# ISTITUTO NAZIONALE DI FISICA NUCLEARE

**Sezione di Milano**

# TUNNELING THROUGH TWO SUCCESSIVE BARRIERS AND THE HARTMAN (SUPERLUMINAL) EFFECT[†]

Vladislav S. Olkhovsky
*Institute for Nuclear Research, Kiev-03028; Research Center "Vidhuk", Kiev, Ukraine*
e-mail: olkhovsk@stenos.kiev.ua

Erasmo Recami
*Facoltà di Ingegneria, Università statale di Bergamo, Dalmine (BG), Italy*
*INFN – Sezione di Milano, Milan, Italy*
and
*D.M.O./FEEC and C.C.S., State University at Campinas, Campinas, S.P., Brazil*
e-mail: Erasmo.Recami@mi.infn.it

and

Giovanni Salesi
*INFN – Sezione di Milano, Milan, Italy; and*
*Facoltà di Ingegneria, Università statale di Bergamo, 24044 – Dalmine (BG), Italy*
e-mail: Giovanni.Salesi@ct.infn.it

## Abstract

We study the phenomenon of one-dimensional non-resonant tunnelling through two successive potential barriers, separated by an intermediate free region $\Re$, by analyzing the relevant solutions to the Schroedinger equation. We find that the total traversal time does *not* depend not only on the barrier widths (the so-called "Hartman effect"), but also on the $\Re$ width: so that the effective velocity in the region $\Re$, between the two barriers, can be regarded as infinite. This agrees with the results known from the corresponding waveguide experiments, which simulated the tunnelling experiment herein considered due to the formal identity between the Schroedinger and the Helmholtz equation.

PACS numbers: 73.40.Gk; 03.65.-w; 03.30.+p; 41.20.Jb; 84.40.Az

In this note we are going to show that —when studying an experimental setup with two rectangular potential barriers (Fig.1)— the (total) phase tunneling time through the two barriers does not depend on the barrier widths *nor on the distance between the barriers*.

Let us consider the (quantum-mechanical) stationary solution for the one-dimensional (1D) tunnelling of a non-relativistic particle, with mass $m$ and kinetic energy $E = \hbar^2 k^2/2m = \frac{1}{2}mv^2$, through two equal rectangular barriers with height $V_0$ ($V_0 > E$) and width $a$, quantity $L - a \geq 0$ being the distance between them. The Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x), \tag{1}$$

where $V(x)$ is zero outside the barriers, while $V(x) = V_0$ inside the potential barriers. In the various regions I ($x \leq 0$), II ($0 \leq x \leq a$), III ($a \leq x \leq L$), IV ($L \leq x \leq L + a$) and V ($x \geq L + a$), the stationary solutions to eq.(1) are the following

$$
\begin{cases}
\psi_{\text{I}} = \text{e}^{+ikx} + A_{1\text{R}}\,\text{e}^{-ikx} & (2\text{a}) \\
\psi_{\text{II}} = \alpha_1\,\text{e}^{-\chi x} + \beta_1\,\text{e}^{+\chi x} & (2\text{b}) \\
\psi_{\text{III}} = A_{1\text{T}}\left[\text{e}^{ikx} + A_{2\text{R}}\,\text{e}^{-ikx}\right] & (2\text{c}) \\
\psi_{\text{IV}} = A_{1\text{T}}\left[\alpha_2\,\text{e}^{-\chi(x-L)} + \beta_2\,\text{e}^{+\chi(x-L)}\right] & (2\text{d}) \\
\psi_{\text{V}} = A_{1\text{T}}\,A_{2\text{T}}\,\text{e}^{ikx}, & (2\text{e})
\end{cases}
$$

where $\chi \equiv \sqrt{2m(V_0 - E)}/\hbar$, and quantities $A_{1\text{R}}$, $A_{2\text{R}}$, $A_{1\text{T}}$, $A_{2\text{T}}$, $\alpha_1$, $\alpha_2$, $\beta_1$ and $\beta_2$ are the reflection amplitudes, the transmission amplitudes, and the coefficients of the "evanescent" (decreasing) and "anti-evanescent" (increasing) waves for barriers 1 and 2, respectively. Such quantities can be easily obtained from the matching (continuity) conditions:

