INFN - Istituto Nazionale di Fisica Nucleare
Sezione di Genova

INFN/TC-88/22
28 Giugno 1988

Self field effects in the critical current measurements of S.C. cables

P. Fabbricatore, P. Fernandes, R. Parodi, R. Vaccarone
INFN - Sezione di Genova, Via Dodecaneso 33, 16146 Genova (Italy)

SELF FIELD EFFECTS IN THE CRITICAL CURRENT MEASUREMENTS OF S.C. CABLES

P. Fabbricatore,P.Fernandes*,R. Parodi and R. Vaccarone
I.N.F.N. Sezione di Genova
Via Dodecaneso 33, 16146 GENOA,ITALY
* I.M.A. C.N.R. Genova

INTRODUCTION

On measuring the critical current of S/C conductors some difficulties arise in the evaluation of the real field at which the critical current is measured.

The field produced by the current flowing in the conductor must be added to the external magnetic field, almost uniform on the conductor width. The resulting magnetic field on the conductor can be highly nonuniform, specially for that cables carrying high current (multi Kamperes cables for High Energy Physics application). In the present work the "Self Field Effect" is studied in detail. The problem of the real field ,at which the critical current measurement is performed, is solved. Furthermore an electrical model is developped (2 Filaments Model) showing the self field influence on the n-value.

BASIC RELATIONS

In this section some useful relations are recalled from literature.

In the region from 5 to 8 Tesla , at a fixed temperature T, the critical current is well described by a linear dependance on the magnetic field:

$$I_c(B) = a_1(T)B_{app} + a_2(T) \tag{1}$$

This equation can be re-written in a more general form considering the empirical scaling formulas found by Lubell[1]. According to these formulae the critical current density is described by:

$$J_c(B, T = 4.2) = J_0\ (1 - a_3 B) \tag{2}$$

where a_3 is a constant with a value of 0.096 in MKS.

The formula for the temperature variation is found to be:

$$J_c(B = const., T) = J'_0(1 - a_4(B)T) \tag{3}$$

Where $a_4 = \frac{1}{T_c(B))} = \frac{1}{9.2(1-\frac{B}{14.5})^{.59}}$; a_4 has a value of 0.174 at B=8 Tesla and 0.139 at B=5 Tesla. From equations (2) and (3) it is possible to find the general relations:

$$J_c(B,T) = J_0\{[1 - (T - T_0)\ F(B,T_0)] - a_3 B\} \tag{4}$$

Where T_0 is a reference temperature and

$$F(B, T_0) = \frac{a_4(1 - a_3 B)}{1 - a_4 T_0} \tag{5}$$

Putting T_0=4.2 K, it results a_3=.096 and consequently F(B,T=4.2) ranges from 0.171 at B=5T to 0.150 at B=8T.

Considering that the critical current measurements are performed in the temperature range 4.2-4.6 K, the maximum error on the critical current value by putting F(B,T=4.2)=F_c=const=0.16 is about 2%. Using this approximation (5) becames:

$$J_c(B,T) = J_0\ [(1 - F_c(T - T_0) - a_3 B] \tag{6}$$

formally equal to (1) if a_1=const=-.096 and $a_2(T)$=1-F_c(T-T_0).

In the following relation (6) will be used under the form

$$J_c(B,T) = J_0[a_2(T) - a_3 B] \tag{7}$$

FIRST ORDER CORRECTION OF SELF FIELD

In order to have a better understanding of the procedure utilized to correct the measured critical current, the analized situation is shown on a graphic (Fig.1).

On the ordinate axis the current fed into the sample is represented, on the ascissa the magnetic field. At a fixed temperature the characteristic curve $I_c=I_c(B)$ is a straight line as given from (6) . At a fixed field the critical current is I_c^*, but the measured value is different due to the self field, that can be represented by the "load line" l_{sf}; the measured critical current is I_{cr} at a real applied field $B_{applied}=B_{external}+B_{self\ field}$. The above discussion can be drawn up analitically as follows. As first approximation the considered self field is the maximum one. It is given by the relation:

$$B_s fm = \frac{I_t}{a_5} \tag{8}$$

being I_t the current carried by the sample and a_5 the slope. From (8):

$$I_t = (B_{app} - B_{ext})a_5 \tag{9}$$

At the intersection between the critical field load line and the curve $I_c=I_c(B)$, it results $I_t=I_c$, then B_{app} and I_{cr} are calculated:

$$B_{app} = \frac{I_0 a_2 + a_5 B_{ext}}{a_5 + a_3 I_0} \tag{10}$$

Where $I_0=J_0$ x (S/C cross section) is used.

$$I_{cr} = \frac{I_c^*}{1 + \frac{a_3 I_0}{a_5}} \tag{11}$$

The percentual correction to obtain I_c^* from I_{cr} is:

$$\epsilon = \frac{I_c^* - I_{cr}}{I_{cr}} \tag{12}$$

From (11) and (12)

$$\epsilon = \frac{a_3 I_0}{a_5} \tag{13}$$

This is an important result because the correction is not depending on the temperature, the applied field or a particular self field value , but only on the critical current vs field slope and the self field slope. It is interesting to perform the calculation of ϵ for a simple geometry as a round cable in a configuration in which the self field is not compensated (hair-pin, long or short sample, coil); in these cases:

$$B_{sfm} = \frac{\mu_0}{2\pi}\frac{I_t}{r_0} \tag{14}$$

r_0 is the outer radius of the conductor. From (13) and (14)

$$\epsilon = \frac{\mu_0}{2\pi}\frac{a_3 I_0}{r_0} \tag{15}$$

Not to loose generality it is convenient to introduce again the characteristic current density $J_0 = \frac{I_0(1-\lambda)}{\pi r_0^2}$ where λ is the $\frac{s.c}{matrix}$ ratio. Then:

$$\epsilon = \frac{\mu_0}{2}\frac{J_0 a_3 r_0}{(1-\lambda)} \tag{16}$$

A reasonable value for $J_0 a_3$ is about 40000 $\frac{A}{cm^2}$, so that for a wire with $\lambda = 1.8$ the self field compensation is $\epsilon = 90 r_0$. For a wire 1 mm in diameter the correction is about 4.5%.

EFFECTIVE FIELD AT THE CONDUCTOR

The following arguments will show that the peak field is not the right value at which the critical current is measured.

The system considered is a round multifilamentary wire, in which the superconducting filaments are placed in a single shell (typical conductor for NMR application).

The filaments are twisted on a diameter d for a characteristic lenght l_p .

A outer field B_{ext} is applied normal to the wire. At this field the self-field produced by the current flowing in the filaments must be added.

For simplicity the self field is supposed to be generated by a current uniformly distributed on the whole cross section of the wire, as shown in Fig.2.

$$B_{app} = [(B_{ext} + B_{self\parallel})^2 + B_{self\perp}^2]^{1/2} \tag{17}$$

Being $B_{self\parallel}$ and $B_{self\perp}$ the components of the self field parallel and normal to the applied field. In the most of the cases the term $B_{self\perp}$ can be neglected. Due to the twist, the term $B_{self\parallel}$ on a single filament changes moving along the wire (say x direction); for a round wire this component can be written:

$$B_{\|} = \Delta B \; sin \frac{2\pi x}{l_p} \tag{18}$$

ΔB is the maximum self field.

Supposing relation (18) to be valid for a certain filament, the same self field component for the opposite filament is:

$$B_{\|} = -\Delta B \; sin \frac{2\pi x}{l_p} \tag{19}$$

On a twist each filament experiences the same field variation starting from different values. We consider the filament in which the maximum self field occurs at $x=l_p/4$ i.e. that filament for which the field is described by (18).

We now suppose that the resistance per unit lenght is described by:

$$R = \alpha I^n \tag{20}$$

The critical value occurs at I_c, where V_c is measured

$$R_c = \alpha \; I_c^n \tag{21}$$

Introducing the reduced current $i=I/I_c$, from (20) and (21) it results:

$$R = R_c \; i^n \tag{22}$$

From the equation (6) :

$$I_c(B,T) = I_0 \; [a_2(T) - a_3 B] \tag{23}$$

Having defined $I_0=J_0$ x (S/C cross section). The field B is written as $B=B_{ext}+B_{\|}$ then it results:

$$I_c(B,T) = I_0[a_2 - a_3 B_{ext} - a_3 \Delta B sin \frac{2\pi x}{l_p}] \tag{24}$$

From which:

$$I_c = I_0(a_2 - a_3 \; B_{ext})(1 - \frac{a_3 \; \Delta B}{a_2 - a_3 \; B_{ext}} \; sin \frac{2\pi x}{l_p}) \tag{25}$$

Designing $K = \frac{a_3 \; \Delta B}{a_2 - a_3 \; B_{ext}}$ the critical current at the applied field can be referred to the critical current at the external field:

$$I_c(B_{app}, T) = I_c(B_{ext}, T)(1 - K\ sin\frac{2\pi x}{l_p}) \tag{26}$$

We remark that if the self field is considered constant, from (12) and (26) it results

$$\epsilon = \frac{K}{1-K} \tag{27}$$

i.e. the constant K is just about the percentual correction to the critical current at the maximum self field. From (26) and (22) it is possible to calculate the integrated voltage on a twist pitch ;this value corresponds to the measured voltage per unit lenght:

$$V_{meas} = \frac{1}{l_p}\int_0^{l_p} R(x) I\ dx \tag{28}$$

Substituting the relation for R :

$$V_{meas} = \frac{1}{l_p}\int_0^{l_p} \frac{V_c i^{(n+1)}}{(1 - K\ sin\frac{2\pi x}{l_p})^n}\ dx \tag{29}$$

V_c is defined as the critical voltage per unit lenght measured at $I=I_c(B_{ext},T)$so that it is a constant. Changing to $y=\frac{2\pi x}{l_p}$ (29) becames

$$V_{meas} = \frac{V_c i^{(n+1)}}{2\pi}\int_0^{2\pi} \frac{1}{(1 - K\ sin\ y)^n}\ dy \tag{30}$$

The function

$$G(K) = \int_0^{2\pi} \frac{1}{(1 - K\ sin\ y)^n}\ dy \tag{31}$$

is shown in fig.3 for several values of field and self field. The self consistence is guaranteed considering that in the limiting situation $B_{self\ field}$=0 (then K=0), so it results G(K)=2 π and the measured voltage is just the same measured for a single filament.

