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USING A PENNING TRAP TO WEIGH AN ANTIPROTON

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ABSTRACT

In this paper a new method for measuring the Earth gravitational acceleration g on antiprotons is proposed. The g value is obtained by measuring the gravity induced shift of the center of the radial orbits of antiprotons stored in a Penning trap. This shift is a measurable effect for very low energy ($\simeq \mu eV$) particles injected near the axis of a big Penning trap having the magnetic field perpendicular to the g direction. A possible experimental setup is described and many sources of errors are analyzed. A comparison with the time of flight method, originally proposed to measure g on antiprotons (and now under experimental investigation by an international collaboration), indicates some advantage of the present proposal. The experimental feasibility of this new measure method relies in the practical limit that it will be possibile to achieve on the minimum antiproton energy and in the control of the radial particle coordinates before the injection in the g sensitive trap.

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1. Introduction

The equivalence principle is a cornerstone of general relativity, and more generally, of all metric theories of gravity. Different versions of this principle, restricting their applications to different classes of phenomena, are presently considered [1]. The weak equivalence principle (WEP) applies to mechanical quantities, whereas the Einstein equivalence principle includes WEP and all non-gravitational phenomena.

Tests of WEP began with experiments of Galileo, achieved a significant level of precision with Eotvos experiments [2] and recently showned that different macroscopic bodies fall in a gravitational field with the same acceleration within one part in 10^{11} [3].

Evidence that matter in the form of elementary particles still obeys WEP is far less conclusive. Although Eotvos type experiments strongly suggest that neutrons, protons and electrons obey WEP with a precision of one part in 10^{8} [4], very few direct tests (with quite low precision) have been performed with massive elementary particles [5],[6]. Moreover the gravitational behaviour of antimatter has never been experimentally investigated: WEP has never been tested directly with stable and massive antiparticles like positrons or antiprotons. A measurement of the gravitational acceleration of antimatter could provide further evidence for WEP and useful informations for contemporary research in fundamental physics. Relevant issues include the low energy behaviour of gravity in unified field theories and open problems related to matter-antimatter asymmetry in the universe (a broad scenario is outlined in [7]).

In this paper we describe a new method for measuring the acceleration g of antiprotons subjected to the gravitational field of the Earth. The experimental techniques which have been proposed to measure the gravitational force on antiprotons (or on charged particles) are essentially of two types. One is known as TOF (time of flight technique) and has already been employed in order to measure the value of the gravitational force acting on electrons [6]. The value of g is obtained by measuring the distribution of the transit times of particles which drift along a vertical tube where all the non gravitational forces can be neglected. In order to reach the top of the tube, where they can be detected, entering particles must have a minimum velocity. Therefore the time distribution for upward launching will show a cutoff time t_c given by

$$t_c = \sqrt{\frac{2L}{g}} \tag{1}$$

where L is the length of the drift tube. The value of g is calculated from the above relation after t_c has been obtained by fitting the measured time distribution near the point of cutoff. This method has also been proposed in order to measure the gravitational force on antiprotons (particles will be extracted from the LEAR machine at CERN) [8] and the experimental research on this subject is now actively pursued [9].

The other approach takes advantage of the fact that the combined action of the gravitational field and a steady transverse magnetic field produces a drift motion of the centre of the cyclotron orbits of the charged particles which is easily related to the magnitude of the gravitational force. An application of this principle to the determination of g for antiprotons had originally been proposed by some of the authors and it is briefly described in a letter adressed to the PSSC committee at CERN [10]. A description of a modified experimental research on this subject is now actively pursued [9].

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The present article is a study of the experimental feasibility of a measurement of the g of antiprotons by following the second of this principles and implementing it inside a Penning trap.

2. First principles of the method

The easiest way to describe the principle of this method is to consider an idealized situation in which a particle of mass m and charge q is moving under the influence of the force of gravity and the force produced by a uniform magnetic field $\vec{B}_g$ directed along the z axis and perpendicular to $\vec{g}$. The resulting motion of the centre of the cyclotron oscillations is the superposition of a uniform rectilinear motion along z and of a drift motion along the direction of $\vec{g} \wedge \vec{B}_g$ with constant speed v_d given by

$$v_d = \frac{mg}{qB_g} \tag{2}$$

As a consequence, a measurement of the displacement Δ of the centre of the cyclotron orbits

$$\Delta = \frac{mg}{qB_g}\Delta t \tag{3}$$

in a given interval of time Δt, can provide a value for g. Unfortunately the value of v_d is so small that even for a measurement at the level of a few % one needs values of Δt which correspond to drift tubes that are too long to be considered in practice. A possibile solution to this problem could be to confine the motion of particles along the z direction, by using 'the walls' produced by an electrostatic potential superimposed to the magnetic field. However Maxwell equations prescribes that a confining electrostatic field $\in_z$ along z has also radial components $\in_x$, $\in_y$ (whose existance has been ignored in [11]). The effect of these components cannot be neglected because they cause an additional drift motion of the centre of the cyclotron oscillations. With the simplest choice one can take

$$\in_z = -\frac{k}{q}z \tag{4}$$

Then for the radial components one gets

$$\in_x = \frac{k}{2q}x \tag{5}$$

$$\in_y = \frac{k}{2q}y \tag{6}$$

This fields configuration, that is a constant magnetic field along the z axis and the electric field described by (4) (5) (6), is exactly the field configuration used in a Penning trap (see fig. 1). This is a well known device largely used to confine charged particles [12] [13].

