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Localized Superluminal Solutions to Maxwell Equations Propagating Along a Normal-Size Waveguide[(†)]

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Abstract

We show that localized (non–evanescent) solutions to Maxwell equations exist, which propagate without distortion along normal waveguides with Superluminal speed.

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1. – Introduction: Localized solutions to the wave equations

Already in 1915 Bateman[1] showed that Maxwell equations admit (besides of the ordinary planewave solutions, endowed in vacuum with speed c) of wavelet-type solutions, endowed in vacuum with group-velocities $0 \leq v \leq c$. But Bateman's work went practically unnoticed. Only few authors, as Barut et al.[2], followed such a research line; incidentally, Barut et al. constructed even a wavelet-type solution travelling with Superluminal group-velocity[3] $v > c$.

In recent times, however, many authors discussed the fact that all (homogeneous) wave equations admit solutions with $0 < v < \infty$: see, e.g., Donnelly & Ziolkowski[4], Esposito[4], Vaz & Rodrigues[4]. Most of those authors confined themselves to investigate (sub- or Super-luminal) *localized* non-dispersive solutions in vacuum: namely, those solutions that were called "undistorted progressive waves" by Courant & Hilbert. Among localized solutions, the most interesting appeared to be the so-called "X-shaped" waves, which —predicted to exist even by Special Relativity in its extended version[5]— had been mathematically constructed by Lu & Greenleaf[6] for acoustic waves, and by Ziolkowski et al.[7], and later Recami[8], for electromagnetism.

Let us recall that such "X-shaped" localized solutions are Superluminal (i.e., travel with $v > c$ in the vacuum) in the electromagnetic case; and are "super-sonic" (i.e., travel with a speed larger than the sound-speed in the medium) in the acoustic case. The first authors to produce X-shaped waves *experimentally* were Lu & Greenleaf[9] for acoustics, and Saari et al.[10] for optics.

Notwithstanding all that work, still it is not yet well understood what solutions (let us now confine ourselves to Maxwell equations and to electromagnetic waves) have to enter into the play in many experiments.

2. – About evanescent waves

Most of the experimental results, actually, did not refer to the abovementioned localized, sub- or Super-luminal, solutions, which in vacuum are expected to propagate rigidly (or almost rigidly, when suitably truncated). The experiments most after fashion are, on the contrary, those measuring the group-velocity of *evanescent waves*[cf., e.g., refs.11,12]. In fact, both Quantum Mechanics[13] and Special Relativity[5] had predicted tunnelling wavepackets (tunnelling photons too) and/or evanescent waves to be Superluminal.

For instance, experiments[12] with evanescent waves travelling down an undersized *waveguide* revealed that evanescent modes are endowed with Superluminal group-velocities[14].

A problem arises in connection with the experiment in Fig.1, with two "barriers" 1 and 2 (i.e., segments of *undersized* waveguide). In fact, it has been found that *for suitable frequency bands* the wave coming out from barrier 1 goes on having a practically infinite speed, and crosses the intermediate *normal-sized* waveguide 3 in zero time[15]. Even if this can be theoretically understood by looking at the relevant transfer function (see the computer simulations, based on Maxwell equations only, in refs.[16,17]), it is natural to wonder *what are* the solutions to Maxwell equations that can travel with Superluminal speed in a *normal* waveguide (where one normally meets ordinary propagating —and not evanescent— modes)...

Namely, the dispersion relation in undersized guides is $\omega^2 - k^2 = -\Omega^2$, so that the standard formula $v \simeq \mathrm{d}\omega/\mathrm{d}k$ yields a $v > c$ group-velocity[17,18]. However, in normal guides the dispersion relation becomes $\omega^2 - k^2 = +\Omega^2$, so that the same formula yields values $v < c$ only.

We are going to show that actually localized solutions to Maxwell equations propagating with $v > c$ do exist even in normal waveguides; but their group-velocity v *cannot* be given[#1] by the approximate formula $v \simeq \mathrm{d}\omega/\mathrm{d}k$. One of the main motivations of the present note is just contributing to the clarification of this question.

3. – About some localized solutions to Maxwell equations.

