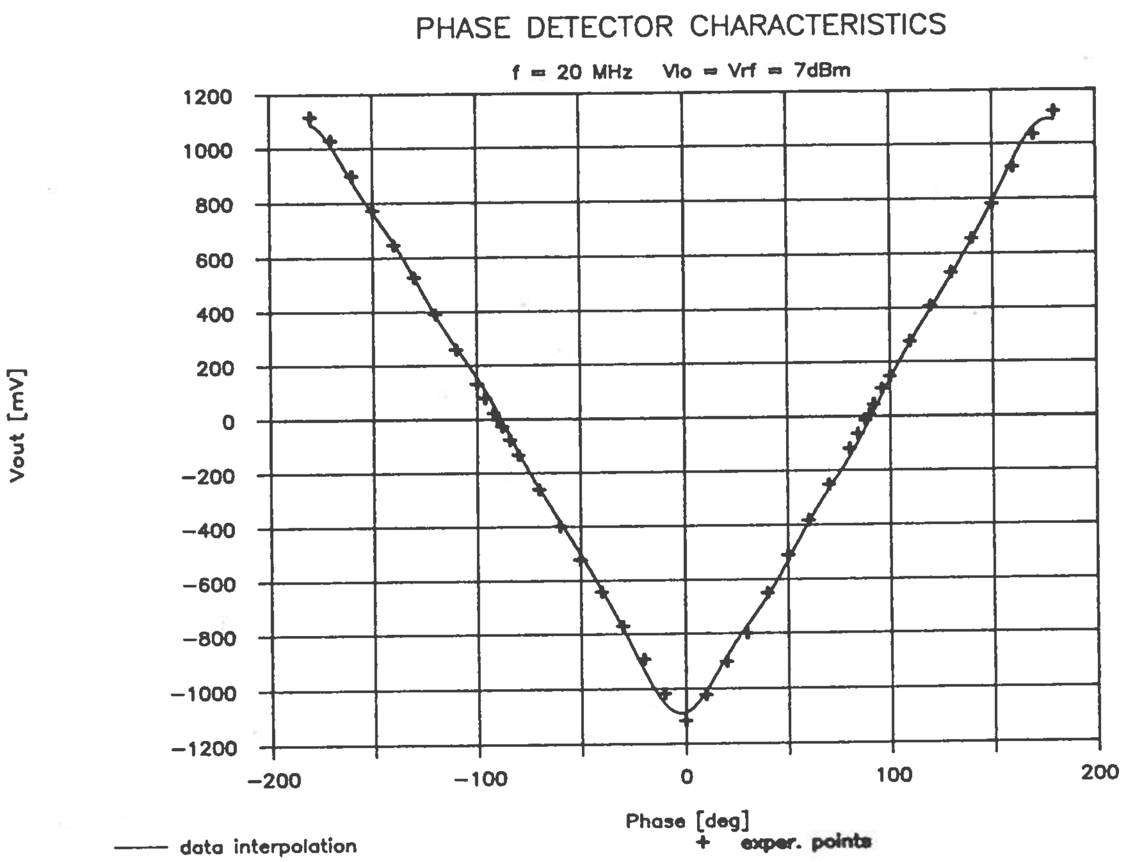

Fig. 6: DBM experimental characteristic for $k \simeq 1$

We can conclude that from k=0 to k=1 the linearity range is increased by a factor of two; hence, using equal amplitudes for LO and RF signals, the device linearity for phase detection is considerably improved.
The experimental characteristic reported in Fig. 6) shows this linearization effect when the two input signals have the same amplitude (k=1).

Appendix

In this appendix we want to show how to get Eqs. 20 and 21 from Eq. 19, that express the DBM output voltage $V_{IF}(\omega_o t, \Phi)$, when comparable input levels are employed.
Let us consider the dc output level, i.e. the mean value of the IF voltage over a whole period of the output signal itself.
Looking at the Eq. 19) it's easy to demonstrate that the IF waveform has a frequency twice the value of the input frequency ω_o, i.e. :

$$V_{IF}(\omega_o t + \pi, \Phi) = V_{IF}(\omega_o t, \Phi) \tag{27}$$

Then the IF dc level is given by:

$$V_o(\Phi) = \frac{1}{\pi} \cdot \int_{\omega_o t}^{\omega_o t + \pi} V_{IF}(\omega_o t, \Phi) d(\omega_o t) \tag{28}$$