The motion of a charged particle stored in a Penning trap is fully discussed in literature [12]; in the present paper we will focus our attention on the possibility of using such a device to weigh charged particles, expecially antiprotons. The motion of a charged particle stored in a Penning trap and influenced by the gravity force is easily found to be very similar to the motion without gravity. The axial and radial motion are fully decoupled and the gravity force does not influence the axial motion. This is an harmonic motion with frequency Ω_z

$$\Omega_z = \sqrt{\frac{k}{m}} = \sqrt{\frac{2qV_g}{mZ_0^2}} \tag{7}$$

where Z_0 is a geometrical trap dimension defined in fig. 1 and V_g is the depth of the axial potential well depending in a simple way on the potentials applied to the trap electrodes.

The gravity force only affects the radial motion. This is still described, as in a Penning trap where the gravity effect is ignored, by the superposition ot two oscillations - the cyclotron motion with frequency Ω_c and the slower magnetron motion with frequency Ω_m - but the relevant result is that the gravity force causes a shift of the centre (equilibrium point) of these radial oscillations with respect to the geometrical centre of the trap. The new equilibrium position is a point C where the gravity force is balanced by the electrical force. Referring to fig. 1 C has coordinates $(C_g, 0, 0)$ where

$$C_g = 2\frac{mg}{k} \tag{8}$$

The particle radial motion $X(t)$ and $Y(t)$ is described by the relation

$$X(t) + iY(t) = (X_m + iY_m)e^{-i\Omega_m t} + (X_c + iY_c)e^{-i\Omega_c t} + C_g \tag{9}$$

where

$$\Omega_m = \frac{\Omega_z^2}{2\Omega_c} \tag{10}$$

$$\Omega_c = \frac{qB_g}{m} - \Omega_m \tag{11}$$

The quantity R_m and R_c

$$R_{m,c} = \sqrt{X_{m,c}^2 + Y_{m,c}^2} \tag{12}$$

are called magnetron and cyclotron radius and their values are related to the particle initial position x_0 and y_0 and velocity v_{x0} and v_{y0}

$$R_m = \sqrt{\frac{(v_{y0} + \Omega_c x_0 - \Omega_c C_g)^2 + (\Omega_c y_0 - v_{x0})^2}{(\Omega_c - \Omega_m)^2}} \tag{13}$$

$$R_c = \sqrt{\frac{(v_{y0} + \Omega_m x_0 - \Omega_m C_g)^2 + (\Omega_m y_0 - v_{x0})^2}{(\Omega_c - \Omega_m)^2}} \tag{14}$$

The description of the radial motion of a particle stored in a Penning trap and influenced by the gravity force perpendicular to the magnetic field, can be summarized saying that the guiding centre of the cyclotron motion describes a circle of radius R_m centered around $C = (C_g, 0, 0)$. Fig 2 clearly shows how the radii R_m and R_c are related to the initial conditions of a particle that, before beeing captured into a Penning trap, travels along the z axis in the same magnetic field which is present inside the trap. Before entering the trap (left side of fig. 2) the particle cyclotron motion has radius $\tilde{r}_c$ and is centered aroun the point A whose radial coordinates are (a_x, a_y). The right side of fig. 2 shows the radial particle orbit after the capture into the trap. The two geometrical parameters R_m and R_c indicated in fig. 2 are given by the following formulas

$$R_c \simeq \frac{m\sqrt{v_{x0}^2 + v_{y0}^2}}{qB} = \tilde{r}_c \tag{15}$$

$$R_m^2 = (a_x - C_g)^2 + a_y^2 \tag{16}$$

that are obtained from (13) and (14) using

$$x_0 = a_x + r_c cos(\tilde{\theta}_c) \tag{17}$$

$$y_0 = a_x - r_c sin(\tilde{\theta}_c) \tag{18}$$

$$v_{x0} = -\frac{qB_g}{m} r_c sin(\tilde{\theta}_c) \tag{19}$$

$$v_{y0} = -\frac{qB_g}{m} r_c cos(\tilde{\theta}_c) \tag{20}$$

where $\tilde{\theta}_c$ is a proper phase and by recalling that the values of the magnetic field and the electric potentials can be properly chosen so that

$$\Omega_m << \Omega_c \tag{21}$$

The effect of the force of gravity is completely negligible at typical experimental conditions for Penning traps ($Z_0 \simeq cm$, $V_0 \simeq$ some $Volts$). However a Penning trap can allow to weigh an antiproton if the well depth gets close to the value of the difference in gravitational potential energy and if the trap dimensions are large enough. Explicitly, by using (7) and (8), one finds

$$C_g = \frac{mgZ_0^2}{qV_g} \tag{22}$$

that is

$$C_g = 10^{-1}\frac{Z_0^2(m^2)}{V_g(\mu V)} \quad (m) \tag{23}$$

A measurement of g can therefore be obtained from that of C_g. This in turn can be obtained measuring the radial coordinates of many trapped particles (by extracting the particles along the z axis of the trap and sending them to impinge on a position sensitive detector) in two different conditions, namely

- after a vey short time following the capture into the trap;
- after a time T_g equal to one half of the magnetron period $T_m = (2\pi)/\Omega_m$ after the capture into the trap (other choices of the T_g value are possible)

The mean value of the first group of measurement should give $< X >= 0$ and $< Y >= 0$ and then define the position of the point O' in fig. 2. The standard deviations around that values measure the fluctuations due to the unknown values of the variables a_x, a_y and $\tilde{r}_c$ of different particles. The mean value of the results of the second group of measurements define a point placed at the distance equal to $2 < R_m >= 2C_g$ from O'. These data should fluctuate around the mean value with the same standard deviation of the first group. In this way the value of C_g is measured together with its statistical error is measured.

