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COMPLEX–BARRIER TUNNELLING TIMES(*)

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ABSTRACT – In this paper we calculate the analytic expression of the *phase time* for the scattering of an electron off a complex square barrier. As is well known the (negative) imaginary part of the potential takes into account, phenomenologically, the absorption. We investigate the Hartman-Fletcher effect, and find that it is suppressed by the presence of a (non negligible) imaginary potential. In fact, when a sufficiently large absorption is present, the asymptotical transmission speed is finite. Actually, the tunnelling time does increase linearly with the barrier width. A recent optical experiment seems to be in agreement with our theoretical expectation.

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1 – INTRODUCTION.

In recent times the longstanding question of the tunnelling times has acquired new urgency because of the recent experimental results claiming for superluminal tunnelling speeds. The problem of defining tunnelling times has a long history, since it arose in the forties and fifties[1],[2],[3] simultaneously with the fundamental problem of introducing *time* as a quantum mechanical observable and, in particular, of a definition (in Quantum Mechanics) of the collision durations. Furthermore, a corpuscular picture of tunnelling is very hard to be realized because of the lack of a direct classical limit for the sub-barrier particle paths and velocities. Nevertheless, various typical definitions for the time spent by a particle in the classically forbidden regions have been proposed[4],[5]; we underline that the differences among them are not only merely formal but, on the contrary, they are a consequence of different views, physical interpretations and experimental expectations. Let us quote, incidentally, the most important definitions of the tunnelling times:

a) the dwell time[4],[5],[6]: i.e., the time spent inside the barrier, averaged over all the incoming particles,with no distinction between transmission and reflection channels;

b) the local "Larmor times"[6]: i.e., the traversal time as measured by the spin precession of the tunnelling particle in a uniform infinitesimal magnetic field;

c) the "complex time approach"[5],[7]: a quantum extension —via path integral averages over the classical paths— of the classical complex time spent by the particle in the scattering process;

d) the "Buttiker-Landauer" times[5],[6],[8]: namely interaction times of the particle with a time modulated barrier;

e) the so called "spatial approaches"[5],[9], based on the probabilistic quantum standard interpretation of the flux densities $J[x,t]$ involved during transmission and reflection;

f) the ordinary "phase times",[1],[2],[4],[5],[10] or group delays: i.e.,the times taken by quasi-monochromatic wave packets to appear on the other side of the barrier, as given by the stationary phase approximation.

Before going on, let us stress the important fact that many of those theoretical pictures do imply the so-called Hartman-Fletcher effect[10], that is to say the surprising occurrence, for sufficiently opaque barriers, of *tunnelling delays independent of the barrier width.* Namely, those delays imply traversal mean group velocities larger than the light speed in vacuum.

Recent optical experiment seem to confirm the exitence of superluminal tunnelling speeds[11]. They can be grouped in 3 main classes:

1) evanescent wave propagation in a low dielectric costant region, separating two regions of higher dielectric costant or, similarly, in optical devices allowing for frustrated total internal reflection;

2) propagation of a gaussian light pulse through an anomalous dispersion medium;

3) evanescent microwaves in a wave guide below cut-off.

Our proposal in this paper is studying the tunnelling time and the Hartman-Fletcher effect in the presence of absorption. To that purpouse we introduce a complex square potential (with a negative immaginary part); such non-real potentials are customary in scattering theory and in nuclear physics (where they are named *optical potentials*). We shall show that the non reality of the effective hamiltonian (related to the existence of other interaction channels) does in general destroy the Hartman-Fletcher effect. Superluminal speeds can be achieved only if the imaginary part of the hamiltonian is chosen sufficiently small. Let us notice that these theoretical predictions are in agreement with the experimental results obtained by Nimtz et al. at Cologne[12] employing evanescent microwaves in absorptive wave guides below cut-off. In this context, the most suitable and natural theoretical approach for the evaluation of the *global* tunnelling times and of the mean tunnelling speeds is perhaps the most "direct" one, the phase time approach already mentioned in f): thus, in the following we are going to adopt such a theoretical approach.

2 – ANALYTIC CALCULATION OF THE PHASE TIME

Following the ordinary procedures employed in refs.[10], the group delay for quasi-monochromatic packets in the stationary phase approximation, is given by:

$$\delta\tau(E) = \hbar \frac{\partial}{\partial E}(\arg A_{\rm T}) \tag{1a}$$

where E is the incoming particle energy, $A_{\rm T}$ is the (complex) transmission amplitude and $\hbar$ is the reduced Planck constant. The global tunnelling time is given, as usual, by the sum of the semiclassical traversal and delay time:=

$$\tau = \tau_c + \delta\tau(E) \tag{1b}$$

In order to find out the analytic expression for $A_{\rm T}$ we have to solve the (stationary) Schroedinger equation with a potential different from zero only in the interval $(0, a)$; namely:

$$V(x) = V_0 - iV_1, \quad x\epsilon(0, a)\,.$$

Let us observe that:

$$\psi_{\rm I} \equiv \psi_{\rm in} + \psi_{\rm R} = e^{ikx} + B_{\rm I}e^{-ikx}$$

and

$$\psi_{\rm III} \equiv \psi_{\rm T} = A_{\rm III}e^{ikx}$$

as in the real potential case (with $k^2 = 2mE/\hbar^2$), while $\psi_{\rm II}$ is obtained by solving the Schroedinger equation in the barrier region:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0 + iV_1)\psi = 0 \tag{3}$$

