ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Bari

INFN/TC–99/13
28 Luglio 1999

SIMULATION RESULT COMPARISON ON HALO FORMATION BETWEEN PARTICLE–CORE MODEL AND A MULTIPARTICLE CODE

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Abstract

Space–charge dominated beams can induce chaotic behavior of particle trajectories leading to halo formation on the beam spot. This causes particle losses along the beam transportation that must be minimized. The fractional losses must be kept below 10^{-7}/m.. This is a very low threshold to check with standard multiparticle codes. To study this kind of problems a particle–core model (PCM) is commonly used. However in this model some approximations are made and then its results need to be checked. In this paper, an inspection on the results of a particular PCM is done by using a multiparticle code with a new particle simulation approach that allows a tight comparison between their results

PACS.: 29.27.Bd; 29.27.Eg; 41.85.–p

Submitted to Nucl. Inst.and Meth. in Phys. Res. A

Published by ***SIS–Pubblicazioni***
Laboratori Nazionali di Frascati

1 INTRODUCTION

In these last years, growing interest has been addressed, from the international scientific community, on the possible applications of high intensity ion beams. Among them, just as examples, we can mention the energy amplifier proposed by C. Rubbia [1] and the transmutations of radioactive waste [2]. However high intensity beam transport poses problems those need to be faced and solved. In particular, a halo formation has been observed around high intensity beams, during the transport, that leads to particle losses. For high current and energy beams the lost particles produce radio activation in the structures and the related radiation can damage the accelerator components. Furthermore the radioactivation makes the accelerator maintenance very difficult and expensive. Because of these problems, it becomes very important to study halo formation mechanism in the beam.
Multiparticle codes could be very helpful to this aim but it is very difficult to study this kind of phenomena by using the standard multiparticle codes because the lost particle fraction, along the transport, must be kept below 10^{-7}/m [3]. This means that the code should use a number of particles of the order of 10^{+6}, in the simulations, to appreciate this kind of lost fractions. This number is very high and very powerful calculators are needed to handle them. In fact, in these kinds of simulations, the space charge force effect, because of the high intensity beam used, becomes a very important issue.

Recently a new calculation technique, called Particle Core Model (PCM) [4], has been introduced to study the halo formation without using of very powerful computers.

The PCM solves the beam envelope (or rms) equation for a continuous beam that is used as a model for the core of the beam. The core can be mismatched so that its radius, taken at the exit of the transport period cell, will oscillate. The halo particles are represented by test particles, which oscillate through the core, influenced by linear external focusing field and the non–linear space charge fields of the core. The model allows one to study the dynamics of the test particles.

Although the PCM calculations allow to study the halo formation this is done by assuming some approximations and then their influence on the PCM results should be checked by a comparison between its results and those given by a multiparticle code that do not have this kind of approximation.

The main approximations assumed in PCM will be shortly recalled in the following with the reference on the PCM described in ref. [5],

The envelope or, in some case, the rms equation, considers a constant particle distribution that is not physical. In fact, in fig. 1 it is shown as an initial uniform, in space, particle distribution evolves after about 150 periodic cells (corresponding to a few tens of plasma periods), because of the space charge effect. Furthermore, the space charge force, seen by the test particles near the edge in the PCM model, is incorrect. In fact, in the envelope equation, the particle distribution is assumed with a sharp edge, while, in the real life, it has a tail with a length equal to the Debye length λ_D [6] (see again fig.1). Another

PCM approximation that can be mentioned is that the coupling, due to space charge forces, between the two transverse planes is completely neglected.

In this paper a comparison test between the PCM results and the simulations of the multiparticle code PARMT, modified to follow directly the results of PCM calculations, will be carried out to check the PCM reliability.