3. Experimental set-up

Last paragraph showned that the g measurement, with the method that we are proposing, requires to inject very low energy ($\simeq \mu eV$) antiprotons as near as possible to the axis of a big ($Z_0 \simeq 1 \quad m$) Penning trap and then, after a proper time is elapsed, to extract them and to measure their radial coordinates.

A possible schematic rapresentation of the experimental apparatus is shown in fig 3. Three different traps are used:

- the first one, that will be called the storage trap, is used to store a large number of antiprotons with thermal energy of 4.2 K ($\simeq 350\ \mu eV$);

- the second trap, that will be called the cooling trap, is aimed to repeatedly cool down to energies of a few μeV, very small bunches of antiprotons extracted from the storage trap;
- the last trap (having bigger dimensions) is used to perform the gravity measurement and we will call it g trap.

Recent experiments at CERN have demonstrated the possibility to capture 10^4 - 10^6 antiprotons in a Penning trap after the particles have been extracted from LEAR [9] [14]. Antiprotons have subsequently been cooled down to 4.2 K by using electron and resistive cooling. The storage trap of our apparatus should perform in such a fashion.

Due to space charge effects, the volume occupied by a big number of particles in this trap would be relatively high. To reduce the dimension of the radial orbits of the particles before injecting them in the g trap, a few particles at a time will be transferred from the storage to the cooling trap. The goal is to perform in this second trap the following operations:

a) reduce the number of trapped particles to one by expelling the rest;
b) reduce the magnetron radius of the remaining particle to the minimum possible value;
c) reduce the particle energy to the optimum value for injection in the g trap.

Techniques utilized in order to perform operations a) and b) are very well known and have already been employed with success. To detect and to measure the number of particles present in the cooling trap one can proceed as described in [15]. In order to decrease the dimensions of the magnetron orbits it is possible to use the sideband cooling technique, which has been studied and has already been employed in several experiments [12]. This technique can be used in particular to couple the cyclotron motion with the magnetron motion while keeping the cyclotron motion at low temperature: the final equilibrium condition should be

$$r_c = r_m \tag{24}$$

where r_c and r_m are the cyclotron and the magnetron radii in the cooling trap. Therefore if the limiting energy E_l of the cyclotron motion is 4.2 K, the values of the radii at equilibrium should be

$$r_c \simeq r_m \simeq \sqrt{2}\sqrt{\frac{m}{q}}\frac{\sqrt{E_l(eV)}}{B_c} \tag{25}$$

where B_c is the magnetic field in the cooling trap. For values of B_c of the order of a few *Teslas* one obtains radii corresponding to fractions of 1 μm.

To achieve goal c), that is further reduction of the antiprotons energy to values of the order $\simeq 1\mu eV$, we plan to use adiabatic cooling by decreasing both the value of the confining potential of the cooling trap V_i and of the magnetic field B_c. If V_i is reduced adiabatically to the final value V_f, the initial axial energy E_{zi} decreases to E_{zf}, where

$$E_{zf} = E_{zi}\sqrt{\frac{V_f}{V_i}} \tag{26}$$

For energies in the range relevant to the experiment, the confining condition

$$E_{zf} < qV_f \tag{27}$$

is always fulfilled by starting with V_i at several hundreds $Volts$ and E_{zi} at about 350 μeV.

If the magnetic field B_c is decreased adiabatically, the action integrals corresponding to the radial motion are conserved. Their values are E_c/ω_c and E_m/ω_m where E_c and E_m are the energies of the cyclotron and magnetron motion respectively and ω_m and ω_c are the magnetron and cyclotron frequencies in the cooling trap. Therefore the radial energy of the particle decreases almost linearly when the value of the magnetic field goes from B_c to the final value B_g. The cyclotron and magnetron radii increase according to the relations

$$r_{mf} = r_{mi}\sqrt{\frac{B_c}{B_g}} \tag{28}$$

$$r_{cf} = r_{ci}\sqrt{\frac{B_c}{B_g}} \tag{29}$$

where $r_{mf,cf}$ and $r_{mi,ci}$ are the values of the radii when the magnetic field is B_g and B_c respectively. Once the adiabatic cooling is completed, the magnetic field in the second trap will be the same as in the g trap.

It is to be noted that reducing the value of the magnetic field increases the uncertainty of the measurement (see (15) and (16)), however this adjustment is necessary in order to properly set the length of the period T_m of the magnetron motion in the final trap and therefore the value of the measuring time T_g. T_m is directly proportional to the value of the magnetic field as can be seen by considering (7),(10),(11) and (21)

$$T_m = \frac{2\pi}{\Omega_m} \simeq 2\pi \ 10^6 \ \frac{B(Tesla) \ Z_0^2(m^2)}{V_b(\mu V)} \ (sec) \tag{30}$$

An upper bound for the period of time available to perform the measurement is set by the typical frequency at which significantly perturbing events can occur. Major disturbances are in principle connected to processes which can alter the particle trajectory, increase its energy or cause its loss. The confining period is basically limited by the processes of annihilations of the antiprotons with the molecules of the residual gas. However experimental data reported in [14] show that antiprotons of energy corresponding to 4.2 K can remain confined for several months. More serious problems can be caused by processes which can increase the energy of the antiprotons: in particular, an increase of the radial energy corresponds to an increase of the cyclotron radius and consequently tends to decrease the precision of the measurements; furthermore, heating of the axial motion can cause the loss of the particle.