Let us start by considering localized solutions to Maxwell equations in vacuum. A *theorem* by Lu et al.[19] showed how to start from a solution holding in the *plane* (x, y) for constructing a threedimensional solution rigidly moving along the z-axis with Super-

[#1] Let us recall that the group-velocity is well defined only when the pulse has a clear bump in space; but it can be calculated by the approximate, elementary relation $v \simeq \mathrm{d}\omega/\mathrm{d}k$ *only* when some extra conditions are satisfied (namely, when ω as a function of k is also clearly bumped).

luminal velocity v. Namely, let us assume that $\psi(\boldsymbol{\rho};t)$, with $\boldsymbol{\rho} \equiv (x,y)$, is a solution of the 2-dimensional homogeneous wave equation:

$$\left(\partial_x^2 + \partial_y^2 - \tfrac{1}{c^2}\partial_t^2\right) \; \psi(\boldsymbol{\rho};t) \; = \; 0 \; . \tag{1}$$

By applying the *transformation* $\boldsymbol{\rho} \rightarrow \boldsymbol{\rho} \; \sin\theta$; $\quad t \rightarrow t - (\cos\theta/c)\, z$, the angle θ being fixed, with $0 < th < \pi/2$, one gets[19] that $\psi(\boldsymbol{\rho} \; \sin\theta;\; t - (\cos\theta/c)\, z)$ is a solution to the threedimensional homogeneous wave-equation

$$\left(\boldsymbol{\nabla}^2 - \tfrac{1}{c^2}\partial_t^2\right) \; \psi\Big(\boldsymbol{\rho} \; \sin\theta;\; t - \tfrac{\cos\theta}{c}\, z\Big) = 0 \; . \tag{2}$$

The mentioned theorem holds for the vacuum case, and in general is not valid when introducing boundary conditions. However we discovered that, in the case of a bidimensional solution ψ valid on a circular domain of the (x,y) plane, such that $\psi = 0$ for $|\boldsymbol{\rho}| = 0$, the transformation above leads us to a (three-dimensional) *localized* solution rigidly travelling with Superluminal speed $v = c/\cos\theta$ inside a *cylindrical waveguide*; even if the waveguide radius r will be no longer a, but $r = a/\sin\theta > a$. We can therefore obtain an undistorted Superluminal solution propagating down cylindrical (metallic) *waveguides* for each (2-dimensional) solution valid on a circular domain. Let us recall that, as well-known, any solution to the scalar wave equation corresponds to solutions of the (vectorial) Maxwell equations (cf., e.g., ref.[8] and refs. therein).

For simplicity, let us put the origin O at the center of the circular domain $\mathcal{C}$, and choose a 2-dimensional solution that be axially symmetric $\psi(\rho;t)$, with $\rho = |\boldsymbol{\rho}|$, and with the initial conditions $\psi(\rho; t=0) \; = \; \phi(\rho)$, and $\partial\psi/\partial t \; = \; \xi(\rho)$ at $t = 0$.

Notice that, because of the transformations

$$\rho \Longrightarrow \rho \; \sin\theta \tag{3a}$$

$$t \Longrightarrow t \; - \; \frac{\cos\theta}{c}\, z \; , \tag{3b}$$

the more the initial $\psi(\rho;t)$ is localized at $t = 0$, the more the (threedimensional) wave $\psi(\rho \; \sin\theta;\; t - (\cos\theta/c)z$ will be localized around $z = vt$. It should be also emphasized that, because of transformation (3b), the velocity c goes into the velocity $v = c/cos\theta > c$.

Let us start with the formal choice

$$\phi(\rho) \;=\; \frac{\delta(\rho)}{\rho} \; ; \qquad \xi(\rho) \equiv 0 \; . \tag{4}$$

In cylindrical coordinates the wave equation (1) becomes

$$\left(\frac{1}{\rho}\,\partial_\rho \rho \partial_\rho \;-\; \frac{1}{c^2}\,\partial_t^2\right)\; \psi(\rho;t) \;=\; 0 \; , \tag{1'}$$

which exhibits the assumed axial symmetry. Looking for factorized solutions of the type $\psi(\rho;t) \;=\; R(\rho){\cdot}T(t)$, one gets the equations $\partial_t^2 T = -\omega^2 T$ and $(\rho^{-1}\partial_\rho + \partial_\rho^2 + \omega^2/c^2)R = 0$, where the "separation constant" ω is a real parameter, which yield the solutions