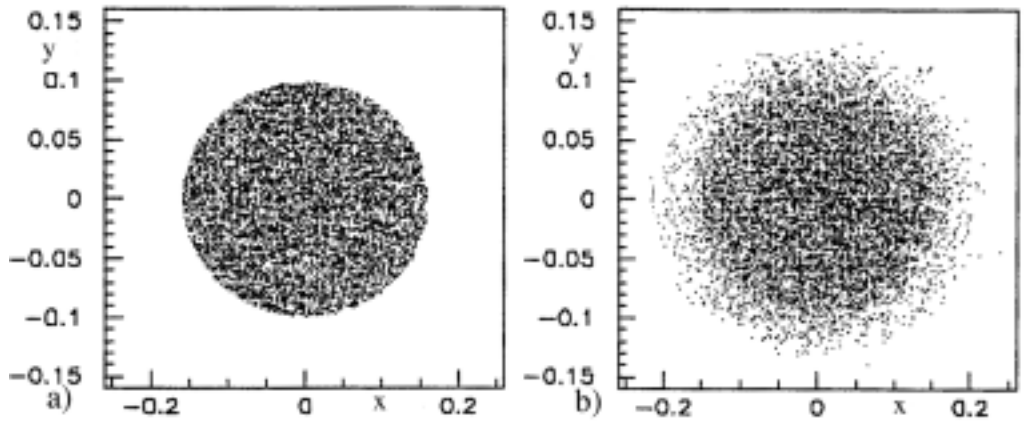


Fig. 1 – Particle distribution in the xy plane: a) initial (KV) distribution, b) distribution after 150 periodic cells. Notice the shielding process from the external focusing force has already occurred and the tail of length λ_D is visible on the beam spot of b). x an y are in cm.

2 SIMULATIONS

PARMT is a Monte Carlo program that can transport an ion beam through a system of optic elements by using the matrix method [7]. It has different techniques for the space charge calculation. Among them there are the 'Particle to Particle' and the 'Fast Poisson Solver' (FPS) technique that can give, with a high precision, the electric field due to the particle distribution step by step along the beam transport. In the 'particle to particle' technique the electric field is directly computed from the Coulomb law:

$$\vec{E}(\vec{x}) = \frac{q}{4\pi\varepsilon_0} \sum_i \frac{\vec{x} - \vec{x}_i}{\left|\vec{x} - \vec{x}_i\right|^3}$$

where $\rho\, \vec{x}_i$ gives the particle position. In this case the time needed for space charge calculation increases exponentially with the number of the particle considered. High intensity beams impose to consider a very high number of particles and in these conditions the calculation time becomes too long. A high intensity ion beam requires, in fact, the FPS technique that computes the space charge electric field by solving the Poisson equation:

$$\mathrm{div}\vec{E}(\vec{x}) = \frac{1}{\varepsilon_0}\rho(\vec{x})$$

where $\rho(\vec{x})$ is the beam charge density distribution. The main features of this kind of computation are shortly given in the following.

A mesh with a size d is superimposed on the beam. The particle charges are distributed among their neighboring mesh nodes. In this way the problem of solving the Poisson equation is reduced to solve a linear system of finite difference equations which can be solved with Fast Fourier Transform (FFT). The computing time, in this case, is mainly determined by the number of mesh points. However there is the constraint that if $\lambda D << d$ the simulations will present some unphysical instability called 'aliasing' [8].

To compare directly the PCM with the PARMT results some modifications have been carried out in the PARMT code. Mainly, two new features have been introduced:

1) an input file with the initial test particles coordinates that can be read, if required, by the main program;
2) N output files, with N number of test particles, where the test particle phase space coordinates along the transport in the periodic cells (Np) are stored.

The PCM results given in ref. [5] will be compared to our PARMT simulations and then some comment on this comparison will be given.

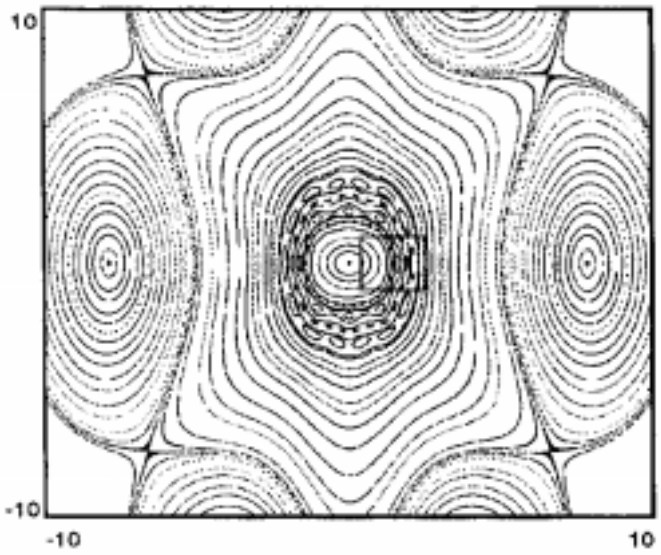
The simulations are carried out on a FODO cell period of length L=80 cm. The other input parameter considered are total transverse emittance, $\varepsilon_x=\varepsilon_y= 1 \times 10^{-6}$ m r; single particle phase advance, $\sigma_o=60.7°$; the space charge parameter (as defined in ref [5]) $\xi = 4 \times 10^{-6}$ (corresponding to I= 95mA) leads to a phase advance $\sigma=30.4°$. The test particles used in the PARMT calculations are 40. Although this number is lower then that used in the PCM calculations, their initial coordinates are taken in the same region and this should give the same behavior. The initial coordinates of the used test particles are $x_1=0.1$cm, $x_2=0.15$cm, $x_3=0.2$cm and so on until to $x_{40}=2$cm while the other test particle coortdinates are all zero.