Elastic collisions with the molecules of the residual gas (its temperature is 4.2 K for a cryogenic apparatus) constitute the major contribution to the heating of the antiprotons (other processes will be discussed in the following paragraph). If an elastic collision with the residual gas changes the antiproton energy by an amount ΔE, the conditions for particle confinement read

$$E_{zg} + \Delta E < qV_g \qquad (31)$$

where $E_{zg} \simeq E_{zf}$ is the particle axial energy in the g trap (see relation (26)) and

$$\Delta E << E_{cg} \qquad (32)$$

where E_{cg} is the cyclotron energy in the g trap.

The mean time τ between two successive collisions with Helium atoms when the exchange energy is ΔE has been evaluated in [16]. For an apparatus at 4.2 K we have

$$\tau = 168 \ \frac{10^{-12}}{P(torr)} \Delta E(\mu eV)^{1/4} \ \ sec \qquad (33)$$

The condition that has to be fulfilled is therefore

$$\tau \gg T_g \qquad (34)$$

with ΔE satisfying both (31) and (32). From eq. (33),(34) and (30) it can be inferred that the magnetic field in the g trap cannot be the same as in the storage trap (in order to achieve an efficient electron cooling, the value of the magnetic field in the storage trap has to be approximately 6 *Tesla*). Feasible experimental conditions can be obtained if the value of the magnetic field in the g trap is of the order of a few *gauss*. The resulting magnetron periods are of the order of hundreds of seconds and are therefore compatible with condition (34) when the temperature is 4.2 K.

A possible set of working conditions that in our opinion could be used to perform the measurement appear in table 1 (see also next paragraph). Real practical values will depend mainly on the experimental limits that will be possible to achieve on V_g and T_g.

The statistical error $\left(\frac{\Delta g}{g}\right)_s$ which affects the measurement of g is related to the statistical error of C_g, therefore it is largely dominated by the uncertainty in the initial conditions of the particles at the instant of injection into the g trap. The measurement of the radial position of N particles yields

$$\left(\frac{\Delta g}{g}\right)_s = \left(\frac{\Delta C_g}{C_g}\right)_s = \frac{(r_{mf} + r_{cf})}{\sqrt{N}} \frac{q}{m} \frac{V_b}{g Z_0^2} \qquad (35)$$

where r_{mf} and r_{cf} defined by (28) (29) are related to a_y, a_y and $\tilde{r}_c$ previously introduced by

$$r_{cf} \simeq \tilde{r}_c \qquad (36)$$

$$r_{mf} \simeq \sqrt{a_x^2 + a_y^2} \tag{37}$$

Relations (36) and (37) obviously hold assuming ideal conditions of transfer from the cooling trap to the g trap. By using the data of table 1 we get

$$\left(\frac{\Delta g}{g}\right)_s \simeq \frac{3\%}{\sqrt{N}} \tag{38}$$

This result shows that the method we propose is very sensitive and allows, in principle, to perform a 1% measurement by using a very small number of particles (in order to reach the same level of precision by using the TOF technique one needs to accumulate a much higher statistics: 10^6 particles taken from a thermal distribution with a temperature $T = 4.2\ K$). The final precision of the g measurement will be largely dominated by systematic errors. As we shall see in the next paragraph some of them, in the actual experimental conditions, should alter the measure by amount comparable with C_g. It is the unavoidable to perform a differential measure calibrating the experimental apparatus with H^- ions. The following paragraph will be devoted to the discussion of many effects that are the main sources of errors. Other possible sources of errors, including the details of the tranfer processes from trap to trap and the apparatus devoted to the position detection, will not be discussed.

4. Sources of errors and disturbances for the measurement

In this paragraph we will discuss what we expect to be
1) the main problems in obtaining antiprotons with the projected energy and orbit dimensions;
2) the main factors which can affect the final uncertainty of the measurement if the requirements of point 1) are fulfilled.

We will often refer to [16], where an analysis of the sources of error for a gravity measurement on antiprotons with the TOF technique is presented. Several considerations therein expressed are also pertinent in this context.

The sources of errors and disturbances that will be considered here (the interaction of the antiprotons with the residual gas has already been discussed) are the following:
- stray electric fields due to the patch effect;
- vertical electric fields (in the g trap) induced by thermal gradients and gravity;
- forces resulting from the image charge;
- precision of the electric and magnetic fields in the measuring trap.
- any process which can heat up the particle and push it back to 4.2 K.

4.1 Patch effect

Among the forces which can affect the final result of the measurement, one of the most important are the stray electric fields due to the patch effect. Such an electric field is present in the vicinity of any conductor and is due to the potential gradient which results from the spatial variation of the work function along its surface. This variation may be due to the random orientation of the crystal domains of the surface. The average value of this electric field is zero but fluctuations can be detected on a small scale. In particular, along the axis of a cylinder of radius R_0, the rms value of the radial field component is ϵ_{pr}^{rms} [17]

$$\epsilon_{pr}^{rms} = 0.2\frac{w}{R_0^2}\Phi^{rms} \tag{39}$$

while the fluctuactions ϵ_{pz}^{rms} of the axial component are described by

$$\epsilon_{pz}^{rms} = 0.4\frac{w}{a^2}\Phi^{rms} \tag{40}$$

In the above relations w is the typical dimension of the patches and Φ^{rms} describes the fluctuations of the potential along the axis.

