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UNIFIED TIME ANALYSIS OF PHOTON AND (NONRELATIVISTIC) PARTICLE TUNNELLING, AND THE SUPERLUMINAL GROUP-VELOCITY PROBLEM[1]

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Abstract

A unified approach to the time analysis of tunnelling of nonrelativistic particles is presented, in which Time is regarded as a quantum-mechanical observable, canonically conjugated to Energy. The validity of the Hartman effect (independence of the Tunnelling Time of the opaque barrier width, with Superluminal group velocities as a consequence) is verified for *all* the known expressions of the mean tunnelling time.

Moreover, the analogy between particle and photon tunnelling is suitably exploited. On the basis of such an analogy, an explanation of some recent microwave and optics experimental results on tunnelling times is proposed. Attention is devoted to some aspects of the causality problem for particle and photon tunnelling.

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1 Introduction.

The study of tunnelling started with the discovery of α-decay, which was followed by Lord Rutherford's investigations and, in 1928, by Gamow's quantum-mechanical description[1]. Much later, from the Fifties onwards, tunnelling experiments in solid state physics (such as those with tunnelling junctions[2], tunnelling diodes[3] and tunnelling microscopes[4]) were performed and theoretically analyzed.

The study of the *tunnelling times* has a long history too. The problem of the definition of a tunnelling time was mentioned at the beginning of the Thirties[5,6]. Then, it remained almost ignored until the Fifties or Sixties, when it was faced the more general question of defining a quantum-collision duration[7-18]: a question that, in its turn, had been put aside since the Twenties, after Pauli's works[19] stressing the impossibility of introducing a self-adjoint operator for Time in quantum mechanics. Among the first attempts to regard time as a quantum-mechanical observable, let us recall refs.[20-26] and, in particular, the clarification that such a problem received during the Seventies and Eighties in refs.[27-29]. Reviews about time as a quantum observable (which results to be a maximal hermitian operator, even if it is not selfadjoint) canonically conjugated to energy, can be found in refs.[29-31]. [Let us mention that a series of new papers recently appeared[32-39, and refs. therein], examining the properties of the time operator in quantum mechanics: however, all such papers seem to ignore the Naimark theorem[40] which is on the contrary an essential mathematical basis for refs.[27-31].] *

Recently, developments in various fields of physics and especially the advent of high-speed electronic devices, based on tunnelling processes, revived the interest in the tunnelling time analysis, whose relevance had been previously apparent in nuclear physics only (α-radioactivity and, afterwards, nuclear sub-barrier fission, fusion, proton-radioactivity, etc.). So that, in recent years, a number of theoretical reviews appeared[41-49]. With regard to experiments on tunnelling times, the great difficulty with actual measurements for particles was due to the too small values of the related tunnelling times (see, for instance, refs.[50-55]). Till when the *simulation* of particle tunnelling by microwave and laser-light tunnelling allowed some very interesting measurements[56-60] of "tunnelling times"; such a simulation being based on the known mathematical analogy between particle and photon tunnelling: Which becomes evident when comparing[61-65] the stationary Schroedinger equation, in presence of a barrier, with the stationary Helmholtz equation for an electromagnetic wavepacket in a waveguide.

In the more interesting *time-dependent* case, however, the Schroedinger and Helmholtz equations are no longer identical: a problem that was left open, and that one cannot forget. Another question that has to be faced is the introduction of an operator for Time in quantum mechanics and in quantum electrodynamics. Below (in Sects.9-10) we shall

*The Naimark theorem (1940) states that a non-orthogonal spectral decomposition of a maximal hermitian operator can be approximated, with a weak *convergence*, by an orthogonal spectral decomposition with any desired accuracy degree.

tackle with such problems, as well as with the physical interpretation of some photon tunnelling experiments.

Returning to the question of the theoretical definition of the tunnelling time for particles, there is not yet a general agreement about such a definition[41-49]; some reasons being the following: (i) The problem of defining tunnelling times is closely connected with the more general definition of the quantum-collision duration, and therefore with the fundamental fact that Time in some cases is just a parameter (like x), but in some other cases is a *(quantum) physical observable* (like $\hat{x}$); (ii) The motion of particles inside a potential barrier is a quantum phenomenon, that till now has been devoid of any direct classical limit; (iii) There are essential *differences* among the initial, boundary and external conditions assumed within the various definitions proposed in the literature; those differences have not been analyzed yet.

Following ref.[49], we can divide the existing approaches into a few groups, based — respectively— on: 1) a time-dependent description in terms of wavepackets; 2) averages over a set of kinematical paths, whose distribution is supposed to describe the particle motion inside a barrier; 3) the introduction of a new degree of freedom, constituting a physical clock for the measurements of tunnelling times. Separately, it stands by itself the dwell time approach. The latter has *ab initio* the presumptive meaning of the time during which the incident flux has to be maintained, to provide the accumulated particle storage in the barrier[9,49]. The *first group* contains the so-called phase times (firstly mentioned in [7,8] and applied to tunnelling in refs.[66,67]), the times related to the motion of the wavepacket spatial centroid (considered for generic quantum collisions in refs.[17,18] and in particular for tunnelling in [68,69]), and finally the Olkhovsky–Recami (OR) method[48,70,71] (based on the generalization of the time durations defined for atomic and nuclear collisions in refs.[11,29,30]), which adopts averages over fluxes pointing in a well-defined direction *only*, and has recourse to a quantum operator for Time.

The *second group* contains methods utilizing the Feynman path integrals[72-75], the Wigner distribution paths[76,77], and the Bohm approach[78].

The methods with a Larmor clock[79] or an oscillating barrier[80,81] pertain to the *third group*.

In our opinion, basic self-consistent definitions of tunnelling durations (mean values, variances, and so on) should be worked out in a way similar to the one followed when defining in quantum mechanics other physical quantities (like distances, energies, momenta, etc.): namely, by utilizing the properties of time as a quantum observable. One ought then choose a time *operator*, canonically conjugated to the energy operator; and take advantage of the equivalence between the averages performed in the time and in the energy representation,[†] with adequate weights (measures). For such definitions, it is obviously necessary to abandon any descriptions in terms of plane waves, and to have rather re-

[†]An equivalence still following from the Naimark[40] theorem!

course to wavepackets and to the time-dependent Schroedinger equation: As it is actually typical in the quantum collision theory (see, e.g., the third one of refs.[10]). Afterwards, one will finally operate within the framework of conventional quantum mechanics; and, within this framework, it will be possible to show (as we shall do) that every known definition of tunnelling time is —at least in some suitable asymptotic regions— either a *particular case* of the most general definition, or a definition valid (not for tunnelling but) for some accompanying process.

The necessary formalism, and the consequent definitions, will be introduced in Sect.2 below. In Sects.3÷5 we shall briefly compare one another the various existing approaches. In Sect.6 we shall discuss some peculiarities of the tunnelling evolution. In Sect.7 we shall show the Hartman–Fletcher effect to be valid for all the known expressions of the mean tunnelling times. The short Sect.8 will present a new "two-phase description" of tunnelling, which is convenient for media without absorption and dissipation, as well as for Josephson junctions. In Sect.9 we investigate the analogies between the (time-dependent) Schroedinger equation, in presence of a quantum barrier, and the (time-dependent) Helmholtz equation for an electromagnetic wavepacket in a waveguide, and discuss the "tunnelling" experiments with microwaves. In Sect.10 we go on to study the tunnelling times in the optical "tunnelling" experiments based on frustrated total internal reflection. In Sect.11, a short note follows on the reshaping (and reconstruction) phenomena, in connection with a possible formulation of the principle of "relativistic causality" which is valid also when the tunnelling velocities are actually Superluminal. Finally, in Sect.12, some conclusions are presented, together with some prospective considerations for the near future.

2 A quantum operator for Time as the starting point for defining the tunnelling durations. The OR formalism.

We confine ourselves to the simple case of particles moving only along the x-direction, and consider a time-independent barrier located in the interval $(0, a)$: See Fig.1, in which a larger interval, (x_i, x_f), containing the barrier region, is also indicated. [We shall call *region* II the barrier region, *region* I the (initial) one on its left, and *region* III the (final) one on its right]. Following the known definition of duration of a collision —set forth firstly in ref.[11], then in [27,29,30] and afterwards generalized in [48,71]— we can eventually define the mean value $\langle t_{\pm}(x) \rangle$ of the time t at which a particle passes through position x (*travelling in the positive or negative direction, respectively*:

$$\langle t_{\pm}(x) \rangle = \frac{\int_{-\infty}^{\infty} t J_{\pm}(x,t) \mathrm{d}t}{\int_{-\infty}^{\infty} J_{\pm}(x,t) \mathrm{d}t} \tag{1}$$

and the variance $\mathrm{D} \equiv \sigma^2$ of that time distribution:

$$\mathrm{D}\, t_{\pm}(x) = \frac{\int_{-\infty}^{\infty} t^2 J_{\pm}(x,t)\mathrm{d}t}{\int_{-\infty}^{\infty} J_{\pm}(x,t)\mathrm{d}t} - [\langle t_{\pm}(x)\rangle]^2 \ , \tag{2}$$

$J_{\pm}(x,t)$ representing the *positive* or *negative values*, respectively, of the probability flux density $J(x,t) = \mathrm{Re}[(i\hbar/m)\Psi(x,t)\partial\Psi^{\dagger}(x,t)/\partial x]$ of a wavepacket $\Psi(x,t)$ evolving in time;[‡] namely $J_{\pm}(x,t) \equiv J\,\Theta(\pm J)$. Let us repeat that, with appropriate averaging weights, the (canonically conjugate) time and energy representations are equivalent in the sense that: $\langle ... \rangle_t = \langle ... \rangle_E$. Below, for the sake of simplicity, we shall omit the index t in all expressions for $\langle ... \rangle_t$. Let us also re-emphasize that the mentioned equivalence is a consequence of the existence in quantum mechanics of a unique operator for time: which, even if not self-adjoint (i.e., with a uniquely defined but non-orthogonal spectral decomposition)[19,20], is however (maximal) *hermitian*; it is represented by the time variable t in the $t-$representation for square-integrable space-time wavepackets, and, in the case of a continuum energy spectrum, by $-i\hbar\partial/\partial E$ in the $E-$representation, for the Fourier-transformed wavepackets (provided that point $E = 0$ is eliminated[29b], i.e. for wavepackets with moving back-tails and, of course, nonzero fluxes; one can notice that states with zero energy E would not play any role, anyway, in collision experiments).[27-31]

Let us stress that this Olkhovsky-Recami (OR) approach is just a direct consequence of conventional quantum mechanics. From the ordinary probabilistic interpretation of $\rho(x,t)$ and from the well-known continuity equation

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial J(x,t)}{\partial x} = 0 \ ,$$

it follows *also in this (more general) case* that the two weights w_+ and w_-

$$w_+(x,t) = J_+(x,t) \ \left[\int_{-\infty}^{\infty} J_+(x,t)\ \mathrm{d}t\right]^{-1}$$

$$w_-(x,t) = J_-(x,t) \ \left[\int_{-\infty}^{\infty} J_-(x,t)\ \mathrm{d}t\right]^{-1} \ ,$$

can be regarded as the probabilities that our "particle" passes through position x during a unit time–interval centred at t (in the case of forward and backward motion, respectively).

[‡]Let us mention that one could measure the quantities $J_{\pm}$, at least in principle, via the following experimental set-up: (i) for measuring J_+, one can have recourse to two detectors, the first one measuring the incident flux J_{in}, while the second one —sufficiently far away, but still located before the barrier— measures the same incoming flux (at the new position) in delayed coincidence with the former measurement; analogously, (ii) to measure J_-, the first detector will measure the reflected flux J_{R}, while the second one measures the same (reflected) flux in *advanced* coincidence with the former.

Actually, for those time intervals for which $J = J_+$ or $J = J_-$, one can rewrite the continuity equation as follows:

$$\frac{\partial \rho_>(x,t)}{\partial t} = -\frac{\partial J_+(x,t)}{\partial x}$$

$$\frac{\partial \rho_<(x,t)}{\partial t} = -\frac{\partial J_-(x,t)}{\partial x} \ ,$$

respectively. These relations can be considered as formal definitions of $\partial\rho_>/\partial t$ and $\partial\rho_</\partial t$. Let us now integrate them over time from $-\infty$ to t; we obtain:

$$\rho_>(x,t) = -\int_{-\infty}^{t} \frac{\partial J_+(x,t')}{\partial x}\,\mathrm{d}t'$$

$$\rho_<(x,t) = -\int_{-\infty}^{t} \frac{\partial J_-(x,t')}{\partial x}\,\mathrm{d}t'$$

with the initial conditions $\rho_>(x,-\infty) = \rho_<(x,-\infty) = 0$. Then, let us introduce the quantities

$$N_>(x,\infty;t) \equiv \int_x^{\infty} \rho_>(x',t)\,\mathrm{d}x' = \int_{-\infty}^{t} J_+(x,t')\,\mathrm{d}t' \ > 0$$

$$N_<(-\infty,x;t) \equiv \int_{-\infty}^{x} \rho_<(x',t)\,\mathrm{d}x' = -\int_{-\infty}^{t} J_-(x,t')\,\mathrm{d}t' \ > 0 \ ,$$

which have the meaning of probabilities for our "particle" to be located at time t on the semi-axis (x,∞) or $(-\infty,x)$ respectively, as functions of the flux densities $J_+(x,t)$ or $J_-(x,t)$, provided that the normalization condition $\int_{-\infty}^{\infty}\rho(x,t)\mathrm{d}x = 1$ is fulfilled. The r.h.s.'s of the last couple of equations have been obtained by integrating the r.h.s.'s of the above expressions for $\rho_>(x,t)$ and $\rho_<(x,t)$ and by adopting the boundary conditions $J_+(-\infty,t) = J_-(-\infty,t) = 0$. Now, by differentiating $N_>(x,\infty;t)$ and $N_<(-\infty,x;t)$ with respect to t, one obtains:

$$\frac{\partial N_>(x,\infty,t)}{\partial t} = J_+(x,t) > 0$$

$$\frac{\partial N_<(x,-\infty,t)}{\partial t} = -\,J_-(x,t) > 0 \ .$$

Finally, from our last four equations one can infer that:

$$w_+(x,t) = \frac{\partial N_>(x,\infty;t)/\partial t}{N_>(x,-\infty;\infty)}$$

$$w_-(x,t) = \frac{\partial N_<(x,-\infty;t)/\partial t}{N_<(-\infty,x;\infty)} \ ,$$

which justify the aforementioned probabilistic interpretation of $w_+(x,t)$ and $w_-(x,t)$. Let us notice, incidentally, that our approach does *not* assume any ad hoc postulate.

Our previous OR formalism is therefore enough for defining mean values, variances (and other "dispersions") related with the duration distributions of various collisions, including tunnelling. For instance, for transmissions from region I to region III we have

$$\langle \tau_{\rm T}(x_i,x_f)\rangle = \langle t_+(x_f)\rangle - \langle t_+(x_i)\rangle \tag{3}$$

$$\mathrm{D}\,\tau_{\rm T}(x_i,x_f) = \mathrm{D}\,t_+(x_f) + \mathrm{D}\,t_+(x_i) \tag{4}$$

with $-\infty < x_i \leq 0$ and $a \leq x_f < \infty$. For a mere *tunnelling* process, one has

$$\langle t_{\rm tun}(0,a)\rangle = \langle t_+(a)\rangle - \langle t_+(0)\rangle \tag{5}$$

and

$$\mathrm{D}\,\tau_{\rm tun}(0,a) = \mathrm{D}\,t_+(a) + \mathrm{D}\,t_+(0) \ . \tag{6}$$

For penetration (till a point x_f inside the barrier region II), similar expressions hold for $\langle \tau_{\rm pen}(x_i,x_f)\rangle$ and $\mathrm{D}\,\tau_{\rm pen}(x_i,x_f)$, with $0 < x_f < a$. For reflections at generic points, located in regions I or II, with $x_i \leq x_f < a$, one has

$$\langle \tau_{\rm R}(x_i,x_f)\rangle = \langle t_-(x_f)\rangle - \langle t_+(x_i)\rangle \tag{7}$$

and

$$\mathrm{D}\,\tau_R(x_i,x_f) = \mathrm{D}\,t_-(x_f) + \mathrm{D}\,t_+(x_i) \tag{8}$$

Let us repeat that these definitions hold within the framework of conventional quantum mechanics, *without* introducing any new physical postulates.