Still looking at Eq. 19 it is also evident that:

$$V_{IF}(\omega_o t, \Phi \pm \pi) = -V_{IF}(\omega_o t, \Phi) \tag{29}$$

Then it is possible to limit the calculation inside the $[0, \pi]$ range of the Φ variable and subsequently to extend the result at the remaining Φ values.
The $V_{IF}(\omega_o t, \Phi)$ plot changes abruptly around the 4 points called α_1, α_2, α_3 and α_4, as can be see in Fig. 4.
The 4 values are solutions of the following trigonometrical equations:

$$\begin{aligned} &\frac{k\cos(\alpha + \Phi)}{\cos\alpha} = -1 \Rightarrow \tan\alpha = \frac{1 + k\cos\Phi}{k\sin\Phi} \Rightarrow \\ &\Rightarrow \alpha_1 = \arctan\frac{1 + k\cos\Phi}{k\sin\Phi} \qquad \alpha_3 = \alpha_1 + \pi \end{aligned} \tag{30}$$

$$\frac{k\cos(\alpha+\Phi)}{\cos\alpha} = 1 \Rightarrow \tan\alpha = -\frac{1-k\cos\Phi}{k\sin\Phi} \Rightarrow$$
$$\Rightarrow \alpha_2 = \pi - \arctan\frac{1-k\cos\Phi}{k\sin\Phi} \qquad \alpha_4 = \alpha_2 + \pi \tag{31}$$

Now calculating the integral in the Eq. 28, using the interval $[\alpha_1, \alpha_3]$ as integration range, we obtain:

$$V_o(\Phi) = (A/\pi)\cdot\left[\int_{\alpha_1}^{\alpha_2}\cos\alpha\, d\alpha + k\int_{\alpha_2}^{\alpha_1+\pi}\cos(\alpha+\Phi)d\alpha\right] =$$
$$= -(A/\pi)[k\sin(\alpha_2+\Phi) + k\sin(\alpha_1+\Phi) + \sin\alpha_1 - \sin\alpha_2] = -(A/\pi)[k\sin\alpha_2\cdot \tag{32}$$
$$\cdot\cos\Phi + k\cos\alpha_2\sin\Phi + k\sin\alpha_1\cos\Phi + k\cos\alpha_1\sin\Phi + \sin\alpha_1 - \sin\alpha_2] =$$
$$= -(A/\pi)[-\sin\alpha_2(1-k\cos\Phi) + \sin\alpha_1(1+k\cos\Phi) + k\sin\Phi(\cos\alpha_1 + \cos\alpha_2)]$$

Directly from Eqs. 27 and 28 with simple trigonometrical manipulations, we obtain:

$$\sin\alpha_1 = \frac{1+k\cos\Phi}{\sqrt{1+k^2+2k\cos\Phi}} \quad \cos\alpha_1 = \frac{k\sin\Phi}{\sqrt{1+k^2+2k\cos\Phi}}$$
$$\sin\alpha_2 = \frac{1-k\cos\Phi}{\sqrt{1+k^2-2k\cos\Phi}} \quad \cos\alpha_2 = -\frac{k\sin\Phi}{\sqrt{1+k^2-2k\cos\Phi}} \tag{33}$$

Then, taking into account Eq. 30, Eq. 32 becomes:

$$V_o(\Phi) = -(A/\pi)\Big[\frac{(1+k\cos\Phi)^2}{\sqrt{1+k^2+2k\cos\Phi}} - \frac{(1-k\cos\Phi)^2}{\sqrt{1+k^2-2k\cos\Phi}} + (k\sin\Phi)^2\cdot$$
$$\cdot\left(\frac{1}{\sqrt{1+k^2+2k\cos\Phi}} - \frac{1}{\sqrt{1+k^2-2k\cos\Phi}}\right)\Big] =$$
$$= -(A/\pi)\left(\frac{1+k^2+2k\cos\Phi}{\sqrt{1+k^2+2k\cos\Phi}} - \frac{1+k^2-2k\cos\Phi}{\sqrt{1+k^2-2k\cos\Phi}}\right) = \tag{34}$$
$$= -\frac{2}{\pi}kA\frac{\sqrt{1+k^2+2k\cos\Phi} - \sqrt{1+k^2-2k\cos\Phi}}{2k} = \frac{2}{\pi}\cdot A_R\cdot g_k(\Phi)$$

with:

$$g_k(\Phi) \equiv -\frac{\sqrt{1+k^2+2k\cos\Phi} - \sqrt{1+k^2-2k\cos\Phi}}{2k}$$

that is Eq. 21, which we had to demonstrate in this appendix.

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