The patch effect is a major source of disturbance for both the cooling and the measurement procedure. In the g trap a radial component of the patch electric field introduces a force which superimposes on the force of gravity. If the particles move in a region which is close to the axis of the trap we can assume that the field ϵ_{pr} is approximately constant and that its effect is to cause a displacement C_p of the centre of the radial orbits which adds up to the displacement produced by the force of gravity. The ratio between these displacements is

$$\frac{C_g}{C_p} = \frac{mg}{q\,\epsilon_{pr}^{rms}} = 5\ \ 10^{-5}\frac{R_0^2(m^2)}{w\Phi^{rms}(cm\ \ V)} \tag{41}$$

From the above relation it can be noted that with large values of R_0 ($R_0 \simeq 1m$) a relative precision of the order of 1 can be obtained using the lowest value which have ever been measured for $w\Phi^{rms}$ ($w\Phi^{rms} < 10^{-4}$ $cm\ Volts$ [18].) At this time reasearch is in progress to investigate coating techniques which are expected to reduce the value of $w\Phi^{rms}$ down to $5 - 10\ 10^{-8}$ $Volts\ cm$ [19] [20]. The achievement of such goal would allow to reach a precision better than 10^{-3}.

In the cooling trap the mean value of the electric field due to patch effect is much higher than the mean field in the g trap since the dimensions of the cooling trap are of the order of 1 cm (dimensions cannot be increased for the resistive cooling to be be effective).

The particle trajectory in the g trap is influenced by the patch effect in the cooling trap because stray electric fields produce random displacements of the centre of the radial oscillations in the cooling trap, which in turn reflect as indeterminacies in the position of the antiprotons at the moment they enter the g trap. As a consequence, a systematic error in the determination of C_g is introduced as described in the following. If δ_{px} and δ_{py} represent the displacement of the centre of the radial oscillations in the cooling trap after the adiabatic cooling process we have

$$\delta_{px} \simeq \delta_{py} = \frac{\epsilon_{pr}^{rms}}{V_f} z_0^2 \tag{42}$$

that is

$$\delta_{px} \simeq \delta_{py} = 0.2w \frac{\Phi^{rms}}{V_f} \frac{z_0^2}{r_0^2} \tag{43}$$

where r_0 and z_0 are the dimensions of the cooling trap.

The radial coordinates of the antiprotons at the instant they leave the cooling trap can be parametrized as

$$x = r_{mf} cos(\theta_m) + r_{cf} cos(\theta_c) + \delta_{px} \tag{44}$$

$$y = -r_{mf} sin(\theta_m) - r_{cf} sin(\theta_c) + \delta_{py} \tag{45}$$

By assuming that the transfer takes place in a homogeneous magnetic field we can write $X(T_g)$, $Y(T_g)$ as functions of C_g,r_{mf},r_{cf},δ_{px}, δ_{py}, θ_m and θ_c. Since the phases θ_m and θ_c are randomly distributed, if we neglect the difference between the cyclotron frequencies and qB/m, we find that the mean values of the radial coordinates in the gv trap are

$$< X(T_g) >= (\delta_{px} - C_g) \;\; cos(\Omega_m T_g) + \delta_{py} \;\; sin(\Omega_m T_g) + C_g \tag{46}$$

$$< Y(T_g) >= (\delta_{px} - C_g) \;\; sin(\Omega_m T_g) + \delta_{py} \;\; cos(\Omega_m T_g) \tag{47}$$

These relations show how the displacement of the center of the radial oscillations in the cooling trap influence the result of the C_g measurement. However by measuring $< X(T_g) >$ and $< Y(T_g) >$ for different values of T_g and by fitting the data with (46) and (47), the values of δ_{px}, δ_{py} , C_g and Ω_m can be separately obtained.

Another way to account for the systematic error described by δ_{px} and δ_{py} is to take measurements with different values of V_g (C_g depends on V_g but δ_{px} and δ_{py} do not).

All these considerations hold if the particle is confined near the centre of the trap.

Table 1 reports possible values of δ_{px} and δ_{py} obtainable with the other data listed in the table using the actual limit on the patch effect field and possible future limits.

4.2 Thermoelectric fields

Problems related to the presence of thermoelectric fields are discussed in detail in ref. [16]. If the temperature of the surface of a metal is not uniform along a direction s, the electric field near the surface is $\in_T$

$$\in_T = \left(\frac{1}{q}\frac{\partial W}{\partial T} + S\right)\frac{dT}{ds} \tag{48}$$

where $\frac{\partial W}{\partial T}$ is the variation of the work function with temperature and S is the thermoelectric coefficient. Typical values for S are of the order of $1\mu eV/K$. The term $\frac{\partial W}{\partial T}$ is presently not well understood. In particular, it has been observed that its value is strongly affected by gas adsorbtion (expecially helium); as a consequence the rate of variation of W with respect to temperature can be extremely high (up to a few mV/K) [16].

For the experiment that we propose, temperature gradients along the axis of the g trap do not interfere with gravity measurements. In fact, the only effect of the resulting field is to displace the centre of the axial oscillations. Temperature variations along the x direction produce electric fields which overlap to gravity and are therefore a source of systematic error. Their effect is to displace the centre of the radial oscillations in both the cooling and the g traps as well as along the transfer channel which connects the two traps.