In the asymptotic cases, when $|x_i| >> a$, it is:

$$\langle \tau_{\rm T}^{\rm as}(x_i,x_f)\rangle = \langle t(x_f)\rangle_{\rm T} - \langle t(x_i)\rangle_{\rm in} \tag{3a}$$

and

$$\langle \tau_{\rm T}^{\rm as}(x_i,x_f)\rangle = \langle \tau_{\rm T}(x_i,x_f)\rangle + \langle t_+(x_i)\rangle - \langle t(x_i)\rangle_{\rm in} \tag{9}$$

where $\langle ... \rangle_{\rm T}$ and $\langle ... \rangle_{\rm in}$ are averages over the fluxes corresponding to $\psi_{\rm T} = A_{\rm T} \exp(ikx)$ and to $\psi_{\rm in} = \exp(ikx)$, respectively. For initial wavepackets of the form

$$\Psi(x,t) = \int_0^\infty G(k-\overline{k}) \exp\left(i(kx - Et)/\hbar\right) {\rm d}k$$

(where $E = \hbar^2 k^2/2m$; $\int_0^\infty |G(k-\overline{k})|^2 {\rm d}E = 1$; $G(0) = G(\infty) = 0$; $k > 0$; $\overline{k}$ being the value corresponding to the peak[§] of G) and for sufficiently small energy (or momentum) spreads, when

$$\int_0^\infty \upsilon^n |G A_{\rm T}|^2 {\rm d}E \approx \int_0^\infty \upsilon^n |G|^2 {\rm d}E$$

with $n = 0, 1$; $\upsilon \equiv \hbar k/m$, one gets:

$$\langle \tau_{\rm T}^{\rm as}(x_i, x_f) \rangle = \langle \tau_{\rm T}^{\rm Ph}(x_i, x_f) \rangle_E \ , \tag{10}$$

where

$$\langle ... \rangle_E = \int_0^\infty {\rm d}E \upsilon |G(k-\overline{k})|^2 ... / \int_0^\infty \upsilon |G|^2 {\rm d}E \ .$$

The quantity

$$\tau_{\rm T}^{\rm Ph}(x_i, x_f) = (1/\upsilon)(x_f - x_i) + \hbar \, {\rm d}(arg A_{\rm T})/{\rm d}E \tag{11}$$

is the transmission phase time obtained by the stationary-phase approximation. In the same approximation, and when it is small the contribution of ${\rm D}\, t_+(x_i)$ to the variance ${\rm D}\, \tau_{\rm T}(x_i, x_f)$ (that can be realized for sufficiently *large* energy spreads, i.e. for spatially short wavepackets), we obtain:

$${\rm D}\, \tau_{\rm T}(x_i, x_f) = \hbar^2 [\langle (\partial |A_{\rm T}|/\partial E)^2 \rangle_E / \langle |A_{\rm T}|^2 \rangle_E \tag{12}$$

In the opposite case of very *small* energy spreads, i.e., quasi-monochromatic particles, the expression (12) becomes just that part of ${\rm D}\, t_+(x_f)$ and ${\rm D}\, t_{\rm T}(x_i, x_f)$ which is due to the barrier presence.

In the limit $|G|^2 \to \delta(E - \overline{E})$, when it is $\overline{E} \equiv \hbar^2 \overline{k}^2/2m$, the equation (10) does yield the ordinary phase time, without averaging. For a rectangular barrier with height V_0 and $\kappa a >> 1$ (where $\kappa \equiv [2m(V_0 - E)]^{1/2}/\hbar$), the expressions (10) and (12) for $x_i = 0$ and $x_f = a$ transform, in the same limit, into the well-known expressions

$$\tau_{\rm tun}^{\rm Ph} \to \frac{2}{\upsilon k} \tag{11a}$$

(see ref.[66], and also [48,49]), and

[§]For real tunnelling, with under-barrier energies, one should actually multiply the weight amplitude G by a cutoff function, which in the case of a rectangular barrier with height V_0 is simply $\Theta(E - V_0)$.

$$(\mathrm{D}\,\tau_{\mathrm{tun}}^{\mathrm{Ph}})^{1/2} \to \frac{ak}{v}\ , \tag{12a}$$

respectively. It should be noticed that our eq.(12a) *coincides* with one of the Larmor times[79] and with the Büttiker-Landauer time[80], as well as with the imaginary part of the complex time in the Feynman path-integral approach (see also ref.[82]).

Recently G.Nimtz stressed the importance of the simple relation (11a), that he heuristically verified, and called it a "universal property" of tunnelling times. Actually, eqs.(11a) and (12a) can strongly help clarifying many of the current discussions about tunnelling times. Let us add, incidentally, that recent theoretical work by Abolhasani and Golshani[71], which regards the OR approach as giving the most natural definition for a transmission time within the standard interpretation of quantum mechanics, conludes that the best times that could be obtained in Bohmian mechanics are *the same* as OR's.

For a real weight amplitude $G(k-\overline{k})$, when $\langle t(0)\rangle >_{\mathrm{in}} = 0$, from (9) we obtain

$$\langle \tau_{\mathrm{tun}}(0,a)\rangle = \langle \tau_{\mathrm{tun}}^{\mathrm{Ph}}\rangle - \langle t_{+}(0)\rangle\ . \tag{13}$$

By the way, if the experimental conditions are such that only the positive-momentum components of the wavepackets are recorded, i.e., $\Lambda_{\mathrm{exp},+}\Psi(x_i,.t) = \Psi_{\mathrm{in}}(x_i,.t)$, quantity $\Lambda_{\mathrm{exp},+}$ being the projector onto the positive-momentum states, then for any x_i in the range $(-\infty,0)$ and x_f in the range (a,∞) it will be:

$$\langle \tau_{\mathrm{T}}(x_i,x_f)\rangle_{\mathrm{exp}} = \langle \tau_{\mathrm{T}}^{\mathrm{Ph}}(x_i,x_f)\rangle_E \tag{10a}$$

and

$$\langle \tau_{\mathrm{tun}}(0,a)\rangle_{\mathrm{exp}} = \langle \tau_{\mathrm{tun}}^{\mathrm{Ph}}\rangle_E\ , \tag{13a}$$

since $\langle t(0)\rangle_{\mathrm{exp}} = \langle t(0)\rangle_{\mathrm{in}}$.

The main criticism, by the authors of refs.[49,64] and also [78,83], of any approach to the definition of tunnelling times in which a spatial or temporal averaging over moving wavepackets is adopted, invokes the lack of a causal relationship between the incoming peak or "centroid" and the outgoing peak or "centroid". It was already clear in the Sixties (see, for instance, ref.[18]) that such criticism is valid only when finite (not asymptotic) distances from the interaction region are considered. Moreover, that criticism applies more to attempts like the one in ref.[69] (where it was looked for the evolution of an incoming into an outgoing peak), than for our definitions of collision, tunnelling, transmission, penetration, reflection (etc.) durations: In fact, our definitions for the mean duration of any such processes do *not* assume that the centroid (or peak) of the incident wavepacket directly evolves into the centroid (or peak) of the transmitted and reflected packets. Our definitions are simply *differences* between the mean times referring to the passage of the final and initial wavepackets through the relevant space-points, regardless

of any intermediate motion, transformation or reshaping of those wavepackets... At last, for each collision (etc.) process as a whole, we shall be able to test the causality condition.

Actually, there is no a single general formulation of the causality condition, which be necessary and sufficient for all possible cases of collisions (both for nonrelativistic and relativistic wavepackets). The simplest (or strongest) nonrelativistic condition implies the non-negativity of the mean durations. This is, however, a sufficient but not necessary causality condition.[¶] Negative times (advance phenomena) were revealed even near nuclear resonances, distorted by the nonresonant background (see, in particular, ref.[30]); similarly, "advance" phenomena can occur also at the beginning of tunnelling (see Sect.6 below).

Generally speaking, a complete causality condition should be connected not only with the mean time duration, but also with other temporal properties of the considered process. For example, the following variant could seem to be more realistic: <<The difference $t_{\rm A}^{\rm eff}(x_i, x_f) = t_f^{\rm eff} - t_i^{\rm eff}$, between the effective arrival-instant of the flux at x_f and the effective start-instant of the flux at x_i, is to be non-negative (where A = T, pen, tun,...)>>; where the effective instants are defined as $t_f^{\rm eff} \equiv \langle t(x_f)\rangle + \sigma[t(x_f)]$, and $t_i^{\rm eff} \equiv \langle t(x_i)\rangle - \sigma[t(x_i)]$, the standard deviations being of course $\sigma[t(x_f)] = [{\rm D}\, t(x_f)]^{1/2}$; $\sigma[t(x_i)] = [{\rm D}\, t(x_i)]^{1/2}$; so that:

$$t_{\rm A}^{\rm eff}(x_i, x_f) \equiv t_f^{\rm eff} - t_i^{\rm eff} \;=\; \langle t(x_f)\rangle - \langle t(x_i)\rangle + \sigma[t(x_f)] + \sigma[t(x_i)] \; .$$

But this condition too is sufficient but not necessary, because often wavepackets are represented with infinite and not very rapidly decreasing forward-tails... More realistic formulations of the causality condition for wavepackets (with very long tails) will be presented in Sect.8.

[¶]In fact, let us recall that: (i) all the ordinary causal paradoxes seem to be solvable[84] within Special Relativity, when it is *not* restricted to subluminal motions only; (ii) nevertheless, whenever it is met an object $\mathcal{O}$ travelling at Superluminal speed, negative contributions ought to be expected to the tunnelling times[85]: and this should not to be regarded as unphysical[84]. In fact, whenever the *object* $\mathcal{O}$ *overcomes* the infinite speed with respect to a certain observer, it will afterwards appear to the same observer as its *anti*-object $\overline{\mathcal{O}}$ travelling in the opposite *space* direction[84]. For instance, when going on from the lab to a frame $\mathcal{F}$ moving in the *same* direction as the particles or waves entering the barrier region, the objects $\mathcal{O}$ penetrating through the final part of the barrier (with almost infinite speed[86]) will appear in the frame $\mathcal{F}$ as anti-objects $\overline{\mathcal{O}}$ crossing that portion of the barrier *in the opposite space–direction*[84]. In the new frame $\mathcal{F}$, therefore, such anti-objects $\overline{\mathcal{O}}$ would yield a *negative* contribution to the tunnelling time: which could even result, in total, to be negative. What we want to stress here is that the appearance of such negative times is predicted by Relativity itself, on the basis of the ordinary postulates[84-86]. From the theoretical point of view, besides refs.[85,86,84], see also refs.[87]. From the (quite interesting!) experimental point of view, see refs.[88].

3 The meaning of the mean dwell time.

As it is known[89] (see also ref.[71]), the mean dwell time can be presented in two equivalent forms:

$$\langle \tau^{\rm Dw}(x_i, x_f)\rangle = \frac{\int_{-\infty}^{\infty} {\rm d}t \int_{x_i}^{x_f} |\Psi(x,t)|^2 {\rm d}x}{\int_{-\infty}^{\infty} J_{\rm in}(x_i,t){\rm d}t} \tag{14}$$

and

$$\langle \tau^{\rm Dw}(x_i, x_f)\rangle = \frac{\int_{-\infty}^{\infty} tJ(x_f,t){\rm d}t - \int_{-\infty}^{\infty} tJ(x_i,t){\rm d}t}{\int_{-\infty}^{\infty} J_{\rm in}(x_i,t){\rm d}t} \ , \tag{14$'$}$$

with $-\infty < x_i \leq 0$ and $a \leq x_f < \infty$. Let us observe that in the first definition, eq.(14), of the mean dwell time, in integrating over t it is used a weight different from the one introduced by us in Sect.2. Let us comment on the meaning of the weight function (the "measure"). Taking into account the relation $\int_{-\infty}^{\infty} J_{\rm in}(x_i,t){\rm d}t = \int_{-\infty}^{\infty} |\Psi(x,t)|^2 {\rm d}x$, which follows from the continuity equation, one can easily see that the weight of eq.(14) is $dP(x,t) = |\Psi(x,t)|^2 {\rm d}x \ / \ \int_{-\infty}^{\infty} |\Psi(x,t)|^2 {\rm d}x$, which has the well-known quantum-mechanical meaning of probability for a particle to be *localized*, or to *dwell*, in the spatial region $(x, x+{\rm d}x)$ at the instant t, independently of the motion direction. Then, the integrated quantity $P(x_1,x_2;t) = \int_{x_1}^{x_2} |\Psi(x,t)|^2 {\rm d}x \ / \ \int_{-\infty}^{\infty} |\Psi(x,t)|^2 {\rm d}x$, has the meaning of probability of finding the particle inside the spatial interval (x_i, x_f) at the instant t (see also ref.[90]).

The equivalence of relations (14) and (14') is a consequence of the continuity equation which links the probabilities associated with the two processes: "dwelling inside" and "passing through" the interval (x_i, x_f). However, we can note that the applicability of the integrated weight $P(x_1, x_2; t)$ for the time analysis (in contrast with the space analysis) is limited, since it allows calculating the mean dwell times only, but not their variances.

Taking into account that $J(x_i,t) = J_{\rm in}(x_i,t) + J_{\rm R}(x_i,t) + J_{\rm int}(x_i,t)$ and $J(x_f,t) = J_{\rm T}(x_f,t)$ (where $J_{\rm in}$, $J_{\rm R}$ and $J_{\rm T}$ correspond to the wavepackets $\Psi_{\rm in}(x_i,t)$, $\Psi_R(x_i,t)$ and $\Psi_{\rm T}(x_f,t)$, which have been constructed in terms of the stationary wave functions $\psi_{\rm in}$, $\psi_{\rm R} = A_{\rm R}\exp(-ikx)$ and $\psi_{\rm T}$, respectively), and that for $J_{\rm int}$ (originating from the interference between $\Psi_{\rm in}(x_i,t)$ and $\Psi_{\rm R}(x_i,t)$) it holds

$$J_{\rm int}(x,t) = {\rm Re}(i\hbar/m)[\Psi_{\rm in}(x,t)\ \partial\Psi^*_{\rm R}(x,t)/\partial x + \Psi_{\rm R}(x,t)\ \partial\Psi^*_{\rm in}(x,t)/\partial x]$$

and

$$\int_{-\infty}^{\infty} J_{\rm int}(x_i,t){\rm d}t = 0$$

we eventually obtain the interesting relation

$$\langle \tau^{\rm Dw}(x_i, x_f)\rangle = \langle T\rangle_E \langle \tau_{\rm T}(x_i, x_f)\rangle + \langle R(x_i)\rangle_E \langle \tau_{\rm R}(x_i, x_f)\rangle \tag{15}$$

with $\langle T\rangle_E = \langle |A_{\rm T}|^2 \upsilon\rangle_E / \langle \upsilon\rangle_E$, $\langle R(x_i)\rangle_E = \langle R\rangle_E + \langle r(x_i)\rangle$, $\langle R\rangle_E = \langle |A_{\rm R}|^2 \upsilon\rangle_E / \langle \upsilon\rangle_E$, $\langle T\rangle_E + \langle R\rangle_E = 1$, and with

$$\langle r(x)\rangle = \frac{\int_{-\infty}^{\infty}[J_+(x,t) - J_{\rm in}(x,t)]{\rm d}t}{\int_{-\infty}^{\infty} J_{\rm in}(x_i,t){\rm d}t} \ .$$

We stress that $\langle r(x)\rangle$ is negative and tends to 0 when x tends to $-\infty$.