The phase space test particle trajectories given by PCM calculations, for the matched case, is given in fig.la). In fig. 1b) are shown the PARMT results that can be compared with those of fig. 1a).

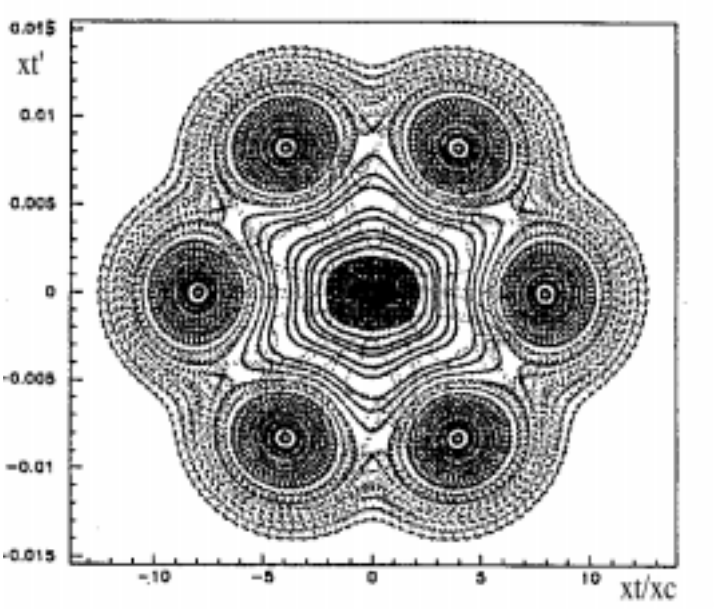
The more external phase space test particle trajectories are very similar in both cases. However, very different test particle trajectories can be observed in the core region (near the beam). In fact in this region, PCM calculations give trajectories of circular shape while PARMT simulations give very complex trajectories.

This difference is mainly due to the strong coupling induced by the high space charge forces between the two transverse phase space planes (xx' and yy'), existing in the core region.

In the PARMT, although a uniform initial distribution has been used (see fig. 1a), it will change because of the space charge effect, as already said and shown in fig. 1b. In fact, the space charge forces induce a particle redistribution to shield the particles inside the beam core from the focusing forces. At the end of this process the final distribution is of the type shown in fig 1b.

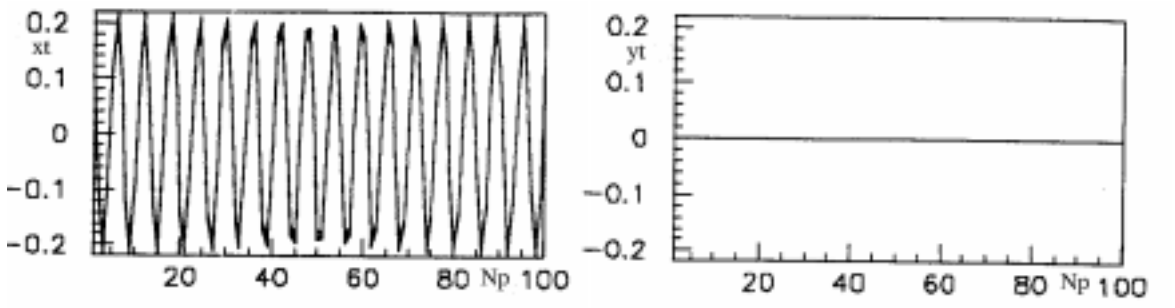


a)