$$
\begin{cases}
\psi_{\text{I}}(0) = \psi_{\text{II}}(0) & (3\text{a}) \\
\left.\dfrac{\partial\psi_{\text{I}}}{\partial x}\right|_{x=0} = \left.\dfrac{\partial\psi_{\text{II}}}{\partial x}\right|_{x=0} & (3\text{b})
\end{cases}
$$

$$
\begin{cases}
\psi_{\text{II}}(a) = \psi_{\text{III}}(a) & (4\text{a}) \\
\left.\dfrac{\partial\psi_{\text{II}}}{\partial x}\right|_{x=a} = \left.\dfrac{\partial\psi_{\text{III}}}{\partial x}\right|_{x=a} & (4\text{b})
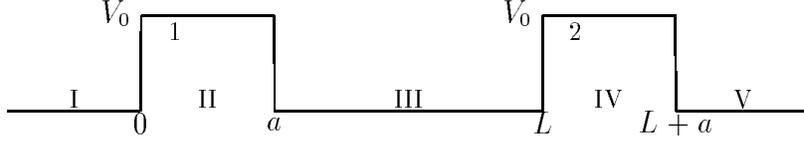\end{cases}
$$

2

Figure 1: The tunneling process, through two successive potential barriers, considered in this paper. We show that the (total) phase tunneling time through the two barriers does not depend on the barrier widths *nor on the distance between the barriers*.

$$\begin{cases} \psi_{\mathrm{III}}(L) = \psi_{\mathrm{IV}}(L) & (5\mathrm{a}) \\ \dfrac{\partial \psi_{\mathrm{III}}}{\partial x}\bigg|_{x=L} = \dfrac{\partial \psi_{\mathrm{IV}}}{\partial x}\bigg|_{x=L} & (5\mathrm{b}) \end{cases}$$

$$\begin{cases} \psi_{\mathrm{IV}}(L+a) = \psi_{\mathrm{V}}(L+a) & (6\mathrm{a}) \\ \dfrac{\partial \psi_{\mathrm{IV}}}{\partial x}\bigg|_{x=L+a} = \dfrac{\partial \psi_{\mathrm{V}}}{\partial x}\bigg|_{x=L+a} & (6\mathrm{b}) \end{cases}$$

Equations (3-6) are eight equations for our eight unknowns ($A_{1\mathrm{R}}$, $A_{2\mathrm{R}}$, $A_{1\mathrm{T}}$, $A_{2\mathrm{T}}$, $\alpha_1$, $\alpha_2$, $\beta_1$ and $\beta_2$). First, let us obtain the four unknowns $A_{2\mathrm{R}}$, $A_{2\mathrm{T}}$, $\alpha_2$, $\beta_2$ from eqs.(5) and (6) in the case of opaque barriers, i.e., when $\chi a \to \infty$:

$$\begin{cases}
\alpha_2 \longrightarrow e^{ikL}\,\dfrac{2ik}{ik-\chi} & (7a) \\[2mm]
\beta_2 \longrightarrow e^{ikL-2\chi a}\,\dfrac{-2ik(ik+\chi)}{(ik-\chi)^2} & (7b) \\[2mm]
A_{2R} \longrightarrow e^{2ikL}\,\dfrac{ik+\chi}{ik-\chi} & (7c) \\[2mm]
A_{2T} \longrightarrow e^{-\chi a}e^{-ika}\,\dfrac{-4ik\chi}{(ik-\chi)^2} & (7d)
\end{cases}$$