When $\Psi_{\rm in}(x_i,t)$ and $\Psi_R(x_i,t)$ are well separated in time, i.e. $\langle r(x)\rangle = 0$, one obtains the simple well-known[33] *weighted average rule:*

$$\langle \tau^{\rm Dw}(x_i,x_f)\rangle = \langle T\rangle_E \ \langle \tau_T(x_i,x_f)\rangle + \langle R\rangle_E \ \langle \tau_{\rm R}(x_i,x_f)\rangle \tag{16}$$

For a rectangular barrier with $\kappa a >> 1$ and quasi-monochromatic particles, the expressions (15) and (16) with $x_i = 0$ and $x_f = a$ transform into the known expressions

$$\langle \tau^{\rm Dw}(x_i,x_f)\rangle = \langle \hbar k/\kappa V_0\rangle_E \ , \tag{15a}$$

(were we took account of the interference term $\langle r(x)\rangle$), and

$$\langle \tau^{\rm Dw}(x_i,x_f)\rangle = \langle 2/\kappa\upsilon\rangle_E \ , \tag{16a}$$

(where the interference term $\langle r(x)\rangle$ is now equal to 0).

When $A_{\rm R} = 0$, i.e. the barrier is transparent, the mean dwell time (14),(14') is automatically equal to

$$\langle \tau^{\rm Dw}(x_i,x_f)\rangle = \langle \tau_{\rm T}(x_i,x_f)\rangle \ . \tag{17}$$

It is not clear, however, how to define *directly* the variances of the dwell-time distributions. The approach proposed in ref.[91] seems rather artificial, with its abrupt switching on of the initial wavepacket. It is possible to define the variances of the dwell-time distributions *indirectly*, for example by means of relation (15), when basing ourselves on the standard deviations $\sigma(\tau_{\rm T})$, $\sigma(\tau_{\rm R})$ of the transmission-time and reflection-time distributions.

4 A brief analysis of the Larmor and Büttiker-Landauer "clock" approaches

One can realize that the introduction of additional degrees of freedom as "clocks" may distort the true values of the tunnelling time. The Larmor clock uses the phenomenon of the change of the spin orientation (the Larmor precession and spin-flip) in a weak homogeneous magnetic field superposed to the barrier region. If initially the particle is polarized in the x direction, after the tunnelling its spin gets small y and z components. The Larmor times $\tau^{\rm La}_{y,{\rm T}}$ and $\tau^{\rm La}_{z,{\rm T}}$ are defined by the ratio of the spin-rotation angles [on

their turn, defined by the $y-$ and $z-$ spin components] to the (precession and rotation) frequency[13,14,79]. For an opaque rectangular barrier with $\kappa a >> 1$, the two expressions were obtained:

$$\langle \tau^{\rm La}_{y,{\rm tun}} \rangle = \langle \tau^{\rm Dw}(x_i, x_f) \rangle = \langle \hbar k / \kappa V_0 \rangle_E \tag{18}$$

and

$$\langle \tau^{\rm La}_{z,{\rm tun}} \rangle = \langle ma/\hbar k \rangle_E \ . \tag{19}$$

In refs.[48,82] it was noted that, if the magnetic field region is infinitely extended, the expression (18) just yields —after having averaged over the small energy spread of the wavepacket— the phase tunnelling time, eq.(11a).

As to eq.(19), it refers in reality not to a rotation, but to a *jump* to "spin-up" or "spin-down" (spin-flip), together with a Zeeman energy-level splitting[49,79]. Due to the Zeeman splitting, the spin component parallel to the magnetic field corresponds to a higher tunnelling energy, and hence the particle tunnels preferentially to that state. This explains why the tunnelling time $\tau^{\rm La}_{z,{\rm tun}}$ entering eq.(19) depends only on the absolute value $|A_{\rm T}|$ (or rather on ${\rm d}|A_{\rm T}|/{\rm d}E$), and coincides with expression (12a).

The Büttiker-Landauer clock[49,80,81] is connected with the oscillation of the barrier (absorption and emission of "modulation" quanta), during tunnelling. Also in this case one can realize (for the same reasons as for $\langle \tau^{\rm La}_{z,{\rm tun}} \rangle$) that the coincidence of the Büttiker-Landauer time with eq.(12a) is connected with the energy dependence of $|A_{\rm T}|$.

5 A short analysis of the kinematical-path approaches

The Feynman path-integral approach to quantum mechanics was applied in [72-75] to evaluate the mean tunnelling time (by averaging over all the paths that have the same beginning and end points) with the complex weight factor $\exp[iS(x(t))/\hbar]$, where S is the action associated with the path $x(t)$. Such a weighting of the tunnelling times implies the appearance of real and imaginary components[49]. In ref.[72] the real and imaginary parts of the complex tunnelling time were found to be equal to $\langle \tau^{\rm La}_{y,{\rm tun}} \rangle$ and to -$\langle \tau^{\rm La}_{z,{\rm tun}} \rangle$, respectively. An interesting development of this approach, its instanton version, is presented in ref.[75]. The instanton-bounce path is a stationary point in the Euclidean action integral. Such a path is obtained by analytic continuation to imaginary time of the Feynman-path integrands (which contain the factor $\exp(iS/\hbar)$). This path obeys a classical equation of motion inside the potential barrier with its sign reversed (so that it actually becomes a well). In ref.[75] the instanton bounces were considered as real physical processes. The bounce duration was calculated in real time, and was found to be in good agreement with the one evaluated via the phase-time method. The temporal density of

bounces was estimated in imaginary time, and the obtained result —in the phase-time approximation limit— coincided with the tunnelling-time standard deviation (as given by eq.(12)). Here one can see a manifestation of the equivalence (in the phase-time approximation) of the Schroedinger and Feynman representations of quantum mechanics.

Another definition of the tunnelling time is connected with the Wigner path distribution[76,77]. The basic idea of this approach, reformulated by Muga, Brouard and Sala, is that the tunnelling-time distribution for a wavepacket can be obtained by considering a classical ensemble of particles with a certain distribution function, namely the Wigner function $f(x,p)$: so that the flux at position x can be separated into positive and negative components:

$$J(x) = J^+(x) + J^-(x) \tag{20}$$

with $J^+ = \int_0^\infty (p/m) f(x,p) dp$ and $J^- = J - J^+$. They formally obtained the same expressions (3) and (5), for the transmission, tunnelling and penetration durations, as in the OR formalism, provided that $J^\pm$ replaces our $J_\pm$. The dwell time decomposition, then, takes the form

$$\langle \tau^{\rm Dw}(x_i,x_f)\rangle = \langle T\rangle_E \, \langle \tau_{\rm T}(x_i,x_f)\rangle + \langle R_{\rm M}(x_i)\rangle_E \, \langle \tau_R(x_i,x_f)\rangle \tag{21}$$

with $R_{\rm M}(x) = \int_0^\infty |J^-(x,t)| {\rm d}t$. Asymptotically, $\langle R_{\rm M}(x)\rangle$ tends to our quantity $\langle R\rangle_E$ and eq.(21) takes the form of the known "weighted average rule" (16).

One more alternative is the stochastic method for wavepackets in ref.[92]. It also leads to real times, but its numerical implementation is not trivial[93].

In ref.[83] the Bohm approach to quantum mechanics was used to choose a set of classical paths which do not cross each other. The Bohm formulation, on one side, can be regarded as equivalent to the Schroedinger equation[94], while on the other side can perhaps provide a basis for a nonstandard interpretation of quantum mechanics[49]. The expression obtained in ref.[83] for the mean dwell time is not only positive definite but also unambiguously distinguishes between transmitted and reflected particles:

$$\tau^{\rm Dw}(x_i,x_f) = \int_0^\infty {\rm d}t \int_{x_i}^{x_f} |\Psi(x,t)|^2 {\rm d}x = T\,\tau_{\rm T}(x_i,x_f) + R\,\tau_{\rm R}(x_i,x_f) \tag{22}$$

with

$$\tau_{\rm T}(x_i,x_f) = \int_0^\infty {\rm d}t \int_{x_i}^{x_f} |\Psi(x,t)|^2 \Theta(x-x_{\rm c}) {\rm d}x/T \tag{23}$$

$$\tau_{\rm R}(x_i,x_f) = \int_0^\infty {\rm d}t \int_{x_i}^{x_f} |\Psi(x,t)|^2 \Theta(x_{\rm c}-x) {\rm d}x/T \tag{24}$$

where T and R are the mean transmission and reflection probability, respectively. The "bifurcation line" $x_{\rm c} = x_{\rm c}(t)$, which separates the transmitted from the reflected trajectories, is defined through the relation

$$T = \int_{-\infty}^{\infty} \mathrm{d}t |\Psi(x,t)|^2 \Theta(x - x_\mathrm{c}) \mathrm{d}x \ . \tag{25}$$

Let us add that two main differences exist between this (Leavens') and our formalism: (i) a difference in the temporal integrations (which are $\int_0^\infty \mathrm{d}t$ and ($\int_{-\infty}^{\infty} \mathrm{d}t$, respectively), that sometimes are relevant; and (ii) a difference in the separation of the fluxes, that we operate "by sign" (cf. eqs.(1),(2)) and here it is operated by the line x_c:

$$J(x,t) = [J(x,t)]_\mathrm{T} + [J(x,t)]_\mathrm{R} \tag{26}$$

with $[J(x,t)]_\mathrm{T} = J(x,t)\ \Theta(x - x_\mathrm{c}(t))$, $[J(x,t)]_\mathrm{R} = J(x,t)\ \Theta(x_\mathrm{c}(t) - x)$.

6 Characteristics of the tunnelling evolution

The results of the calculations presented in ref.[71], within the OR formalism, show that: (i) *the mean tunnelling time does not depend on the barrier width a for sufficiently large a* ("Hartman effect"); (ii) the quantity $\langle \tau_\mathrm{tun}(0,a) \rangle$ decreases when the energy increases; (iii) the value of $\langle \tau_\mathrm{pen}(0,x) \rangle$ rapidly increases for increasing x near $x = 0$ and afterwards tends to saturation (even if with a very slight, continuous increasing) for values near $x = a$; and (iv) at variance with ref.[95], no plot for the mean penetration time of our wavepackets presents any interval with negative values[‖], nor with negative slop for increasing x.

In Fig.2 the dependence of the values of $\langle \tau_\mathrm{tun}(0,a) \rangle$ on a is presented for gaussian wave packets

$$G(k - \overline{k}) \equiv C \exp[-(k - \overline{k})^2/(2\,\Delta k)^2]$$

and rectangular barriers with the same parameters as in ref.[95]: namely, $V_0 = 10$ eV; $\overline{E} = 2.5$, 5, and 7.5 eV with $\Delta k = 0.02$ $\mathring{A}^{-1}$ (curves 1a, 2a, 3a respectively); and $\overline{E} = 5$ eV with $\Delta k = 0.04$ $\mathring{A}^{-1}$ and 0.06 $\mathring{A}^{-1}$ (curves 4a, 5a, respectively). On the contrary, the curves for $\langle t_+(a) \rangle$, corresponding to different energies and different Δk, are all practically superposed to the single curve 6. Moreover, since $\langle \tau_\mathrm{tun}^\mathrm{Ph} \rangle$ depends only very weakly on a, the quantity $\langle \tau_\mathrm{tun}(0,a) \rangle$ depends on a essentially through the term $\langle t_+(0) \rangle$ (see curves 1b÷5b).

Let us emphasize that all these calculations show that $\langle t_+(0) \rangle$ assumes *negative* values (see also [96]). Such "acausal" time-advance is a result of the interference between the incoming waves and the waves reflected by the barrier forward edge: It happens that the reflected wavepacket cancel out the back edge of the incoming-wavepacket, and the larger the barrier width, the larger is the part of the incoming-wavepacket back edge which is

[‖]See, however, footnote[‖)]

extinguished by the reflected waves, up to the saturation (when the contribution of the reflected-wavepacket becomes almost constant, independently of a). Since all $\langle t_+(0)\rangle$ are negative, eq.(13) yields that the values of $\langle \tau_{\rm tun}(0,a)\rangle$ are always positive and larger than $\langle \tau_{\rm tun}^{\rm Ph}\rangle$. In connection with this fact, it is worthwhile to note that the example with a classical ensemble of two particles (one with a large above-barrier energy and the other with a small sub-barrier energy), presented in ref.[93], does not seem to be well-grounded, not only because that tunnelling is a quantum phenomenon without a direct classical limit, but, first of all, because ref.[93] overlooks the fact that the values of $\langle t_+(0)\rangle$ are negative.

Let us mention that the last calculations by Zaichenko[96] (with the same parameters) have shown that such time-advance is noticeable also *before* the barrier front (even if *near* the barrier front wall, only). He found also negative values of $\langle \tau_{\rm pen}(x_i,x_f)\rangle$, for instance, for $x_i=-a/5$ and x_f in the interval 0 to $2a/5$: but this result too is not acausal, because the last equation of Sect.2 (for example) is fulfilled in this case.

7 On the general validity of the Hartman–Fletcher effect (HE)

We called[48] "Hartman–Fletcher effect" (or for simplicity "Hartman effect", HE) the fact that for opaque potential barriers the mean tunnelling time does not depend on the barrier width, so that for large barriers the effective tunnelling–velocity can become arbitrarily large. Such effect was first studied in refs.[66,67] by the stationary-phase method for the one-dimensional tunnelling of quasi-monochromatic nonrelativistic particles; where it was found that the *phase tunnelling time*

$$\tau_{\rm tun}^{\rm Ph}=\hbar\,{\rm d}(\arg A_{\rm T}+ka)/{\rm d}E \tag{27}$$

(which equals the *mean tunnelling time* $\langle\tau_{\rm tun}\rangle$ when it is possible to neglect the interference between incident and reflected waves outside the barrier[48]) was independent of a. In fact, for a rectangular potential barrier, it holds in particular that $A_{\rm T}=4ik\kappa[(k^2-\kappa^2){\rm D}_-+2ik\kappa{\rm D}_+]^{-1}\exp[-(\kappa+ik)a]$, with ${\rm D}_\pm=1\pm\exp(-2\kappa)$, and that $\tau_{\rm tun}^{\rm Ph}=2/(\upsilon\kappa)$ when $\kappa a>>1$.

Now we shall test the validity of the HE for all the other theoretical expressions proposed for the mean tunnelling times. Let us first consider the *mean dwell time* $\langle\tau_{\rm tun}^{\rm Dw}\rangle$, ref.[89], the mean Larmor time $\langle\tau_{y,{\rm tun}}^{\rm La}\rangle$,[79,13] and *the real part* of the complex tunnelling time obtained by averaging over *the Feynman paths* ${\rm Re}\tau_{\rm tun}^{\rm F}$, ref.[72], which all equal $\hbar k/(\kappa V_0)$ in the case of quasi-monochromatic particles and opaque rectangular barriers: One can immediately see[61] that also in these cases there is *no* dependence on the barrier width, and consequently the HE is valid. As to the OR nonrelativistic approach,

developed in refs.[48,71,96], the validity of the HE for the mean tunnelling time can be inferred directly from the expression

$$\langle t_{\rm tun}\rangle = \langle t_+(a)\rangle - \langle t_+(0)\rangle = \langle \tau_{\rm tun}^{\rm Ph}\rangle_E - \langle t_+(0)\rangle \ ; \tag{28}$$

it was moreover confirmed by the numerous calculations performed in the same set of papers[48,71,96] for various cases of gaussian wavepackets (see also Sect.6 above).