Being G(K)$\geq 2\pi$, a lower value of the critical current is measured :

$$I_{c\ new} = \frac{V_{meas}}{R_c} = I_{c\ old}\ (\frac{2\pi}{G(K)})^{\frac{1}{n}} \tag{32}$$

It is interesting to calculate the critical field B_{eff} at which $I_{c\ new}$ is measured. This field must be greater than B_{ext} and can be written as a sum of the applied field and an effective self field $\Delta B^{\star}$:

$$B_{eff} = B_{ext} + \Delta B^{\star} \tag{33}$$

From (26),(32):

$$I_c(B_{eff}) = I_c(B_{ext})(1 - K^{\star}) \tag{34}$$

Where $K^{\star} = \frac{a_3}{a_2 - a_3} \frac{\Delta B^{\star}}{B_{ext}}$ From (32) and (33) it results:

$$K^{\star} = 1 - (\frac{2\pi}{G(K)})^{\frac{1}{n}} \tag{35}$$

and $\Delta B^{\star}$ can be calculated:

$$\Delta B^{\star} = \frac{1}{a_3}[1 - \frac{2\pi}{G(K)}^{\frac{1}{n}}](a_2 - a_3 B_{ext}) \tag{36}$$

In Fig.4 the effective self field vs the maximum self field is shown.

From these considerations it is found that the self field correction $\epsilon^{\star}$ depends on the external field and the self field values; it is given from (27):

$$\epsilon^{\star} = \frac{K^{\star}}{1 - K^{\star}} = [\frac{G(K)}{2\pi}]^{\frac{1}{n}} - 1 \tag{37}$$

It must be remarked that while relation (29) is valid only for a round cable, because the sine dependance, the results (31) to (37) are general.

EFFECTS DUE TO THE CURRENT TRANSFER

In this section we will investigate the effects due to exchanges of current between filaments. For simplicity the system under consideration is the previous one (one shell of filaments) with only two filaments. The basic idea is that the filament in the zone at higher field (the more resistive) can give current to the other filament (less resistive). The situation is reversed moving along the wire; after half a twist the opposite condition occurs.

Indeed the results that will be obtained are also valid for a cable composed by 2 strands, because the system is composed by 2 interacting objects, phisically caracterized by having a critical current I_c and a n-value . To study this situation, the system can be simulated using a network as in Fig.5. The footindex 1 or 2 relate to the two filaments; the position is x for the considered loop, $x - \Delta x$ *and* $x + \Delta x$ for the two neighbouring rings of the chain. The filament denoted by '1' is characterized by having the maximum self field at x=$\frac{l_p}{4}$; its resistance per unit lenght is (from 22) :

$$R_1(x) = \frac{V}{I} = R_c \, (\frac{i_1}{1 - K \, sin\frac{2\pi x}{l_p}})^n \tag{38}$$

The opposite filament has a resistance per unit lenght:

$$R_2(x) = \frac{V}{I} = R_c \, (\frac{i_2}{1 + K \, sin\frac{2\pi x}{l_p}})^n \tag{39}$$

The transverse electrical resistance for a single element $\Delta \; x$ is $R_t = \frac{\rho_t \; d_l}{\Delta x \; d_t}$; we introduce a shape factor $\delta = \frac{d_l}{d_t}$ equal to unity for a round cable and a value greater than zero for a Rutherford cable with B_{ext} normal to the wide face (δ equal to the ratio between larger and shorter side).

Applying the Kirchoff's laws the following equations are written:

$$I_1(x - \Delta x) = I_1(x) + I_{t12}(x) \tag{40}$$

$$I_2(x - \Delta x) = I_2(x) + I_{t21}(x) \tag{41}$$

$$I_1(x) = I_1(x + \Delta x) + I_{t12}(x + \Delta x) \tag{42}$$

$$I_2(x) = I_2(x + \Delta x) + I_{t21}(x + \Delta x) \tag{43}$$

$$\begin{aligned} R_1(x) \; \Delta x \; I_1(x) \; + \; R_t \; [I_{t12}(x + \Delta x) \; - \; I_{t21}(x + \Delta x)] \; - \\ R_2(x) \; \Delta x \; I_2(x) \; + \; R_t \; [I_{t21}(x) \; - \; I_{t12}(x)] = 0 \end{aligned} \tag{44}$$

From (40) to (44):

$$\begin{aligned} R_1(x) \; \Delta x \; I_1(x) \; - \; R_2(x) \; \Delta x \; I_2(x) \; - \\ R_t \; [[(I_1(x + \Delta x) - I_1(x)) \; - (I_1(x) - I_1(x - \Delta x))] \; - \\ [(I_2(x + \Delta x) - I_2(x)) \; - (I_2(x) - I_2(x - \Delta x))] = 0 \end{aligned} \tag{45}$$

For $\Delta x \rightarrow 0$, putting $I_1(x) + I_2(x) = I(x)$ =constant and assuming $\frac{d^2 I_1}{dx^2} = -\frac{d^2 I_2}{dx^2}$ (45) becames:

$$2 \; \rho_t \delta \; \frac{d^2 I_1}{dx^2} - [R_1(x) \; + \; R_2(x)] \; I_1(x) \; + \; R_2(x) \; I(x) = 0 \tag{46}$$

Substituting in (46) the expressions for R(x) as in (38) and (39), then rearranging:

$$\frac{d^2 I_1}{dx^2} - \frac{\rho_c}{2\,\rho_t \delta\, A_{sc}} \frac{[\, i_1^{n+1}\,(1 + K\, sin\frac{2\pi x}{l_p})^n \;-\; i_2^{n+1}\,(1 - K\, sin\frac{2\pi x}{l_p})^n]}{(1 - K^2\, sin^2 \frac{2\pi x}{l_p})^n} = 0 \quad (47)$$

The reduced current i_2 can be replaced by $i - i_1$ ($i = I/I_c$) and then, to symmetrize the equation a new variable "f" is introduced so that:

$$i_1 = i/2 \;-\; f \quad and \quad i_2 = i/2 \;+ f \qquad (48)$$

The new variable represents the quantity for which the current flowing trought a filament is different from half of whole current. Furthermore an effective cross section is introduced $A_{ef} = \frac{2 A_{sc} \rho_t \delta}{\rho_c}$ .Re-writing (48):

$$\frac{d^2 f}{dx^2} + \frac{1}{A_{ef}} (\frac{i}{2})^{n+1}$$

$$\frac{[\, (1 - \frac{2\,f}{i})^{n+1}\,(1 + K\, sin\frac{2\pi x}{l_p})^n \;-\; (1 + \frac{2\,f}{i})^{n+1}\,(1 - K\, sin\frac{2\pi x}{l_p})^n]}{(1 - K^2\, sin^2 \frac{2\pi x}{l_p})^n} = 0 \qquad (49)$$

This non-linear differential equation can be solved numerically, but for a better understanding of the physical aspects of the problem , an analytic solution can be tried with some approximation.

For K≪ 1 it is possible to develope in a Taylor serie the terms:

$$(1 \pm K\; sin\frac{2\pi x}{l_p})^n \; and \; (1 - K^2\; sin\frac{2\pi x}{l_p})^n$$

and stopping at the first 2 terms, they are changed to:

$$(1 \pm K\; nsin\frac{2\pi x}{l_p}) \; and \; (1 - n\; K^2\; sin\frac{2\pi x}{l_p})$$

The denominator of right side of (49) can be changed completely with unity.

The same can be made for the term:

$$(1 \pm \frac{2\,f}{i})^{n+1}$$

This means that relatively little current exchanges are allowed .

Using these approximations and re-arranging (49) is changed to:

$$\frac{d^2 f}{dx^2} - \frac{2}{A_{ef}} (\frac{i}{2})^n (n+1) f + \frac{2}{A_{ef}} (\frac{i}{2})^{n+1} K n \sin \frac{2 \pi x}{l_p} = 0 \tag{50}$$

A harmonic solution can be found putting the limiting conditions:

$$f(x=0) = f(x=l_p) = 0 \tag{51}$$

which means that no transfer current is between filaments in the regions where they experience the same field.Resulting:

$$f = \frac{\frac{2}{A_{ef}} \frac{i}{2}^{n+1} K n}{\frac{2}{A_{ef}} \frac{i}{2}^n (n+1) + \frac{4 \pi^2}{l_p^2}} \sin \frac{2 \pi x}{l_p} \tag{52}$$

From which the reduced current in the filament 1 is found:

$$i_1 = \frac{i}{2} (1 - \frac{\frac{2}{A_{ef}} \frac{i}{2}^n K n}{\frac{2}{A_{ef}} \frac{i}{2}^n (n+1) + \frac{4 \pi^2}{l_p^2}} \sin \frac{2 \pi x}{l_p}) \tag{53}$$

It is now possible calculate the voltage drop along a twist :

$$V = \frac{1}{l_p} \int_0^{l_p} R_1(x) I_1(x) dx \tag{54}$$

From expression for R_1 as in (38) and from (54) it results:

$$V = \frac{V_c}{l_p} (\frac{i}{2})^{n+1} \int_0^{l_p} (1 - \frac{\frac{2}{A_{ef}} (\frac{i}{2})^n K n}{\frac{2}{A_{ef}} (\frac{i}{2})^n (n+1) + \frac{4\pi^2}{l_p^2}} \sin \frac{2 \pi x}{l_p})^{n+1} \frac{dx}{(1 - K \sin \frac{2\pi x}{l_p})^n} \tag{55}$$