Thus, we get for $\psi_{\rm II}$ the following expression:

$$\psi_{\rm II} = A_{\rm II}e^{ik_{\rm II}x} + B_{\rm II}e^{-ik_{\rm II}x} \tag{4}$$

where

$$k_{\rm II} \equiv \sqrt{2m(E - V_0 + iV_1)}/\hbar \tag{5a}$$

Let us notice that eq.(5a) implies that $k_{\rm II}$ is a complex quantity:

$$k_{\rm II} \equiv \xi + i\mu \tag{5b}$$

where ξ, μ are real numbers.

In the $E < V_0$ case we get:

$$\xi = (\sqrt{m}/\hbar)\sqrt{\sqrt{(E-V_0)^2 + V_1^2} - (V_0 - E)} \tag{6a}$$

and:

$$\mu = (\sqrt{m}/\hbar)\sqrt{\sqrt{(E-V_0)^2 + V_1^2} + (V_0 - E)} \tag{6b}$$

Imposing the continuity boundary conditions for ψ and its derivative, i.e.:

$$\psi_{\rm I}(0) = \psi_{\rm II}(0)\,; \quad \psi_{\rm II}(a) = \psi_{\rm III}(a)\,; \quad \psi'_{\rm I}(0) = \psi'_{\rm II}(0)\,; \quad \psi'_{\rm II}(a) = \psi'_{\rm III}(a)\,,$$

we get, after some algebra, the expression for $A_{\rm T} \equiv A_{\rm III}$:

$$A_{\rm T} = \frac{4kk_{\rm II}e^{ik_{\rm II}a}e^{-ika}}{k^2 + k_{\rm II}^2(1 - e^{2ik_{\rm II}a}) + 2kk_{\rm II}(1 + e^{2ik_{\rm II}a})} \tag{7a}$$

Since we are interested in determining $\phi \equiv \arg A_{\rm T}$ let us express $A_{\rm T}^{-1}$ in agebraic form, e.g.:

$$A_{\rm T}^{-1} = \frac{Ax + By + C\omega}{2kk_{\rm II}k_{\rm II}^*} + i\frac{Cr - Ax + By}{2kk_{\rm II}k_{\rm II}^*} \tag{7b}$$

where:

$$A \equiv \xi(k^2 + k_{\rm II}k_{\rm II}^*) \tag{8a}$$
$$B \equiv \mu(k_{\rm II}k_{\rm II}^* - k^2) \tag{8b}$$
$$C \equiv 2kk_{\rm II}k_{\rm II}^* \tag{8c}$$
$$x \equiv \sin\xi a \, \cosh\mu a \tag{8d}$$
$$y \equiv \cos\xi a \, \sinh\mu a \tag{8e}$$
$$\omega \equiv \cos\xi a \, \cosh\mu a \tag{8f}$$
$$r \equiv -\sin\xi a \, \sinh\mu a \tag{8g}$$

Thus we may write:

$$\phi \equiv \arg A_{\rm T} = \arctan\left(\frac{-Cr + Ax - By}{C\omega + Ay + By}\right) - ka. \tag{9a}$$

After some manipulations, quantity ϕ can be written as follows:

$$\phi = \arctan(\frac{\tanh \mu a(C - B\cot \xi a) + A}{\cot \xi a(C + A\tanh \mu a) + B}) - ka. \tag{9b}$$

According to eq. (1b) the tunnelling time is:

$$\tau(E) = \frac{a}{v} + \hbar \frac{\partial}{\partial E}(\arg A_{\mathrm{T}}) = \hbar \frac{\partial}{\partial E}(\arg A_{\mathrm{T}} + ka) \tag{10}$$

where $v = \hbar k/m$.

By inserting eq. (9b) into eq. (10) we get, after some elaborations (see the Appendix), the rather complicate expression:

$$\tau = \frac{n}{d} \tag{11}$$

where

$$n \equiv \sin 2\xi a[(-am\mu/\hbar^2\rho^2)(C^2 - B^2 - A^2) + (A'C - AC')] \; + \; \cos 2\xi a[2BCam\mu/(\hbar^2\rho^2)] + \\ + \sinh 2\mu a[(am\xi/\hbar^2\rho^2)(A^2 + B^2 + C^2) + (BC' - B'C)] \; + \; (2am\xi AC/\hbar^2\rho^2)\cosh 2\mu a + \\ + 2(A'B - AB')(\sin^2 \xi a + \sinh^2 \mu a) \tag{11a}$$

and

$$d \equiv 2(A^2 + B^2)(\sin^2 \xi a + \sinh^2 \mu a) + 2AC \sinh 2\mu a + 2BC \sin 2\xi a + 2C^2(\cos^2 \xi a + \sinh^2 \mu a). \tag{11b}$$