Let us now consider, by contrast, *the second Larmor time*[69]

$$\tau_{z,{\rm tun}}^{\rm L} = \frac{\hbar[\langle(\partial|A_{\rm T}|/\partial E)^2\rangle}{\langle|A_{\rm T}|^2\rangle]^{1/2}} \ , \tag{29}$$

the *Büttiker-Landauer time* $\tau_{\rm tun}^{\rm B-L}$,[80] and *the imaginary part* of the complex tunnelling time ${\rm Im}\tau_{\rm tun}^{\rm F}$,[72] obtained within the *Feynman approach*, which too are equal to eq.(29): They all become equal to $a\mu/(\hbar\kappa)$, i.e., they all are *proportional* to the barrier width a, in the opaque rectangular-barrier limit;[61] so that the HE is *not* valid for them! However, it was shown in ref.[48] that these last three times are *not mean times*, but merely *standard deviations* (or "mean square fluctuations") of the tunnelling-time distributions, because they are equal to $[{\rm D}_{\rm dyn}\tau_{\rm tun}]^{1/2}$, where ${\rm D}_{\rm dyn}\tau_{\rm tun}$ is that part of ${\rm D}\,t_+(x_f)$ [or analogously of ${\rm D}\,t_{\rm T}(x_i,x_f)$] which is due to the barrier presence and is defined by the simple equation ${\rm D}_{\rm dyn}\ \tau_{\rm tun} = {\rm D}\,\tau_{\rm tun} - {\rm D}\,t_+(0)$, where ${\rm D}\tau_{\rm tun} = \langle\tau_{\rm tun}^2\rangle - \langle\tau_{\rm tun}\rangle^2$ and $\langle\tau_{\rm tun}^2\rangle = \langle[t_+(a) - \langle t_+(0)\rangle]^2\rangle + {\rm D}\,t_+(0)$. In conclusion, the former three times are not connected with the peak (or group) velocity of the tunnelling particles, but with the *spread* of the tunnelling velocity distributions.

All these results are obtained for transparent media (without absorption and dissipation). As it was theoretically demonstrated in ref.[97] within nonrelativistic quantum mechanics, the HE vanishes for barriers with high enough absorption. This was confirmed experimentally for electromagnetic (microwave) tunnelling in ref.[98].

The tunnelling through potential barriers with dissipation will be examined elsewhere.

Here let only add a comment. From some papers[99], it seems that the integral penetration time, needed to cross a portion of a barrier, in the case of a very long barrier starts to increase again —after the plateau corresponding to infinite speed— proportionally to the distance. This is due to the contribution of the above-barrier frequencies (or energies) contained in the considered wavepackets, which become more and more important as the tunnelling components are progressively damped down. In this paper, however, we refer to the behaviour of the *tunnelling* (or, in the classical case, of the evanescent) waves.

8 The Two-phase description of tunnelling

Let us mention also a new description of tunnelling which can be convenient for transparent media, and also for Josephson junctions. In such a representation the transmission and reflection amplitudes have been rewritten[100,61] (for the same external boundary conditions in Fig.1) in the form

$$A_{\rm T} = i\,{\rm Im}\,(\exp(i\varphi_1))\exp(i\varphi_2 - ika), \quad A_{\rm R} = {\rm Re}\,(\exp(i\varphi_1))\exp(i\varphi_2 - ika)\ , \qquad (30)$$

where the phases φ_1 and φ_2 are typical parameters for the description of a two-element monodromic matrix S, or of a two-channel collision matrix S; with elements $S_{00} = S_{11} = A_{\rm T}$ and $S_{01} = S_{10} = A_{\rm R}$ and with the unitarity condition $[i, j, k = 0, 1]$

$$\Sigma_{j=0}^{1} S_{ij} S_{jk}^{*} \ = \ \delta_{ik}\ .$$

In particular, for rectangular potential barriers it is $\varphi_1 = \arctan\{2\sigma/[(1+\sigma^2)\sinh(\kappa a)]\}$, and $\varphi_2 = \arctan\{\sigma\sinh(\kappa a)/[\sinh^2(\kappa a/2) - \sigma^2\cosh^2(\kappa a/2)]\}$, with $\sigma = \kappa/k$ and $\kappa^2 = \kappa_0^2 - k^2$, it being $\kappa_0 = [2\mu V_0]^{1/2}/\hbar$. In terms the the phases φ_1 and φ_2, the expressions for $\tau_{\rm tun}^{\rm Ph}$ and $\tau_{z,{\rm tun}}^{\rm L} = \tau_{\rm tun}^{\rm B-L}$ acquire the following form:

$$\tau_{\rm tun}^{\rm Ph} = \hbar\frac{\partial(\arg A_{\rm T} + ka)}{\partial E} = \hbar\frac{\partial(\varphi_2)}{\partial E}\ ; \quad \tau_{z,{\rm tun}}^{\rm L} = \tau_{\rm tun}^{\rm B-L} = \hbar\frac{\partial\varphi_1}{\partial E}\,\cot(\varphi_1)\ . \qquad (31)$$

So, one can see that in the opaque barrier limit the phases φ_2, or φ_1, enter into the play only when the considered *times* are dependent on a, or independent of a, respectively.

For the times $\langle\tau_{\rm tun}^{\rm Dw}\rangle = \langle\tau_{z,{\rm tun}}^{\rm L}\rangle$, one obtains in this formalism a complicated expression, which can be represented[61] only in terms of *both* φ_1 and φ_2.

In the presence of absorption, both phases become complex and hence the formulae (31) become much more lengthy, and in general depend on a with a violation of the HE, in accordance with refs.[97,98].

9 Time-dependent Scrödinger and Helmholtz equations: Similarities and distinctions between their solutions.

The formal analogy is well-known between the (time-independent) Schroedinger equation in presence of a potential barrier and the (time-independent) Helmholtz equation for a wave-guided beam; this was the basis for regarding the evanescent waves in suitable ("undersized") waveguides as *simulating* the case of tunnelling photons. We want here to study analogies and differences between the corresponding *time-dependent* equations.

Let us mention, incidentally, that a similar analysis for the relativistic particle case was performed for instance in refs.[101,102].

Here we shall deal with the comparison of the *solutions* of the time-dependent Schroedinger equation (for nonrelativistic particles) and of the time-dependent Helmholtz equation for electromagnetic waves. In the time-dependent case such equations are no longer mathematically identical, since the time derivative appear at the fist order in the former and at the second order in the latter. We shall take advantage, however, of a similarity between the probabilistic interpretation of the wave function for a quantum particle and for a classical electromagnetic wavepacket (cf., e.g., refs.[103]); this will be enough for introducing identical definitions of the mean time instants and durations (and variances, etc.) in the two cases (see also refs.[104,105]).

Concretely, let us consider the Helmholtz equation for the case of an electromagnetic wavepacket in the hollow rectangular waveguide, with an "undersized" segment, depicted in Fig.3 (with cross section $a \times b$ in its narrow part, it being $a < b$), which was largely employed in experiments with microwaves[56]. Inside the waveguide, the time-dependent wave equation for any of the vector quantities $\vec{A}, \vec{E}, \vec{H}$ is of the type

$$\Delta \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = 0 \tag{32}$$

where $\vec{A}$ is the vector potential, with the subsidiary gauge condition $\mathrm{div}\vec{A} = 0$, while $\vec{E} = -(1/c)\partial \vec{A}/\partial t$ is the electric field strength, and $\vec{H} = \mathrm{rot}\vec{A}$ is the magnetic field strength. As is known (see, for instance, refs.[106-108]), for boundary the conditions

$$\begin{aligned} E_y = 0 \quad & \text{for } z = 0 \text{ and } z = a \\ E_z = 0 \quad & \text{for } y = 0 \text{ and } y = b \end{aligned} \tag{33}$$

the monochromatic solution of eq.(32) can be represented as a superposition of the following waves (for definiteness we chose TE-waves):

$$\begin{aligned} E_x &= 0 \\ E_y^{\pm} &= E_0 \sin(k_z z) \cos(k_y y) \exp[i(\omega t \pm \gamma x)] \\ E_z^{\pm} &= -E_0 (k_y/k_z) \cos(k_z z) \sin(k_y y) \exp[i(\omega t \pm \gamma x)] \ , \end{aligned} \tag{34}$$

with $k_z^2 + k_y^2 + \gamma^2 = \omega^2/c^2 = (2\pi/\lambda)^2$, $k_z = m\pi/a$, $k_y = n\pi/b$, m and n being integer numbers. Thus:

$$\gamma = 2\pi[(\frac{1}{\lambda})^2 - (\frac{1}{\lambda_c})^2]^{1/2}, \ (\frac{1}{\lambda_c})^2 = (\frac{m}{2a})^2 + (\frac{n}{2a})^2 \tag{35}$$

where γ is real ($\gamma = \mathrm{Re}\ \gamma$) if $\lambda < \lambda_c$, and λ is imaginary ($\lambda = \mathrm{Im}\ \lambda$) if $\lambda > \lambda_c$. Similar expressions for λ were obtained for TH-waves[56,107].

Generally speaking, any solution of eq.(32) can be written as a wavepacket constructed from monochromatic solutions (34), analogously to what hold for any solution of the time-dependent Schroedinger equation. Without forgetting that in the first-quantization scheme, a probabilistic single-photon wave function can be represented[103,109] by a wavepacket for $\vec{A}$: which in the case of plane waves writes for example

$$\vec{A}(\vec{r},t) = \int_{k>0} \frac{\mathrm{d}^3\vec{k}}{k} \vec{\chi}(\vec{k})\ \exp\left(i\vec{k}\cdot\vec{r} - ikct\right) \tag{36}$$

where $\vec{r} \equiv (x,y,z)$; $\vec{\chi}(\vec{k}) = \sum_{i=1}^{2} \chi_i(\vec{k})\ \vec{e_i}(\vec{k})$; $\vec{e_i}\cdot\vec{e_j} = \delta_{i,j}$; $\vec{e_i}\cdot\vec{k} = 0$; $i,j = 1,2$ (or $i,j = y,z$ if $\vec{k}\cdot\vec{r} = k_x x$); $k = |\vec{k}|$; $k = \omega/c$; and $\chi_i(\vec{k})$ is the amplitude for the photon to have momentum $\vec{k}$ and polarization $\vec{e_i}$, so that $|\chi_i(\vec{k})|^2 d\vec{k}$ is then proportional to the probability that the photon have a momentum between $\vec{k}$ and $\vec{k} + d\vec{k}$ in the polarization state $\vec{e_i}$. Though it is not possible to localize a photon in the direction of its polarization, nevertheless, *for one-dimensional propagation* it is possible to use the space-time probabilistic interpretation of eq.(36) along the axis x (the propagation direction)[109,105]. This can be realized from the following. Usually one does not have recourse directly to the probability density and probability flux density, but rather to the the *energy density* s_0 and the *energy flux density* s_x; they however do *not* constitute a 4-dimensional vector, being components of the energy-momentum tensor. Only in two dimensions their continuity equation[103] is Lorentz invariant!; we can write down it (for *one space dimension*) as:

$$\partial s_0/\partial t + \partial s_x/\partial x = 0\ , \tag{37}$$

where

$$s_0 = [\vec{E}^*\cdot\vec{E} + \vec{H}^*\cdot\vec{H}]/8\pi, \quad s_x = c\ \mathrm{Re}[\vec{E}^*\cdot\vec{H}]_x/2\pi \tag{38}$$

and the axis x is the motion direction (i.e., the mean momentum direction) of the wavepacket (36). As a normalization condition one can identify the integrals over space of s_0 and s_x with the mean photon energy and the mean photon momentum, respectively. With this normalization, which bypasses the problem of the impossibility of a direct probabilistic interpretation in space of eq.(36), we can define *by convention* as

$$\rho_{\mathrm{em}}\,\mathrm{d}x = \frac{S_0\,\mathrm{d}x}{\int S_0\,\mathrm{d}x}, \quad S_0 = \int s_0\,\mathrm{d}y\mathrm{d}z \tag{39}$$

the probability density for a photon to be localized in the one-dimensional space interval $(x, x+\mathrm{d}x)$ along the axis x at time t, and as

$$J_{\mathrm{em},x}\,\mathrm{d}x = \frac{S_x \mathrm{d}x}{\int S_x \mathrm{d}x}, \quad S_x = \int s_x\,\mathrm{d}y\mathrm{d}z \tag{40}$$

the flux probability density for a photon to pass through point x (i.e., through the plane orthogonal at x to the x-axis) during the time interval $(t, t+\mathrm{d}t)$; on the analogy of the probabilistic quantities ordinarily introduced for particles. The justification, and convenience, of such definitions are also supported by the coincidence of the wavepacket group velocity with the velocity of the energy transportation, which was established for electromagnetic plane-waves packets in the vacuum; see, e.g., ref.[110]. For a definition of group velocity in the case of evanescent waves, see Appendix B in ref.[111].

In conclusion, the solution (36) of the time-dependent Helmholtz equation (for relativistic electromagnetic wavepackets) is quite similar to the plane-wave packet solution of the time-dependent Schroedinger equation (for non-relativistic quantum particles), with the following differences:

(i) the space-time probabilistic interpretation of eq.(36) is valid only in the one-dimensional space case, at variance with the Schroedinger case. It is interesting that the same conclusion holds for waveguides or transparent media, when reflections and tunnellings can take place; in particular, for the waveguides depicted in Fig.3, and for optical experiments (with frustrated total reflection)[51,52] in the case, e.g., of a double prism arrangement;

(ii) the energy-wavenumber relation for non-relativistic particles (corresponding to selfadjoint, linear Hamiltonians) is quadratic: for instance, in vacuum it is $E = \hbar^2 k^2/2m$; this leads to the fact that wavepackets do always *spread*. By contrast, the energy-wavenumber relation for photons in the vacuum is linear: $E = \hbar ck$; and therefore there is *no* spreading.

On the analogy of conventional nonrelativistic quantum mechanics, one can define from eq.(40) the mean time at which a photon passes through point (or plane) x as[48,105]:

$$\langle t(x)\rangle = \int_{-\infty}^{\infty} t\, J_{\mathrm{em},x}\mathrm{d}t = \frac{\int_{-\infty}^{\infty} t\, S_x(x,t)\mathrm{d}t}{\int_{-\infty}^{\infty} S_x(x,t)\mathrm{d}t} \tag{41}$$

(where, with the natural boundary conditions $\chi_i(0) = \chi_i(\infty) = 0$, we can use in the energy $E = \hbar ck$ representation the same time operator already adopted for particles in nonrelativistic quantum mechanics; and hence one can prove the equivalence of the calculations of $\langle t(x)\rangle$, $\mathrm{D}\,t(x)$, etc., in both the time and energy representations).

In the case of fluxes which change their signs with time we can introduce also for photons, following refs.[48,71], the quantities $J_{\mathrm{em},x,\pm} = J_{\mathrm{em},x}\ \Theta(\pm J_{\mathrm{em},x})$ with the same physical meaning as for particles. Therefore, suitable expressions for the mean values and variances of propagation, tunnelling, transmission, penetration, and reflection durations can be obtained in the same way as in the case of nonrelativistic quantum mechanics for particles (just by replacing J with J_{em}). In the particular case of quasi-monochromatic wavepackets, by using the stationary-phase method (under the same boundary conditions

considered in Sect.2 for particles), we obtain for the photon *phase tunnelling time* the expression

$$\tau^{\rm Ph}_{\rm tun,em} = \frac{2}{c\kappa_{\rm em}} \tag{42}$$

,

for $L\kappa_{\rm em} >> 1$, quantity L being the length of the undersized waveguide (cf. Fig.3). Eq.(42) is to be compared with eq.(11a). From eq.(42) we can see that when $L\kappa_{\rm em} > 2$, the effective tunnelling velocity

$$v^{\rm Ph}_{\rm tun,em} = \frac{L}{\tau^{\rm Ph}_{\rm tun,em}} \tag{43}$$

is Superluminal, i.e., larger than c. This result agrees with all the known experimental results performed with microwaves (cf., e.g., refs. [56,65,98]).

