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DETECTION OF TURN-TO-TURN INSULATION FAILURES IN THE TOROIDAL ATLAS COILS AFTER COLD MASS ASSEMBLING

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1. - INTRODUCTION

In the field of electrical machine coil test the PJ is a well known method: with this procedure the coil under test is pulsed by means of the discharge of a capacitor.

The current and voltage waveforms at the coil ends are typically damped oscillations. The presence of a shorted turn strongly modifies the discharge parameters i.e.: the period, the amplitude and the damping coefficient of the oscillation. In particular the period decreases considerably while the current amplitude and damping coefficient increase.

The short circuit in a coil is easily detected if it is possible to compare the discharge behaviour of two or several identical coils.

The detection of a single turn short circuit becomes hard if the coil is fully embedded in a metallic structure of very low resistance. In this case the discharge parameters of an intact coil are strongly affected so that the presence of a single turn short circuit could modify them of imperceptible quantities. Moreover the discharge parameters of identical coils can be affected by little changes of the coupling between the coils and the metallic structure so that their comparison could leave unresolved the problem of the presence or not of short circuits.

This is exactly the situation of the ATLAS coils after their integration in the supporting structure (casing).

The aim of this work is to verify the sensitivity of the method of capacitor discharge when the coils are strongly coupled with the casing.

The theoretical analysis of the discharge process, when the coil (or double pancake) is inside the casing, results complicate because it requires an adequate evaluation of the electrical parameters of the casing and the coil. In particular the self inductance of the casing is depending by its complex structure (several sections weakly coupled among them) and by the induced current distribution inside the bulk of the casing, moreover the resistance of the coil is depending by the magnetic coupling and by the frequency of the discharge.

In the following the analysis of the discharge will be carried out by means of simple RLC circuit in the case of intact and isolated coils (Fig. 1), by means of two coupled circuits (R_1 L_1 C and R_2 L_2) in the case of coil and short circuit or coil and casing (Fig.2) and by means of three coupled circuits (R_1 L_1 C - R_2 L_2 – R_3 L_3) in the case of coils, casing and short circuit (Fig.3).

2. - FIRST CIRCUIT ANALYSIS

As shown in Fig. 1, C is the capacitor capacitance, Q(t) the charge on its armatures, L_x and R_x are respectively the self-inductance and the resistance of an intact double pancake (L_x=L_g and R_x=R_g) or coil (L_x=L_b ed R_x=R_b).

The discharge process is described by the following differential equation:

$$L_x\, dI_1/dt + R_x\, I_1 + Q(t)/C = 0 \qquad (1)$$

The solution of the equation (1) is represented by:

$$I_1(t) = A_1 e^{-\gamma t} \sin(\omega_0 t + \varphi) \qquad (2)$$

where:

$$\gamma = R_x/(2\, L_x) \qquad \omega_0 = [1/(L_x C) - R_x^2/(4 L_x^2)]^{1/2} \qquad (2bis)$$

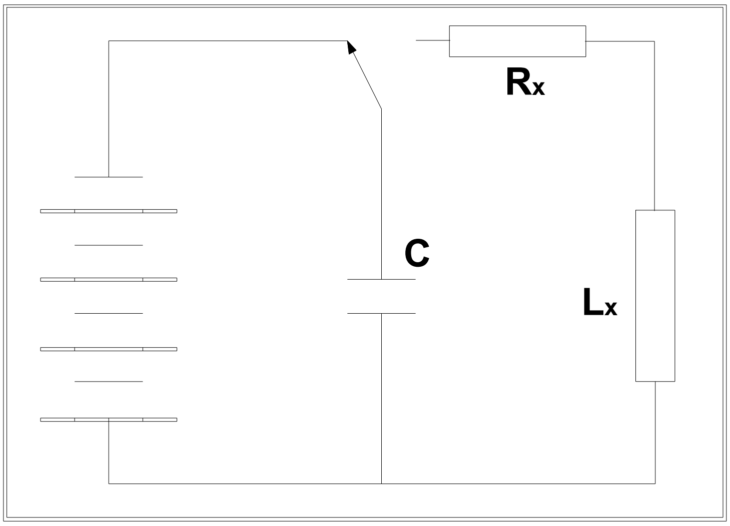

FIG. 1 - Electric diagram for the discharge on an intact and isolated element (double pancake or coil)

The constant A_1 and φ are determined by the initial conditions:

$$I_1(t = 0) = 0 \qquad - L_x\, (dI_1/dt)_{t=0} = Q/C \qquad (3)$$

with the following result:

$$A_1 = Q/(\omega_0 L_x\, C) \qquad \varphi = 0 \qquad (4)$$

In particular if $R_x << 2(L_x/C)^{1/2}$ the amplitude of the oscillation is given by $A_1 = Q/(L_x\, C)^{1/2}$. The measure of the period T ($T=2\pi/\omega_0$) and the damping coefficient γ allows to determine the self inductance L_x and the resistance R_x of the coil under test.

3. - SECOND CIRCUIT ANALYSIS

Indicated with L_x ($L_x = L_g$ for double pancake and $L_x = L_b$ for the coil) and with L_y (L_y=L_s) for the shorted turn e $L_y = L_c$ for the casing) the self inductances of both items present in the electric diagram, with M_{xy} their mutual inductance and with R_x ed R_y their equivalent resistances, the behaviour of the electrical circuit, shown in Fig. 2, is described with the following differential equations:

$$L_x\, dI_1/dt + M_{xy}\, dI_2/dt + R_x I_1 + Q/C = 0 \qquad (5)$$

$$L_y dI_2/dt + M_{xy}\, dI_1/dt + R_y I_2 = 0 \qquad (6)$$

where I_1 is the current intensity in the double pancake (or coil), and I_2 (with polarity opposite to I_1) is the current intensity in the shorted turn (or in the casing).

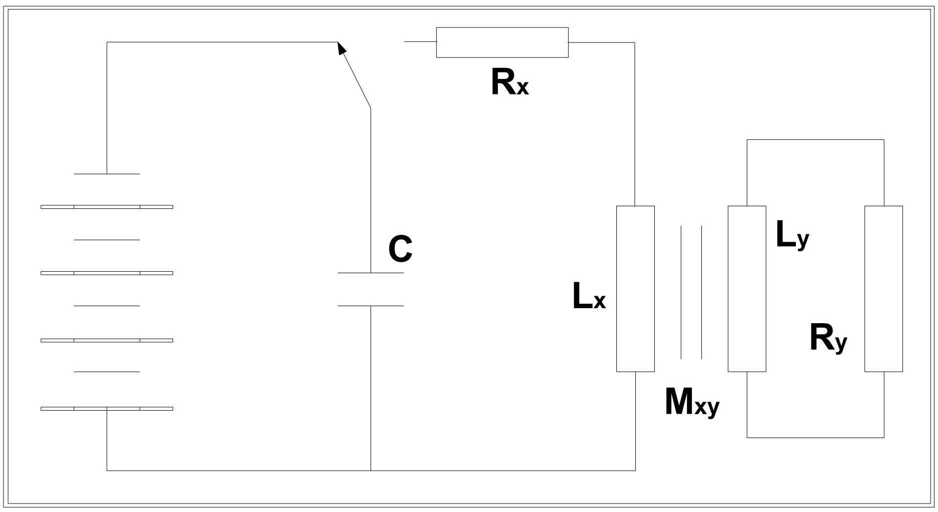

FIG. 2 - Electric diagram for the discharge on a double pancake (or coil) with shorted turn or intact double pancake (or coil) inside the casing.

The two differential equations (5) and (6) can be reduced to a single differential equation bringing from equation (5) the expressions of dI_2/dt and I_2 and substituting them in the equation (6):

$$\square_1 \, d^3I_1/dt^3 + \square_2 \, d^2I_1/dt^2 + \square_3 \, dI_1/dt + \square_4 \, I_1 = 0 \qquad (7)$$

where:

$$\square_1 = L_xL_y - M_{xy}^2 \qquad \square_2 = L_yR_x + L_xR_y$$

$$\square_3 = L_y/C + R_xR_y \qquad \square_4 = R_y/C \qquad (7\ \text{bis})$$

The solution of the differential equation (7) is given by:

$$I_1(t) = A_2 \, e^{x_1 t} + B_2 e^{x_2 t} + C_2 e^{x_3 t} \qquad (8)$$

where x_1, x_2, x_3 are the roots (in general two real and one complex) of the third order algebraic equation:

$$\square_1 \, x^3 + \square_2 \, x^2 + \square_3 \, x + \square_4 = 0$$

(9)

By applying Tartaglia method [(1)] and by defining the following parameters (expressed as a function of the algebraic coefficients (7 bis)) :

$$p = -(1/3) \, \square_2^2/\square_1^2 + \square_3/\square_1 \qquad (10)$$

$$q = (2/27) \, \square_2^3/\square_1^3 - (1/3) \, \square_2\square_3/\square_1^2 + \square_4/\square_1 \qquad (11)$$

$$r = \square_2/(3\square_1) \qquad (12)$$

the three roots of the equation (9) are obtained and expressed by:

$$x_{s+1} = \mathbf{U}^s \, u + \mathbf{U}^{3-s} \, v - r \qquad \text{con } s = 0,1,2 \qquad (13)$$

being:

$$u = [-q/2+(q^2/4+p^3/27)^{1/2}]^{1/3} \qquad (14)$$

$$v = [-q/2-(q^2/4+p^3/27)^{1/2}]^{1/3} \qquad (15)$$

where $\mathbf{U}$ represents one of the two complex roots [$\mathbf{U} = (-1 \pm (3)^{1/2} \, i)/2$].