Then, we may obtain the other four unknowns $A_{1R}$, $A_{1T}$, $\alpha_1$, $\beta_1$ from eqs.(3) and (4), again in the case $\chi a \to \infty$; one gets for instance that:

$$A_{1T} = -e^{-2\chi a}\,\frac{4i\chi k}{(\chi-ik)^2}\,A \tag{8}$$

where

$$A \equiv \frac{2\chi k}{2\chi k\,\cos k(L-a)\,+\,(\chi^2-k^2)\,\sin k(L-a)} \tag{9}$$

results to be *real*; and where, it must be stressed,

$$\delta \equiv \arg\left(\frac{ik+\chi}{ik-\chi}\right)$$

is a quantity that *does not depend* on $a$ or on $L$. This is enough for concluding that the phase tunnelling time (see, for instance, refs.[1-3])

$$\tau^{\text{ph}}{}_{\text{tun}} \equiv \hbar\,\frac{\partial\arg\left[A_{1T}A_{2T}e^{ik(L+a)}\right]}{\partial E} = \hbar\,\frac{\partial}{\partial E}\,\arg\left[\frac{-4ik\chi}{(ik-\chi)^2}\right] \tag{10}$$

*does not depend* on $L+a$ (it being actually independent both of $a$ and of $L$).

This result does *not only* confirm the so-called "Hartman effect"[1,3] for the two single barriers —i.e., the independence of the tunnelling time from the barrier widths—, but it does *also* extend such an effect by implying the total tunnelling time to be independent even of $L$ (see Fig.1)! This can be regarded as a further, impressive evidence of the fact that the quantum systems behave as non-local.

The tunnelling-time independence from the width ($a$) of each one of the two barriers is itself a generalization of the Hartman effect, and can be a priori understood —following refs.[4,5]— on the basis of the reshaping phenomenon which takes place inside a barrier.

4

With regard to the even more interesting tunnelling-time independence from the distance $L - a$ between the two barriers, it can be understood on the basis of the interference *between the waves* outcoming from the first barrier (region II) and traveling in region III *and the waves* reflected from the second barrier (region IV) back into the same region III. Such an interference has been shown[3] to cause an "advancement" (i.e., an effective acceleration) on the incoming waves; a phenomenon quite similar to the *advancement* expected even in region I. Namely, passing to the wavepacket language, we noticed in ref.[3] that the arriving wavepacket does interfere with the reflected waves that start to be generated as soon as the packet forward tail reaches the (first) barrier edge; in such a way that (already before the barrier) the backward tail of the initial wavepacket decreases —for destructive interference with those reflecred waves— at a larger degree than the forward one. This simulates an increase of the average speed of the entering packet: hence, the effective (average) flight-time of the approaching packet from the source to the barrier does decrease.

So, the phenomena of reshaping and "advancement" (inside the barriers and to the left of the barriers) can qualitatively explain why the tunnelling-time is independent of the barrier widths and of the distance between the two barriers. It remains impressive, nevertheless, that in region III —where no potential barrier is present, the current is non-zero and the wavefunction is oscillatory,— the effective speed (or group-velocity) is practically *infinite*. After some straightforward but rather bulky calculations, one can moreover see that the same effects (i.e., the independence from the barrier widths and from the distances between the barriers) are still valid for any number of barriers, with different widths and different distances between them.

Finally, let us mention that the known similarity between photon and (nonrelativistic) particle tunneling[3-7] implies our previous results to hold also for photon tunnelling through successive "barriers": For example, for photons in presence of two successive band gap filters, or two suitable gratings. Experiments should be easily realizable; while indirect experimental evidence seems to come from papers as [8].

At the classical limit, the (stationary) Helmholtz equation for an electromagnetic wavepacket in a waveguide is known to be mathematically identical to the (stationary)

5

Schroedinger equation for a potential barrier;* so that, for instance, the tunnelling of a particle through and under a barrier can be simulated[3-7,9-11] by the traveling of evanescent waves along an undersized waveguide. Therefore, the results of this paper are to be valid also for electromagnetic wave propagation along waveguides with a succession of undersized segments (the "barriers") and of normal-sized segments. This agrees with calculations performed, within the classical realm, directly from Maxwell equations[10,11], and has already been confirmed by a series of "tunnelling" experiments with microwaves: see refs.[9] and particularly [12].

6

---

*These equations are however different (due to the different order of the time derivative) in the time-dependent case. Nevertheless, it can be shown that they still have in common classes of analogous solutions, differing only in their spreading properties[3,7].