$$\begin{aligned} T \;&=\; A\,\cos\omega t \;+\; B\,\sin\omega t \\ R \;&=\; C\;J_0(\frac{\omega}{c}\,\rho) \; , \end{aligned} \tag{5}$$

where quantities A, B, C are real constants, and J_0 is the ordinary zero-order Bessel function (we disregarded the analogous solution $Y_0(\omega\rho/c)$ since it diverges for $\rho = 0$). Finally, by imposing the boundary condition $\psi = 0$ at $\rho = a$, one arrives at the base solutions

$$\psi(\rho;t) \;=\; J_0(\frac{k_n}{a}\rho)\;\; (A_n\,\cos\omega_n t \;+\; B_n\,\sin\omega_n t) \;\; ; \qquad \lambda \equiv \frac{\omega}{c}a \; , \tag{6}$$

the roots of the Bessel function being

$$\lambda_n = \frac{\omega_n a}{c} \; .$$

The general solution for our bidimensional problem (with our boundary conditions) will therefore be the Fourier-type series

$$\Psi_{\rm 2D}(\rho;t) \;=\; \textstyle\sum_{n=1}^{\infty} J_0(\frac{\lambda_n}{a}\rho)\;\; (A_n\,\cos\omega_n t \;+\; B_n\,\sin\omega_n t) \;\; . \tag{7}$$

The initial conditions (4) imply that $\sum A_n J_0(\lambda_n \rho/a) \; = \; \delta(\rho)/\rho$, and $\sum B_n J_0(\lambda_n \rho/a) \; = \; 0$, so that all B_n must vanish, while $A_n \; = \; 2[a^2 J_1^2(\lambda_n)]^{-1}$; and eventually one gets:

$$\Psi_{\rm 2D}(\rho;t) \;=\; \sum_{n=1}^{\infty} \left(\frac{2}{a^2 J_1^2(\lambda_n)} \right) \; J_0(\frac{\lambda_n}{a}\rho) \; \cos\omega_n t \; , \tag{8}$$

where $\omega_n = \lambda_n c/a$.

Let us explicitly notice that we can pass from such a formal solution to more physical ones, just by considering a finite number N of terms. In fact, each partial expansion will satisfy (besides the boundary condition) the second initial condition $\partial_t \psi = 0$ for $t = 0$, while the first initial condition gets the form $\phi(\rho) = f(\rho)$, where $f(\rho)$ will be a (well) localized function, but no longer a delta-type function. Actually, the "localization" of $\phi(\rho)$ increases with increasing N. We shall come back to this point below.

4. – Localized waves propagating Superluminally down (normal-sized) waveguides.

We have now to apply transformations (3) to solution (8), in order to pass to threedimensional waves propagating along a cylindrical (metallic) waveguide with radius $r = a/\sin\theta$. We obtain that Maxwell equations admit in such a case the solutions

$$\Psi_{\rm 3D}(\rho,z;t) \;=\; \sum_{n=1}^{\infty} \left(\frac{2}{a^2 J_1^2(\lambda_n)} \right) \; J_0(\frac{\lambda_n}{a}\rho\sin\theta) \; \cos\left[\frac{\lambda_n\cos\theta}{a} \, (z - \frac{c}{\cos\theta}t) \right] \tag{9}$$

where $\omega_n = \lambda_n c/a$, which are sums over different propagating modes.

Such solutions propagate, down the waveguide, rigidly with Superluminal velocity[#2] $v = c/\cos\theta$. Therefore, (non-evanescent) solutions to Maxwell equations exist, that are waves propagating undistorted along *normal* waveguides with Superluminal speed (even if in normal-sized waveguides the dispersion relation for each mode, i.e. for each term of the Fourier-Bessel expansion, is the ordinary "subluminal" one, $\omega^2/c^2 - k_z^2 = +\Omega^2$).

[#2] Let us stress that each eq.(9) represents a *multimodal* (but *localized*) propagation, as if the geometric dispersion compensated for the multimodal dispersion.

It is interesting that our Superluminal solutions travel rigidly down the waveguide: this is at variance with what happens for truncated (Superluminal) solutions[7-10], which travel almost rigidly only along their finite "field depth" and then abruptly decay.