Considering that, for physical reasons the current amplitude behaviour would be a damped oscillation it is easy to foresee that one root will be real and negative whilst the other two will be complex and conjugate. Consequently the term $q^2/4 + p^3/27$ must be positive and the quantities “u” and “v” are real. With these premises the three roots of algebraic expression would be written in the following form:

$$x_1 = u + v - r \tag{16}$$

$$x_2 = -(u + v)/2 - r + i\,(3)^{1/2}\,(u - v)/2 \tag{17}$$

$$x_3 = -(u + v)/2 - r - i\,(3)^{1/2}\,(u - v)/2 \tag{18}$$

Therefore the equation (8) becomes:

$$I_1(t) = A_2 e^{\alpha t} + e^{\beta t}\,[B_2 e^{\,i\omega_1 t} + C_2 e^{\,-i\omega_1 t}] \qquad \text{(8 bis)}$$

where: $\alpha = u + v - r$ $\quad \beta = -(u + v)/2 - r$ $\quad \omega_1 = (3)^{1/2}\,(u - v)/2$ (18 bis)

The equation (8 bis) can be written again in the form:

$$I_1(t) = A_2 e^{\alpha t} + D_2 e^{\beta t} \cos\,[\omega_1 t + \varphi] \qquad \text{(18 ter)}$$

where $B_2 = D_2 \cos \varphi$ and $C_2 = D_2 \sin \varphi$ (19)

The conditions for the calculation of A_2, D_2 and φ are the following:

a) the current intensity I_1 at time $t = 0$ must be null, from the equation (8 bis) the first relation becomes :

$$A_2 + D_2 \cos \varphi = 0 \tag{20}$$

b) the voltage at the ends of the coil under test at time $t = 0$ must be $\Delta V = -Q/C$. From the equation system (5) and (6), by taking into account the condition a) also valid for current I_2, it is possible to obtain:

$$L_x\, dI_1/dt + M_{xy}\, dI_2/dt + Q/C = 0 \qquad \text{(5 bis)}$$

$$L_y dI_2/dt + M_{xy}\, dI_1/dt = 0 \qquad \text{(6 bis)}$$

By computing dI_2/dt from equation (6 bis) and replacing it in (5 bis) the following relation is obtained:

$$[(L_x L_y - M_{xy}^{\,2})/L_y]\, dI_1/dt = -Q/C \tag{21}$$

and finally the second relation:

$$[(L_xL_y - M_{xy}^2)/L_y]\;[A_2\square + D_2(\square\cos\square - \square\,\mathrm{sen}\,\square)] = -\,Q/C \qquad (22)$$

c) the integral of current intensity in the interval $t = 0$ and $t = \infty$, that must be equal to the charge Q, gives the third relation:

$$\int_0^\infty I_1(t)\,dt = -A_2/\square - D_2[\square\cos\square + \square\,\mathrm{sen}\,\square] / (\square^2 + \square^2) = Q \qquad (23)$$

The values of A_2, D_2 e $\square$, obtained from the initial conditions, are given by:

$$\mathrm{tg}\,\square = \{\square - \square - [1/\square - \square/(\square^2 + \square_1^2)] / (\square_{xy} C)\} / \{\square_1\,[1-1/[(\square^2 + \square_1^2)\,\square_{xy} C]]\} \qquad (24)$$

$$A_2 = -\,Q / [\square_{xy} C\,(\square - \square\,\square\,\square_1\,\mathrm{tg}\square)] \qquad (25)$$

$$D_2 = +\,Q\,(1 + \mathrm{tg}^2\,\square)^{1/2} / [\square_{xy} C\,(\square - \square - \square_1\,\mathrm{tg}\,\square)] \qquad (26)$$

where : $\square_{xy} = [(L_xL_y - M_{xy}^2)/L_y]$

The calculation of the definite integral (23) and the quantities A_2, D_2 and $\mathrm{tg}\square$ is reported in Appendix A.

The time evolution of current intensity I_2 (in the short circuited turn or casing) are well described by the same differential equation (7). Therefore the solution is given by:

$$I_2(t) = A'_2\,e^{\square t} + D'_2\,e^{\square t}\cos(\square_1 t + \square') \qquad (27)$$

where the damping coefficients $\square$ e $\square$ and the angular frequency $\square_1$ are identical to those ones of the current intensity $I_1(t)$. The amplitudes A'_2, D'_2 and the phase $\square'$ can be obtained from the initial conditions:

a') $I_2(0) = 0$ (28)

b') $(dI_2/dt)_{t=0} = (M_{xy}/L_y)\,(dI_1/dt)_{t=0} = Q/(\square'_{xy} C)$ where $\square'_{xy} = (L_xL_y - M_{xy}^2)/M_{xy}$ (29)

c') $\int_0^\infty I_2(t)\,dt = 0$ (30)

By exploiting the previous equations by means of (27) the following relations are obtained:

a'') $A'_2 + D'_2\cos\square' = 0$ (28 bis) b”) $A'_2 \square + D'_2(\square \cos \square' - \square_1 \text{ sen } \square') = Q/(\square'_{xy} C)$ (29 bis)

c”) $A'_2/\square - D'_2 (\square \cos \square' + \square_1 \text{ sen } \square')/(\square^2 + \square_1^2)$ (30 bis)

and finally:

$\text{tg } \square' = [(\square^2 + \square_1^2)/\square - \square]/\square_1$ (31)

$A'_2 = - Q / [\square'_{xy} C (\square - \square - \square_1 \text{ tg } \square')]$ (32)

$D'_2 = + Q (1 + \text{tg}^2 \square')^{1/2} / [\square'_{xy} C (\square - \square - \square_1 \text{ tg } \square')]$ (33)

Whereas the current intensity of the double pancake (or coil) can be easily measured, the current intensity $I_2(t)$ cannot be measured, nevertheless it is necessary to evaluate it in order to determine its effect in the resistance of the double pancake (or coil) (see paragraph 5b).

4. - THIRD CIRCUIT ANALYSIS.

The third circuit , whose diagram is presented in Fig. 3, describes the configuration with a double pancake (or coil) installed in the casing with one or more turn-to-turn short circuited .

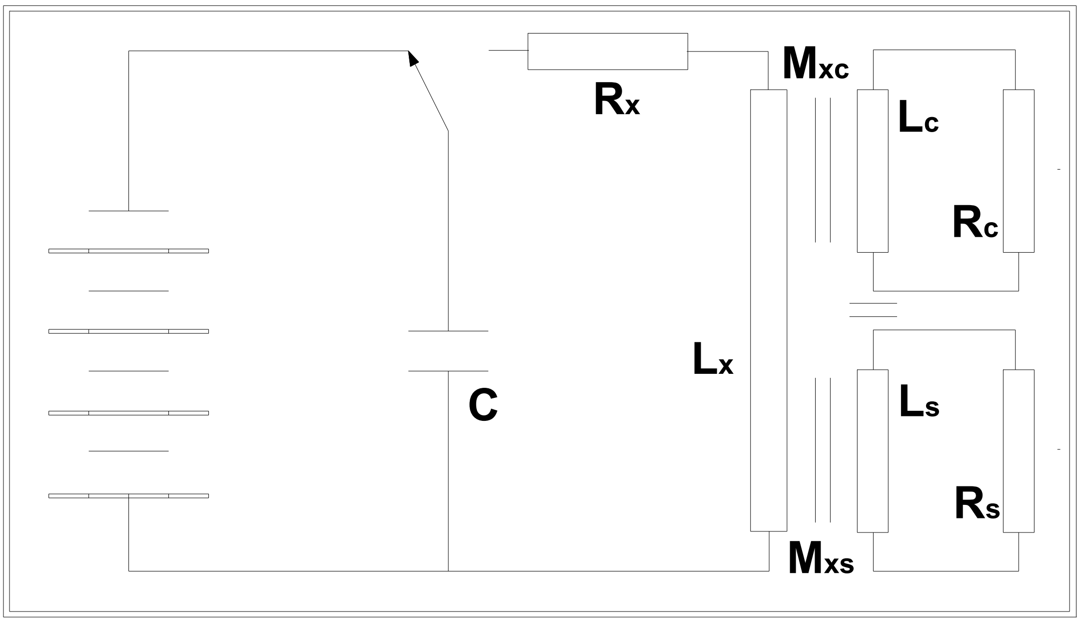

FIG. 3 - Electric diagram for the discharge in a double pancake (or coil) installed in the casing with one or more turns short circuited.

The capacitor discharge in the coil is described by the following differential equations:

$$L_x\, dI_1/dt + M_{xc}\, dI_2/dt + M_{xs}\, dI_3/dt + R_x I_1 + Q/C = 0 \qquad (34)$$

$$L_c dI_2/dt + M_{xc}\, dI_1/dt + M_{cs}\, dI_3/dt + R_c I_2 = 0 \qquad (35)$$

$$L_s dI_3/dt + M_{xs}\, dI_1/dt + M_{cs}\, dI_2/dt + R_s I_3 = 0 \qquad (36)$$

where:
L_x, L_c, L_s are the self-inductances respectively of the intact double pancake (or coil) , the casing, and the short circuit;
M_{xc}, M_{xs}, M_{cs} are the mutual inductances respectively between the double pancake (or coil) and the casing, the double pancake (or coil) and the short circuited turns, the casing and short circuited turns;
R_x, R_c, R_s are the resistances of the double pancake (or coil), the casing and the short circuited turns;
I_1, I_2, I_3 are the current intensities in the double pancake (or coil), the casing and short circuited turns.

The solution of the system of equations (34÷36) with the analytical procedure is quite complicate, thus a numerical solution has been preferred. The steps to get a first order system of differential equations, solvable with the Runge Kutta method, are reported in Appendix B.

5. - CALCULATION OF THE PHYSICAL CHARACTERISTICS OF THE CIRCUIT ELEMENTS.

For the analysis and comparison of the circuits response it is necessary to evaluate very carefully the self-inductances, the mutual-inductances and resistances of the circuit components.

a) *Self and mutual induction coefficients*

A quick evaluation of the induction coefficients is done with the same procedure used during the design of the ATLAS toroid[2]. The specific flux (per unit length) produced by the longer sides of the double pancake (or coil) considered of infinite length has been calculated and corrected with a proper coefficient (□=1.10) to compensate this approximation[3].