Having stated that little current exchange are allowed between the two filaments we can approximate:

$$i_1^{n+1} = \frac{i}{2} [1 - (n+1) \frac{\frac{2}{A_{ef}} (\frac{i}{2})^n K n}{\frac{2}{A_{ef}} \frac{i}{2}^n (n+1) + \frac{4 \pi^2}{l_p^2}} \sin \frac{2 \pi x}{l_p}] \tag{56}$$

So that the voltage per unit lenght is:

$$V = \frac{V_c}{2\pi} \frac{i}{2}^{n+1} [G_1(K) - G_2 \frac{\frac{2}{A_{ef}} (\frac{i}{2})^n K (n+1) n}{\frac{2}{A_{ef}} (\frac{i}{2})^n (n+1) + \frac{4\pi^2}{l_p^2}}] \tag{57}$$

$G_1(K)$ is just G(K) previously described, while $G_2(K)$ is:

$$G_2 = \int_0^{2\pi} \frac{sin\ y}{(1 - K\ sin\ y)^n}\ dy \qquad (58)$$

Where the new variable $y = \frac{2\pi\ x}{l_p}$ has been introduced

Some interesting considerations arise from (58)

(1) If no current transfer between filaments (or strands) occurs due to a high ρ_t value, the term of (58) multiplied to G_2 vanishes and the current dependance of the voltage is that described by (29).

(2) The current transfer acts towards a reduction of the voltage drop ; the reduction is increasing with the current so that there is also an effect on the n-value.

(3) There is a further variation of the self field correction now given by:

$$\epsilon^{**} = [\frac{G_1(K) - A(K, n, l_p, A_{ef})G_2(K)}{2\pi}]^{\frac{1}{n}} - 1 \qquad (59)$$

APPLICATIONS OF THE DEVELOPED MODELS

Summarizing the obtained results, there are three levels for the self field correction:

1) The maximum self field is considered

In this case the self field correction is field and temperature indipendent;it is defined by relation (13). The application field is every superconducting wire or cable.The correction is over extimated.

2) The effective self field is considered.

The self field correction depends on the values of the external and self fields applied. For a cable having a single shell of strands (Rutherford type) or filaments (NMR wire) the correction is given from (37) The function G(K) is described in (31) for a round cable.

3) Transfers of current between filaments are considered

In this case the self field correction depends on several phisical quantity.The n-value is changed. For a 2 filaments (strands) cable the self field correction is represented by relation (59)

Using these models an attempt is made to describe the behaviour of the cable used in the dipoles for the HERA ring, considering as starting point the characteristics of the single strand.

The cable has a trapezoidal shape, with height 10 mm and bases 1.67,1.28 mm. It is compoused by 24 strands 0.84 mm in diameter ,critical current as in Table I and a n-value of about 40. The evaluation of the field requires a description of the experimental set-up. In a first stage we consider a single straight cable, placed in a field normal to the wide face . In Fig.6 the iso-field lines at the cable are shown; the external field is 5 Tesla, the transport current 8000 A. The maximum self field is .53 Tesla so that the self field slope is $a_5 = 1.50910^4$ A/Tesla. The slope I_0a_3 is directly found by Table I to be 24 x 105=2550 A/Tesla.From (13) it is found that the self field correction at maximum field $\epsilon = 16.8\%$ To evaluate the effective self field, the function G(K) must be calculated. In Fig. 7 the $B_{||}$ component of the self field is shown (tick line),compared with a function (thin line) $B=B_{||\ max} sin^3(\frac{2\pi x}{l_p} + \psi)$ being ψ a phase due to the trapezoidal geometry. In the next calculations the sine function approximation is used, so that for a fixed experimental set-up only the maximum self field is required to be known . Furthermore no physical meaning is lost putting $\psi = 0$. In Fig.8 the function G(K) for this cable is shown, while in Fig.9 the calculated effective self field is shown.

At a fixed external field the maximum applied self field must be calculated using (10), then the effective self field is drawn from Fig.9 and the self field correction is calculated either by (13) with a_5 slope calculated from effective self field, or directly from (37) using G(K) of Fig.8. In Table II the maximum self field,the effective one and the correction are shown for different values of the external field. In the last step the transfer current effects are taken into account. In order to have a comparison with experimental results , the calculation is performed not only for the single straight cable but also for a real experimental configuration[2,3] in which the maximum self field is .47 Tesla at 8000 A ($a_5 = 1.7\ 10^5$).

Equation (49) is solved numerically using a routine of NAG library (D02GAF).

In Fig.10 the foreseen Voltage-current characteristic for the straight is shown and compared with the situations of (a) no self field and (b) effective self field.In Fig.11 the plot LogV vs. LogI is shown in the three cases. The interesting result is that the n-value (the slope of curves) at the beginning of the transition is just equal to the one of the single strand, but increasing the current it is lowered.

In TABLE III the values of the self field correction are shown.

In Fig. 12 and 13 the results for the real configuration at an external field of 6.4 Tesla . In Fig.14 the measured characteristic. Some interesting considerations arise from (58)

(1) If no current transfer between filaments (or strands) occurs due to a high ρ_t value, the term of (58) multiplied to G_2 vanishes and the current dependance of the voltage is that described by (29).

(2) The current transfer acts toward a reduction of the voltage drop ; the reduction is increasing with the current so that there is also an effect on the n-value.

(3) There is a further variation of the self field correction now given by:

$$\epsilon^{**} = [\frac{G_1(K) - A(K,n,l_p,A_{ef})G_2(K)}{2\pi}]^{\frac{1}{n}} - 1 \qquad (59)$$

APPLICATIONS OF THE DEVELOPED MODELS

Sumarizing the obtained results, there are three levels for the self field correction:

1) The maximum self field is considered

In this case the self field correction is field and temperature indipendent;it is defined by relation (13). The application field is every superconducting wire or cable.The correction is over extimated.

2) The effective self field is considered.

The self field correction depends on the values of the external and self fields applied. For a cable having a single shell of strands (Rutherford type) or filaments (NMR wire) the correction is given from (37) The function G(K) is described in (31) for a round cable.

3) Transfers of current between filaments are considered

In this case the self field correction depends on several phisical quantity.The n-value is changed. For a 2 filaments (strands) cable the self field correction is represented by relation (59)

Using these models an attempt is made to describe the behaviour of the cable used in the dipoles for the HERA ring, considering as starting point the characteristics of the single strand.

The cable has a trapezoidal shape, with height 10 mm and bases 1.67,1.28 mm. It is compoused by 24 strands 0.84 mm in diameter ,critical current as in Table I and a n-value of about 40. The evaluation of the applied field requires a description of the experimental set-up. In a first stage we consider a single straight cable, placed in a field normal to the wide face . In Fig.6 the iso-field lines at the cable are shown; the external field is 5 Tesla, the transport current 8000 A. The maximum self field is .53 Tesla so that the self field slope is $a_5 = 1.50910^4$ A/Tesla. The slope $I_0 a_3$ is directly found by Table I to be 24 x 105=2550 A/Tesla.From (13) it is found that the self field correction at maximum field $\epsilon = 16.8\%$ To evaluate the effective self field, the function G(K) must be calculated. In Fig. 7 the $B_{||}$ component of the self field is shown (tick line),compared with a function (thin line) $B=B_{||\ max} sin^3(\frac{2\pi x}{l_p} + \psi)$ being ψ a phase due to the trapezoidal geometry. In the next calculations the sine^3 function approximation is used, so that for a fixed experimental set-up only the maximum self field is required to be known . Furthermore no physical meaning is lost putting $\psi = 0$. In Fig.8 the function G(K) for this cable is shown, while in Fig.9 the calculated effective self field is shown.

At a fixed external field the maximum applied self field must be calculated using (10), then the effective self field is drawn from Fig.9 and the self field correction is calculated or by (13) with a_5 slope calculated from effective self field or directly from (37) using G(K) of Fig.8. In Table II the maximum self field,the effective one and the correction are shown for different values of the external field. In the last step the transfer current effects are taken into account. In order to have a comparison with experimental results , the calculation is performed not only for the single straight cable but also for a real experimental configuration[2,3] in which the maximum self field is .47 Tesla at 8000 A ($a_5 = 1.7\ 10^5$).

Equation (49) is solved numerically using a routine of NAG library (D02GAF).

In Fig.10 the foreseen Voltage-current characteristic for the straight is shown and compared with the situations of (a) no self field and (b) effective self field.In Fig.11 the plot LogV vs. LogI is shown in the three cases. The interesting results is that the n-value (the slope of curves) at the beginning of the transition is just equal that one of the single strand, but increasing the current it is lowered. In TABLE III the values of the self field correction are shown.

In Fig. 12 and 13 the results for the real configuration at an external field of 5.81 Tesla and temperature 4.51K In Fig.14 the measured characteristic and in Fig.15 the connected LogV vs LogI curve.

A critical current of 7570 A is observed (foreseen 7975) with a n-value of 17 (22). From the experimental results a confirm comes out for the changing of the n-value; instead it is not clear the difference between the foreseen and measured critical current. There are two possibilities:

1) The effective self field theory does not apply

2) A degradation is occurred from the strands to the cable (5.3%).

A work is in progress to understand if any degradation occurs. Our opinion is that both the effective field theory and transfer current theory are quite good and can give an acceptable background to connect critical current measurements performed in different laboratories.