Finally, let us consider a finite number of terms in eq.(8), at $t = 0$. We made a few numerical evaluations: let us consider the results for $N = 22$ (however, similar results can be already obtained, e.g., for $N = 10$). The first initial condition of eq.(4), then, is no longer a delta function, but results to be the (bumped) bidimensional wave represented in Fig.2.

The threedimensional wave, eq.(9), corresponding to it, i.e., with the same finite number $N = 22$ of terms, is depicted in Fig.3. It is still an exact solution of the wave equation, for a metallic (normal-sized) waveguide with radius $r = a/\sin\theta$, propagating rigidly with Superluminal group-velocity $v = c/\cos\theta$; moreover, it is now a *physical* solution. In Fig.3 one can see its central portion, while in Fig.4 it is shown the space profile along z, for $t =$ const., of such a propagating wave.

5. – A re-derivation of our results from the standard theory of waveguide propagation.

Lu's theorem is certainly a very useful tool to build up localized solutions to Maxwell equations: actually, it can be used to get a variety of solutions, eq.(9) being just the simplest example. Nevertheless, due to the novelty of our previous results, it may be worthwhile to outline an alternative derivation of them which can sound more familiar.

For the sake of simplicity, let us limit ourselves to the domain of TM (transverse magnetic) modes. When a solution in terms of the longitudinal electric component, E_z, is sought, one has to deal with the *simple* boundary condition $E_z = 0$. We shall look, moreover, for axially symmetric solutions (i.e., independent of the azimuthal variable φ). [Such choices could be easily generalized, just at the cost of increasing the mathematical complexity]. Quantity E_z is then completely equivalent to the scalar variable $\Psi \equiv \Psi_{\rm 3D}$ used in the previous analysis.

Let us try to find out solutions of the form

$$E_z(\rho, z; t) \;=\; C\; Q(\rho)\; \exp\left[i\left(\frac{\omega z \cos\theta}{c} - \omega t\right)\right] \tag{10}$$

where $Q(\rho)$ is assumed to be a function of the radial coordinate ρ only, and C is a normalization constant. Here we call c the velocity of light in the medium filling the cylindrical waveguide, supposing it nondispersive. The angular frequency ω is for the moment arbitrary.

By inserting expression (10) into the Maxwell equation for E_z, one obtains

$$\rho^2 \frac{\mathrm{d}^2 Q(\rho)}{\mathrm{d}\rho^2} + \rho \frac{\mathrm{d}Q(\rho)}{\mathrm{d}\rho} + \rho^2 k_0^2\; \sin^2\theta\; Q(\rho) \;=\; 0\,; \qquad k_0 \equiv \frac{\omega}{c}\;,$$

whose only solution, which be finite on the waveguide axis, is $Q(\rho) = J_0(\rho k_0 \sin\theta)$. By imposing the boundary condition $Q(\rho) = 0$ for $\rho = r$, we get that the acceptable angular frequencies are constrained to be

$$\omega_l \;=\; \frac{c\lambda_l}{r\sin\theta} \tag{11}$$

where λ_l is the l-th zero of the equation $J_0(x) = 0$.

Therefore, assuming an arbitrary parameter θ, we find that, for every mode supported by the waveguide and labelled by the index l, there is just *one* frequency at which the assumed dependence (10) on z and t is physically realizable. Let us show such a solution to be the standard one known from classical electrodynamics. In fact, by inserting the allowable frequencies ω_l into the complete expression of the mode, we have:

$$E_z^l(\rho, z; t) \;=\; C\; J_0(\frac{\rho\lambda_l}{r})\; \exp\left[i\left(\frac{\omega_l z \cos\theta}{c} - \omega_l t\right)\right]\;. \tag{12}$$

But the generic solution for (axially symmetric) TM_{0l} modes in a cylindrical metallic waveguide is:[20]

$$E_z^{\mathrm{TM}_{0l}}(\rho, z; t) \;=\; C\; J_0(\frac{\rho\lambda_l}{r})\; \exp\left[i\left(\beta(\omega_l)z - \omega_l t\right)\right]$$

with the dispersion relation $\beta^2(\omega_l) = (\omega_l/c)^2 - (\lambda_l/r)^2$. By identifying $\beta(\omega) \equiv \omega_l \cos\theta/c$, as suggested by eq.(12), and remembering the expression for ω_l given by eq.(11), the ordinary dispersion relationship is gotten. We have therefore verified that every term in the expansion (9) is a solution to Maxwell equations not different from the usual one.