This procedure allows the calculation of the self and mutual-inductances of the double pancake (or coil) and short circuited turns with an accuracy of few percent, considered that the distribution of induced currents during the capacitor discharge produces small variation of self and mutual inductance calculated for continuous transport currents. This accuracy seems enough for the aim of our analysis that is based on the comparison of discharge parameters of intact coil and short circuited coil, rather than their absolute values.

As already written the calculation of the casing self and mutual-inductances is quite complicate, therefore we have evaluated the variation range of these parameters and we have considered these values that make more critical the detection of a short circuit.

The analytical calculation of these quantities are reported in Appendix C. The results are listed in Tables I and II.

Table I
Induction coefficients of an intact double pancake or coil

Item	**L(H)**	**M(H)**	**k**
Double pancake	0.145	M_{gc} = (1.52 ÷ 3.69) E-03	0.80 ÷ 0.97
Coil	0.521	M_{bc} = (2.89 ÷ 7.0) E-03	0.80 ÷ 0.97
Coil anti-inductive	0.060	M_{bc} = (0.98 ÷ 2.38) E-03	0.80 ÷ 0.97
Casing	(2.5 – 10) E-05		

Table II
Induction coefficients of double pancake or coil with one short circuited turn

Item	**L(H)**	**M(H)**	**k**
Double pancake	0.140	M_{gc} = (1.50 ÷3.63) E-03 M_{gs} = 2.12 E-03	0.80 ÷ 0.95 0.67
Coil	0.512	M_{bc} = (2.87 ÷ 6.97) E-03 M_{bs} = 3.54 E-03	0.80 ÷ 0.95 0.58
Coil anti-inductive	0.059	M_{bc} = (0.97 ÷ 2.35) E-03 M_{bs} = 1.19 E-03	0.80 ÷ 0.95 0.58
Casing	(2.5 ÷10) E-05	M_{cs} = (3.2 ÷ 8.2) E-05	0.80 ÷ 0.95
Short circuited turn	7.17 E-05		

b) Resistances of the components

The resistance values are obtained from the power dissipated in the different elements. The power is the sum of two terms one linked to current flowing in the circuit from the capacitor discharge or from the variation of magnetic flux coupled with the element under test; the other term linked to the current loops induced in the conductor body itself. Therefore we get the equivalent resistance R_t of a circuit element, with the approximation of a stationary process, like a sum of a resistance R_i related to the transport current and an equivalent resistance R_e related to the current induced in the conductor body.

The resistance R_i of a rectangular conductor with height "h", width "a" and lenght "L" is equal to:

$$R_i = \rho L/(2\,\delta h) \qquad (37)$$

where "ρ" is the material resistivity, "h" the conductor side parallel to the field B and "δ" the conductor thickness where the transport current is confined because of the skin effect:

$$\delta = (\omega\mu/2\rho)^{-1/2} \qquad (38)$$

where "ω is the transport current angular frequency and "μ" the material permeability.
If $\delta \geq a/2$ the resistance R_i in practice is equivalent to the d.c. resistance ($R_i=\rho L/ah$).

The equivalent resistance R_e is given by the ratio of the power W_e (produced by the Joule effect of the current induced in the conductor body) to the square of the transport current I_{eff} [(4)]:

$$R_e = W_e/I^2_{eff} = [2(2\mu^2\delta/\rho)^{-1}\int B_o^{\,2}\, dS]/I^2_{eff} \qquad (39)$$

where dS = Ldh is the conductor lateral side surface, where the induced currents are flowing, and B_o is the amplitude of oscillating magnetic field, produced by transport current only nearby the surface S. We can assume B_o constant along the lateral side of the conductor and variable with respect to its position inside the double pancake (or coil) as follows:

$$B_o^{\,j} = \alpha\,(\mu_o I/2\pi)\,[(1-2j/N)/a \sum_{n=1}^{N} (1/n) + N/D] \qquad j = 1,\ldots,N \qquad (40)$$

where N is the number of turns of a single pancake (N=30), α=2 in case of double pancake and α=4 in case of coil, "a" is the shorter side of a cable of rectangular cross section, " D " is the average distance between the two long sides of the double pancake (or coil).

By taking into account that $\mu \sim \mu_o$ from the relations (38), (39) e (40) the following expression for the resistance R_e is obtained:

$$R_e = (\alpha/2\pi)^2 \sum^{K} (\mu_o\,\omega\,\rho/2)^{1/2}\,[(1-2j/N)/a \sum^{N} (1/n) + N/D]^{\,2}\,(hL)\,(I/I_{eff})^2 \qquad (41)$$

j=1 n=1

where the summation on index "j" is extended to K=1, in case of single turn, to K = 30 (or 60) in case of double pancake (or coil).

The relations (37) and (41) show that the total resistance of one element of the circuit increases with the angular frequency "ω" of the discharge current and is related to the capacitance "C" used in the discharge circuit.

For the double pancake (or coil) the ratio $(I/I_{eff})^2$ assumes the value 2 so that $R_e \gg R_i$ while for a single turn and casing the term $(I/I_{eff})^2$ assumes very low values (in this case the real current circulating in the examined element is quite higher than the current I generating the magnetic field B_o) consequently the total resistance of these two elements is mainly determined by R_i.

The values of $R_t = R_e + R_i$ for the double pancake, for the coil, for a single turn and for the casing at three different values of the capacity "C" of the condenser are reported in Table III. We notice that the average value of the total resistance of one turn reported is coincident with R_i, while for the casing resistance, in order to take into account the structure complexity, is reported a reasonable range of values.

Table III
Electrical resistance of the circuit elements

Item	C = 50 μF	C = 500 μF	C = 5 mF
Double pancake	R_t = 3.92 Ω	R_t = 2.32 Ω	R_t = 1.42 Ω
Coil	R_t = 8.22 Ω	R_t = 4.86 Ω	R_t = 2.96 Ω
Coil anti-inductive	R_t = 4.78 Ω	R_t = 2.83 Ω	R_t = 1.72 Ω
Single turn	R_t = 2.2 mΩ	R_t = 2.2 mΩ	R_t = 2.2 mΩ
Casing	R_t = 0.24 ÷ 0.96 mΩ	R_t = 0.133 ÷ 0.53 mΩ	R_t = 0.075 ÷ 0.3 mΩ

6. - SELECTION OF THE DISCHARGE CAPACITOR

The selection of a proper test capacitor takes into account three different values (C=50 μF, 500 μF , 5 mF) and the discharge parameters are compared in case of an intact coil (diagram of Fig. 1) and in case of a coil with a short circuited turn (diagram of Fig. 2).

For the calculation the values of self and mutual inductances reported in the Tables I, II and III have been used.

The waveform of the discharge current for the three selected capacitors, charged at 400 Volt, are reported in the figures 4, 5 and 6. The current in the intact and isolated coil and the current in the isolated coil with one short circuited turn are indicated respectively with I_{int} and I_{sh}.

The main discharge parameters (the period T and the damping coefficient □) and their percent variations for three different values of the capacitance C are reported in Table IV.

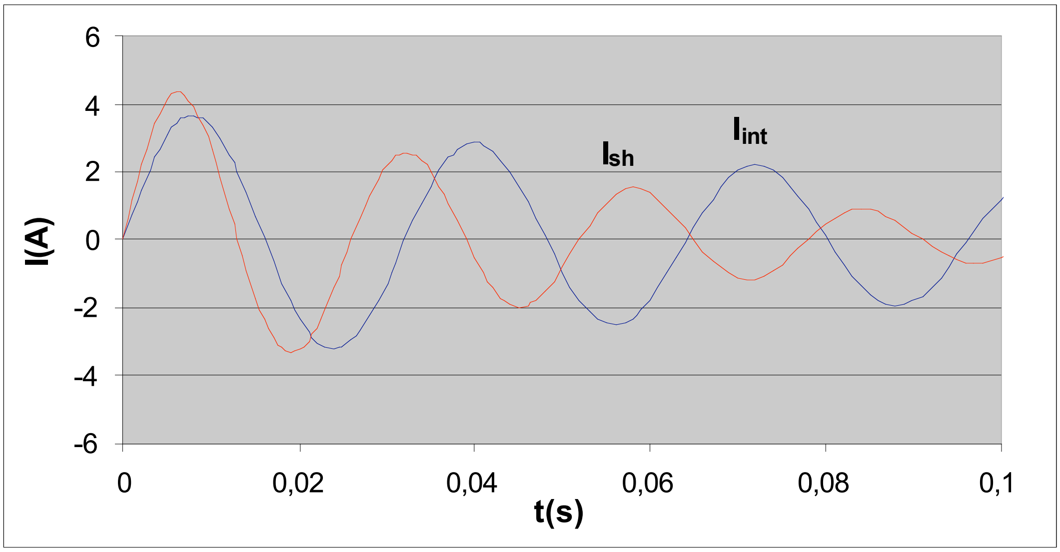

FIG. 4 – Discharge waveforms with C=50 F in an isolated and intact coil (I_{int}) and in the isolated coil with one short circuited turn (I_{sh}).