REFERENCES

1 M.S.Lubell "Empirical scaling formulas for critical current and critical field for commercial NbTi" IEEE Trans.Mag. vol Mag-19 NO. 3 MAY 1983

2 P.Fabbricatore, A.Matrone, A.Parodi, R.Parodi, C.Salvo and R.Vaccarone "MA.RI.S.A., a test facility for reaserch in applied superconductivity" contributed paper to ICEC12 Southampton July 12-15 1988

3 P.Fabbricatore,R.Parodi and R.Vaccarone "Self field effects in the critical current measurements of Hera cables" Internal report INFN/TC-88/7

TABLE I :Critical current values			
B (T)	5,5	6,5	7,5
I (A)	450	340	240

TABLE II: Effective s.f. and correction			
B external Tesla	applied sf Tesla	effec.sf Tesla	correction %
5.5	0.606	0.402	11.02
6.0	0.534	0.343	10.64
6.5	0.460	0.295	10.56

TABLE III S.F. correction			
Field (T)	5.50	6.00	6.50
Corr. (%)	9.45	9.52	9.65

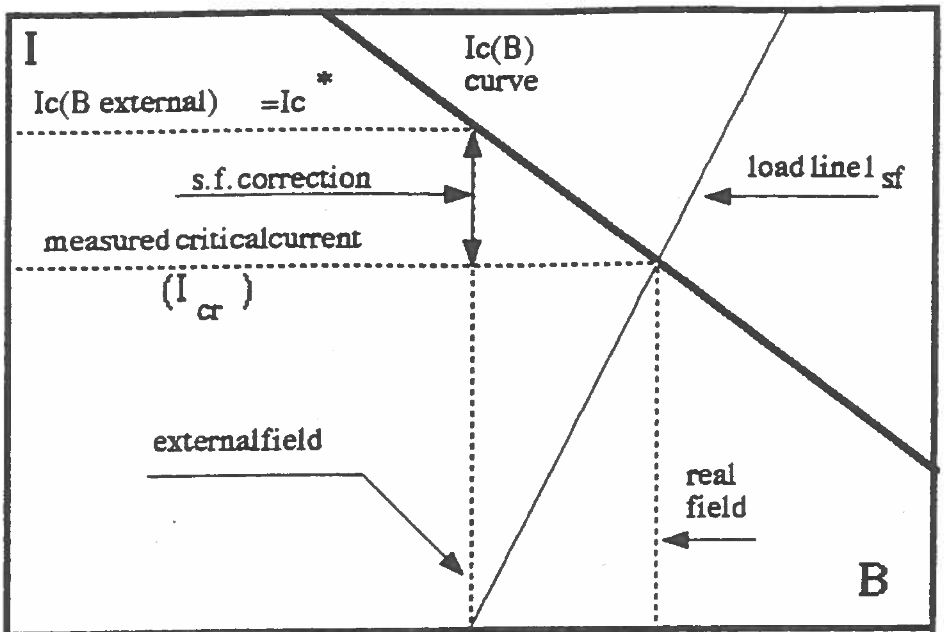


Fig. 1 Schematic view of self field correction

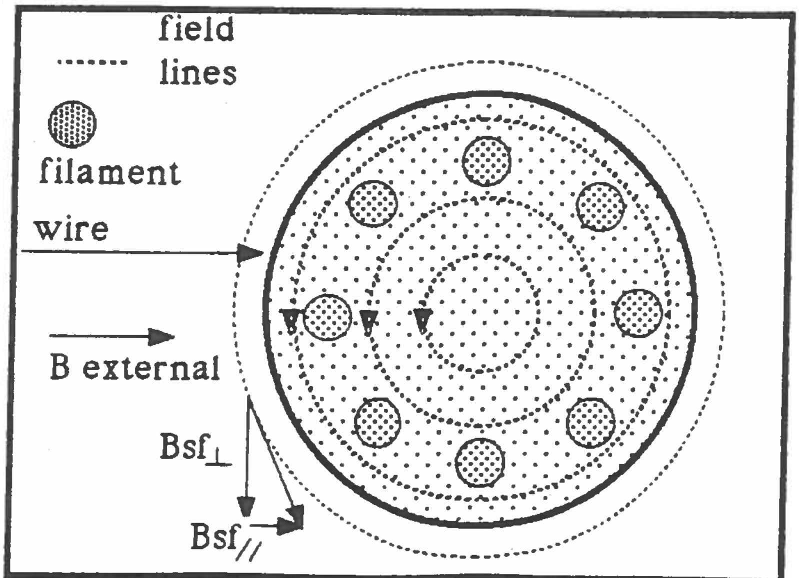


Fig.2 The considered system

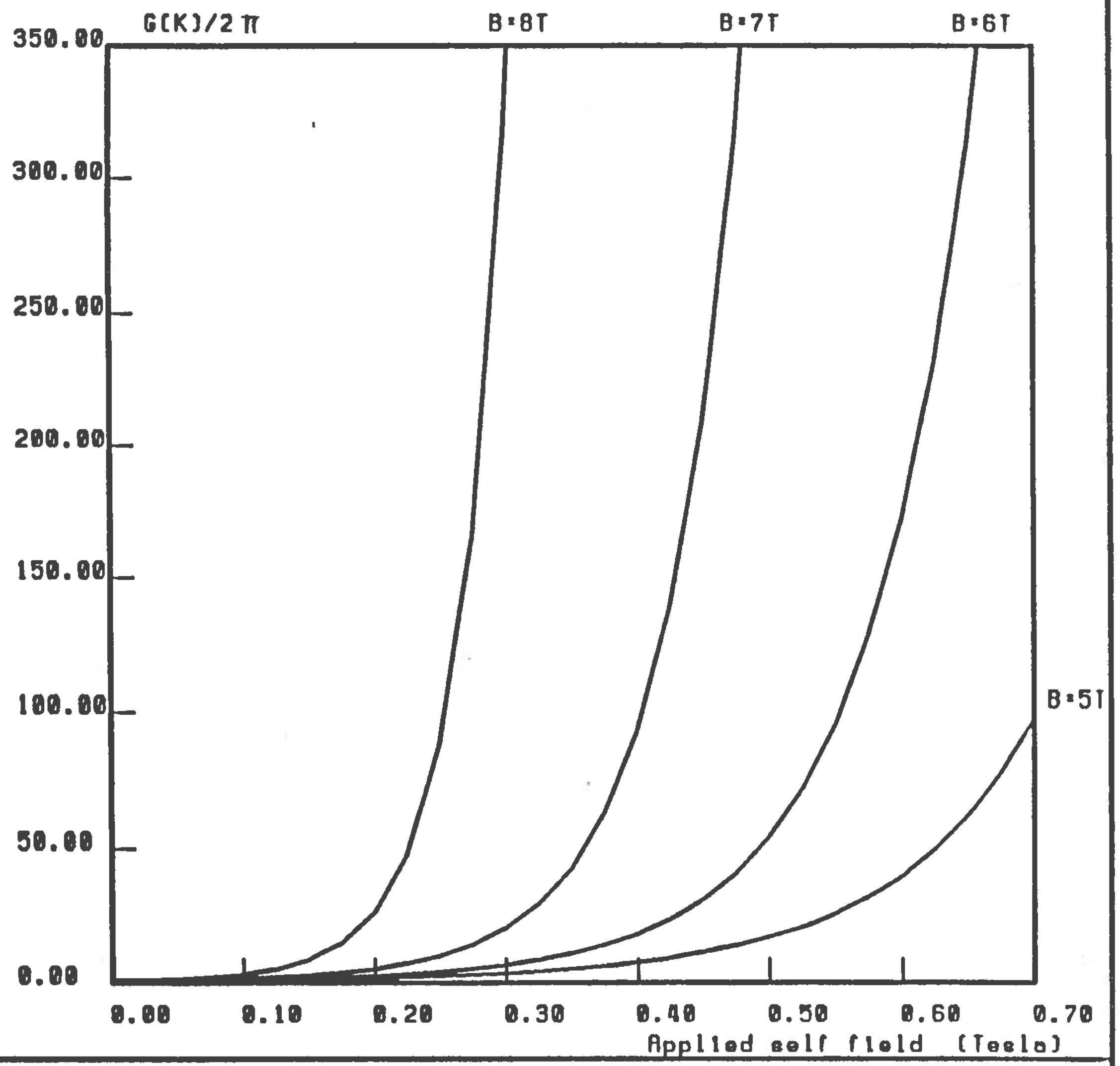


FIG.3 SPECIAL FUNCTION G(K)