10 Tunnelling times in frustrated total internal reflection experiments

Some results of optical experiments with tunnelling photons were described in ref.[60a], where it was considered the scheme here presented in Fig.4a. A light beam passes from a dielectric medium into an air slab with width a. For incidence angles i greater than the critical angle $i_{\rm c}$ of total internal reflection, most of the beam is reflected, and a small part of it tunnels through the slab. Here tunnelling occurs in the x direction, while the wavepacket goes on propagating in the z direction. Its peak, which is emerging from the second interface, has undergone a temporal shift, which is equal to the mean phase tunnelling time $\langle\tau^{\rm Ph}_{\rm tun}$, and a spatial shift D along z. Since it is natural to assume that the propagation velocity v_z along y is uniform during tunnelling, then

$$D = v_z\langle\tau^{\rm Ph}_{\rm tun}\rangle \tag{44}$$

so that the mean phase time can be simply obtained by measuring D.

Since tunnelling imposes also a change in the mean energy (or wavenumber) of a wavepacket, and the plane wave components with smaller incident angle are better transmitted than those with larger incidence angles, then the emerging beam suffers an angular deviation δi, that can be interpreted as a beam mean-direction rotation during tunnelling. And hence, by taking into account formulas (12)-(12a) and Sect.4, we can conclude that δi and the quantity $\langle\tau^{\rm La}_{z,\rm tun}\rangle = ({\rm D}\,\tau^{Ph}_{\rm tun})^{1/2}$ are proportional to each other:

$$\delta i = \Omega\langle\tau^{\rm La}_{z,\rm tun}\rangle \tag{45}$$

where Ω is the rotation frequency which was calculated in ref.[60a]. So, the time $\langle \tau^{\rm La}_{z,{\rm tun}} \rangle = ({\rm D}\, \tau^{Ph}_{\rm tun})^{1/2}$ too can be simply obtained by measuring δi. Both these times characterize the intrinsic properties of the tunnelling process, under the conditions imposed on the wavepackets (which were described in Sect.2).

Let us remark that the conclusion of the authors of ref.[60a] about the fact that the mean phase time $\langle \tau^{\rm Ph}_{\rm tun} \rangle$ was inadequate as a definition of the tunnelling time is not true, because they describe the wave function in the air slab by the evanescent term $\exp(-\kappa x)$ only, instead of considering the superposition $\alpha \exp(-\kappa x) + \beta \exp(\kappa x)$ of *evanescent and anti-evanescent* waves. It is important to recall that such a superposition of decreasing and increasing waves —normally used in the case of particle tunnelling— is necessary to obtain a resulting non-zero flux!.[48]

With such a correction, one can see that the very small values of $\langle \tau^{\rm Ph}_{\rm tun} \rangle$ (about 40 fs) obtained in the experiment[60a] for $a = 20\ \mu$ imply for the tunnelling photon a *Superluminal peak velocity* of about $5{\cdot}10^{10}$ cm/s.

But in the double-prism arrangement, it was predicted by Newton, and preliminarly confirmed 250 years later by F.Goos and H.Hänchen, that the reflected and transmitted beams are also spatially shifted with respect to what expected from geometrical optics (cf. Fig.4b). Recent rather interesting esperiments have been performed by Haibel et al.[60c], who discovered a strong dependence of the mentioned shift on the beam width and especially on the incidence angle.

11 A remark on reshaping

The Superluminal phenomena, observed in the experiments with tunnelling photons and evanescent electromagnetic waves[56-60], generated a lot of discussions on relativistic causality[112-120]. This revived an interest also in similar phenomena that had been previously observed in the case of electromagnetic pulses propagating in dispersive media[88,121,122]. On the other side, it is well-known since long that the wavefront velocity (well defined when the pulses have a step-function envelope or at least an abruptly raising forward edge) cannot exceed the velocity of light c in vacuum[108,123]. Even more, the (Sommerfeld and Brillouin) *precursors* —that many people, even if not all, believe to be necessarily generated together with any signal generation— are known to travel exactly at the speed c in any media (for a recent approach to the question, see ref.[124]). Such phenomena were confirmed by various theoretical methods and in various processes, including tunnelling[102,112-113,125]. Discussions are presently going on about the question whether the signal velocity has to do with the previous speed c of with the group velocity.[125,124] Another point under discussion is whether the shape of a realistic wavepacket must possess, or not, an abruptly raising forward edge.[102,115-118].

A simple way of understanding the problem, in a "causal" manner, might consist in explaining the Superluminal phenomena during tunnelling as simply due to a "reshaping",

with attenuation, of the pulse, as already attempted (at the classical limit) in refs.[100-102]: namely, the later parts of an incoming pulse are preferentially attenuated, in such a way that the outcoming peak appears shifted towards earlier times even if it is nothing but a portion of the incident pulse forward tail[57]. In particular, the following scheme is compatible with the usual idea of causality: If the overall pulse attenuation is very strong and, during tunnelling, the leading edge of the pulse is less attenuated than the trailing edge, then the time envelope of the outcoming (small) flux can stay totally beneath the initial temporal envelope (i.e., the envelope of the initial pulse in the case of free motion in vacuum).[116-120] And, if $A_{\rm T}$ depends on energy much more weakly than the initial wavepacket weight-factor, then the spectral expansion, *and hence the geometrical form*, of the transmitted wavepacket will be practically the same as the spectral expansion, *and the form*, of the entering wavepacket (*reshaping*). By contrast, if the dependence of $A_{\rm T}$ on energy is not weak, the pulse form and width can get strongly modified (*"reconstruction"*).

The very definition of causality seems to be in need of some careful revision[126]. Various, possible (*sufficient but not necessary)* "causality conditions" have been actually proposed in the literature. For our present purposes, let us mention that an acceptable, more general causality condition (allowing the time envelope of the final flux, $J_{\rm fin}$, to arrive at a point $x_f \geq a$ even earlier than that of the initial pulse) might be for example the following one:

$$\int_{-\infty}^{t} [J_{\rm in}(x_f,\tau) - J_{\rm fin,+}(x_f,\tau)]d\tau \geq 0 \ , \quad -\infty < t < \infty \ ; x_f \geq a \ . \tag{46}$$

It simply requires that, during any (upper limited) time interval, the *integral* final flux (along any direction) does not exceed the *integral* "initial" flux which *would* pass through the same position x_f in the case of free motion; although one can find finite values of t_1 and t_2 $(-\infty < t_1 < t_2 < \infty)$ such that $\int_{t_1}^{t_2}[J_{\rm in}(x_f,\tau) - J_{\rm fin,+}(x_f,\tau)]d\tau < 0$.

But other conditions for causality can of course be proposed; namely:

$$\frac{\int_{-\infty}^{t_0} tJ_{\rm fin,+}(x_f,\tau)d\tau}{\int_{-\infty}^{t_0} J_{\rm fin,+}(x_f,\tau)d\tau} - \frac{\int_{-\infty}^{t_0} tJ_{\rm in}(x_f,\tau)d\tau}{\int_{-\infty}^{t_0} J_{\rm in}(x_f,\tau)d\tau} \geq 0 \ , \tag{46a}$$

where t_0 is the instant corresponding to the intersection (*after* the final-peak appearance) of the time envelopes of those two fluxes. Relation (46a) simply means that there is a *delay* in the (time averaged) appearance at a certain point $x_f > a$ of the forward part of the final wavepacket, with respect to the (time averaged) appearance of the forward part of the initial wavepacket in the case that it freely moved (in vacuum). Conditions (46) and (46a) are rather general.

It is curious that, without violating such causality conditions, a piece of information, by means of a (low-frequency) modulation of a (high-frequency) carrying wave, can be transmitted —even if with a strong attenuation— with a *Superluminal* group velocity.

12 Tunnelling through successive barriers

Let us finally study the phenomenon of one-dimensional non-resonant tunnelling through two successive opaque potential barriers[128], separated by an intermediate free region $\mathcal{R}$, by analyzing the relevant solutions to the Schroedinger equation. We shall find that the total traversal time does *not* depend not only on the opaque barrier widths (the so-called "Hartman effect"), but also on the $\mathcal{R}$ width: so that the effective velocity in the region $\mathcal{R}$, between the two barriers, can be regarded as infinite. This agrees with the results known from the corresponding waveguide experiments, which simulated the tunnelling experiment herein considered due to the formal identity between the Schroedinger and the Helmholtz equation.

Namely, in this Section we are going to show that —when studying an experimental setup with two rectangular opaque potential barriers (Fig.5)— the (total) phase tunneling time through the two barriers does depend neither on the barrier widths *nor on the distance between the barriers.*[128]

Let us consider the (quantum-mechanical) stationary solution for the one-dimensional (1D) tunnelling of a non-relativistic particle, with mass m and kinetic energy $E = \hbar^2 k^2/2m = \frac{1}{2}mv^2$, through two equal rectangular barriers with height V_0 ($V_0 > E$) and width a, quantity $L-a \geq 0$ being the distance between them. The Schrödinger equation is

$$-\frac{\hbar^2}{2m}\,\frac{\partial^2}{\partial x^2}\,\psi(x) \;+\; V(x)\,\psi(x) \;=\; E\,\psi(x)\,, \tag{47}$$

where $V(x)$ is zero outside the barriers, while $V(x) = V_0$ inside the potential barriers. In the various regions I ($x \leq 0$), II ($0 \leq x \leq a$), III ($a \leq x \leq L$), IV ($L \leq x \leq L+a$) and V ($x \geq L+a$), the stationary solutions to eq.(47) are the following

$$\left\{\begin{array}{lcll} \psi_{\rm I} &=& {\rm e}^{+ikx} + A_{1{\rm R}}\,{\rm e}^{-ikx} & (48{\rm a}) \\ \psi_{\rm II} &=& \alpha_1\,{\rm e}^{-\chi x} + \beta_1\,{\rm e}^{+\chi x} & (48{\rm b}) \\ \psi_{\rm III} &=& A_{1{\rm T}}\,\left[{\rm e}^{ikx} + A_{2{\rm R}}\,{\rm e}^{-ikx}\right] & (48{\rm c}) \\ \psi_{\rm IV} &=& A_{1{\rm T}}\,\left[\alpha_2\,{\rm e}^{-\chi(x-L)} + \beta_2\,{\rm e}^{+\chi(x-L)}\right] & (48{\rm d}) \\ \psi_{\rm V} &=& A_{1{\rm T}}\,A_{2{\rm T}}\,{\rm e}^{ikx}\,, & (48{\rm e}) \end{array}\right.$$

where $\chi \equiv \sqrt{2m(V_0 - E)}/\hbar$, and quantities $A_{1{\rm R}}$, $A_{2{\rm R}}$, $A_{1{\rm T}}$, $A_{2{\rm T}}$, α_1, α_2, β_1 and β_2 are the reflection amplitudes, the transmission amplitudes, and the coefficients of the "evanescent" (decreasing) and "anti-evanescent" (increasing) waves for barriers 1 and 2, respectively. Such quantities can be easily obtained from the matching (continuity) conditions:

$$\left\{\begin{array}{lcll} \psi_{\rm I}(0) &=& \psi_{\rm II}(0) & (49{\rm a}) \\ \left.\dfrac{\partial\psi_{\rm I}}{\partial x}\right|_{x=0} &=& \left.\dfrac{\partial\psi_{\rm II}}{\partial x}\right|_{x=0} & (49{\rm b}) \end{array}\right.$$

$$\left\{ \begin{array}{rcl} \psi_{\rm II}(a) & = & \psi_{\rm III}(a) \\ \left.\dfrac{\partial\psi_{\rm II}}{\partial x}\right|_{x=a} & = & \left.\dfrac{\partial\psi_{\rm III}}{\partial x}\right|_{x=a} \end{array} \right. \begin{array}{l} \text{(50a)} \\ \text{(50b)} \end{array}$$

$$\left\{ \begin{array}{rcl} \psi_{\rm III}(L) & = & \psi_{\rm IV}(L) \\ \left.\dfrac{\partial\psi_{\rm III}}{\partial x}\right|_{x=L} & = & \left.\dfrac{\partial\psi_{\rm IV}}{\partial x}\right|_{x=L} \end{array} \right. \begin{array}{l} \text{(51a)} \\ \text{(51b)} \end{array}$$

$$\left\{ \begin{array}{rcl} \psi_{\rm IV}(L+a) & = & \psi_{\rm V}(L+a) \\ \left.\dfrac{\partial\psi_{\rm IV}}{\partial x}\right|_{x=L+a} & = & \left.\dfrac{\partial\psi_{\rm V}}{\partial x}\right|_{x=L+a} \end{array} \right. \begin{array}{l} \text{(52a)} \\ \text{(52b)} \end{array}$$

Equations (49-52) are eight equations for our eight unknowns ($A_{1\rm R}$, $A_{2\rm R}$, $A_{1\rm T}$, $A_{2\rm T}$, α_1, α_2, β_1 and β_2). First, let us obtain the four unknowns $A_{2\rm R}$, $A_{2\rm T}$, α_2, β_2 from eqs.(51) and (52) in the case of *opaque* barriers, i.e., when $\chi a \to \infty$:

$$\left\{ \begin{array}{ll} \alpha_2 \longrightarrow {\rm e}^{ikL}\,\dfrac{2ik}{ik-\chi} & \text{(53a)} \\ \beta_2 \longrightarrow {\rm e}^{ikL-2\chi a}\,\dfrac{-2ik(ik+\chi)}{(ik-\chi)^2} & \text{(53b)} \\ A_{2\rm R} \longrightarrow {\rm e}^{2ikL}\,\dfrac{ik+\chi}{ik-\chi} & \text{(53c)} \\ A_{2\rm T} \longrightarrow {\rm e}^{-\chi a}{\rm e}^{-ika}\,\dfrac{-4ik\chi}{(ik-\chi)^2} & \text{(53d)} \end{array} \right.$$

Then, we may obtain the other four unknowns $A_{1\rm R}$, $A_{1\rm T}$, α_1, β_1 from eqs.(49) and (50), again in the case $\chi a \to \infty$; one gets for instance that:

$$A_{1\rm T} \;=\; -{\rm e}^{-2\chi a}\,\frac{4i\chi k}{(\chi-ik)^2}\;A \qquad \text{(54)}$$

where

$$A \equiv \frac{2\chi k}{2\chi k\;\cos k(L-a)\;+\;(\chi^2-k^2)\;\sin k(L-a)}$$

results to be *real*; and where, it must be stressed,

$$\delta \equiv \arg\left(\frac{ik+\chi}{ik-\chi}\right)$$

is a quantity that *does not depend* on a and on L. This is enough for concluding that the phase tunnelling time (see, for instance, refs.[42,48,66,67,70,128])

$$\tau^{\rm ph}{}_{\rm tun} \equiv \hbar \, \frac{\partial \arg \left[A_{1\rm T} A_{2\rm T} {\rm e}^{ik(L+a)} \right]}{\partial E} \; = \; \hbar \, \frac{\partial}{\partial E} \, \arg \left[\frac{-4ik\chi}{(ik-\chi)^2} \right] , \qquad (55)$$

while depending on the energy of the tunnelling particle, *does not depend* on $L + a$ (it being actually independent both of a and of L).

This result does *not only* confirm the so-called "Hartman effect"[48,66,67,70,128] for the two opaque barriers —i.e., the independence of the tunnelling time from the opaque barrier widths—, but it does *also* extend such an effect by implying the total tunnelling time to be independent even of L (see Fig.5): something that may be regarded as a further evidence of the fact that quantum systems seem to behave as non-local.[128-131,88,87,71,48] It is important to stress, however, that the previous result holds only for non-resonant (nr) tunnelling: i.e., for energies far from the resonances that arise in region III due to interference between forward and backward travelling waves (a phenomemon quite analogous to the Fabry-Pérot one in the case of classical waves). Otherwise it is known that the general expression for (any) time delay τ near a resonance at $E_{\rm r}$ with half-width Γ would be $\tau = \hbar\Gamma[(E-E_{\rm r})^2 + \Gamma^2]^{-1} + \tau_{\rm nr}$.