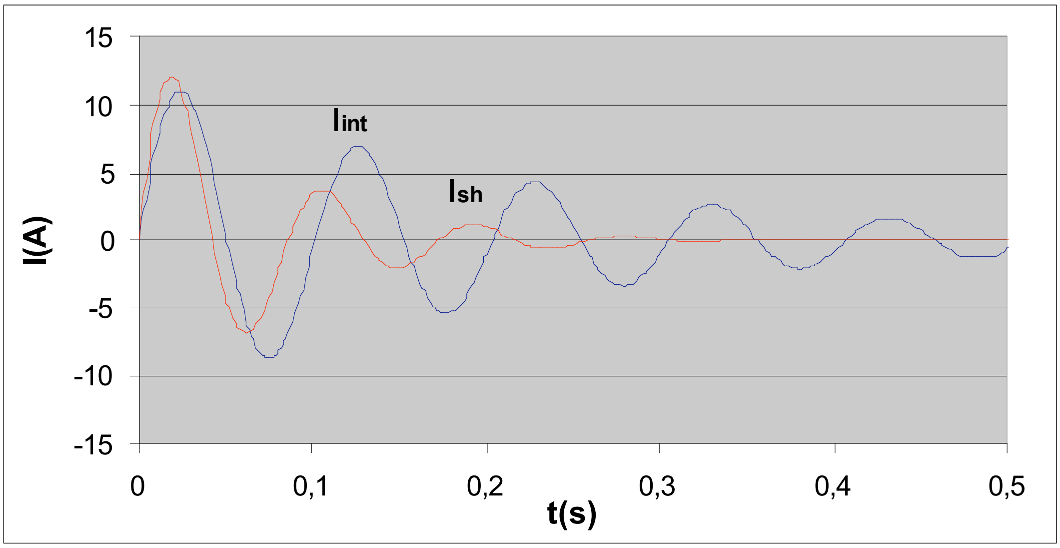

**FIG. 5 – Discharge waveforms with C=500 F in an isolated and intact coil (I_{int})
and in the isolated coil with one short circuited turn (I_{sh}).**

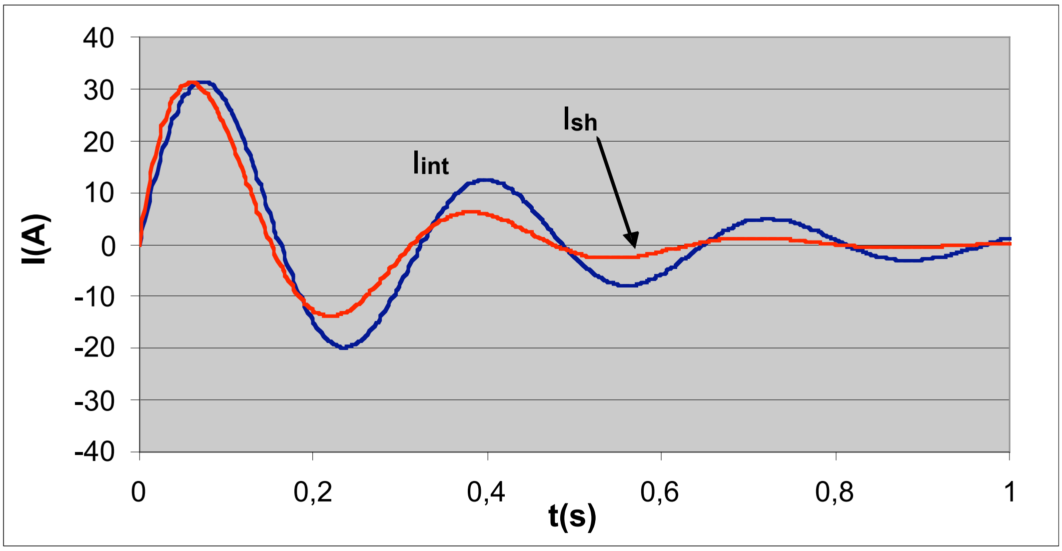

FIG. 6 – Discharge waveforms with C=5 mF in an isolated and intact coil (I_{int}) and in the isolated coil with one short circuited turn (I_{sh}).

The analysis of the plots and data of Table IV show wide differences in the discharge behaviour of the three capacitors with reference to the oscillating period and damping coefficient.

In the following paragraphs only circuits with the higher capacitor (5mF) will be considered. In this way the sensitivity of the method in the worse conditions can be verified but in the same time the measurements are less influenced by the effect of the parasitic capacitance of the system and finally the designer will have wider possibility in the choice of the data acquisition system (i.e. the possibility to choose the proper discharge period correlated with the switch opening speed, data acquisition speed, sensitivity and full scale of the measuring instruments, etc).

Table IV

Discharge parameters of an intact coil and of a coil with one short circuited turn

C	Item	T (ms)	□(%)	□ (s^{-1})	□(%)
50 □F	Intact Coil Coil with short (R_s = 2.2 m□)	32.09 26.0	 - 19.0	- 7.88 - 20.0	 + 154
500 □F	Intact Coil Coil with short (R_s = 2.2 m□)	101.7 86.6	 - 14.8	- 4.66 -13.80	 + 196
5 mF	Intact Coil Coil with short (R_s = 2.2 m□)	324.0 323.6	 - 0.12	- 2.84 - 5.08	 + 79

7. - ANALYSIS OF THE DISCHARGE IN AN ISOLATED AND INTACT DOUBLE PANCAKE AND IN A DOUBLE PANCAKE WITH ONE SHORT CIRCUITED TURN.

In Figs. 7 and 8 the time evolutions of the capacitor currents (C = 5 mF) are plotted for an intact double pancake, for a coil with one short circuited turn and for an anti-inductive coil (intact and with one short circuited turn). These plots must be compared with those of Fig. 6 related with a coil discharge. In the same figures the current behavior in the shorted turn is reported. The values of the discharge parameter of a coil, anti-inductive coil and double pancake are reported in Table V for comparison.

Table V
Discharge parameters in the coil and the double pancake (C = 5 mF)

Item	**T(ms)** □(%)	□(s^{-1}) □(%)	I_{max}(A) □(%)
Coil intact	324.0	- 2.84	31.5
	- 0.12	+ 79	- 1.0
Coil with short circ. turn	323.6	- 5.08	31.2
Coil anti-induct. intact	112.3	- 14.33	79.8
	- 8.8	+ 93	+ 8.8
Coil anti-induct. with short circ. turn	102.5	- 27.6	86.8
Double pancake intact	170.7	- 4.90	60.8
	- 7.9	+210	+5.8
Double pancake with short circ. turn	157.3	- 15.2	64.3

As expected, the detection of a short circuited turn is easier if the discharge is performed on the double pancake rather than on the whole coil (in both configurations: normal and anti-inductive).

This method of analysis is always applicable (even if the double pancakes are installed in the casing) so that it will be adopted as a standard test procedure.

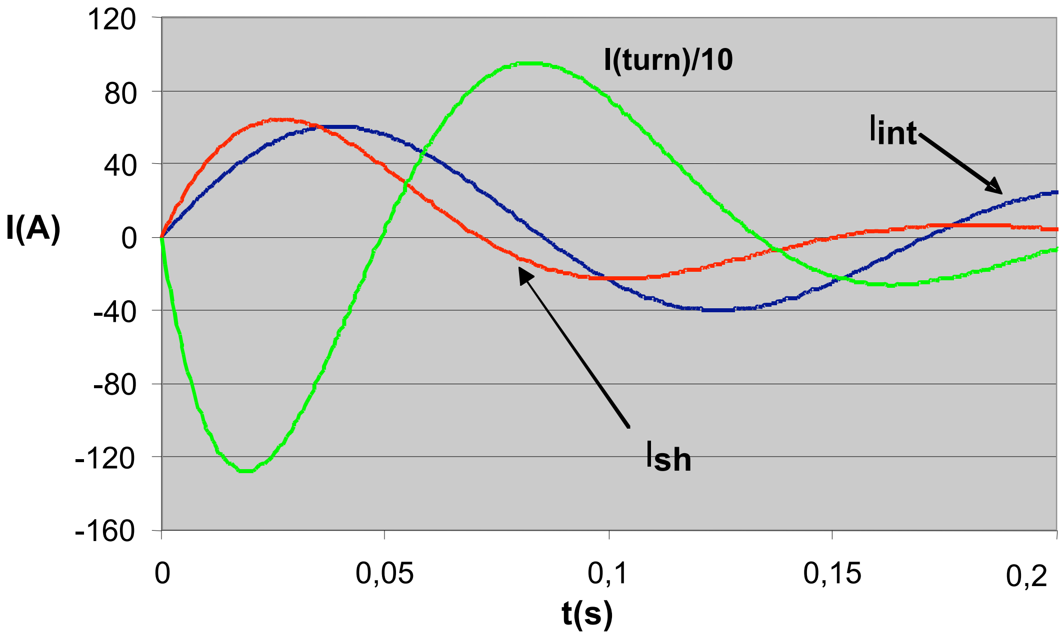

FIG. 7 - Time evolutions of the discharge currents with C=5 mF in an intact and isolated double pancake (I_{int}), in an isolated double pancake with one short circuited turn (I_{sh}) and in the short circuited turn (I_{turn}) (in this case I_{turn} is ten times lower than the actual one).