Since we want to check the occurrence of the Hartman-Fletcher effect, we are interested in studying the opaque barrier limit. That is, we are interested in the asymptotic condition:

$$ak_{\mathrm{II}}k^*_{\mathrm{II}} >> 1 \tag{12}$$

which yields:

$$\tau^{\mathrm{asy}} = \frac{m\xi a}{\hbar(\xi^2 + \mu^2)} \tag{13}$$

that is, τ^{asy} is directly proportional to the barrier width. In other words the mean tunnel speed v_l is asymptotically :

$$v_l \equiv \frac{a}{\tau^{\rm asy}} = \frac{\hbar(\xi^2+\mu^2)}{m\xi}. \tag{14}$$

Thus, we do not obtain the Hartman-Fletcher effect and the saturation of the transmission times anylonger, but a limiting speed as it is shown in eq. (14). Nevertheless, as we can deduce from (14), for sufficiently small values of ξ,i.e., in the case of low absorption, we get:

$$v_l \longrightarrow \infty$$

so that superluminal tunnelling velocities are not *a priori* forbidden.

4 – Appendix

Our starting point will be the analytical expression for ϕ, already written in (9b). Thus, we can calculate the global tunnelling time (10) and obtain that in the general case:

$$\frac{\partial}{\partial E}(\phi + ka) = \frac{n}{d}$$

where

$$n \equiv (1/\cosh^2\xi a)\{\sin\xi a\cos\mu a[a\mu'(C^2-B^2-A^2)+(A'C-AC')] + a\mu' BC(\sin^2\xi a-\cos^2\xi a)\} +$$
$$+ \tanh\mu a[a\xi'(A^2+B^2+C^2)+(BC'-B'C)] +$$
$$+ a\xi' AC(1+\tanh^2\mu a) + (A'B-AB')(\sin^2\xi a+\tanh^2\mu a\cos^2\mu a)$$

and

$$d \equiv (A^2+B^2)(\sin^2\xi a+\tanh^2\mu a\cos^2\xi a) + 2AC\tanh\mu a + C^2(\cos^2\xi a+\sin^2\xi a\tanh^2\mu a) +$$
$$+ \frac{BC\sin 2\xi a}{\cosh^2\mu a}\,,$$

symbol $'$ meaning derivation with respect to the energy.

At this point, we can check our complicated formula in the special case $V_1 = 0$. Let us observe that the assumption $V_1 = 0$ implies:

$$\xi = 0,\quad \xi' = 0,\quad \mu' = -m/\hbar^2(1/\mu)\ ;\quad k' = m/\hbar^2(1/k)\ ;$$

and also:

$$A = 0\ ;\quad A' = 0\ ;\quad B = \mu^3 - \mu k^2\ ;\quad B' = 3\mu^2\mu' - \mu' k^2 - 2\mu k k'\ ;$$

$$C = 2k\mu^2 \quad C' = 2\mu^2 k' + 4\mu\mu' k \ .$$

After some manipulations, we get the expression for the phase time in the particular case $V_1 = 0$:

$$\frac{\partial}{\partial E}(\phi + ka)_{V_1=0} = \frac{(am/\hbar^2)2k^2\mu(\mu^2 - k^2) + (m/\hbar^2)(2mV_0/\hbar^2)^2)\sinh 2\mu a}{k\mu[(2m/\hbar^2)V_0^2 \sinh \mu a^2 + 4k^2\mu^2} \ .$$

If we define $(2mV_0/\hbar^2)^2 \equiv k_0^2$ and $D \equiv [(2m/\hbar^2)V_0^2 \sinh \mu a^2 + 4k^2\mu^2]$, we realize that our expression is identical with eq.(12a) of ref.[5]. After this check, we can now go back to the general expression (11) of the tunnelling time which —after further algebra— can be written:

$$\tau = \frac{n}{d}$$

where now

$$n \equiv \sin 2\xi a[(-am\mu/\hbar^2\rho^2)(C^2 - B^2 - A^2) + (A'C - AC')] + \cos 2\xi a[2BCam\mu/(\hbar^2\rho^2)] +$$
$$+ \sinh 2\mu a[(am\xi/\hbar^2\rho^2)(A^2 + B^2 + C^2) + (BC' - B'C)] + (2am\xi AC/\hbar^2\rho^2)\cosh 2\mu a +$$
$$+ 2(A'B - AB')(\sin^2 \xi a + \sinh^2 \mu a)$$

and

$$d \equiv 2(A^2 + B^2)(\sin^2 \xi a + \sinh^2 \mu a) + 2AC \sinh 2\mu a + 2BC \sin 2\xi a +$$
$$+ 2C^2(\cos^2 \xi a + \sinh^2 \mu a) \ .$$

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