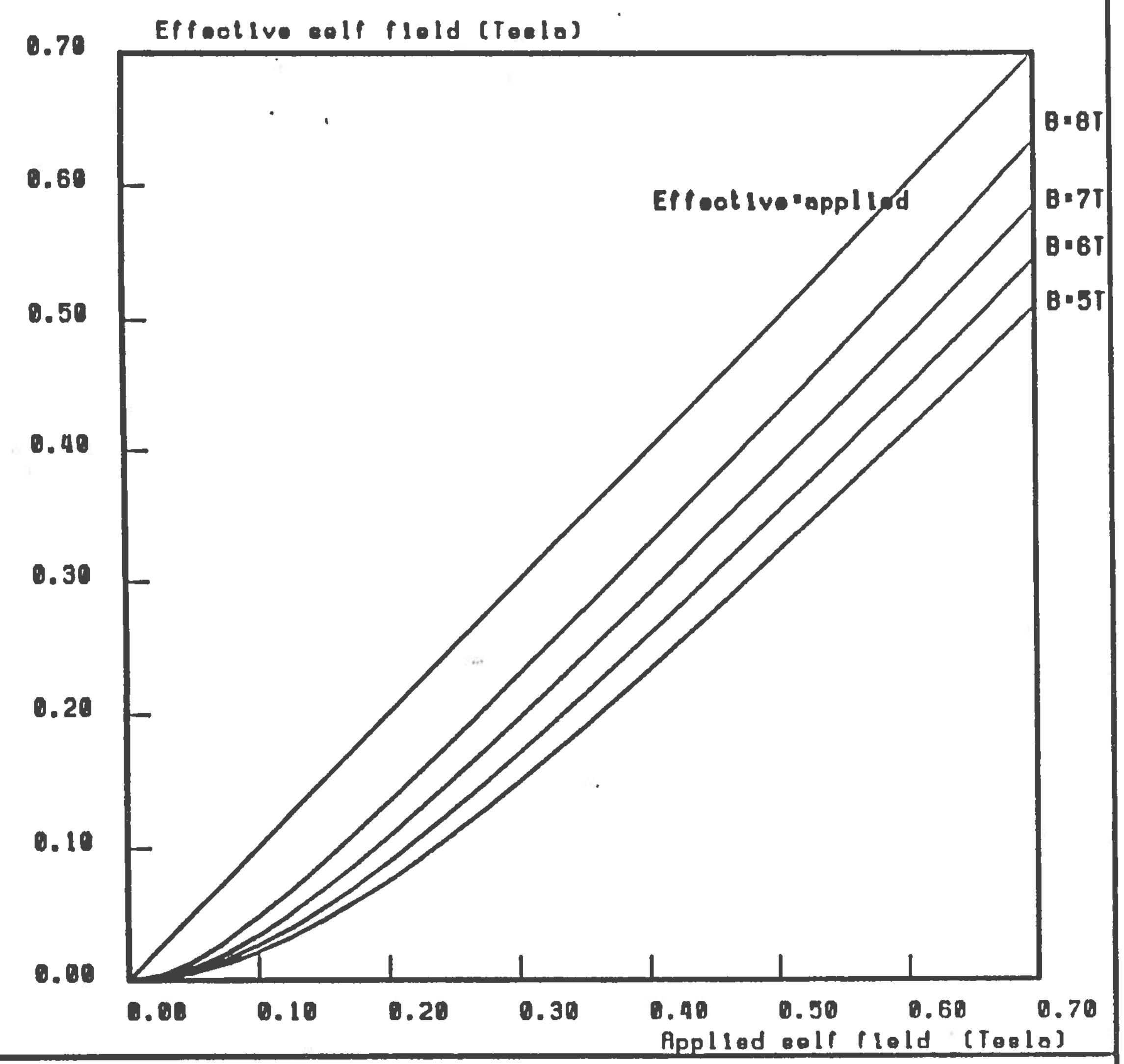


FIG.4 EFFECTIVE SELF FIELD

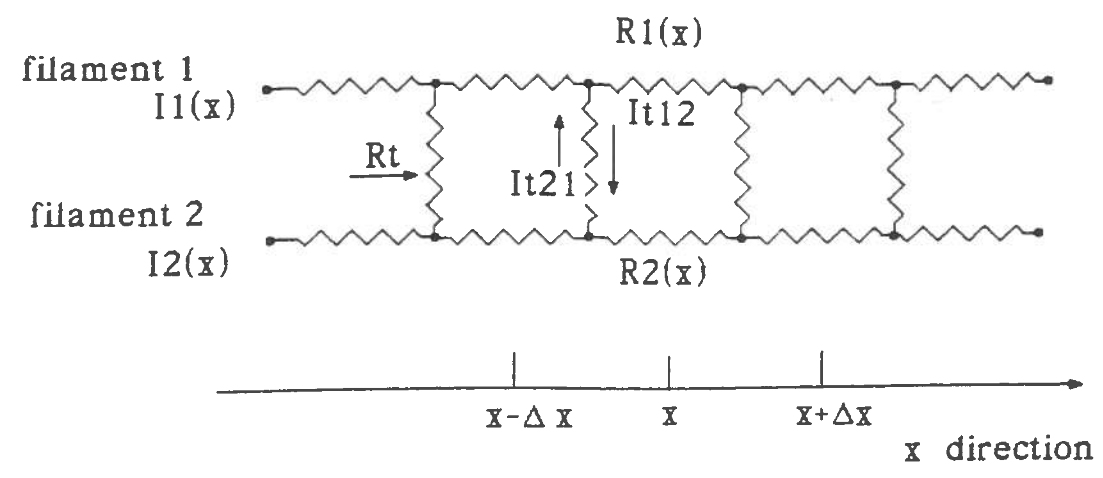


Fig.5 Simulation network for current transfer

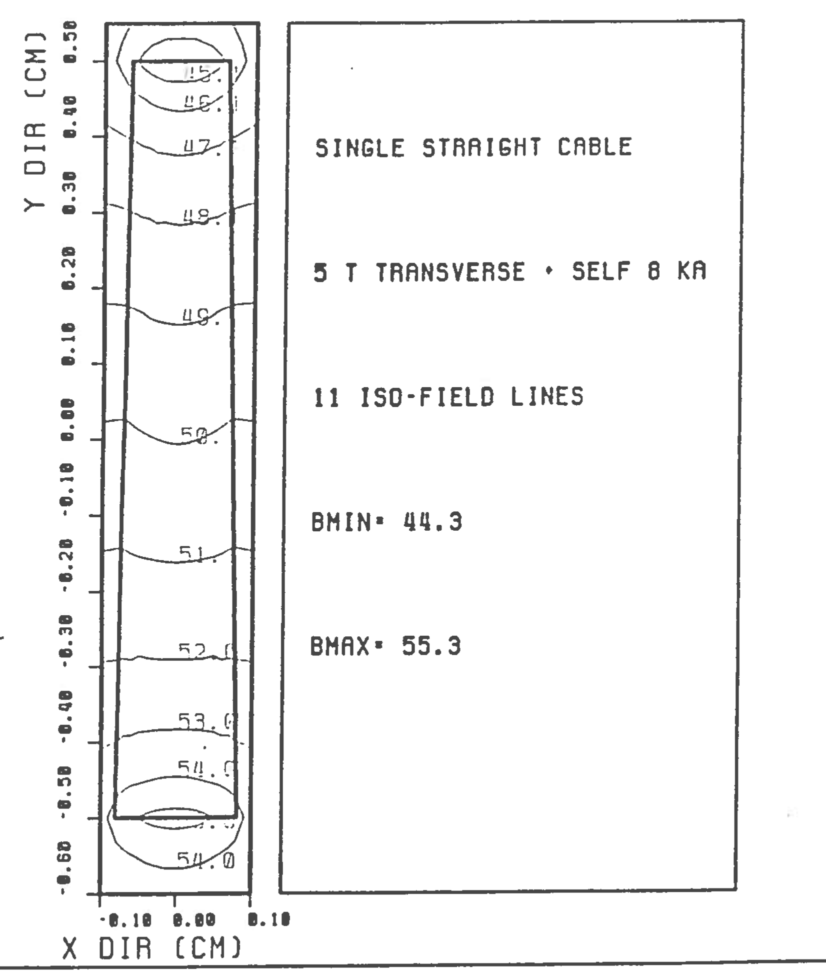