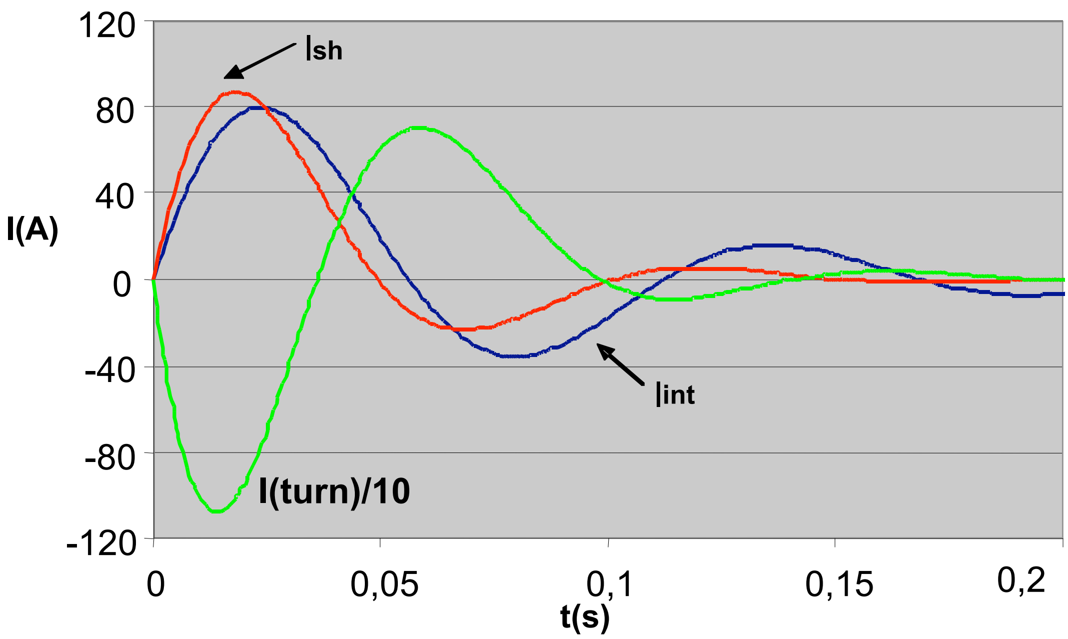

FIG. 8 - Time evolution of the discharge current in the anti-inductive coil : I_{int} (current in the intact and isolated coil) - I_{sh} (current in the coil with one short circuited turn) - I_{turn} (current in the short circuited turn).

8.- DISCHARGE ANALYSIS ON AN INTACT DOUBLE PANCAKE INSTALLED IN THE CASING.

For this analysis the electric diagram of Fig. 2 has been used by assuming that the casing is represented by a single turn coil coupled with the double pancake.

The large size (wide cross section of the ring obtained by sticking two parts only partially insulated) and the particular shape (presence of “ribs” connecting the long sides of the casing) do not allow a precise estimate of the self inductance and resistance of the casing. For this reason a range of values has been chosen for “ L_c”, “ M_{gc}” and “ R_c”, in order to evaluate analytically the effects produced by the casing on the discharge parameters and to find the sensitivity limits for the detection of a single short circuited turn.

In order to choose properly the inductive and resistive parameters of the casing the damping coefficient □ and the period T=2□/□□ (equations (18 bis)) have been calculated as a function of the coupling coefficient k, defined by:

$$k = M_{gc}/(L_g L_c)^{1/2} \qquad (42)$$

for a set of values of L_c and R_c. The results are plotted in Figs. 9 and 10.

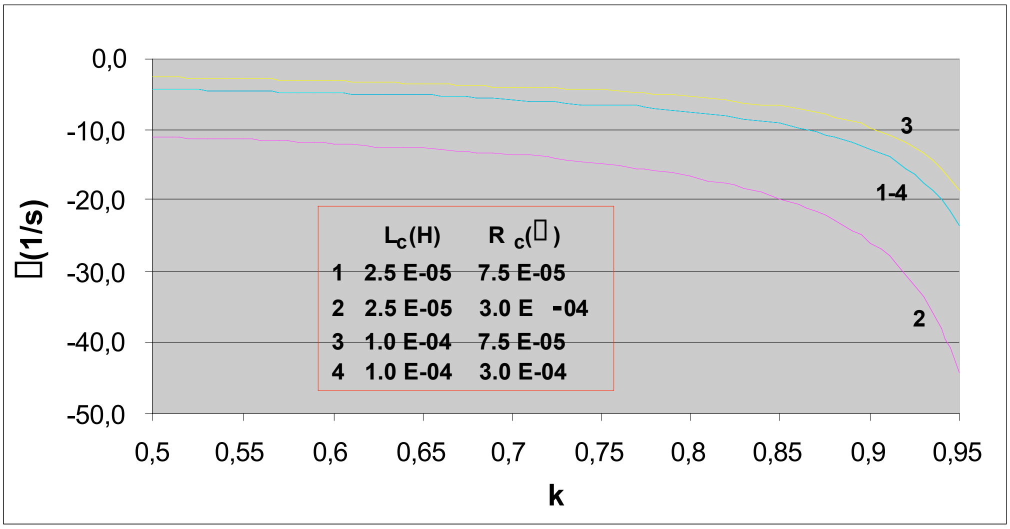

FIG. 9 - Damping coefficient □ as function of coupling coefficient k for boundary values of L_c ed R_c.

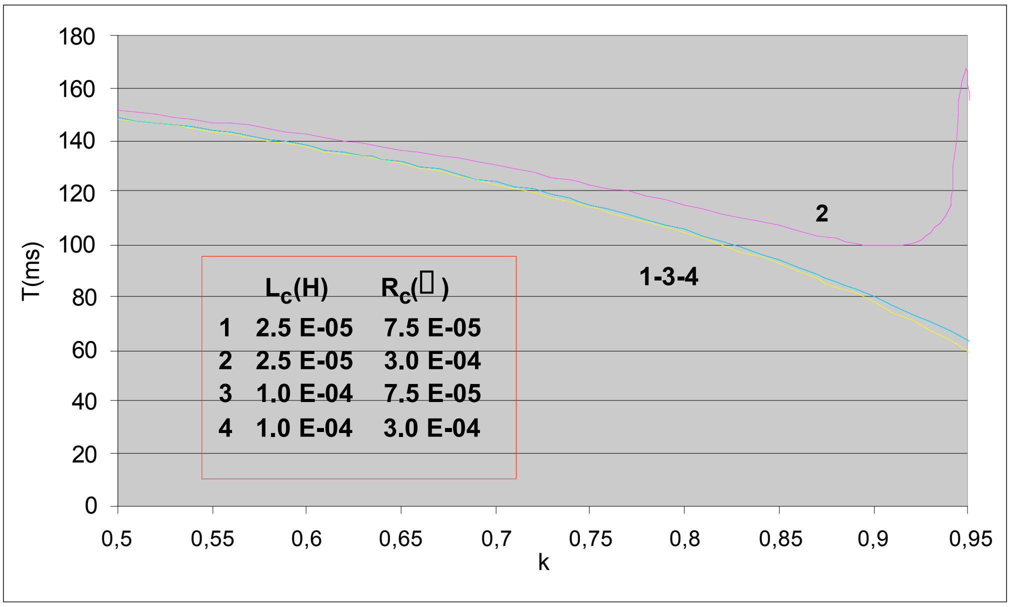

FIG. 10 - Period T as a function of coupling coefficient k for boundary values of L_c ed R_c.

As shown in Fig. 9 the damping effect, in the considered range, reaches the maximum value (curve 2) when the self inductance assumes the value L_c=2.5 10^{-5} H (minimum in the range) and the resistance the value R_c=3.0 10^{-4} (maximum in the range) and grows up quickly in the interval 0.9<k<0.95. While we have the minimum damping effect (curve 3) when L_c assumes the upper limit of the range values L_c=1.0 10^{-4} H and R_c the lower limit of the range values R_c=7.5 10^{-5} .

Fig. 10 shows that the period decreases from T=170.7 ms (discharge period for an intact and isolated (k =0) double pancake) to T ~ 60 ms (for k=0.95) for all the couples of L_c and R_c range values except for L_c=2.5 10^{-5} H and R_c=3.0 10^{-4} . For this couple of values the period rises quickly in the range 0.92 ‹ k ‹ 0.95.

The behavior of the damping coefficient as a function of k, L_c ed R_c can be easily understood by considering that the energy transfer from the first inductive element (double pancake) to the coupled inductive element (casing) increases with the coupling coefficient k, and moreover the a.c. current in the casing is mainly determined by the self inductance L_c. Therefore it follows that the power losses in the casing increase for small values of L_c and high values of R_c.

The behavior of the period T, as a function of k, L_c ed R_c, can be understood with an equivalent circuit where the pulsation increases for low values of the coupling coefficient k and decreases for high values of k due to the significant power losses in the casing. In particular when L_c=2.5 10^{-5} H and R_c=3.0 10^{-4} and k increases, the system behavior is quite similar to that one of an RLC circuit in critical damped or over-damped conditions ($\omega^2 \leq 0$).

The discharge parameters T and α for an intact and isolated double pancake and an intact double pancake installed in the casing with different values of L_c and R_c (see insert of Figs. 9 and 10) are listed in Table VI.

Table VI

Discharge parameters of an isolated d. p. and a d. p. installed in the casing

	Isolated double pancake	Double pancake installed in the casing			
	L_g = 0.145 H R_g = 1.42 Ω	L_c =2.5 E-5 H R_c =7.5 E-5 Ω	L_c =2.5 E-5 H R_c =3.0 E-4 Ω	L_c =1.0 E-4 H R_c =7.5 E-5 Ω	L_c =1.0 E-4 H R_c =3.0 E-4 Ω
T (ms)	170.7	64.6	130.6	61.2	64.6
α (s^{-1})	- 4.90	- 23.4	- 44.0	- 18.4	- 23.4

The graphs of Figs. 11 and 12 show the discharge currents in an intact and isolated double pancake and in an intact double pancake installed in the casing when the following parameters are considered:

$$L_c = 2.5\ 10^{-5}\ \mathrm{H} \qquad R_c = 7.5\ 10^{-5}\ \Omega \qquad k = 0.95 \qquad (43)$$
$$L_c = 2.5\ 10^{-5}\ \mathrm{H} \qquad R_c = 3.0\ 10^{-4}\ \Omega \qquad k = 0.95 \qquad (44)$$

In the following the analysis of the discharge process of a double pancake installed in the casing will be done with the three values reported in (43) and based on the following observations:

i) if the inductive and resistive parameters of the casing used produce an excessive damping effect (like in the case of the parameters (44)) it will be enough to decrease the capacity C of a proper factor and the system will return to the conditions similar to that above indicated;

ii) the choice of a k value so high (not corresponding to the value estimated in Appendix C) is needed to take into account the power loss process inside the body of the casing that produces a strong damping effect and a reduction of the period of the current in the double pancake.