The most important effect occurs inside the g trap. The ratio between the displacement of the radial oscillations C_T and C_g is

$$\frac{C_T}{C_g} = \frac{q \in_T}{mg} \tag{49}$$

that is

$$\frac{C_T}{C_g} \simeq 10^4 S(mV/K)\frac{dT}{dx}(K/m) \tag{50}$$

The control of temperature gradients is therefore a crucial part for the success of the experiment. Temperature gradients as low as 10^{-5} K/m have been obtained [16].

4.3 Gravity-induced electric field

A metallic structure placed in a gravitational field, undergoes slight deformations of the crystal lattice. Moreover, the electrons distribution inside the solid, is affected by the presence of a gravitational field. The resultant electric field inside a cylindre of vertical axis, has been widely discussed in literature [16]. The value of this field $\in_i$ is given by

$$\in_i = \frac{m_e g}{q} + \frac{\partial W}{\partial z} \tag{51}$$

where m_e is the mass of the electron and W is the work function of the metal. The first term is due to the electron redistribution, the second, which gives a bigger contribution, is due to the crystal deformation. Recent measurements [21] have shown that the ratio between the force of gravity induced electric fields and the weight of antiprotons is close to 1. A measurement on gold electrodes has yielded a ratio < 0.18. In the apparatus that we propose the axis of the cylinder is perpendicular to the direction of gravity, however we expect that the order of magnitude for the above ratio, will still be given by eq (51). As for other sources of systematic errors it is crucial to calibrate the apparatus with different kind of particles.

4.4 Forces due to image charges

The field created by the image charges induced on the walls of the trap affects the motion of the antiprotons. The force F_{im} near the axis of the trap can be estimated with the following expression (which is exact for a cylinder of infinite length)

$$F_{im} = \frac{C_2}{4\pi\epsilon_0} q^2 \frac{r}{R_0^3} \tag{52}$$

where R_0 is the radius of the apparatus and C_2 is 1.0027 [16]. This force adds up to the one due to the confining fields and therefore alters the value of C_g. The x coordinates of the radial oscillation centre in the g trap becomes C_{im}

$$C_{im} = \frac{mg}{\frac{k}{2} + \frac{4\pi\epsilon_0 mg R_0^3}{q^2}} \tag{53}$$

and the relative variation is

$$\frac{C_g - C_{im}}{C_g} = \frac{q}{4\pi\epsilon_0} \frac{Z_0^2}{R_0^3 V_b} \tag{54}$$

or, by inserting the numerical factors

$$\frac{C_g - C_{im}}{C_g} = \frac{1.44\ \ 10^{-3}}{V_b(\mu V)} \frac{Z_0^2(m^2)}{R_0^3(m^3)} \tag{55}$$

For the given values of the potentials and the geometrical dimensions, this effect introduces a negligeable error.

4.5 Precision of the fields in the g trap

The trajectory described in paragraph 2 has been calculated assuming ideal behaviour of the electric and magnetic fields. In real conditions the effect of manifucture imperfections of the electrodes, non-homogeneous magnetic fields and errors in the alignement between the trap axis and the direction of the magnetic field, are effects whose contribution to the particle trajectory and therefore to the error in the final measurement, have to be evaluated.

A non homogeneous magnetic field, and therefore a gradient of $\vec{B}_g$, introduces a drift motion of the centre of the cyclotron orbits which superimposes on the drift motion due to the force of gravity [22]. The drift velocity v_∇ due to this gradient is given by

$$v_\nabla = (E_\perp + 2E_\parallel)\frac{\vec{B}_g \wedge \vec{\nabla} B_g}{qB_g^3} \tag{56}$$

where $E_\perp$ and $E_\parallel$, for each point in the trajectory, represent respectively the energy of motion transverse and parallel to the local direction of the magnetic field. The tolerance on $\vec{\nabla} B_g$ can be obtained observing that, if $R_m \simeq C_g$, the centre of the cyclotron motion ideally moves with the velocity given by (2) and therefore by requiring that

$$v_\nabla \ll \frac{mg}{qB_g} \tag{57}$$

By using (56) and neglecting the difference between $\vec{B}_g$ and its z component we have

$$\frac{\partial B_g}{\partial s}\frac{1}{B_g} = \frac{mg}{E_\perp + 2E_\parallel} \tag{58}$$

($\frac{\partial B_z}{\partial s}$ stays for either $\frac{\partial B_z}{\partial x}$ or $\frac{\partial B_z}{\partial y}$) that is

$$\frac{\partial B_g}{\partial s}\frac{1}{B_g} = \frac{10^{-3}}{(E_\perp + 2E_\parallel)(\mu eV)} \quad cm^{-1} \tag{59}$$

Therefore for a measurement of g with a precision of 1%, an accuracy of at least $10^{-4} - 10^{-5}$ cm^{-1} is required.