The uncommon feature of our solution (9) is that, given a particular value of θ, the phase-velocity of *all* its terms is always the same, it being independent of the mode index l:

$$v_{\rm ph} = \left[\frac{\beta(\omega_l)}{\omega_l}\right]^{-1} = \frac{c}{\cos\theta} \; .$$

In such a case it is known that the group-velocity of the pulse *equals* the phase-velocity[21]: which in our case is the velocity *tout court* of the localized pulse.

With reference to Fig.5, we can easily see that all the allowed values of ω_l can be calculated by determining the intersections of the various branches of the dispersion relation with a straight line, whose slope depends on θ only. By using suitable combinations of terms, corresponding to different indexes l, as in our eq.(9), it is possible to describe a disturbance having a time-varying profile, as already shown in Figs.2-4 above. The pulse thus displaces itself *rigidly*, with a *velocity* v equal to $v_{\rm ph}$.

It should be repeated that the velocity v (or group-velocity $v_{\rm g} \equiv v$) of the pulses corresponding to eq.(9) is *not* to be evaluated by the ordinary formula $v_{\rm g} \simeq {\rm d}\omega/{\rm d}k$ (valid for quasi-monochromatic signals). This is at variance with the common situation in optical and microwave communications, when the signal is usually an "envelope" superimposed to a carrier wave whose frequency is generally much higher than the signal bandwidth. In *that* case the standard formula for $v_{\rm g}$ yields the correct velocity to deal with (e.g., when propagation delays are studied). Our case, on the contrary, is much more reminiscent of a baseband modulated signal, as those studied in ultrasonics: the very concept of a carrier becomes meaningless here, as the elementary "harmonic" components have widely different frequencies.

Let us finally remark that similar considerations could be extended to all the situations where a waveguide supports several modes. Tests at microwave frequencies should be rather easy to perform; by contrast, experiments in the optical domain would be probably ruled out, at present, by the limited extension of the spectral windows corresponding to not too large attenuations.

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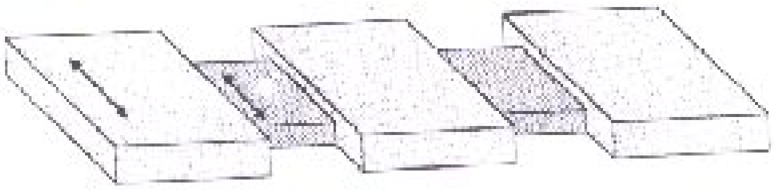

Fig. 1 – The experiment with two “barriers” (i.e., segments of *undersized* waveguide) 1 and 2. It had been found that, for suitable frequency bands, the wave coming out from barrier 1 can go on with practically infinite speed, crossing the intermediate (normal–sized) waveguide 3 in zero time. We show in this paper that localized solutions to the Maxwell equations do actually exist, which travel along a normal waveguide with Superluminal speed.