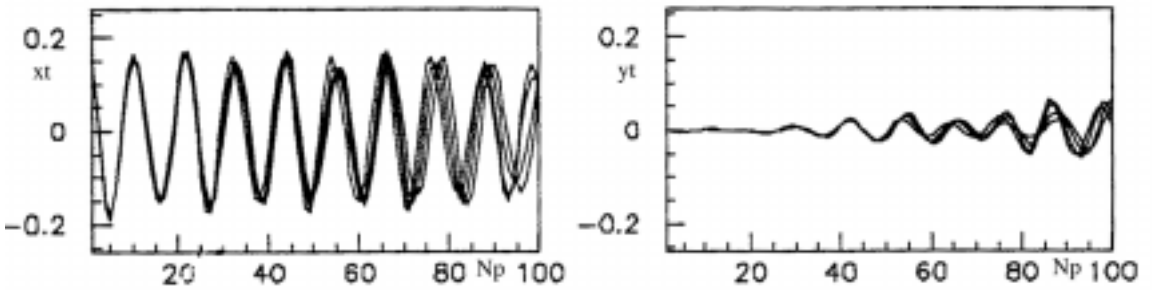


b)

Fig. 2 – Test particle trajectories in the transverse phase space xx' for the matched case: a) results from PCM; b) results from the multiparticle code PARMT. x is in cm x' in rad and xc is the horizontal core size. Notice that in a) x' is normalized to 1 mrad.

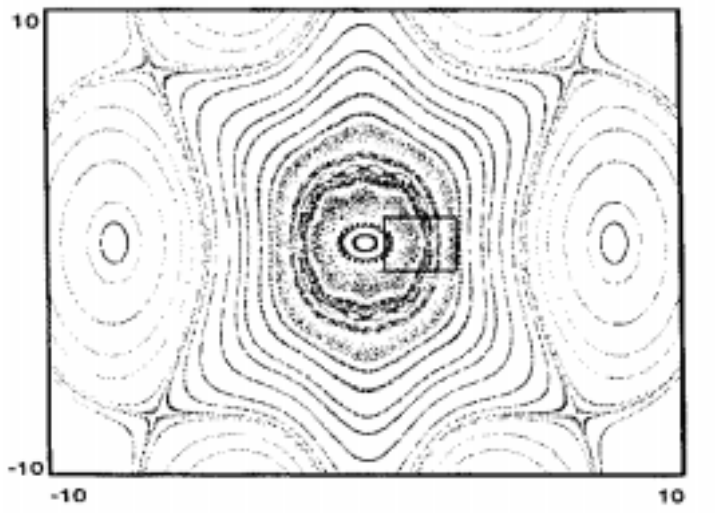


a)

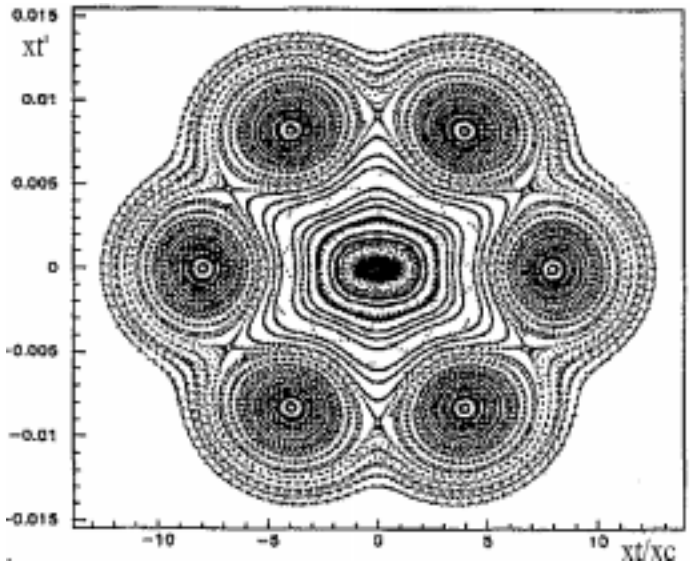


b)

Fig. 3 – Transverse positions, x and y, of the first 4 test particles *vs* the periodic cell numbers Np: a) low current ion beam (I=0.5 mA); b) high current ion beam (I=95 mA). Notice the high coupling between the transverse planes and the tune shifts due to the space charge effect in b).



a)



b)

Fig. 4 – The same of fig. 2 for the mismatched case.