In comparison the following arguments show that one does not need a very high precision in aligning the direction of the magnetic field with the trap axis z. If we call Θ_B the angle between these two directions and rewrite the equations of motion, it is easy to verify that the radial motion is now coupled to the motion along z and that it is described by non-circular orbits. The three frequencies associated with the motion are not the same as those given by equations (7), (10) and (11), however the important result is that the point of equilibrium of the orbits does not change with respect to the ideal case. The coupling between the radial and the axial motion causes the axial energy of a particle to depend on the cyclotron radius. As a consequence, the radial dimensions of the orbits in the g trap increase with the difference of the average energy of the motion along z and the average energy of the radial motion. Since

$$R_c \simeq \frac{mv_\perp}{qB_g} = \frac{m(\vec{v} - \vec{v} \cdot \frac{\vec{B}_g}{B_g})}{qB_g} \tag{60}$$

the change in the radius of the cyclotron motion due to a finite value of Θ_B (with $\Theta_B \ll 1$) is readily obtained

$$\frac{\Delta R_c}{R_c} = -\Theta_B + \frac{E_{zg}}{E_{cg}} \frac{\Theta_B^4}{8} \tag{61}$$

Therefore a misalignment of a few degrees can be tolerated as long as E_{zg} is close enough to E_{cg}.

In order to extract the value of g from a measurement of C_g, it is crucial to minimize non-ideal behaviours of the electric field introduced by imperfections in the shape of the electrodes. We have estimated the order of magnitude of the tolerance in the construction of the electrodes by considering an elliptical shape in place of the ideal circular one. Equations (5) and (6) became

$$\epsilon'_x = \frac{k_x}{2q} x \tag{62}$$

$$\epsilon'_y = \frac{k_y}{2q} y \tag{63}$$

where (referring for simplicity to a Penning trap having electrodes shaped as hyperboloids)

$$k_x = -\frac{4q(V_{ring} - V_{endcap})}{R_X^2 + (1 + (1+\epsilon)^2) Z_0^2} \tag{64}$$

$$k_y = -\frac{4q(V_{ring} - V_{endcap})}{R_X^2 + (1 + (1+\epsilon)^2) Z_0^2} (1+\epsilon)^2 \tag{65}$$

R_X and R_0 are the lengths of the semi-axis of the ellipse and

$$R_X = (1+\epsilon) R_0 \tag{66}$$

Since

$$g = \frac{C_g k_x}{2m} \tag{67}$$

the error in k_x contributes to the error in g

$$\frac{\Delta g}{g} = \frac{\Delta C_g}{C_g} + \frac{\Delta k}{k} \tag{68}$$

By using (59) we get

$$\frac{\Delta k}{k} = \frac{(k_x - k)}{k} = \frac{1 + 2Z_0^2/R_0^2}{(1+\epsilon)^2 + (2 + 2\epsilon + \epsilon^2)Z_0^2/R_0^2} - 1 \qquad (69)$$

If for the sake of simplicity we let $R_0 = Z_0$ (for $\epsilon \ll 1$) we obtain

$$\frac{(k_x - k)}{k} = 1.33\epsilon \qquad (70)$$

Therefore the relative error introduced by axially asymmetric mechanical imperfections is expected to be of the same order of magnitude as the degree of asymmetry in the shape of the electrodes.

Other mechanical imperfections occurring in the manifacture of the electrodes, if they do not brake the azimuthal symmetry, perturbe the harmonic electric potentials by introducing terms which are proportional to even powers of the coordinates. Therefore, near the trap center, the electric potential in polar coordinates takes the form [12]

$$V \propto \sum c_k (r/d)^k P_k(cos\theta) \qquad (71)$$

where

$$d = \frac{1}{2}\left(Z_0^2 + \frac{R_0^2}{2}\right) \qquad (72)$$

and $P_k(cos\theta)$ are the Legendre polinomials. If we neglect terms of order 6 or higher, the potential of an antiproton near the centre of the g trap is

$$qV(x,y,z) = -\frac{qV_0}{4d^2}(r^2 - 2z^2) + \frac{c_4 qV_0}{16d^4}(8z^4 + 3r^4 - 24r^2z^2) + mgx \qquad (73)$$

A a consequence the x coordinate of the point of equilibrium of the radial oscillations has to be modified (with respect to eqn (18)) by adding a term D_4 where

$$\frac{D_4}{C_g} = \frac{3}{2}c_4\left(\frac{C_g}{d}\right)^2 \qquad (74)$$

Typical value of c_4 are of the order of 10^{-1} 10^{-2} [12] and by using "compensating" electrodes this value can be lowered by at least an order of magnitude. Even without the employment of sophisticated techniques the systematic error due to an anharmonic potential can be reduced to a negligible value. However it has to be pointed out that the above estimates apply only if we assume small oscillation amplitudes.

Furthermore anharmonic terms can introduce a statistical error, for the dimensions of radial orbits would also depend on the axial energy.

Finally we observe that a misalignment $\delta'_{x,y}$ between the axis of the cooling trap and the axis of the scale trap, produces an effect which can be described by relations which are formally identical to (46) and (47).

4.6 Sources of heat for antiprotons

Once the adiabatic process of cooling is completed, the temperature of antiprotons will be lower than the temperature of the surrounding environment by some orders of magnitude. It is therefore important to evaluate the typical time scale involved before significant changes of the antiprotons temperature begin to take place. The processes that we considered are the following:

1) elastic collisions whith the residual gas;

2) interaction with thermal radiation at 4.2 K;

3) resistive heating.

Process 1 has already been considered in paragraph 3.

The effect of process 2) can be investigated by considering the scattering between an antiproton with energy corresponding to a temperature $\ll 4.2\ K$ and a photon in equilibrium with the walls of the apparatus at $4.2K$. We assume that each collision increases the energy of an antiproton by an amount $\hbar\omega$ where $\hbar\omega$ is the photon energy sampled from a black body distribution at $4.2\ K$. The cross section for the process is reported in [16]. After the integration over the whole energy spectrum of the thermal radiation, we find that the typical time scale over which an antiproton undergoes significant heating, is much greater than the time needed to perform the gravity measurement.