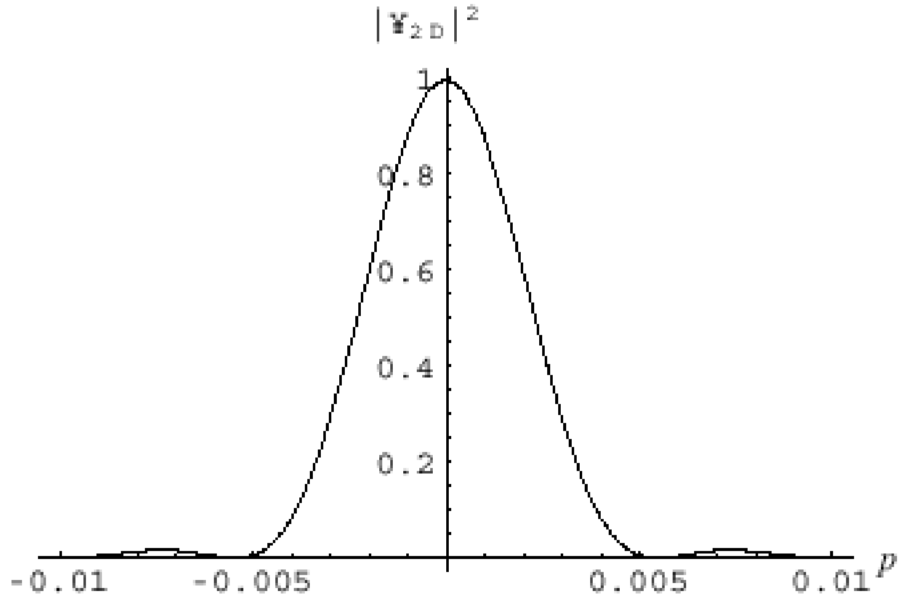

Fig. 2 – Shape of the bidimensional solution of the wave equation valid on the circular domain $\rho \leq a$; a=0.1mm of the (x,y) plane, for t=0, corresponding to the sum of N=22 terms in the expansion (8). It is no longer a delta function, but it is still very well peaked. By choosing it as the initial condition, instead of the first one of eqs. (4), one gets the threedimensional wave depicted in Figs. 3 and 4. The normalization condition is such that $|\Psi_{2D}(p=0; t=0)|^2=1$

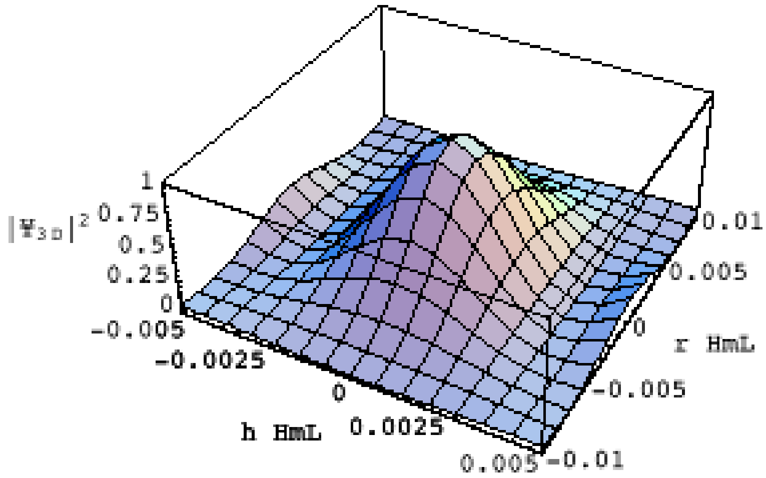

Fig. 3 – The (very well localized) threedimensional wave corresponding to the initial, bidimensional choice in Fig. 2. It propagates rigidly (along the normal–sized circular waeguide with radius r=a/sinθ) with Superluminal speed v=c/cosθ). Quantity η is defined as $\eta \equiv \left(z - \frac{c}{\cos\theta} t \right)$. The Normalization condition is such that $|\Psi_{3D}(\rho=0; \eta=0)|^2=1$.

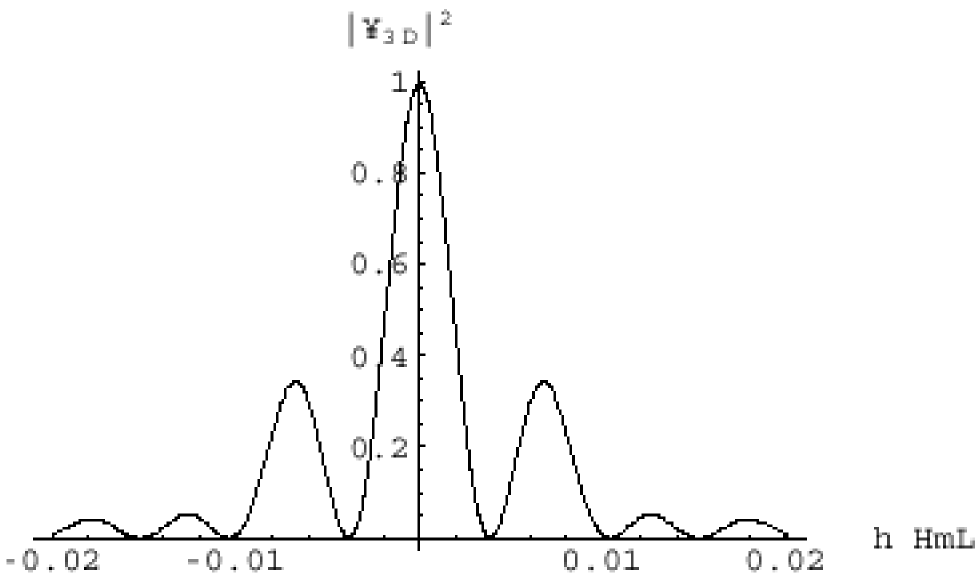

Fig. 4 – The shape along z, at t=0, of the threedimensional wave whose main peak is shown in Fig. 3.