If the space charge forces, involved in the particle redistribution process, are enough high they can induce coupling between the x (horizontal) and y (vertical) phase space planes.

In the PCM calculations, instead, a linear space charge force is assumed, and no coupling is considered between the two transverse phase planes. The coupling found in the PARMT calculations can be put in evidence by fig.3 where the test particle trajectories along the periodic cell transport are shown in the case of high space charge (I=95A) and low space charge forces (I=0.5).

It must be noticed that in the phase space, the case with low current (fig. 3a) gives circular shape trajectories, in the core region, as in PCM results.

The other, more interesting, test that has been done on the PCM results with the multiparticle code PARMT, it is the case in which the beam is mismatched with the FODO periodic cell. An odd mismatch of 10% of the beam size has been used as in ref. [5], The simulation results of the mismatched case are shown in fig.4.

The PCM phase space core trajectories show a substantial difference between the matched and mismatched case. In the mismatched case the core trajectories have lost their circular shape and look like more chaotic. The reason of these very different results can be clarified by the following comment.

When the beam is mismatched with the focusing transport channel its envelope, at exit of the channel, is different from that of the entrance. Then, if we plot the beam envelope at each periodic channel exit (or equivalently the xrms and yrms vs Np, cell period number) an oscillating curve will be obtained with amplitudes that will depend on the entity of the mismatch. The beam envelope oscillations could induce on the test particle phase space trajectories, a chaotic behavior and surely very different trajectories with respect to the matched case. In fact, in this case, the oscillating beam envelope generates very different space charge forces for the test particle with initial coordinates in the region of the beam core where occur the beam envelope oscillations.

In the PARMT simulations, instead, more slight differences between the matched and the mismatched case can be observed. This different behavior between the two types of calculations can be explained by the following considerations. In the mismatched case of the PCM calculations the beam envelope oscillates, as said before, with constant amplitudes, while, in the PARMT simulations, a damping on the beam oscillations has been observed. This damping mechanism is not yet very clear. It can be due to a kind of Landau damping, being, the beam particles, oscillators coupled each other by their space charge. Further studies in this sense are under way. In fig 5, the xrms and yrms (the rms values of the macroparticle transverse positions), are shown as followed by PARMT and the damping effect is visible. To be sure that these rms oscillations are given by the breathing of the whole beam due to the mismatch, the behavior of the beam spot along the periodic cells has been checked, as shown in fig. 6. From that figure it can be noticed as the beam spot oscillates in odd mode, that is, alternatively, in the horizontal and vertical plane as in the xrms and yrms oscillations. Furthermore, from fig.5, it can be seen that the oscillation amplitudes are almost completely damped after about 350 periodic cells and then it practically returns to be in matched conditions. From this considerations we can conclude

that for this reason, the test particle trajectory results between the two cases, matched and unmatched, are very similar.

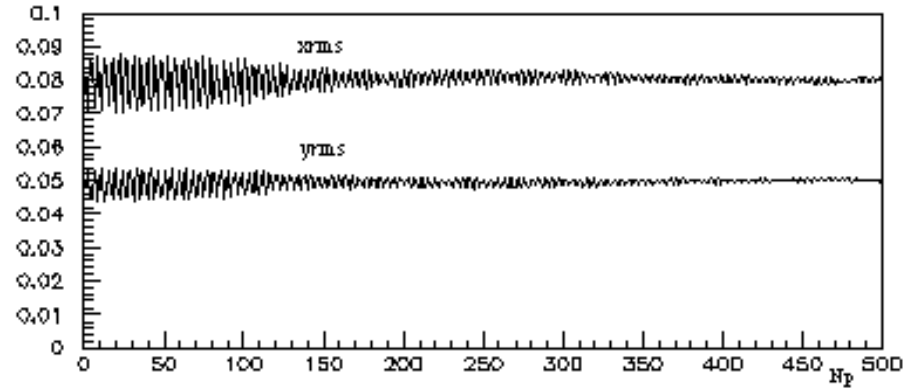


Fig. 5 – The xrms and yrms *vs* the periodic cell number Np for the mismatched case. Xrms and yrms are in cm. Notice the damping effect on the oscillation amplitudes.