The tunnelling-time independence from the width (a) of each one of the two opaque barriers is itself a generalization of the Hartman effect, and can be a priori understood —following refs.[57,62] (see also refs.[64,55])— on the basis of the reshaping phenomenon which takes place inside a barrier.

With regard to the even more interesting tunnelling-time independence from the distance $L-a$ between the two barriers, it can be understood on the basis of the interference *between the waves* outcoming from the first barrier (region II) and traveling in region III *and the waves* reflected from the second barrier (region IV) back into the same region III. Such an interference has been shown[48,70,131] to cause an "advance" (i.e., an effective acceleration) on the incoming waves; a phenomenon similar to the analogous *advance* expected even in region I. Namely, going on to the wavepacket language, we noticed in ref.[48,70,131] that the arriving wavepacket does interfere with the reflected waves that start to be generated as soon as the packet forward tail reaches the (first) barrier edge: in such a way that (already before the barrier) the backward tail of the initial wavepacket decreases —for destructive interference with those reflected waves— at a larger degree than the forward one. This simulates an increase of the average speed of the entering packet: hence, the effective (average) flight-time of the approaching packet from the source to the barrier does decrease.

So, the phenomena of reshaping and advance (inside the barriers and to the left of the barriers) can qualitatively explain why the tunnelling-time is independent of the barrier widths and of the distance between the two barriers. It remains impressive, nevertheless, that in region III —where no potential barrier is present, the current is non-zero and the

wavefunction is oscillatory,— the effective speed (or group velocity) is practically *infinite*. Loosely speaking, one might say that the considerd two-barriers setup is an "(intermediate) space destroyer". After some straightforward but rather bulky calculations, one can moreover see that the same effects (i.e., the independence from the barrier widths and from the distances between the barriers) are still valid for any number of barriers, with different widths and different distances between them.

Finally, let us mention that the known similarity between photon and (nonrelativistic) particle tunneling[48,57,61,62,70,132; see also 55,64,130] implies our previous results to hold also for photon tunnelling through successive "barriers": For example, for photons in presence of two successive band gap filters: like two suitable gratings or two photonic crystals. Experiments should be easily realizable; while indirect experimental evidence seems to come from papers as [129,121].

At the classical limit, the (stationary) Helmholtz equation for an electromagnetic wavepacket in a waveguide is known to be mathematically identical to the (stationary) Schroedinger equation for a potential barrier;** so that, for instance, the tunnelling of a particle through and under a barrier can be simulated[48,70,58-62,86,132-134] by the traveling of evanescent waves along an undersized waveguide. Therefore, the results of this paper are to be valid also for electromagnetic wave propagation along waveguides with a succession of undersized segments (the "barriers") and of normal-sized segments. This agrees with calculations performed, within the classical realm, directly from Maxwell equations[130,86,134,135], and has already been confirmed by a series of "tunnelling" experiments with microwaves: see refs.[58-60,133] and particularly [116,136].

13 Conclusions and prospects

I. A basic physical formalism for determining the collision and tunnelling times for nonrelativistic particles and for photons seems to be now available:
(1) We have found selfconsistent definitions for the mean times and durations of various collision processes (including tunnelling), together with the variances of their distributions. This was achieved by utilizing the properties of *time*, regarded as a quantum observable (in quantum mechanics and in quantum electrodynamics).
(2) Such definitions seem to work rather well, at least for large (asymptotic) distances between initial wavepackets interaction region, and for finite distances between interaction region and final wavepackets. In these cases the phase-time, the clock and the instanton approaches yield results which happen to coincide either with the mean duration or with the standard deviation [square root of the duration-distribution variance] forwarded by

**These equations are however different (due to the different order of the time derivative) in the time-dependent case. Nevertheless, it can be shown that they still have in common classes of analogous solutions, differing only in their spreading properties[48,70,61, and 131].

our own formalism. And the (asymptotic) mean dwell time results to be the average weighted sum of the tunnelling and reflection durations: cf. eq.(16).

Notice that formulae (4), (6), (8) can be rewritten in a unified way (in terms of the mean square time durations) as follows:

$$\langle [\tau_{\rm N}(x_i, x_f)]^2 \rangle = [\langle \tau_{\rm N}(x_i, x_f) \rangle]^2 + {\rm D}\tau_{\rm N}(x_i, x_f) \tag{56}$$

with ${\rm D}\tau_{\rm N}(x_i, x_f) = {\rm D}\, t_{\rm s}(x_f) + {\rm D}\, t_{+}(x_i)$, where N may mean T or tun or pen or R, etc., and s $= +, -$: more precisely, s $= -$ in the case of reflection and s $= +$ in the remaining cases. Relations (56) can be further on rewritten in the following equivalent forms:

$$\langle [\tau_{\rm N}(x_i, x_f)]^2 \rangle = [\langle t_{\rm s}(x_f) - t_{+}(x_i) \rangle]^2 + {\rm D}t_{\rm s}(x_i) \ . \tag{56a}$$

We can now see that the square phase duration $[\langle \tau_{\rm T}^{\rm Ph} \rangle]^2 + {\rm D}\, \tau_{\rm T}^{\rm Ph}$, and the square hybrid time $[\tau_{y,{\rm tun}}^{\rm La})^2 + (\tau_{z,{\rm tun}}^{\rm La})^2]^2$ introduced by Büttiker[79], as well as the square magnitude of the complex tunnelling time in the Feynman path-integration approach, are all *examples of mean square durations*. Notice that the Feynman approach (in the case of its instanton version) and the Büttiker hybrid time (in the case of an infinite extension of the magnetic field) coincide with the mean square phase duration.

By the way, our present formalism has been already applied and tested in the time analysis of nuclear and atomic collisions for which the boundary conditions are experimentally and theoretically assigned in the region, asymptotically distant from the interaction region, where the incident (before collision) and final (after collision) fluxes are well separated in time, without any superposition and interference. And it has been supported by results (see, in particular, refs.[29,30] and references therein) such as:
(i) the validity of a correspondence principle between the time-energy QM commutation relation and the CM Poisson brackets;
(ii) the validity of an Ehrenfest principle for the average time durations;
(iii) the coincidence of the quasi-classical limit of our own QM definitions for time durations (when such a limit exists; i.e. for above-barrier energies) with analogous well-known expressions of classical mechanics;
(iv) the direct and indirect experimental data on nuclear-reaction durations, in the range $10^{-21} \div 10^{-15}$ s, and the compound-nucleus level densities extracted from those data.

Let us mention that for a complete extraction of the time-durations from *indirect* measurements of nuclear-reaction durations it is necessary to have at disposal correct definitions not only of the mean durations but also of the duration variances[30], as provided by our formalism.

At last, let us recall that such a formalism provided also useful tools for resolving some long-standing problems related to the time-energy uncertainty relation[29,30].

II. In order to apply the present formalism to the cases when one considers not only asymptotic distances, but also the region inside and near the interaction volume, we had

to revise the notion of weighted average (or integration measure) in the time representation, by adopting the two weights $J_{\pm}(x,t)\mathrm{d}t$ when evaluating instant and duration mean values, variances, etc., for a moving particle, and the third weight $dP(x,t)$ or $P(x_1,x_2;t)\mathrm{d}t$ when calculating mean durations for a "dwelling" particle. And in terms of these three weights we can express all the different approaches proposed within conventional quantum mechanics, including the mean dwell time, the Larmor-clock times, and the times given by the various versions of the Feynman path-integration approach: Namely, we can put them all into a single *non-contradictory* scheme on the basis of our formalism, even for a particle inside the barrier.

The same three weights can be used also in the analogous quantum-mechanical formalism for the *space* analysis of collision and propagation processes (see also [18]).

III. Our flux separation into J_+ and J_- is not the only procedure to be possible within *conventional quantum mechanis (and quantum electrodynamics)*, although it is the only *non-coherent* flux separation known to us avoiding the introducing of any new postulates. In fact, one can also adopt the "*coherent* wavepacket separation" into positive and negative momenta, which has a clear meaning outside the barrier, but is obtainable only via a mathematical tool like the momentum Fourier expansion inside the barrier. Such a separation can be transformed into an "incoherent flux separation" by exploiting the postulate of the measurement quantum theory about the possibility of describing the measurement conditions in terms of the corresponding projectors: that is to say, of the projectors $\Lambda_{\mathrm{exp},\pm}$ onto positive-momentum and negative-momentum states, respectively [cf. eq.(13a), Sect.2]. There are also flux separation schemes within nonstandard versions of quantum theory (cf., e.g., Sect.5). However, whatever separation scheme we choose, we have to stick to at least two necessary conditions:
(A) each normalized flux component must possess a probabilistic meaning, and
(B) the standard flux expressions, well-known in quantum collision theory, must be recovered in the asymptotically remote spatial regions.

In brief, with regard to the region inside and near a barrier, at least four kinds of separation procedures for the wavepacket fluxes do exist, which satisfy the previous conditions:
(i) The OR separation $J = J_+ + J_-$, with $J_{\pm} = J\ \Theta(\pm J)$, which was obtained from the conventional continuity equation for probability (i.e., from the time-dependent Schroedinger equation) without any new physical postulates or any new mathematical approximations[71]. The asymptotic behaviour, e.g., of the obtained expressions was tested by comparison with other approaches and with the experimental results[48]; see also point (v) below.
(ii) The separation proposed here, i.e., $J = J_{\mathrm{exp},+} + J_{\mathrm{exp},-}$ (quantities $J_{\mathrm{exp},\pm}$ being the fluxes which correspond to $\Lambda_{\mathrm{exp},\pm}(x,t)$, respectively), is also a consequence of the conventional probability continuity equation, provided that it is accepted the wave-function reduction

postulate of ordinary quantum measurement theory. It corresponds to the adoption of "semi-permeable" detectors, which are open for particles arriving from one direction only. The asymptotic behaviour of the expressions, obtained on the basis of this separation, coincides with that yielded by (i).
(iii) Relation (20) was obtained in the Muga-Brouard-Sala approach, within the physically clear "incoherent flux separation" of positive and negative momenta, but with the additional introduction of the Wigner-path distributions.
(iv) Relation (26) was obtained in the Leavens' approach, on the basis of an incoherent flux separation of the trajectories of particles to be transmitted from particles to be reflected, via the introduction of the nonstandard Bohm interpretation of quantum mechanics.

The flux separation schemes (i), (iii) and (iv) yield asymmetric expressions for the mean dwell time near a barrier [equations (15), (21) and (22)-(25), respectively], apparently due to the right-left asymmetry of the boundary conditions: we have incident and reflected wavepackets on the left, and only a transmitted wavepacket on the right. The separation procedure (ii) yields the symmetric expression (16) for the mean dwell time even near a barrier.

IV. In Sect.7 we have shown that (in the absence of absorption and dissipation) the Hartman effect is valid for *all* the mean tunnelling times, while it does not hold only for the quantities that at a closer analysis did not result to be tunnelling times, but rather tunnelling-time *standard deviations*.

Let us recall at this point that only the sum of increasing (evanescent) and decreasing (anti-evanescent) waves corresponds to a non-zero *stationary* flux. Considering such a sum is standard in quantum mechanics, but not when studying evanescent waves (the analogue of tunnelling photons) in classical physics. On the contrary, that sum should of course be taken into account also in the latter case, obtaining non-zero (stationary) fluxes.

In any case, it is interesting to notice that in the *non-stationary* case, even evanescent waves alone, or anti-evanescent waves alone, correspond separately to *non-zero* fluxes. Even more, from the general expression of a non-stationary wave packets inside a barrier, one can directly see that, e.g., *evanescent* waves (considered alone) seem to fill up instantaneously the entire barrier as a whole!; this being a further evidence of the *non-local* phenomena which take place during *sub-barrier* tunnelling. Even stronger examples of non-locality have been met by us in Sect.12 above: cf. eq.(55). Some numerical evaluations[86,98,126], based on Maxwell equations only, showed that analogous phenomena occur for classical evanescent waves in under-sized waveguides ("barriers"), as confirmed by experience. / Let us recall, at last, that even Superluminal localized (non-dispersive, wavelet-type) pulses which are solutions to the Maxwell equations have been constructed[130], which are *not* evanescent but on the contrary propagate without distortion along *normal* waveguides.

V. In connection with Sects.2, 6 and 11, let us recall that the requirement that the values of the collision, propagation, tunnelling duration be positive is a *sufficient* but not necessary causality condition. Therefore we have not got a unique general formulation of the causality principle which is necessary for all possible cases. In Sect.2 and 11 some new formulations of the causality condition heve been by us just proposed.

VI. The phenomena of reshaping, which were dealt with in Sect.9, as well as the "advance" which takes place before the barrier entrance (discussed in Sect.6) are closely connected with the (coherent) superposition of incoming and reflected waves. Moreover, the study of reshaping (or reconstruction) and of the advance phenomenon can be of help, by themselves, in understanding the problems connected with Superluminal phenomena and the definition of signal velocity[115-120].

VII. In the case of tunnelling through two successive opaque barriers (cf. Fig.5), we strongly generalized the Hartman effect, by showing in Sect.12 that *far from resonances* the (total) phase tunneling time through the two opaque barriers —while depending on the energy— is *independent* not only of the barrier widths, but even *of the distance between the barriers*: So that the effective velocity in the free region, between the two barriers, can be regarded as infinite.

VIII. We mentioned in Sect.8 that the two-phase description of tunnelling can be convenient for media without absorption and dissipation, and also for Josephson junctions.

IX. The OR formalism, as presented in this paper, permits in principle to study the time evolution of collisions in the Schroedinger and Feynman representations (which lead, by the way, to the same results). An interesting attempt was undertaken in ref.[140] to a selfconsistent description of a particle motion, by utilizing the Feynman representation and comparing their method with the OR formalism (in its earlier version, presented in ref.[48]), even if skipping the separation $J = J_+ + J_-$.

There is one more possible representation, equivalent to Schroedinger's and Feynman's, for investigating the collision and tunnelling evolution. Let us recall that in quantum theory to the *energy* E there correspond the two operators $i\hbar\partial/\partial t$ and the hamiltonian operator. Their duality is well represented by the Schroedinger equation $H\Psi = i\hbar\partial\Psi/\partial t$. A similar duality does exist in quantum mechanics for *time*: besides the general form $-i\hbar\partial/\partial E$, which is valid for any physical systems (in the continuum energy spectrum case), it is possible to express the time operator $\hat{T}$ (which is hermitian, and also maximal hermitian[27,18,19], even if not self-adjioint) in terms of the coordinate and momentum operators[25,30,141,142], by utilizing the commutation relation $[\hat{T}, \hat{H}] = i\hbar$. So that one can study the collision and tunnelling evolutions via the operator $\hat{T}$ by the analogous equation $\hat{T}\Psi = t\Psi$, particularly for studying he influence of the barrier shape on the tunnelling time[30].

X. In Sect.9 the analogy between tunnelling processes of photons (in first quantization) and of non-relativistic particles has been discussed and clarified, and it was moreover shown that the properties of time as an observable can be extended from quantum mechanics to one-dimensional quantum electrodynamics. On the basis of this analogy, in Sects.9 and 10 a selfconsistent interpretation of the photon tunnelling experiments, described in refs.[56,60], was presented.

XI. At last, let us mention that for discrete energy spectra, the time analysis of the processes (and, particularly, in the case of wavepackets composed of states bound by two well potentials, with a barrier between the wells) is rather diffeent from the time analysis of processes in the continuous energy spectra. For the former, one may use the formalism[30,31] for the time operator in correspondence with a discrete energy spectrum: and the durations of the transitions from one well to the other happen to be given by the Poincaré period $2\pi\hbar/d_{\min}$, where $d_{\min}$ is the highest common factor of the level distances, which is determined by the *minimal level splitting caused by the barrier* and hence depends on the barrier traversal probability at the relevant energies[143].