Fig.6 Field at the conductor

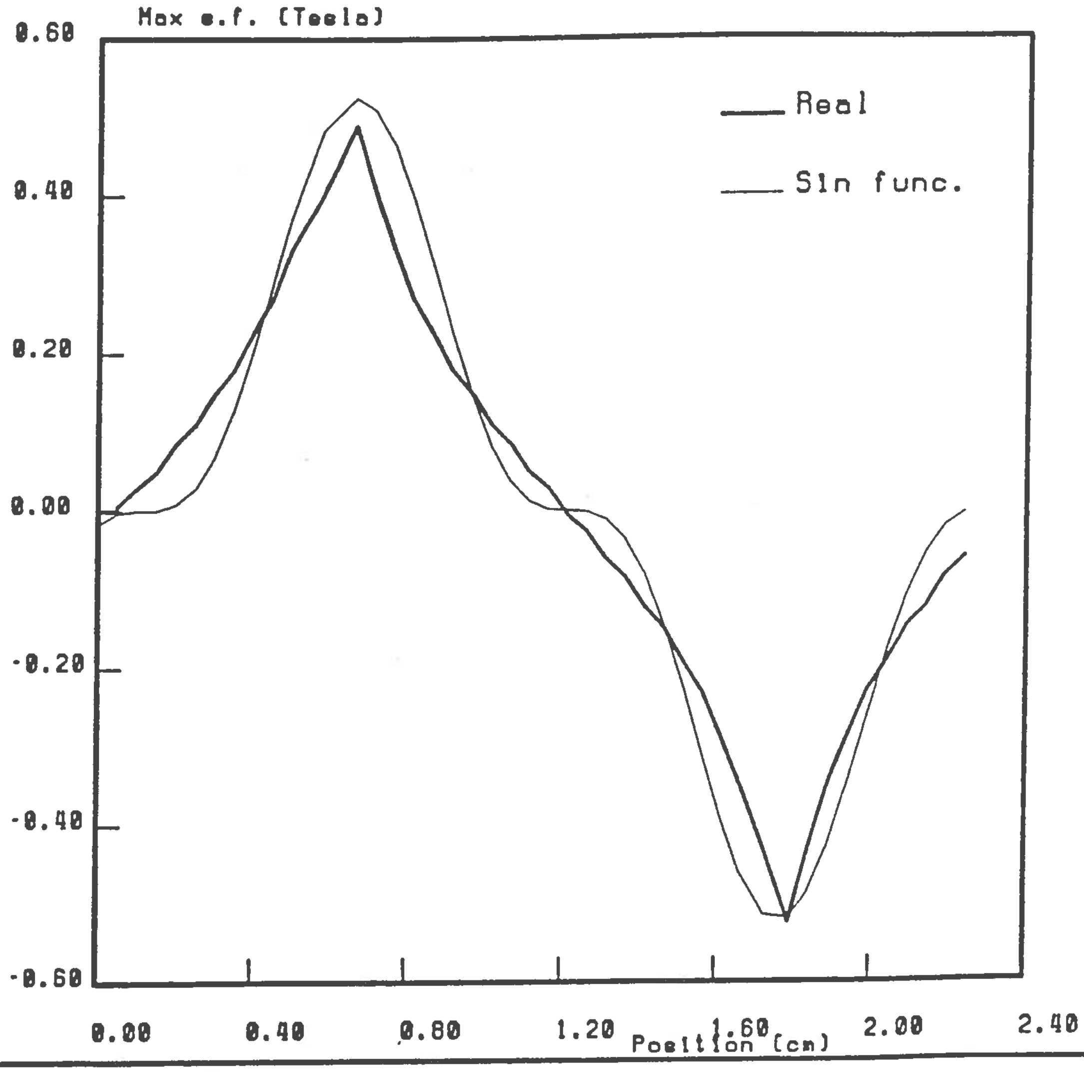


FIG.7 FIELD PROFILE AT THE CONDUCTOR

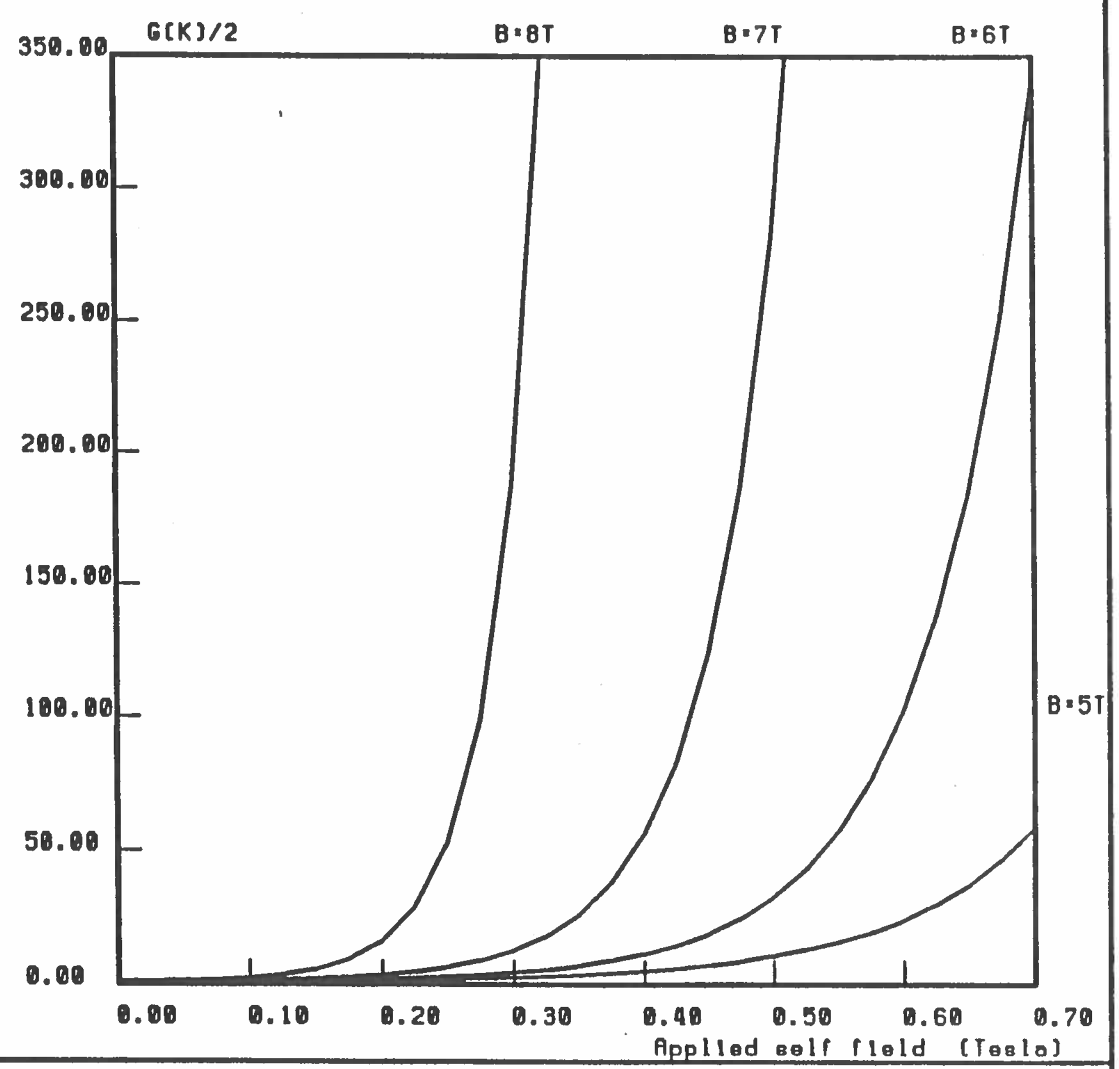


FIG.8 FUNCTION G(K) FOR HERA CABLE

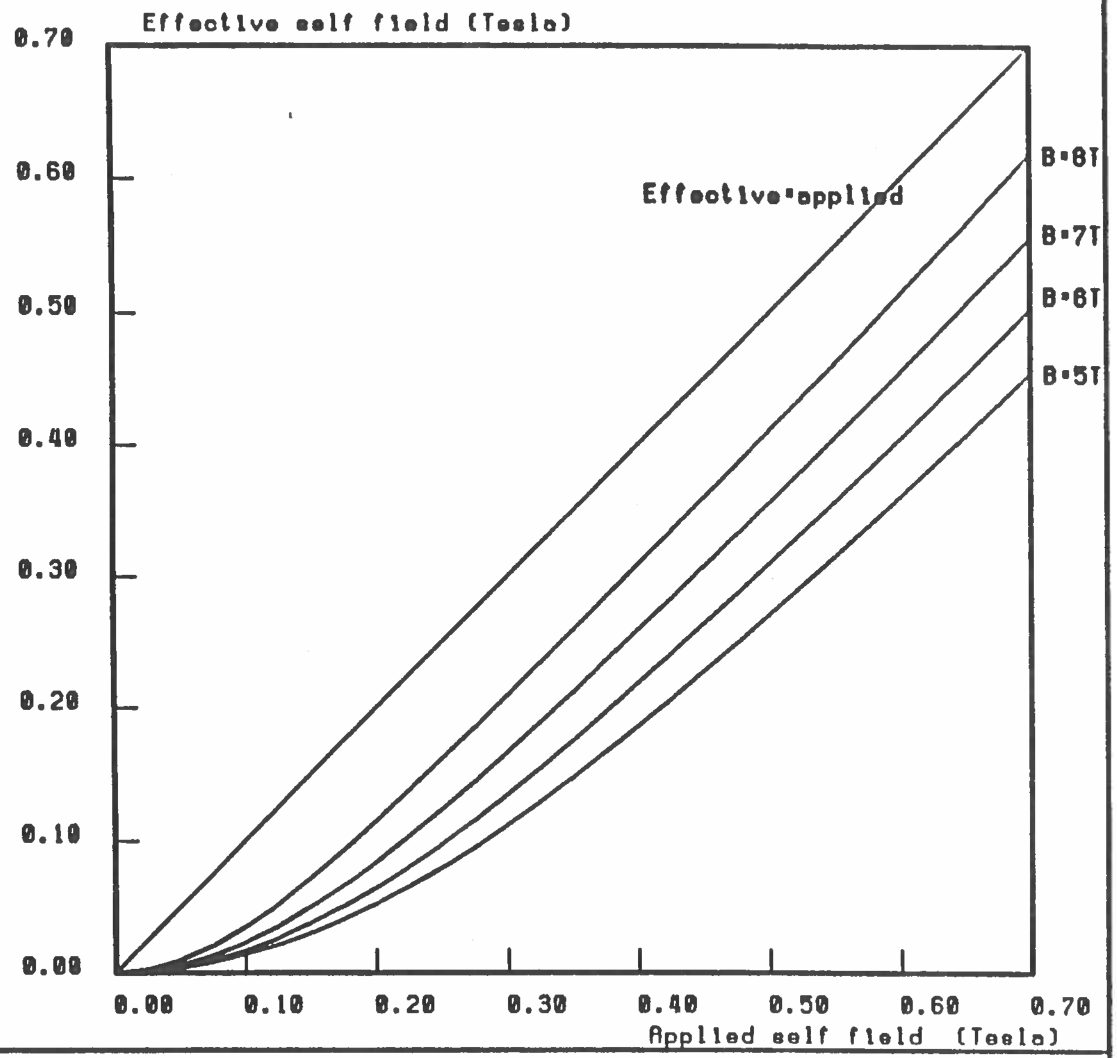