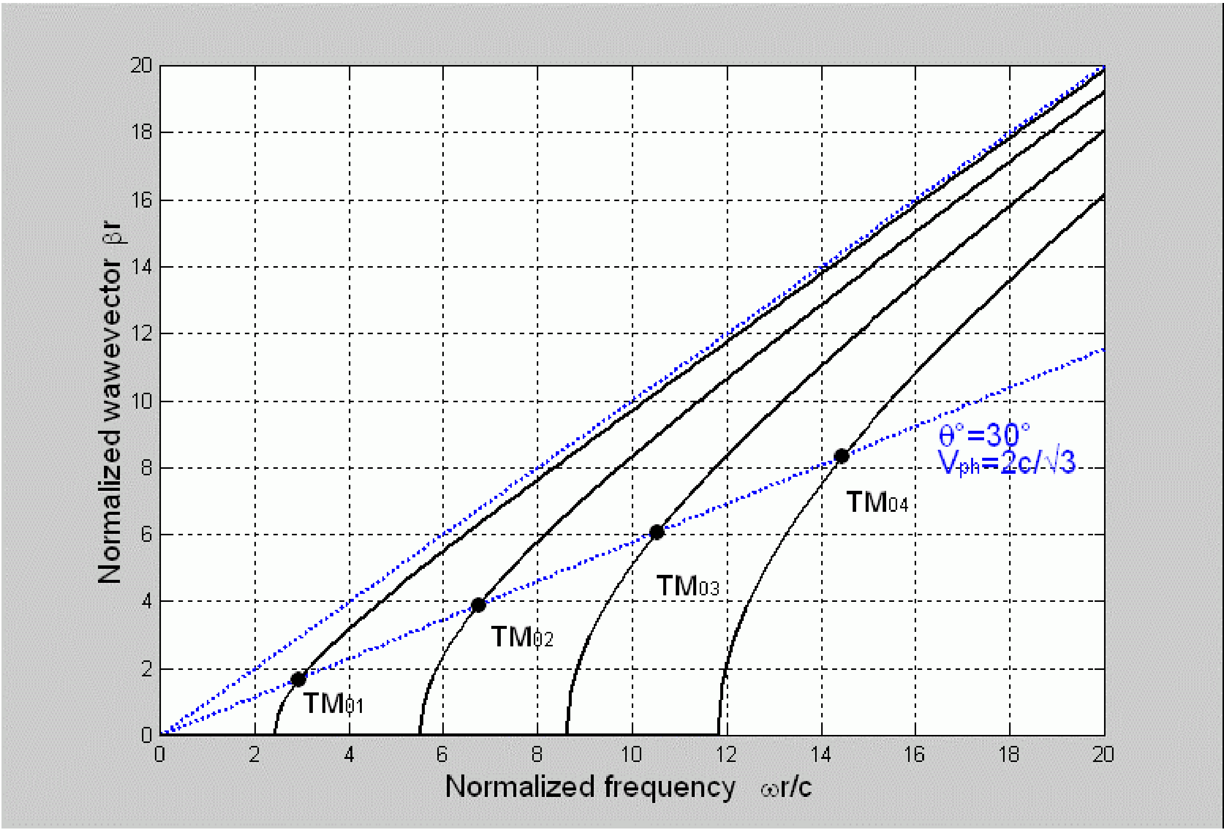

Fig. 5 – Dispersion curves for the symetrical TM_{0l} modes in a perfect cylindrical waveguide, and location of the frequencies whose corresponding modes have equal phase–velocity. [Actually, the phase velocity c/cosθ of all the terms in eq. (9) is always the same, being independent of the mode index l: in such a case it is known that the group–velocity of the pulse equals the phase–velocity: which in our case it the velocity tout court of the localized pulse].