One can expect that the time analysis of more complicated processes, in the quasi-discrete (resonance) energy regions, with two (or more) well-potentials, such as the *photon ot phonon-induced tunnellings from one well to the other*, could be performed by a suitable combination and generalization of the methods elaborated for continuous and discrete energy spectra.

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References

[1] G.Gamow: Z.Phys. **51** (1928)204.

[2] I.Giaver: Phys.Rev.Lett. **5** (1960)147.

[3] L.Esaki: IEEE Trans. **ED-23** (1976) 644.

[4] G.Binder, H.Roehrer, Ch.Gerber and E.Weibel: Appl.Phys.Lett. **40** (1982)37.

[5] E.U.Condon: Rev.Mod.Phys. **3** (1931) 43.

[6] L.A.MacColl: Phys.Rev. **40** (1932) 621.

[7] D.Bohm: *Quantum Theory* (Prentice-Hall; New York, 1952).

[8] E.Wigner: Phys.Rev. **98** (1955) 145.

[9] F.Smith: Phys.Rev. **118** (1960) 349; 119 (1960) 2098.

[10] M.L.Goldberger and K.M.Watson: Phys.Rev. **127** (1962) 2284; M.Froissart, M.L.Goldberger and K.M.Watson: Phys.Rev. **131** (1963) 2820; M.L.Goldberger and K.M. Watson: *Collision Theory* (Wiley; New York, 1964).

[11] T.Ohmura: Progr.Theor.Phys.Suppl. **29** (1964) 108.

[12] R.Fong: Phys.Rev. **B140** (1965) 762.

[13] A.I.Baz': Sov.J.Nucl.Phys. **4** (1967) 182; **5** (1967) 161.

[14] V.F.Rybachenko: Sov.J.Nucl.Phys. **5** (1967) 484.

[15] V.S.Olkhovsky: Nuovo Cimento **48** (1967) 178.

[16] J.M.Jauch and J.P.Marchand: Helv.Phys.Acta **40** (1967) 217.

[17] R.Vedrinski: Nuovo Cimento **B51** (1967) 487.

[18] V.S.Olkhovsky and E.Recami: Nuovo Cimento **A63** (1969) 814.

[19] W.Pauli: in *Handbuch der Physik*, ed. by S.Flugge (Berlin, 1926), v.**5/1**, p.60; *General Principles of Quantum Theory* (Springer; Berlin, 1980).

[20] F.Engelman and E.Fick: Nuovo Cim. Suppl. **12** (1959) 63; Z.Phys. **175** (1963) 271; **178** (1964) 551.

[21] V.N.Paul: Ann.Phys. **9** (1962) 252.

[22] B.A.Lippmann: Phys.Rev. **151** (1966) 1023.

[23] M.Razavy: Nuovo Cimento **B63** (1969) 271.

[24] T.T.Gien: Can.J.Phys. **47** (1969) 278; **48** (1970) 639.

[25] D.M.Rosenbaum: J.Mat.Phys. **10** (1969) 1127.

[26] G.R.Allcock: Ann.Phys.(N.Y.) **53** (1969) 253.

[27] V.S.Olkhovsky, E.Recami and A.I.Gerasimchuk: Nuovo Cimento **A22** (1974) 263; V.S.Olkhovsky and E.Recami: Lett. Nuovo Cimento (first series) **4** (1970) 1165-1175; E.Recami: "A Time-Operator and the Time-Energy Uncertainty Relation", Chapt. 4 in *The Uncertainty Principle and Foundations of Quantum Mechanics*, ed. by W.C.Price & S.S.Chissick (J.Wiley; London, 1977), pp. 21-28. See also V.S.Olkhovsky: Ukrainskiy Fiz. Zhurnal **18** (1973) 1910 [in Russian and Ukrainian].

[28] A.S.Holevo: Rep.Math.Phys. **13** (1978) 379; *Probabilistic and Statistical Aspects of Quantum Theory* (Amsterdam, 1982).

[29] a) V.S.Olkhovsky: Sov.J.Part.Nucl. **15** (1984) 130; b) E.Recami, W.A.Rodrigues Jr. and P.Smrz: Hadronic J. **6** (1983) 1773-1789. See also I.L.Egusquiza and J.G.Muga: Phys. Rev. A61 (1999), issue Dec.9, 1999; J.Leòn, J.Julve, P.Pitamga and F.J.de Urrìes: e-print quant-ph/9903060; J.Leòn: J. Phys. A40 (1997) 4791.

[30] V.S.Olkhovsky: Nukleonika **35** (1990) 99; Atti Accademia Peloritana dei Pericolanti, Sci.Fis.Mat.Nat. **70** (1992) 21.

[31] V.S.Olkhovsky: *Physics of the Alive* **6** (1998) 17; in *Mysteries, Puzzles and Paradoxes in Quantum Mechanics*, ed. by R.Bonifacio (A.I.P.; Woodbury, N.Y., 1999), pp.272-276; V.S.Olkhovsky, E.Recami and A.Agresti: to be published.

[32] P.Bush, M.Grabowski and P.J.Lahti: Phys. Lett. **A191** (1994) 357.

[33] D.H.Kobe and V.C.Aguilera-Navarro: Phys. Rev. **A50** (1994) 933.

[34] P.Blanchard and A.Jadczyk: Helv. Phys. Acta **69** (1996) 613.

[35] N.Grot, C.Rovelli and R.S.Tate: Phys. Rev. **A54** (1996) 4676.

[36] J.León: J. Phys. **A30** (1997) 4791.

[37] Ya.Aharonov, J.Oppenhem, S.Popescu, B.Reznik and W.Unruh: Phys. Rev. **A57** (1998) 4130.

[38] V.Delgado: Phys. Rev. **A59** (1999) 1010.

[39] P.Kochański and K.Wódkiewicz: Phys. Rev. **A60** (1999) 2689.

[40] M.A.Naimark: Izv. Akad. Nauk SSSR Ser. Math. **4** (1940) 277 [in Russian].

[41] S.Collins, D.Lowe and J.R.Barker: J.Phys. **C20** (1989) 6233.

[42] E.H.Hauge and J.A.Støvneng: Rev.Mod.Phys. **61** (1989) 917.

[43] Z.H.Huang, P.H.Cutler, T.E.Feuchtwang et al,: J.Vac.Sci.Techn. **A8** (1990) 186.

[44] M.Büttiker: in *Electronic Properties of Multilayers and Low Dimensional Semiconductors*, ed. by J.Chamberlain, L.Eaves and J.Portal (Plenum; New York, 1990), p.297.

[45] C.R.Leavens and G.Aers: in *Scanning Tunneling Microscopy and Related Methods*, ed. by R.Behm, N.Garcia and H.Rohrer (Kluwer; Dordrecht, 1990), p.59.

[46] A-P.Jauho: in *Quantum Transport in Semiconductors*, ed. by D.Ferry and C.Jacoboni (Plenum; New York, 1991), p.179; in *Hot Carriers in Semiconductor Nanostructures: Physics and Applications*, ed. by J.Shah (Academic; London, 1992), p.121.

[47] M.Jonson: in *Quantum Transport in Semiconductors*, ed. by D.Ferry and C.Jacoboni (Plenum; New York, 1991), p.193.

[48] V.S.Olkhovsky and E.Recami: Phys.Reports **214** (1992) 339; see also ref.[71] below.

[49] R.Landauer and Th.Martin: Rev.Mod.Phys. **66** (1994) 217.

[50] P.Guéret, E.Marclay and H.Meier: Europhys.Lett. **3** (1987) 367; P.Gue'ret, A.Baratoff and E.Marclay: Solid State Commun. **68** (1988) 977.

[51] M.Tsuchiya, T.Matsusue and H.Sakaki: Phys.Rev.Lett. **59** (1987) 2356.

[52] D.Esteve, J.M.Martinis, C.Urbina, C.Turlot, M.H.Devoret, P.Grabert and H.Linkwitz: Phys.Scr. **T29** (1989) 121.

[53] H.Q.Nguen, P.H.Cutler, T.E.Feuchtwang et al.: EEE Trans.Electr.Dev. **36** (1989) 26171; A.A.Lucas, P.H.Cutler, T.E.Feuchtwang et al.: J.Vac.Sci.Techn. **A6** (1988) 461.

[54] R.Landauer: Nature **341** (1989) 567.

[55] A.Ranfagni, D.Mugnai, P.Fabeni and G.P.Pazzi: Appl.Phys.Lett. **58** (1991) 774; A.Ranfagni, D.Mugnai and A.Agresti: Phys.Lett. **A175** (1993) 334.

[56] A.Enders and G.Nimtz: J.Phys.-I (France) **2** (1992) 1693; **3** (1993) 1089; Phys.Rev. **B47** (1993) 9605; Phys.Rev. **E48** (1993) 632; G.Nimtz: in *Tunneling and its Applications* (World Sci.; Singapore, 1997), p.223-237.

[57] A.M.Steinberg, P.G.Kwiat and R.Y.Chiao: Phys.Rev.Lett. **71** (1993) 708; R.Y.Chiao, P.G.Kwiat and A.M.Steinberg: Scient. Am. **269** (1993), issue no.2, p.38.

[58] A.Ranfagni P.Fabeni, G.P.Pazzi and D.Mugnai: Phys.Rev. **E48** (1993) 1453.

[59] Ch.Spielman, R.Szipoecs, A.Stingl and F.Krausz: Phys.Rev.Lett. **73** (1994) 2308.

[60] a) Ph.Balcou and L.Dutriaux: Phys.Rev.Lett. **78** (1997) 851; b) V.Laude and P.Tournois: J.Opt.Soc.Am. B16 (1999) 194; c) A.Haibel, G.Nimtz and A.A.Stahlhofen: "Frustrated total reflection: The double-prism revisited", in press in Phys.Rev. E (December 2000).

[61] J.Jakiel, V.S.Olkhovsky and E.Recami: Physics Letters **A248** (1998) 156-160.

[62] R.Y.Chiao, P.G.Kwiat and A.M.Steinberg: Physica **B175** (1991) 257.

[63] A.Ranfagni, D.Mugnai, P.Fabeni and G.Pazzi: Appl.Phys.Lett. **58** (1991)774; D.Mugnai, A.Ranfagni, R.Ruggeri and A.Agresti: Phys.Rev.Lett.**68** (1992) 259; A.Ranfagni, D.Mugnai and A.Agresti: Phys.Lett. **A175** (1993) 334.

[64] Th.Martin and R.Landauer: Phys.Rev. **A45** (1992) 2611.

[65] G.Nimtz, A.Enders and H.Spieker: J.Phys.-I (France) **4** (1994) 1.

[66] T.E.Hartman: J.Appl.Phys. **33** (1962) 3427.

[67] J.R.Fletcher: J.Phys. **C18** (1985) L55.

[68] E.H.Hauge, J.P.Falck and Fjeldly: Phys.Rev. **B36** (1987) 4203.

[69] R.Landauer and Th.Martin: Solid State Commun. **84** (1992) 115.

[70] V.S.Olkhovsky, E.Recami and A.K.Zaichenko: Solid State Commun. **89** (1994) 31; V.S.Olkhovsky and A.Agresti: in *Tunneling and its Applications* (World Sci.; Singapore, 1997), p.327-355.

[71] V.S.Olkhovsky, E.Recami, F.Raciti and A.K.Zaichenko: J.Phys.-I (France) **5** (1995) 1351. See also M.Abolhasani and M.Golshani: ref.[94].

[72] D.Sokolovski and L.M.Baskin: Phys.Rev. **A36** (1987) 4604.

[73] E.Pollak and W.H.Miller: Phys.Rev.Lett. **53** (1984)115; E.Pollak: J.Phys.Chem. **83** (1985) 1111.

[74] D.Sokolovski and J.N.L.Connor: Phys.Rev. **A44** (1990) 1500; **A47** (1993) 4677.

[75] D.Mugnai and A.Ranfagni: Europhys.Lett. **6** (1988)1; Nuovo Cimento **D14** (1992) 541.

[76] H.W.Lee and M.O.Scully: J.Chem.Phys. **77** (1982) 4609; Found.Phys. **13** (1983) 61.

[77] J.G.Muga, S.Brouard and R.Sala: Phys.Lett. **A167** (1992) 24; S.Brouard, R.Sala and J.R.Muga: Phys.Rev. **A49** (1994) 4312.

[78] C.R.Leavens and G.Aers: in *Scanning Tunnelling Microscopy – III*, ed. by R.Wiesendager and H.-J.Guntherodt (Springer; New York, 1993), p.105; K.Imafuku, I.Ohba and Y.Yamanaka: Phys. Rev. A56 (1997) 1142-1153.

[79] M.Büttiker: Phys.Rev. **B27** (1983) 6178.

[80] M.Büttiker and R.Landauer: Phys.Rev.Lett. **49** (1982) 1739; Phys.Scr. **32** (1985)429; IBM J.Res.Dev. **30** (1986) 451.

[81] A.Pimpale, S.Holloway and R.Smith: J.Phys. **A24** (1991) 3533.

[82] Z.Huang, P.H.Cutler, T.E.Feuchtwang et al.: J.de Phys.,Suppl. 11, **49** (1988) 6-17.

[83] C.R.Leavens: Phys.Lett. **A197** (1995) 88.

[84] E.Recami: Rivista Nuovo Cim. **9** (1986), issue no.6, pp.1-178; E.Recami: Found.Phys. **17** (1987) 239-296; Lett.N.Cim. **44** (1985) 587-593.

[85] See page 1361 in ref.[71]. See also refs.[84].

[86] See the first one of refs.[48] and refs. therein, ref.[71], and A.P.L.Barbero, H.E.Hernández F., and E.Recami: "On the propagation speed of evanescent modes" [electronic LANL archives # physics/9811001], submitted to Phys.Rev.E; E.Recami, H.E.Hernández F., and A.P.L.Barbero: Ann.der Phys. **7** (1998) 764-773; E.Recami, F.Fontana and G.Garavaglia: ref.[110].

[87] R.Y.Chiao, A.E.Kozhekin and G.Kurizki: Phys.Rev.Lett. **77** (1996) 1254; C.G.B.Garret and D.E.McCumber: Phys.Rev. **A1** (1970) 305.

[88] S.Chu and Wong W.: Phys.Rev.Lett. **48** (1982) 738; M.W.Mitchell and R.Y.Chiao: Phys.Lett. **A230** (1997) 133-138; G.Nimtz: Europ.Phys.J.-B (to appear as a Rapid Note); further experiments are being performed at Glasgow [D.Jaroszynski, private communication].

[89] W.Jaworski and D.M.Wardlaw: Phys.Rev. **A37** (1988) 2843.

[90] A.Begliuomini and L.Bracci: private communication.

[91] R.S.Dumont and T.L.Marchioro: Phys.Rev. **A47** (1993) 85.

[92] M.-Q.Chen and M.S.Wang: Phys.Lett. **A149** (1990) 441.

[93] V.Delgado, S.Brouard and J.G.Muga: Solid State Commun. **94** (1995) 979.

[94] M. Abolhasani and M.Golshani: Phys. Rev. A62 (2000), issue June 14, 2000; E.Recami and G.Salesi: Phys.Rev. **A57** (1998) 98-105.

[95] R.C.Leavens: Solid State Commun. **85** (1993) 115; **89** (1994) 37.

[96] A.K.Zaichenko: private communication.

[97] F.Raciti and G.Salesi: J.Phys.-I (France) **4** (1994) 1783. See, however, also S.Baskoutas, A.Jannussis and R.Mignani: J. Phys. **A27** (1994) 2189.

[98] G.Nimtz, H.Spieker and M.Brodowsky: J.Phys.-I (France) **4** (1994) 1379.