FIG.9 EFFECTIVE SELF FIELD FOR HERA CABLE

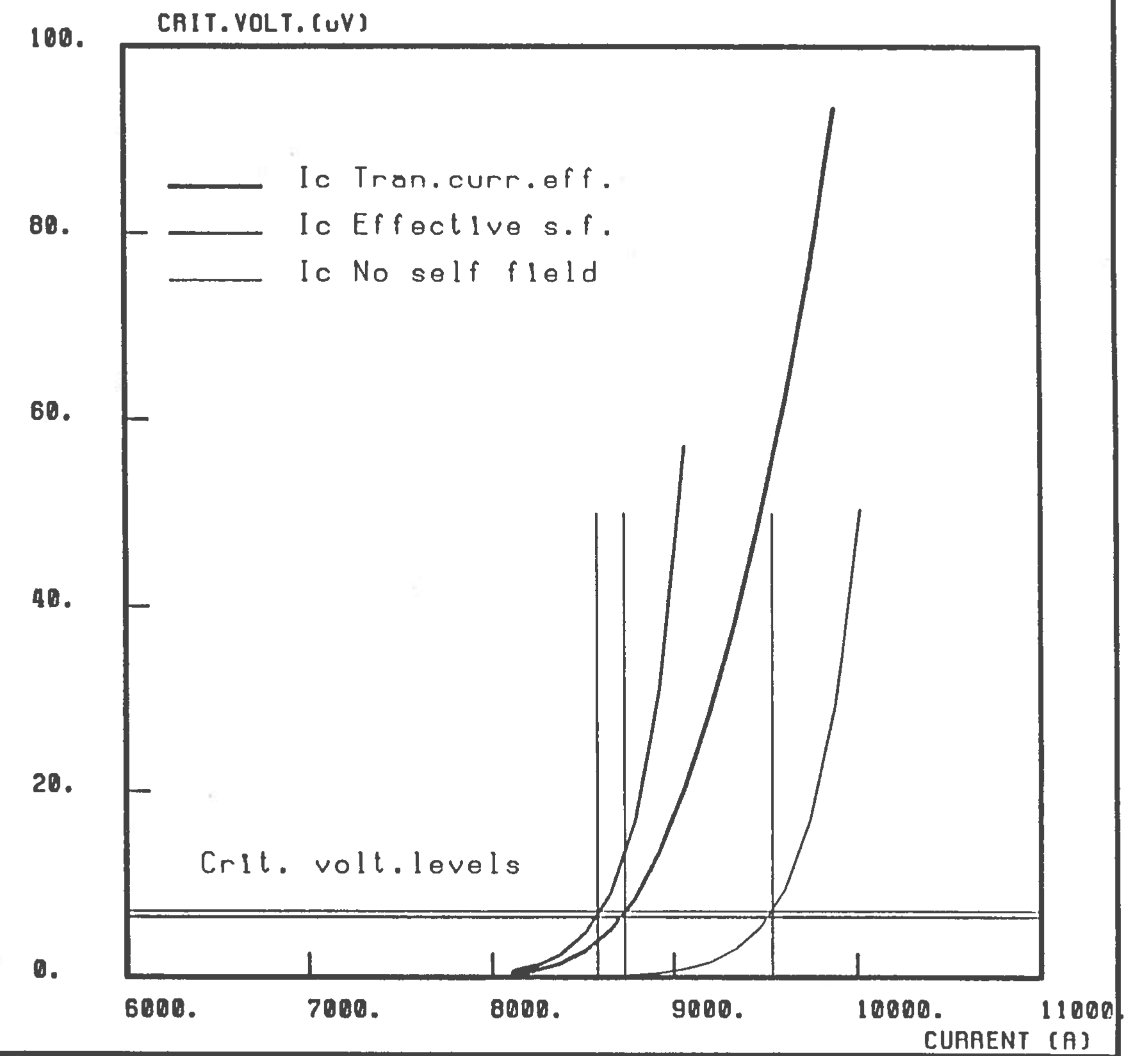


FIG.10 CALCULATED I vs V

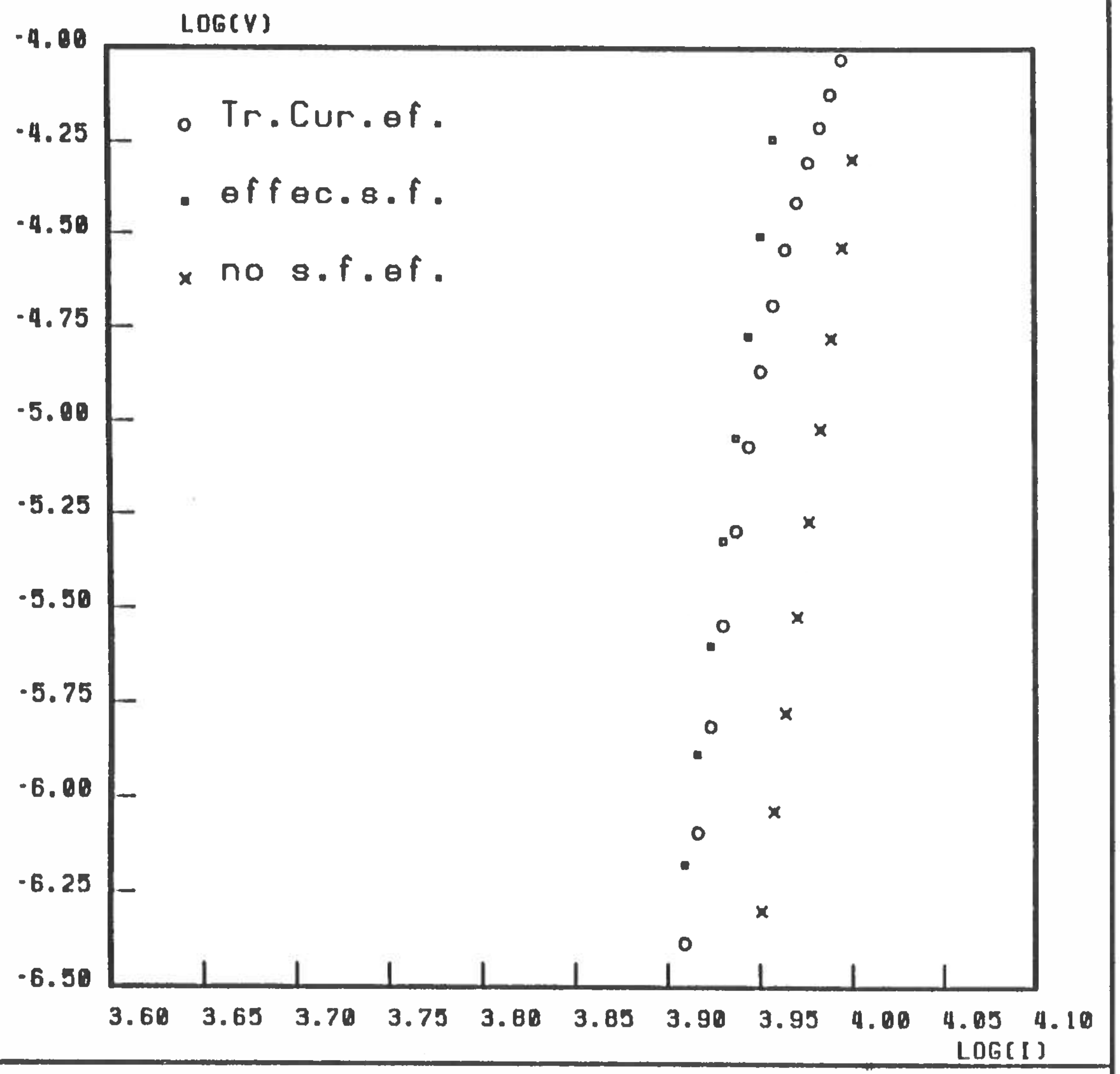


FIG.11 LogV vs LogI (slope is n)