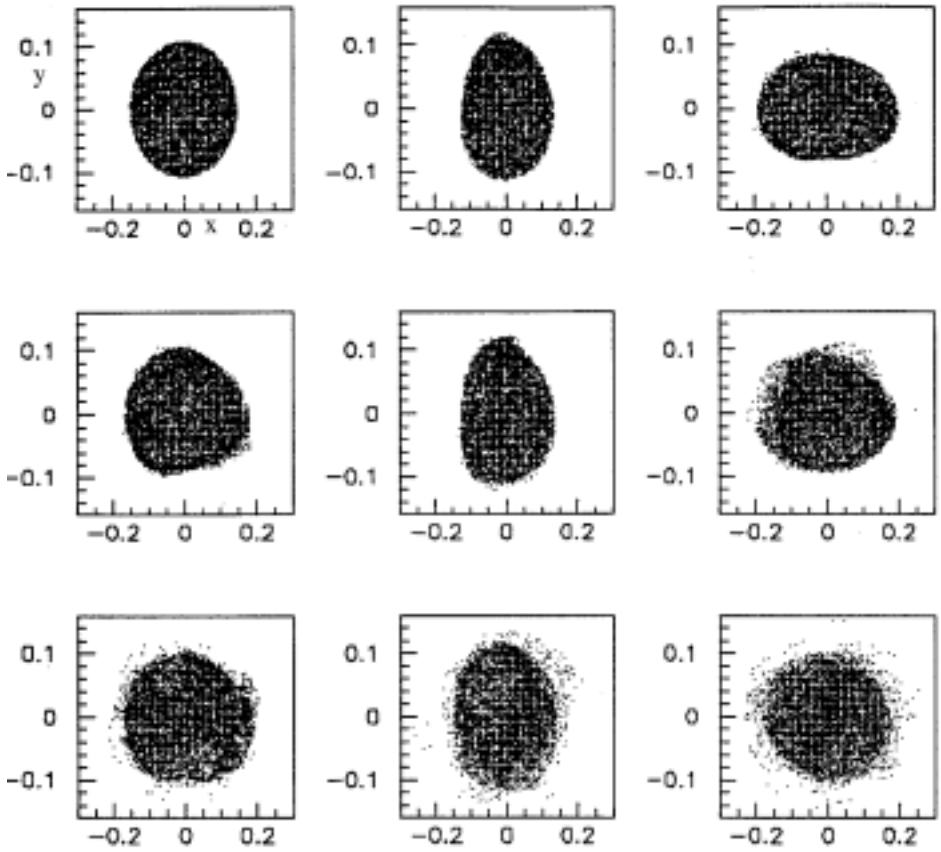


Fig. 6 – Beam spots, each one taken after 10 periodic cell. Notice the beam spot is breathing in the odd mode.

3 CONCLUSION

A continuous beam has been transported through hundreds of FODO periodic cells. The comparison of the simulation results obtained either with PARMT and PCM leads to the following comments. The more external phase space test particle trajectories are practically the same in both the results of PARMT and PCM. The test particle trajectories, in the region near and inside the beam core, are very different in the two types of calculations. This seems due to the space charge coupling between the two transverse planes.

In PARMT the transverse oscillations, due to the breathing of the beam when it is mismatched with the periodic cell, damp very quickly when the space charge is strong, as shown in our simulations. In PCM calculations, instead, we have a constant oscillation of the beam envelope. This means that, in PCM, the test particle close to the breathing beam are influenced in very different way in case of mismatch, as shown by the PCM calculations of the fig.la and 2a, and also stated in ref. [5].

In PARMT, a more slight difference in the test particle trajectories close the beam, between the matched and the mismatched case is found with respect to the PCM calculations.

PCM calculations seem unsuitable to study test particles taken near the beam core region in the case of very high beam current (I=95 mA). Some modification on the PCM envelope equation to take into account the discrepancies with the multiparticle code are then required if one want studies with more accuracy the particle behavior near the beam core.

4 ACKNOWLEDGEMENTS

I wish to thank Dr. A. Pisent for his helpful comment and for furnishing the PCM results of the comparison.

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