[99] M.Abolhasani and M.Golshani: ref.[94]; J.Leòn: (private commun.); V.Petrillo and L.Refaldi: "A time asymptotic expression for the wave function emerging from a quanto-mechanical barrier", subm. for pub.; L.Refaldi: MSc thesis (V.Petrillo, R.Bonifacio, E.Recami supervisors), Phys. Dept., Milan Univ., 2000.

[100] J.Jakiel: PhD Thesis (Inst. Nucl. Phys.; Cracow, 1998). See also ref.[61].

[101] B.Gaveau, T.Jacobson, M.Kac and L.S.Schulman: Phys.Rev.Lett. **53** (1984) 419.

[102] A.Ranfagni and D.Mugnai: Phys.Rev. **E52** (1995) 1128; D.Mugnai, A.Ranfagni, R.Ruggeri, A.Agresti and E.Recami: Phys.Lett. **A209** (1995) 227.

[103] A.I.Akhiezer and V.B.Berestezki: *Quantum Electrodynamics* (FM; Moscow, 1959) [in Russian]; R.Mignani, E.Recami & M.Baldo: Lett. Nuovo Cimento **11** (1974) 568-572.

[104] E.Recami: in *Hadronic Mechanics and Non Potential Interactions*, ed. by M.Mijatovic (Nova Science Publ.; N.Y., 1990), pp. 231-238; E.Recami & M.Evans: in *The Enigmatic Photon - vol.4*, ed. by M.Evans & J.-P.Vigier (Plenum; New York, 1998).

[105] V.S.Olkhovsky: Ukrainskiy Fiz.Zhurnal **28** (1983) 1319 [in Russian and Ukrainian].

[106] J.D.Jackson: *Classical Electrodynamics* (Wiley; 1975), Sect.8.3.

[107] P.M.Morse and H.Feshbach: *Methods of Theoretical Physics*, (McGraw Hill; 1953), part II, chapter 13.

[108] L.Brillouin: *Wave Propagation and Group Velocity* (Academic; New York, 1960).

[109] S.Schweber: *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson & Co.; Evanston, Ill., 1961), Chapter 5.3.

[110] L.B.Felsen and N.Marcuvitz: *Radiation and Scattering of Waves* (Prentice-Hall; New Jersey, 1973), v.**1**, Chapter 1.5.

[111] E.Recami, F.Fontana and R.Garavaglia: Int.J.Mod.Phys. A15 (2000) 2793-2812.

[112] R.Fox, C.G.Kuper and S.G.Lipson: Proc.Roy.Soc.London **A316** (1970) 515.

[113] J.M.Deutch and F.E.Low: Ann.Phys.(N.Y.) **228** (1993) 184; K.Hass, P.Busch: Phys.Lett. **A185** (1994) 9.

[114] M.Ya.Azbel': Solid State Commun. **91** (1994) 439.

[115] W.Heitmann, G.Nimtz: Phys.Lett. **A196** (1994) 154.

[116] G.Nimtz, A.Enders and H.Spieker: J.de Phys.I France **4** (1994) 565; and in *Wave and Particle in Light and Matter*, ed. by A.van der Merwe and A.Garuccio (Plenum; New York, 1993).

[117] A.Steinberg: J.de Physique-I **4** (1994) 1813.

[118] G.Nimtz: J.de Physique-I **4** (1994) 1817.

[119] G.Nimtz and W.Heitman: Progr.Quant.Electr. **21** (1997) 81-108.

[120] R.Y.Chiao and A.M.Steinberg: in *Progress in Optics*, ed.by E.Wolf (Elsevier Science; Amsterdam, 1997), v.**37**, pp.346-405.

[121] C.G.B.Garrett and D.E.McCumber: Phys.Rev. **A1** (1970) 305. See also ref.[129].

[122] M.D.Crisp: Phys.Rev. **A4** (1971) 2104.

[123] A.Sommerfeld: Z.Phys. **8** (1907) 841; **44** (1914) 177.

[124] A.P.L.Barbero, H.E.Hernández F., and E.Recami: in refs.[86].

[125] R.Y.Chiao: "Tunnelling times and superluminality: A tutorial"; G.Nimtz: "Superluminal evanescent modes"; E.Recami: "Superluminal motions in special relativity", all in *Mysteries, Puzzles and Paradoxes in Quantum Mechanics*, ed. by R.Bonifacio (A.I.P.; Woodbury, N.Y., 1999), at p.3, p.14 and p.22, respectively.

[126] See, e.g., P.Caldirola and E.Recami: "Causality and tachyons in Relativity", in *Italian Studies in the Philosophy of Science*, ed. by M.Dalla Chiara (Reidel, Boston, 1980), pp.249-298; E.Recami: "Relativity and beyond", cap. 16 in *Albert Einstein, 1879-1979: Relativity, Cosmology and Quanta*, ed. by F.de Finis & M.Pantaleo (Johnson Rep.Co.; New York, N.Y., 1979), vol. 2, pp.537-597, whose Russian version "Teoriya otnositelnosti i ee obobstchenya" appeared in *"Astrofizika, Kyanti i Teoriya Otnositelnosti*, ed. by F.I.Fedorov (Mir; Moscow, 1982), cap. 4, pp. 53-128. Cf. also E.Recami: "Tachyon mechanics and causality; A systematic thorough analysis of the tachyon causal paradoxes", Found.Phys. **17** (1987) 239-296.

[127] F.Cardone, R.Mignani and V.S.Olkhovsky: J.de Physique-I **7** (1997) 1211.

[128] V.S.Olkhovsky, E.Recami and G.Salesi: "Tunneling through two successive barriers and the Hartman (Superluminal) effect", Report INFN/FM-00/02 (Frascati, 10.02.00) [Lanl Archives # quant-ph/0002022], subm.for pub.

[129] S.Chu and S.Wong: Phys.Rev.Lett. **48** (1982) 738. See also ref.[121].

[130] M.Zamboni-Rached, E.Recami and F.Fontana: "Localized Superluminal solutions to Maxwell equations propagating along a normal-sized waveguide", Report INFN/FM-00/01 (Frascati, 10.03.00) [Lanl Archives # physics/0001039], subm. for pub. Cf. also E.Recami: Physica A252 (1998) 586, and refs. therein.

[131] D.Mugnai, A.Ranfagni, R.Ruggeri, A.Agresti and E.Recami: ref.[102].

[132] Y.Japha and G.Kurizki: Phys.Rev. **A53** (1996) 586; "Causality and interference in electron and photon tunneling", preprint (Weizmann Inst.Sc.; Rehovot, Jan.1996); "Causality and non-locality in 'faster-than-light' photon travel", preprint (Weizmann Inst.Sc.; Rehovot, Sept.1996).

[133] A.Enders and G.Nimtz: J.de Phys.-I **3** (1993) 1089.

[134] H.M.Brodowsky, W.Heitmann and G.Nimtz: Phys. Lett. **A222** (1996) 125.

[135] E.Recami: in *Causality and Locality in Modern Physics*, ed. by G.Hunter, S.Jeffers and J.-P.Vigier (Kluwer; Dordrecht, 1998), p.113.

[136] A.Enders and G.Nimtz: Phys.Rev. **B47** (1993) 9605.

[137] K.W.McVoy, L.Heller, M.Bolsterli: Rev.Mod.Phys. **39** (1967) 245.

[138] A.Anderson: Am.J.Phys. **57** (1989) 230.

[139] J.Jakiel, S.P.Maidanyuk and V.S.Olkhovsky: to appear in Journal of Physical Studies (L'VOV).

[140] G.Iannacone and B.Pellegrini: Phys.Rev. **B49** (1994-I) 16548.

[141] I.Fujiwara: Progr.Theor.Phys. **62** (1979) 1179; **64** (1980) 18.

[142] T. Gotò et al.: Progr.Theor.Phys. **64** (1980) 1; Progr.Theor.Phys. **66** (1981) 1525, 1915.

[143] V.S.Olkhovsky and S.P.Maidanyuk: J. Phys. Studies (L'vov) **3** (1999) 12.

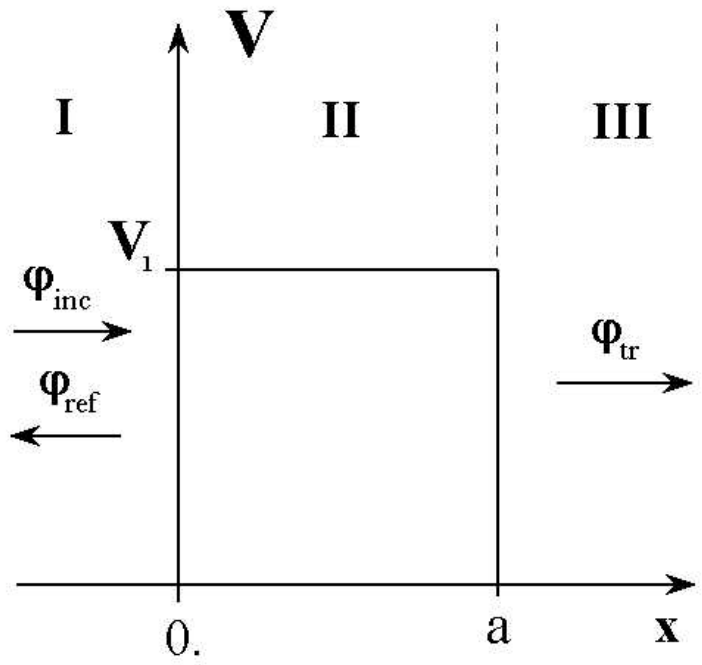
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Fig. 1

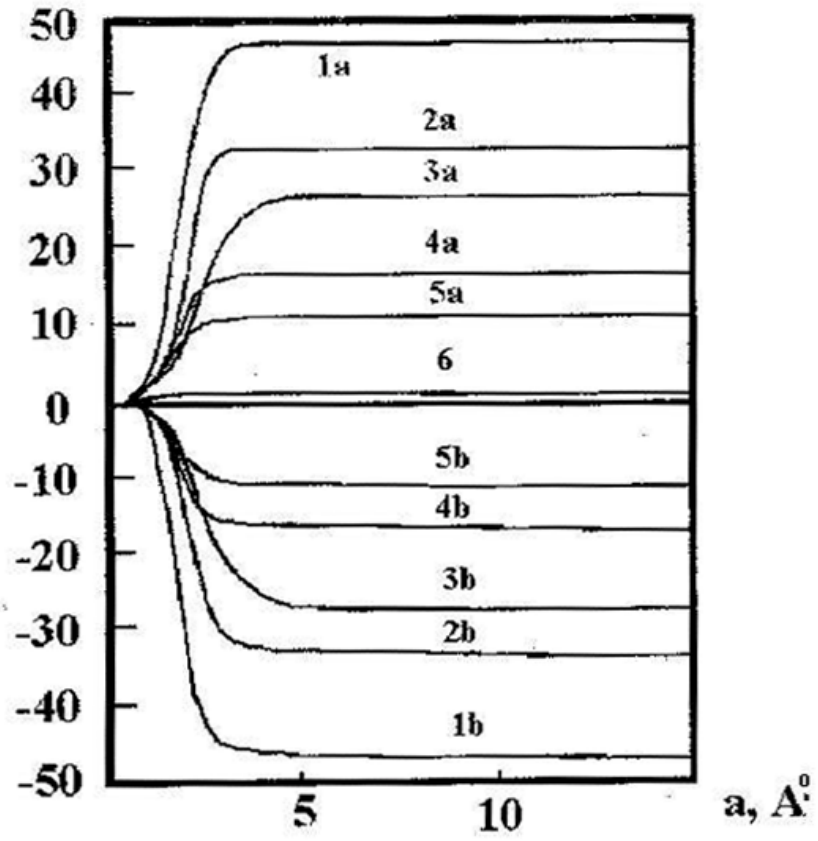
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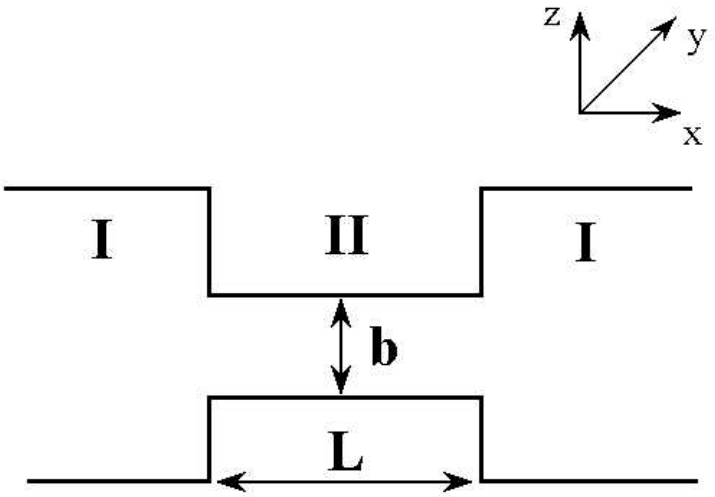
Fig. 2



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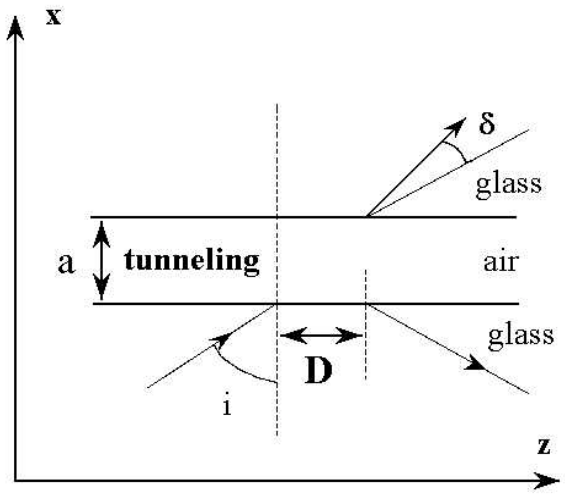

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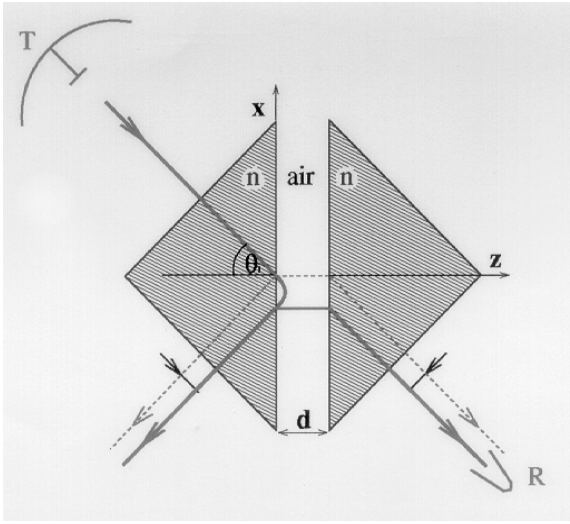

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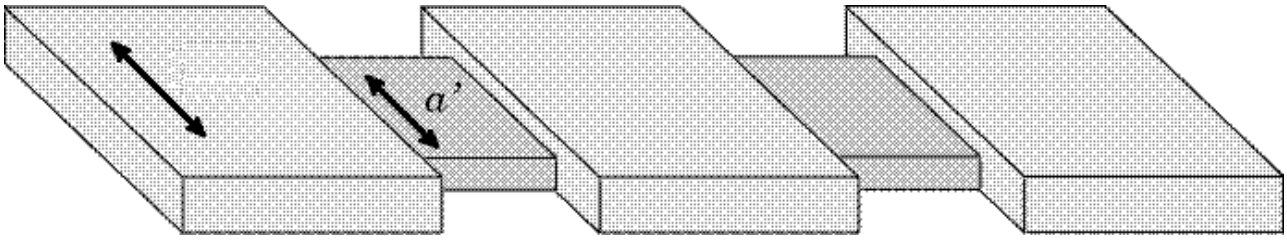

Fig. 5