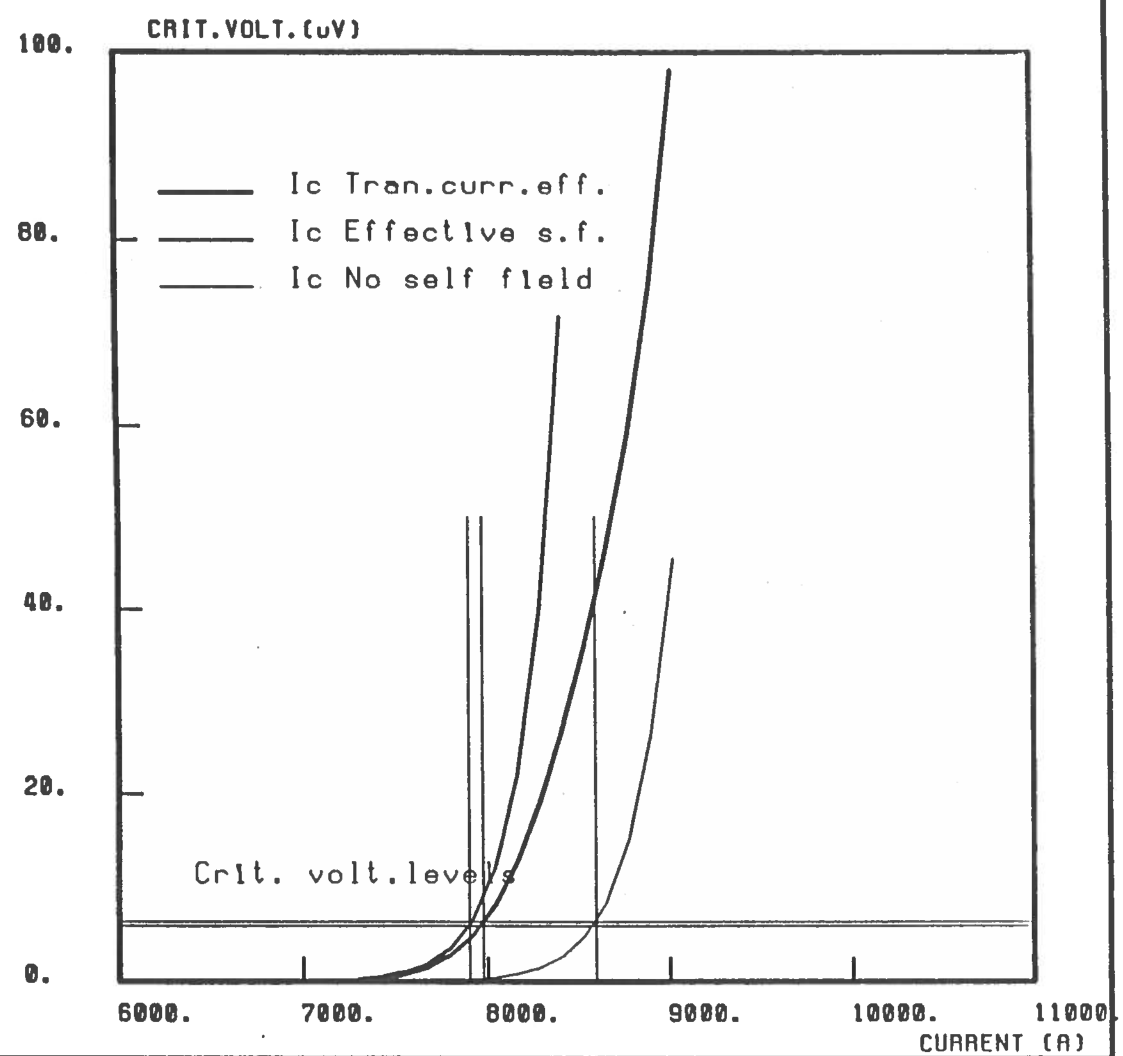


FIG.12 CALCULATED I vs V

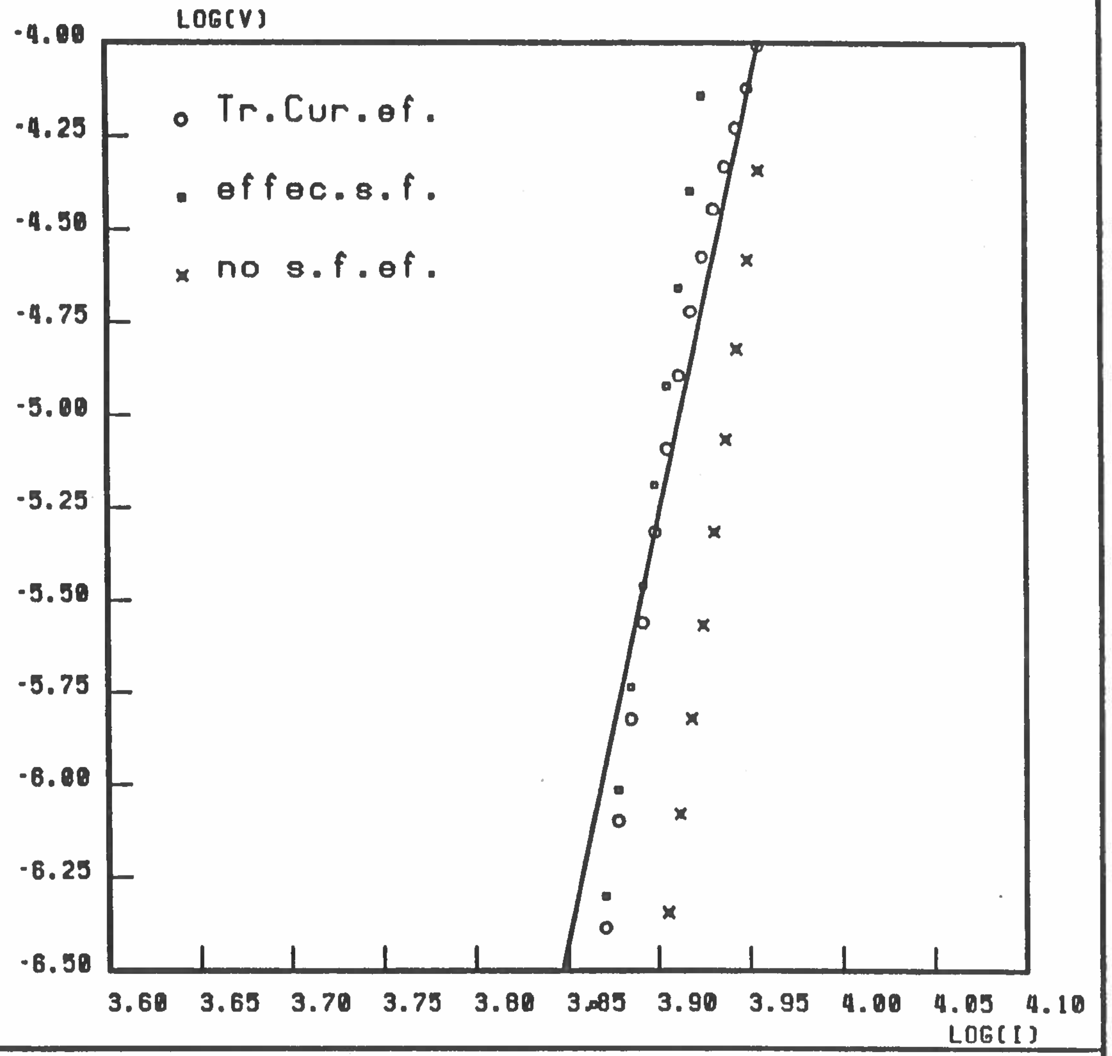


FIG.13 LogV vs LogI (slope is n)

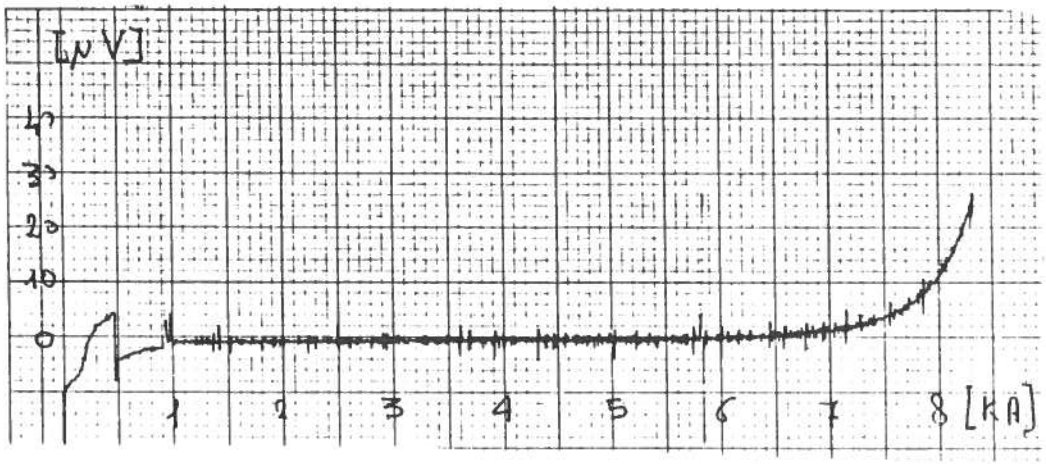
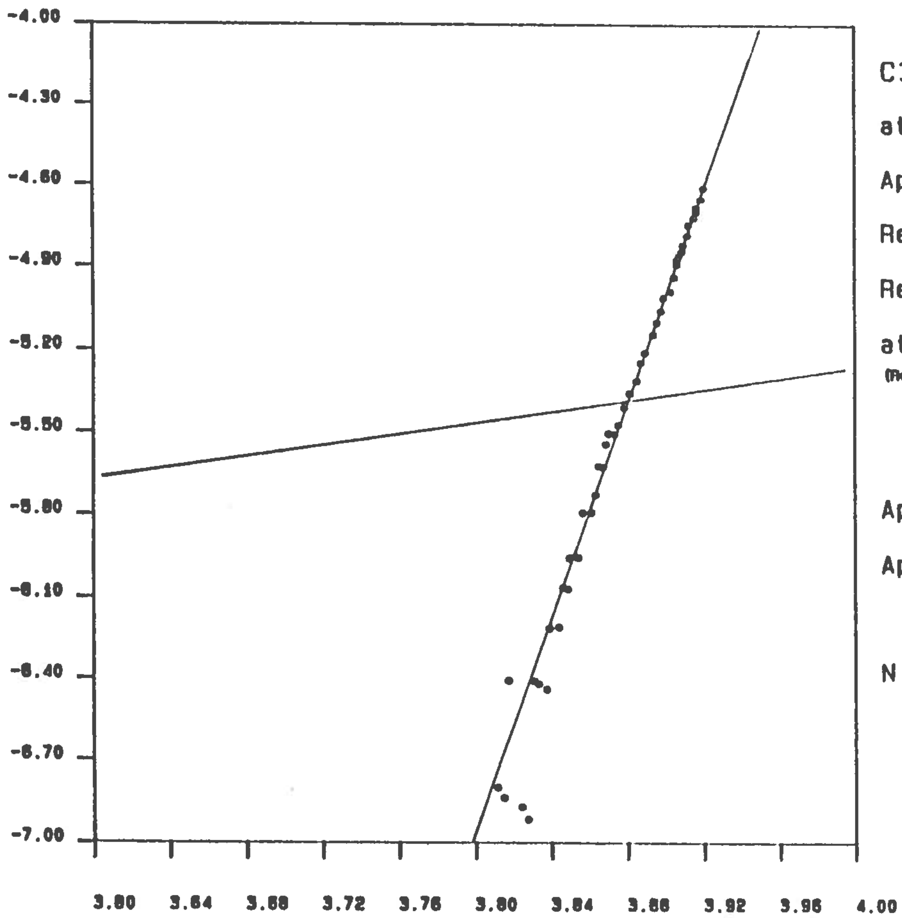


Fig.14 Measured characteristic



C30/17 * 05/02/1988

at meas.Temp= 4.51 (K)

Appl. field = 5.8065 TESLA

Real field = 6.3225 TESLA

Real. Ic = 7564. (A)

at Voltage = 4.10 E-6 V

(Resistive criterium 10e-12 Ohm cm)

Approx. Ic at

Appl. field = 8449. (A)

N AT T=Tmeas B=B real ≃ 17

Fig.15 measured LogV vs LogI