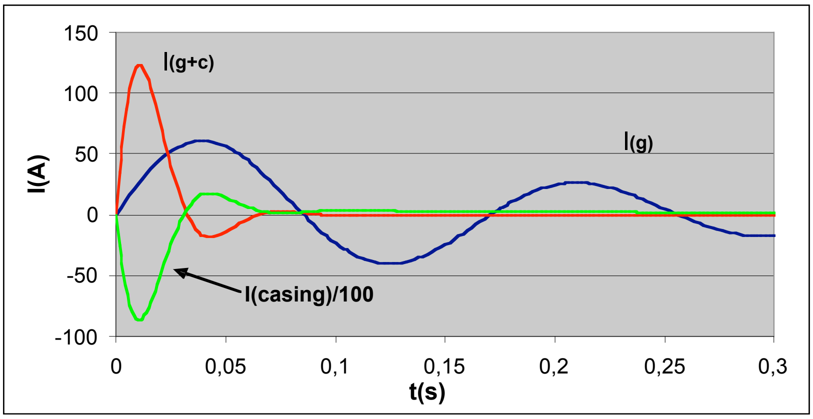

FIG. 11- Current intensity in an intact and isolated double pancake (I(g)), in an intact double pancake installed in the casing (I(g+c)) and in the casing (I(casing)), when the electrical parameters (43) are used.

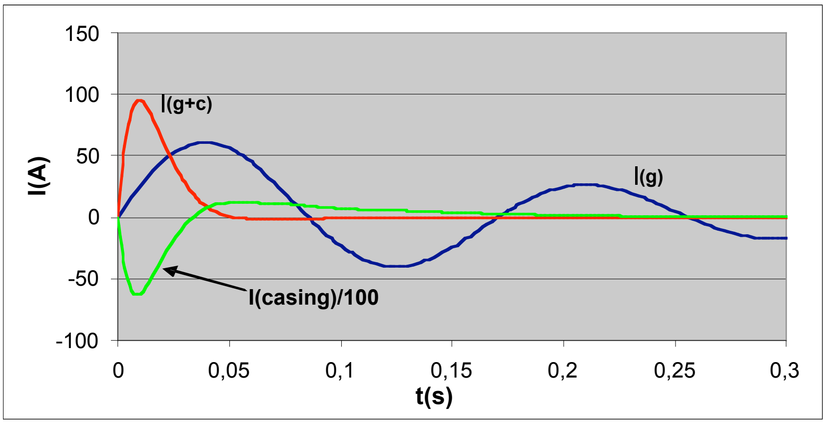

FIG. 12 - Current intensity in an intact and isolated double pancake (I(g)), in an intact double pancake installed in the casing (I(g+c)) and in the casing (I(casing)), when the electrical parameters (44) are used.

9. - ANALYSIS OF THE DISCHARGE IN A DOUBLE PANCAKE WITH A SHORT CIRCUITED TURN AND INSTALLED IN THE CASING.

The integration of the differential equations (34-36) is performed with the Runge Kutta method by using a time integration step Δt = 0.01 ms. The inductance and resistance parameters of the various elements are listed in the Table VII.

Table VII
Parameters of the double pancake, short circuited turn and casing.

Item	L(H)	M(H)	R(□)
Intact double pancake	L_g = 0.145	M_{gc} = 1.80 E-03	R_g = 1.42
Casing	L_c =2.5E-05		R_c =7.5 E-05
Double pancake with short circuited turn	L_g = 0.140	M_{gc} = 1.77 E-03 M_{gs} = 2.1 E-03	R_g = 1.418
Casing	L_c = 2.5E-05	M_{cs} = 2.6 - 3.4 E-05	R_c = 7.5 E-05
Short circuited turn	L_s = 7.17 E-05		R_s = 2.2 E-03

Since the discharge behavior is also controlled by the coupling coefficient (k_{cs}) between casing and short circuited turn, the discharge has been evaluated for the two boundary values of k_{cs} (k_{cs}=0.6 and k_{cs}= 0.8)

The graphs of Figs. 13 and 14 show the discharge currents respectively for k_{cs}=0.6 and k_{cs}= 0.8, while the Table VIII contains the discharge parameters (T e □).

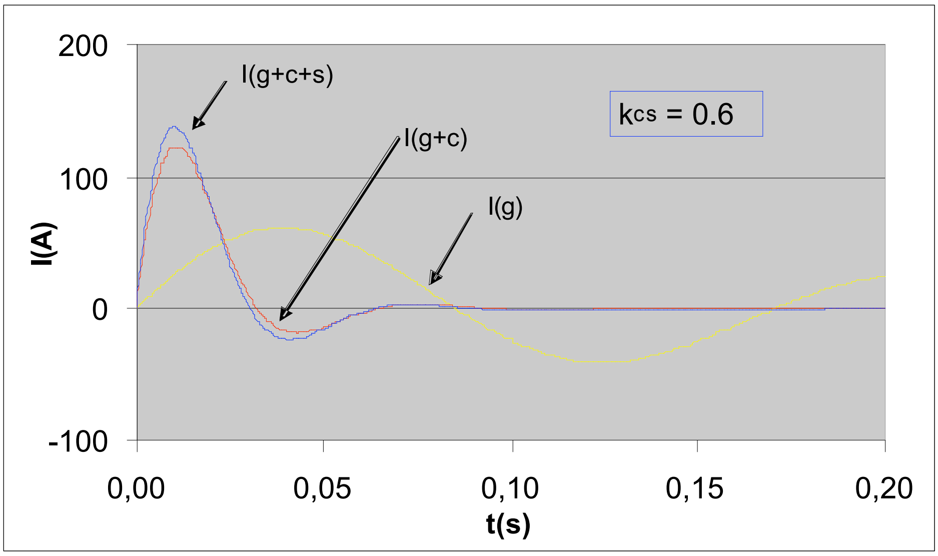

FIG. 13 – Current intensity in an intact and isolated double pancake [I(g)], in an intact double pancake installed in the casing [I(g+c)] and in the double pancake into the casing with one short circuited turn [I(g+s+c)], when k_{cs} = 0.6.

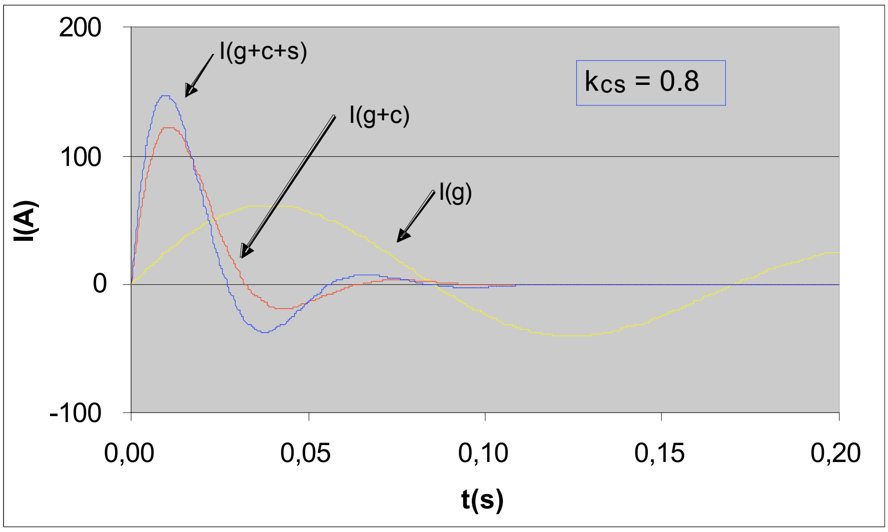

FIG. 14 – Current intensity in an intact and isolated double pancake [I(g)], in an intact double pancake installed in the casing [I(g+c)] and in the double pancake into the casing with one short circuited turn [I(g+s+c)], when k_{cs} = 0.8.

Table VIII
Discharge parameters in a double pancake

System	k_{cs} = 0.6		k_{cs} = 0.8	
	T(ms)	□(s^{-1})*	T(ms)	□(s^{-1})
Double pancake	170.7	- 4.90	170.7	- 4.90
Double pancake + casing	64.6	- 58.8	64.6	- 58.8
Double pancake + casing + turn	63.5	- 57.1	56.3	- 49.1

* Note: the coefficient "□" calculated by the formula □= $\ln(I_2/I_1)/(t_2-t_1)$ is coincident with coefficient □ reported in the text only for the double pancake intact and isolated. It was decided to report the values of □ because this quantity will be experimentally determined, being I_1 the first maximum and I_2 the first minimum of the measured current.

These results show that the most sensible parameter to detect the presence of a single short circuited turn is represented by the current peak (first maximum). The other two parameters (T and □) are not very sensible if the coupling coefficient between the short circuited turn and the casing is low (k_{cs} = 0.6) and become sensible like the current peak when k_{cs} assumes higher values. This work states that the presence of a short circuited turn inside a double pancake or coil, after a careful evaluation of the discharge parameters (period, damping coefficient and current amplitude of the first oscillation), can be detected in spite of the presence of the casing.

10. - CONCLUSIONS

The PJ method has shown enough sensitivity to detect short circuited turn inside the double pancake also if the casing behaves as a strong screen.

The possibility to choose the capacitance “C” of the capacitor in a wide range of values, allows a proper choice on the base of the characteristic of the coils under test and of the speed of data acquisition system.

For the research of possible short circuited turns inside the coils it is suggested to perform the test on the single double pancake.

Obviously the results obtained with this theoretical analysis are only an useful guide for the experimental data analysis, in other words it is not possible to use the discharge parameters hereby calculated and to state the presence or less of a short circuited turn. Moreover we recall that the measured time evolution of the current intensity could differ from the calculated one due to the complexity of the dissipation process inside the casing.

On the basis of this work the following test procedure should consist in the measurement of the current intensity in each double pancake of the coils in these conditions:

a) the two double pancakes are electro-magnetically isolated between them and with the same electromagnetic coupling with the environment,
b) both double pancakes are inside the casing but the measurements will be done on each double pancake separately.