## References

[1]  T.E.Hartman: J. Appl. Phys.**33** (1962) 3427;  J.R.Fletcher: J. Phys. C18 (1985) L55.

[2]  E.H.Hauge and J.A.Støvneng: Rev. Mod. Phys. 61 (1989) 917.

[3]  V.S.Olkhovsky and E.Recami: Phys. Reports 214 (1992) 340;  V.S.Olkhovsky, E.Recami, F.Raciti and A.K.Zaichenko: J. Phys.-I (France) 5 (1995) 1351.  See also V.S.Olkhovsky, E.Recami and J.Jakiel: "A new time-analysis of photon and (non-relativistic) particle tunnelling" (to be subm. for pub.);  D.Mugnai et al.: Phys. Lett. A209 (1995) 227.

[4]  R.Y.Chiao, P.G.Kwiat and A.M.Steinberg:  Physica B175 (1991) 257.  See also Th.Martin and R.Landauer: Phys. Rev. A45 (1992) 2611;  A.Ranfagni, D.Mugnai, P.Fabeni and G.P.Pazzi: Appl. Phys. Lett. 58 (1991) 774.

[5]  A.M.Steinberg, P.G.Kwiat and R.Y.Chiao:  Phys. Rev. Lett. 71 (1993) 708;  R.Y.Chiao, P.G.Kwiat and A.M.Steinberg: Scient. Am. 269 (1993), issue no.2, p.38.

[6]  Y.Japha and G.Kurizki: Phys. Rev. A53 (1996) 586;  "Causality and interference in electron and photon tunneling", preprint (Weizmann Inst.Sc.; Rehovot, Jan.1996);  "Causality and non-locality in 'faster-than-light' photon travel", preprint (Weizmann Inst.Sc.; Rehovot, Sept.1996).

[7]  V.S.Olkhovsky and A.Agresti:  in *Tunneling and its Applications* (World Scient.; Singapore, 1997), pp.327-355;  J.Jakiel, V.S.Olkhovsky and E.Recami: Phys. Lett. A248 (1998) 156.

[8]  S.Chu and S.Wong:  Phys. Rev. Lett. 48 (1982) 738;  see also C.G.B.Garret and D.E.McCumber: Phys. Rev. A1 (1970) 305;  L.Wang: submitted to Nature (2000).

[9]  A.Enders and G.Nimtz: J. Phys.-I (France) 3 (1993) 1089;  A.Ranfagni, P.Fabeni, G.P.Pazzi and D.Mugnai: Phys. Rev. E48 (1993) 1453;  Ch.Spielmann, R.Szipocs, A.Stingl and F.Krausz: Phys. Rev. Lett. 73 (1994) 2308,  Ph.Balcou and L.Dutriaux: Phys. Rev. Lett. 78 (1997) 851;  V.Laude and P.Tournois: J. Opt. Soc. Am. B16 (1999) 194.

[10] A.P.L.Barbero, H.E.Hernández F., and E.Recami: "On the propagation speed of evanescent modes" [Lanl Archives # physics/9811001], to appear in Phys. Rev. E. Cf. also E.Recami, H.E.Hernández F., and A.P.L.Barbero: Ann. der Phys. 7 (1998) 764; H.M.Brodowsky, W.Heitmann and G.Nimtz: Phys. Lett. A222 (1996) 125.

[11] M.Zamboni-Rached, E.Recami and F.Fontana: "Localized Superluminal Solutions to Maxwell Equations propagating along a normal-sized waveguide", Lanl Archives # physics/0001039, subm. for pub.; cf. also E.Recami and F.Fontana: "Special Relativity and Superluminal motions", in press in Int. J. Mod. Phys.; E.Recami: in *Causality and Locality in Modern Physics*, ed. by G.Hunter, S.Jeffers and J.-P.Vigier (Kluwer; Dordrecht, 1998), p.113.

[12] G.Nimtz, A.Enders and H.Spieker: J. de Physique-I 4 (1994) 565; and in *Wave and Particle in Light and Matter*, ed. by A.van der Merwe and A.Garuccio (Plenum; New York, 1993); A.Enders and G.Nimtz: Phys. Rev. B47 (1993) 9605.