Only after a very careful comparison of the period, of the damping coefficient and of the first current peak of each couple of double pancakes it will be possible to detect a short circuited turn.

Obviously this procedure is based in these assumptions:

i) the contact resistance of the short circuit is at least ten times lower than the resistance of a single turn,
ii) it is very improbable that two short circuits, with identical characteristics, happen in the two double pancakes installed in the same casing under test. Furthermore it is also possible to increase the accuracy of the analysis if the data of the 16 double pancakes are compared.
iii) the effects on the discharge parameters due to the differences in electrical characteristics of two double pancakes are very less important of those ones produced by a single turn short circuit. It is easy to verify with the relations (2bis) e (2ter) that the variations of 2% of the self inductance and resistance of the double pancake will produce a maximum variation of the damping coefficient □ of 4% and about 1% of the period T. These percent variations are significantly lower than those ones produced by a short circuited turn.

APPENDIX A

The integral (23) is given by:

$$\int_0^\infty I_1(t)\,dt = \int_0^\infty (A_2 e^{\square t} + D_2 e^{\square t}\cos(\square t + \square))\,dt =$$

$$= \int_0^\infty [A_2 e^{\square t} + D_2 e^{\square t}(\cos \square t \cos \square \; \square \; \text{sen}\, \square t \;\text{sen}\, \square)]\,dt =$$

$$= A_2/\square\,\Big[e^{\square t}\Big]_0^\infty + D_2/(\square^2 + \square^2)\,\Big[e^{\square t}(\square\,\text{sen}\,\square t + \square\cos \square t)\cos \square\Big]_0^\infty$$

$$\square\, D_2/(\square^2 + \square^2)\,\Big[e^{\square t}(\square\,\text{sen}\,\square t \;\square\; \square\cos \square t)\,\text{sen}\,\square\Big]_0^\infty \qquad (\square 1)$$

By considering that □ e □ assume always negative values, the integral (A1) becomes:

$$\int_0^\infty I_1(t)\,dt = -A_2/\square - D_2/(\square^2 + \square^2)\,\Big[\square\cos\square + \square\,\text{sen}\,\square\Big] \qquad (\square 2)$$

The system of the unknowns $A_2, D_2, \square$:

$$A_2 + D_2 \cos \square = 0 \qquad (A3)$$

$$A_2\square + D_2(\square\cos\square - \square\,\text{sen}\,\square) = -Q/(\square_{xy}C) \qquad (A4)$$

$$-A_2/\square - D_2[\square\cos\square + \square\,\text{sen}\,\square] / (\square^2 + \square^2) = Q \qquad (A5)$$

can be easily solved by extracting the quantity A2 by the equation (A3) and by replacing it in the other two equations:

$$D_2(-\square\cos\square + \square\cos\square - \square\,\text{sen}\,\square) = -Q/(\square_{xy}C) \qquad (A6)$$

$$D_2[\cos\square/\square - (\square\cos\square + \square\,\text{sen}\,\square) / (\square^2 + \square^2)] = Q \qquad (A7)$$

From equations (A6) and (A7) it follows:

$$\text{tg}\,\square = \{\square - \square - [1/\square - \square/(\square^2 + \square^2)] / (\square_{xy}\,C)\} / \{\square[1-1/[(\square^2 + \square^2)\,\square_{xy}\,C]]\} \qquad (A8)$$

and by considering that: $\text{sen}\,\square = \text{tg}\,\square / (1 + \text{tg}^2\,\square)^{1/2}$ $\qquad \cos\square = 1 / (1 + \text{tg}^2\,\square)^{1/2}$

it is possible to deduce from the equations (A6) and (A3) the relations for the quantities A_2 e D_2.

APPENDIX B

The system of the differential equations can be written in the following form, where the unknowns are represented by the second derivatives of the three currents I_1, I_2, I_3, and the other quantities (first derivatives and functions I_1, I_2, I_3) represent the known term:

$$L_x\, d^2I_1/dt^2 + M_{xc}\, d^2I_2/dt^2 + M_{xs}\, d^2I_3/dt^2 = - R_x\, dI_1/dt - I_1/C \qquad \text{(B1)}$$

$$M_{xc}\, d^2I_1/dt^2 + L_c d^2I_2/dt^2 + M_{cs}\, d^2I_3/dt^2 = - R_c dI_2/dt \qquad \text{(B2)}$$

$$M_{xs}\, d^2I_1/dt^2 + M_{cs}\, d^2I_2/dt^2 + L_s d^2I_3/dt^2 = - R_s dI_3/dt \qquad \text{(B3)}$$

The solution of the system is given by:

$$d^2I_1/dt^2 = (1/\Delta) \begin{vmatrix} - R_x\, dI_1/dt - I_1/C & + M_{xc} & + M_{xs} \\ - R_c dI_2/dt & + L_c & + M_{cs} \\ - R_s dI_3/dt & + M_{cs} & + L_s \end{vmatrix} \qquad \text{(B4)}$$

$$d^2I_2/dt^2 = (1/\Delta) \begin{vmatrix} + L_x & - R_x\, dI_1/dt - I_1/C & + M_{xs} \\ + M_{xc} & - R_c dI_2/dt & + M_{cs} \\ + M_{xs} & - R_s dI_3/dt & + L_s \end{vmatrix} \qquad \text{(B5)}$$

$$d^2I_3/dt^2 = (1/\Delta) \begin{vmatrix} + L_x & + M_{xc} & - R_x\, dI_1/dt - I_1/C \\ + M_{xc} & + L_c & - R_c dI_2/dt \\ + M_{xs} & + M_{cs} & - R_s dI_3/dt \end{vmatrix} \qquad \text{(B6)}$$

where Δ is the determinant of the matrix:

$$\Delta = \begin{vmatrix} + L_x & + M_{xc} & + M_{xs} \\ + M_{xc} & + L_c & + M_{cs} \\ + M_{xs} & + M_{cs} & + L_s \end{vmatrix} =$$

$$= L_x(L_cL_s - M_{cs}^2) - M_{xc}(M_{xc}L_s - M_{cs}M_{xs}) + M_{xs}(M_{xc}M_{cs} - L_cM_{xs}) \qquad \text{(B7)}$$

The equation system (B1 – B3) becomes:

$$d^2I_1/dt^2 = a_{10}\ I_1 + a_{11}\ dI_1/dt + a_{12}\ dI_2/dt + a_{13}\ dI_3/dt$$

$$d^2I_2/dt^2 = a_{20}\ I_1 + a_{21}\ dI_1/dt + a_{22}\ dI_2/dt + a_{23}\ dI_3/dt \qquad (B8)$$

$$d^2I_3/dt^2 = a_{30}\ I_1 + a_{31}\ dI_1/dt + a_{32}\ dI_2/dt + a_{33}\ dI_3/dt$$

where:

$a_{10} = -\ (1/\square)(L_cL_s - M_{cs}^2)/C$
$a_{11} = -\ (1/\square)R_x(L_cL_s - M_{cs}^2)$
$a_{12} = +(1/\square)\ R_c(M_{xc}L_s - M_{xs}M_{CS})$
$a_{13} = +(1/\square)\ R_s(M_{cs}L_C - M_{xc}M_{cs})$
$a_{20} = +(1/\square)\ (M_{xc}L_s - M_{xs}M_{CS})/C$
$a_{21} = +(1/\square)\ R_x(M_{xc}L_s - M_{xs}M_{CS})$
$a_{22} = -(1/\square)R_c(L_xL_s - M_{xs}^2)$
$a_{23} = +(1/\square)\ R_s(M_{cs}L_x - M_{xs}M_{XC})$
$a_{30} = -(1/\square)\ (M_{xs}L_C - M_{xc}M_{cs})/C$
$a_{31} = +(1/\square)\ R_x(M_{xs}L_C - M_{xc}M_{cs})$
$a_{32} = +(1/\square)\ R_c(M_{cs}L_x - M_{xc}M_{XS})$
$a_{33} = -(1/\square)\ R_s(L_xL_c - M_{xc}^2)$

For the numerical integration with Runge Kutta method the previous system (B8) is transformed in a system of differential equations of the first order. By putting:

$$y_{10} = I_1 \qquad y_{20} = I_2 \qquad y_{30} = I_3$$

the system (B8) becomes:

$$\begin{aligned}
dy_{10}/dt &= y_{11}\\
dy_{20}/dt &= y_{21}\\
dy_{30}/dt &= y_{31} \qquad (B9)\\
dy_{11}/dt &= a_{10}\ y_{10} + a_{11}\ y_{11} + a_{12}\ y_{21} + a_{13}\ y_{31}\\
dy_{21}/dt &= a_{20}\ y_{10} + a_{21}\ y_{11} + a_{22}\ y_{21} + a_{23}\ y_{31}\\
dy_{31}/dt &= a_{30}\ y_{10} + a_{31}\ y_{11} + a_{32}\ y_{21} + a_{33}\ y_{31}
\end{aligned}$$

The initial conditions (for $t = 0$) are□

$$y_{10}(0) = 0 \qquad y_{20}(0) = 0 \qquad y_{30}(0) = 0$$

$$y_{11}(0) = -\ (1/\square)(L_cL_s - M_{cs}^2)\ (Q/C)$$

$$y_{21}(0) = +(1/\square)\ (M_{xc}L_s - M_{xs}M_{CS})(Q/C) \qquad (B10)$$

$$y_{31}(0) = +(1/\square)\ (M_{xs}L_C - M_{xc}M_{cs})(Q/C)$$

APPENDIX C

As specified in the paragraph 5 the self and mutual inductances of the various elements are obtained by calculating the specific flux of the magnetic induction B produced by the current I along two series of straight, parallel and infinitely long conductors. The calculated values are corrected by means of the factor.

$$\square_t = 1.10$$

due to the finite length and the rounded corners of the element.

Each element can be a single loop (casing or short circuited turn) or several loops in one plane (single pancake), in two planes (double pancake) in four planes (coil).

a) Self-inductance L_s of a single turn.

The following value of L_s is obtained by applying the Biot and Savart law:

$$L_s/D_s = \Phi(B)/I\,D_s = 2\,\square_t\,\mu_o \int_{h_s/2}^{a_s} dr/(2\pi r) = \square_t\,(\mu_o/\pi)\,\ln(2a_s/h_s) \qquad \text{(C1)}$$

where D_s is the length of the long side of the loop, a_s the length of the short side and h_s the conductor thickness (for a single loop h_s = 12.0 mm i.e. it is equal to the thickness of the superconductor matrix).

The value of the self inductance is a little different if the considered loop is located in the internal region (D_s = 24.178 m and a_s =4.214 m) or in the external region (D_s = 24.950 m and a_s = 4.974 m) of the pancake. The values of the self inductance are respectively:

$$L_s(\text{int}) = 6.97\ 10^{-5}\ \text{H} \qquad\qquad L_s(\text{ext}) = 7.37\ 10^{-5}\ \text{H}$$

By considering that the value of the self inductance of the loop can change at the maximum of ± 3% in respect to the mean value, the last one will be assumed in the analysis of the discharge:

$$L_s = 7.17\ 10^{-5}\ \text{H}$$

b) Self and mutual inductances of the pancakes in the coil configuration..

The aim of this calculation is to determine the self and mutual inductances of the double pancakes and the self inductance of the whole coil as sum of the self and mutual inductances of the pancakes. The self inductance of the pancake with N turns (see Fig. 1c for the integration parameters) is given by:

N a N b

$$L_{g1}/D_{g1} = \square\, (\square_o/\square)\{N \sum_{j=1} \int_0 dr/[r + b - (j-0.5)h] \;+\; Nb/(a+b) \;+\; \sum_{j=1} \int_{jh} (\square x - \square)\, dx\} \quad (C2)$$

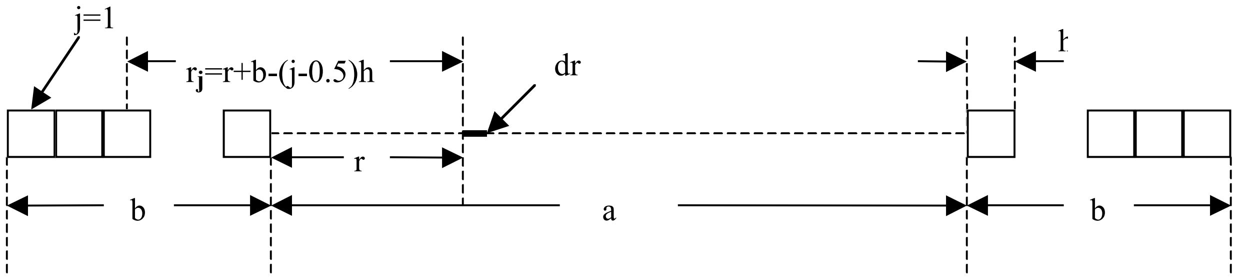

Fig. 1c Sketch of the pancake with the integration parameters.

where:

$$\square = \sum_{j=1}^{N} 1/[(j-0.5)\, h] \qquad\qquad \square = 2\square/b$$

The first term in (C2) represents the contribution of all turns to the magnetic flux inside the internal region (region of size a in Fig. 1c) , the second and third terms represent the contributions to the magnetic flux produced by the currents in the conductor regions (regions of size b in Fig. 1c).

By solving the integrals in (C2) the following result is obtained:

$$L_{g1}/D_{g1} = \square\, (\square_o/\square)\{N \sum_{j=1}^{N} \ln [(a+b-(j-0.5)h)/(b-(j-0.5)h)] + Nb/(b+a) + \sum_{j=1}^{N} [(\square/2)(b^2-j^2h^2) \square(bjh)]\}$$

and by putting:

$N = 30 \quad D_{g1} = 24.564$ m $\quad a = 4.214$ m $\quad b = 0.386$ m $\quad h = 12.87$ mm C(3)

the following value of the self inductance is obtained:

$$L_{g1} = 41.4 \text{ mH} \qquad (C4)$$

The mutual inductance between two pancakes (with the same number N of turns) is approximately given by:

$$M_{gij} = \square_t\, (\mu_o/2\pi)\, D_{g1}\, N^2 \ln [(c^2 + d_{ij}^2)/(b^2/4 + d_{ij}^2)] \qquad \text{(C5)}$$

where $c = a + 2b$, d_{ij} is the distance between the median planes of the two pancakes and the other parameters are defined in Fig. 1c and in the list (C3).
The distances between the median planes of the pancakes are:

$d_{12} = d_{21} = d_{34} = d_{43} = 0.06$ m
$d_{13} = d_{31} = d_{24} = d_{42} = 0.17$ m
$d_{14} = d_{41} = 0.23$ m
$d_{23} = d_{32} = 0.11$ m

so that the following values for the mutual inductances are obtained:

$$\begin{aligned} M_{g12} &= M_{g21} = M_{g34} = M_{g43} = 31.2 \text{ mH} \\ M_{g13} &= M_{g31} = M_{g24} = M_{g42} = 28.8 \text{ mH} \\ M_{g14} &= M_{g41} = 27.3 \text{ mH} \\ M_{g23} &= M_{g32} = 30.3 \text{ mH} \end{aligned} \qquad \text{(C6)}$$

c) Self and mutual inductances of the double pancakes in the coil configuration.

From the previous values (C6) the self and mutual inductances of the double pancakes (when they are assembled in the casing) are deduced:

$$L_g = 2\, L_{g1} + 2M_{g12} = 145.2 \text{ mH} \qquad \text{(C7)}$$

$$M_{gg} = M_{g13} + M_{g24} + M_{g14} + M_{g23} = 115.2 \text{ mH} \qquad \text{(C8)}$$

d) Self inductance of the coil.

From the data (C7) and (C8) the self inductance of the coil is obtained:

$$L_b = 2L_g + 2M_{gg} = 520.8 \text{ mH} \qquad \text{(C9)}$$

This value is in good agreement with that one (L_b = 525 mH) calculated during the design of the ATLAS Toroid [(2)] .
If the two double pancakes are connected anti-inductively the self inductance of the coil is:

$$L'_b = 2L_g - 2\, M_{gg} = 60 \text{ mH} \qquad \text{(C10)}$$

e) Mutual inductance between a single loop and the double pancake or the coil.

In average the mutual inductance of a single loop (short circuited turn) can be calculated as sum of two terms: the first one is due to the N-1 turns of the pancake in which is present the short circuited loop, the second one is due to the N turns of the second pancake:

$$M_{gs} = [(N-1)/N^2]\, L_{g1} + \square\, (\square_o/\square)\, D_{g1} \sum_{j=1}^{N} \int_{h/2}^{a_s} rdr/[(r + (j-0.5)h)^{2} + d^2] \qquad \text{(C11)}$$

where a_s = a+b =4.594 m and d = 0.06 m is the distance between the median planes of the two pancakes.

By solving the integral in (C11) the following expression for the mutual inductance M_{gs} (turn-double pancake) is obtained:

$$M_{gs} = [(N-1)/N^2]\, L_{g1} + \square\, (\square_o/\square)\, D_{g1} \Big\{0.5 \sum_{j=1}^{N} \ln [((a_s+(j-0.5)h)^2+d^2)/(j^2h^2+d^2)] - \sum_{j=1}^{N} (j-0.5)h/d\ [\mathrm{arctg}[a_s+(j-0.5)h/d] - \mathrm{arctg}(jh/d)]\Big\} \qquad \text{(C12)}$$

With the geometrical data of the pancake and loop the mutual inductance assumes the following value:

$$M_{gs} = 2.12\ \text{mH} \qquad \text{(C13)}$$

with a coupling coefficient $k_{gs} = M_{gs}/(L_s L'_g)^{1/2}$ (where L'_g = 140.4 mH is the self inductance of a double pancake with N=59 turns):

$$k_{gs} = 0.67$$

The mutual inductance between a single loop and the coil has been obtained by scaling the data of the pancakes:

$$M_{bs} = 3.54\ \text{mH} \qquad k_{bs} = 0.59$$

f) Self inductance of the casing.

In the calculation of the self inductance the casing has been assumed as a tape of thickness $(w_2\text{-}w_1)$, being $2w_2 = 5.15$ m and $2w_1 = 3.55$ m respectively the external and the internal dimensions of the short side of the casing. Being the average length of the casing $D_c = 24.32$ m, the self inductance is given by:

$$L_c = \square\ (\mu_o/\square)\ D_c/(w_2\text{-}w_1)\{w_2 \ln [(w_2+w_1)/(w_2\text{-}w_1)] + w_1 \ln [(w_2^2\text{-}w_1^2)/4w_1^2]\} \qquad (C14)$$

and by substituting the numerical data the following value is obtained:

$$L_c = 27.9\ \mu H \qquad (C15)$$

This value is probably underestimated because of the approximations used in the calculations so that in the discharge analysis values of L_c in the range $25\ \mu H < L_c < 100\ \mu H$ will be considered.

g) Mutual inductance between casing and double pancake and between casing and turn.

Also in this case only an approximate estimate of the mutual inductances has been carried out.

The values of these mutual inductances are respectively:

$M_{gc} = 1.6$ mH $\qquad k_{gc} = 0.8$

$M_{cs} = 32\ \mu H \qquad k_{cs} = 0.7$

In the discharge analysis the coupling coefficient between the casing and the double pancake will be increased up to a value in the range $0.90 < k_{gc} < 0.97$ in order to increase the screening effect of the casing and to evaluate the limits of sensitivity of the method.

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