Finally we consider the effect of a resistance R connected to an electrode of the trap. A resistive load is produced by the tuned circuits that are employed to detect particles in the cooling trap. The resistance R acts as a generator of random potential across the trap. The power spectrum is related to the equivalent noise temperature T_{eff} by $4KT_{eff}R$ ($T_{eff} \simeq 4.2K$). The effect of the corresponding random electric field is to set a lower limit to the process of resistive cooling if the initial energy of the particles is greater than KT_{eff} or to increase the temperature of the particles if their energy is lower than KT_{eff}.

It is not difficult to demonstrate that the characteristic time of this heating process is identical to the characteristic time τ_r of resistive cooling where

$$\tau_r = \frac{4ml_0^2}{(\alpha q)^2 R} \tag{75}$$

l_0 stands for the radial or axial dimension of the trap and α is a geometrical factor ($\alpha < 1$) [12]. In order to estimate the characteristic time scales involved, we considered the interval of time Δt_2 which is necessary to double the initial energy (here called $E^0_{r,z}$) of the particles. Since the axial and radial energies of the antiprotons increase with time according to

$$E_{r,z}(t) \simeq (E^0_{r,z} - KT_{eff}) \ e^{-\frac{t}{\tau_r}} + KT_{eff} \tag{76}$$

we get

$$\Delta t_2 = 2.8 \ \ 10^9 E^0_{r,z} \ \ (\mu eV)\frac{l_0^2(m^2)}{R(Ohm)} \ \ sec \tag{77}$$

where we assumed $T_{eff} = 4.2K$ and $\alpha = 0.5$. Values of $R \simeq 10^5 \ Ohm$ as for trap connected to a tuned detection circuit would give problems. However this effect is negligeable in the g trap (there is not a detection circuit here) and in the cooling trap during the adiabatic cooling process because the detection circuit is detuned.

5. Conclusions

The new way of measuring the gravitational acceleration of antiprotons that is suggested in this paper is a possible alternative to the TOF method whose possibilities are now under experimental investigation.

The main difficulty of our method lies in the need of preparing into a trap single particles having very low energy and very small cyclotron and magnetron radii, that is a very well defined position for the injection in the g trap. Once this difficult goal has been fulfilled the result of the g measurement is completely independent on the energy and radii distribution of the particles. This is one of the advantage of our method with respect to the TOF method. There the g value is obtained fitting the shape of the measured time distribution near the cutoff after that a given hypothesis on the initial velocity distribution of the particle launched in the drift tube is assumed [8].

Besides, against a longer preparation time of each particle, we get also the big advantage that, in principle, a single measure would give the final result, that is the g value. Repeated measures increase the precision in the usual way that is lowering the error of the single measure by $1/\sqrt{N}$.

The TOF method and the method discussed here also differs for the sensitivity to the patch effect. As discussed in [16] taking into account the patch effect the time of flight t of a particle of velocity v_0 in a vertical tube of length is L is

$$t = \sqrt{\frac{m}{2}} \int_0^L (\frac{mv_0^2}{2} - mgz + qV_p(z))^{-\frac{1}{2}} dz$$

where $V_p(z)$ is the potential on the axis due to the patch effect. From this relation we see that in order to make the measurement possible it is necessary to have $qV_p(z) \ll mgz$.

This because the effect of the patch potential not only causes a sistematic error, as in the method proposed by us, but also strongly influences the statistical error. The rms value of V_p on the tube axis is V_p^{rms}

$$V_p^{rms} = 0.2\frac{w\Phi^{rms}}{a_d} \tag{78}$$

where a_d is the radius of the drift tube. Assuming $w\Phi^{rms} = 10^{-4}$ cm V (see paragraph 4.1) in order to get $qV_p^{rms} \ll mgL$ in a drift tube having the same length to diameter ratio as in [8], it is necessary to have $La_d \gg 2$ m^2. This means $a_d \gg 15$ cm and $L \gg 15$ m. These dimensions are not realistic for the TOF method expecially because of the requirements on the magnetic field strenght and homogeneity. Therefore the feasibility of the TOF experiment strongly relies on the success of the tecniques which can be used to reduce the patch effect. As discussed in paragraph 4 our method would allow to perform a differential g measurement with the value of the patch fields actually measured. Of course every improvement in the lowering the patch fields will make the measurement easier and more precise.

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Tabella 1

V_g	$100\ \mu V$
Z_0	$1\ m$
R_0	$1\ m$
C_g	$1\ mm$
$\left(\frac{\Delta g}{g}\right)_s$	$\frac{3\%}{\sqrt{N}}$
$E_{cg} \simeq E_{cf}$	$0.3\ \mu eV(3.5\ mK)$
$E_{zg} \simeq E_{zf}$	$0.35\ \mu eV(4.2\ mK)$
B_g	$50\ Gauss$
T_g	$314\ sec$
r_0, z_0	$\simeq 1\ cm$
V_i	$100\ V$
V_f	$100\ \mu V$
B_c	$6\ Tesla$
r_{ci}, r_{mi}	$0.4\ \mu m$
r_{cf}, r_{mf}	$15\ \mu m$
δ_{px}, δ_{py}	$2\ mm\ -\ 1 \mu m$
C_g/C_p	$5\ 10^{-1}\ -\ 10^3$

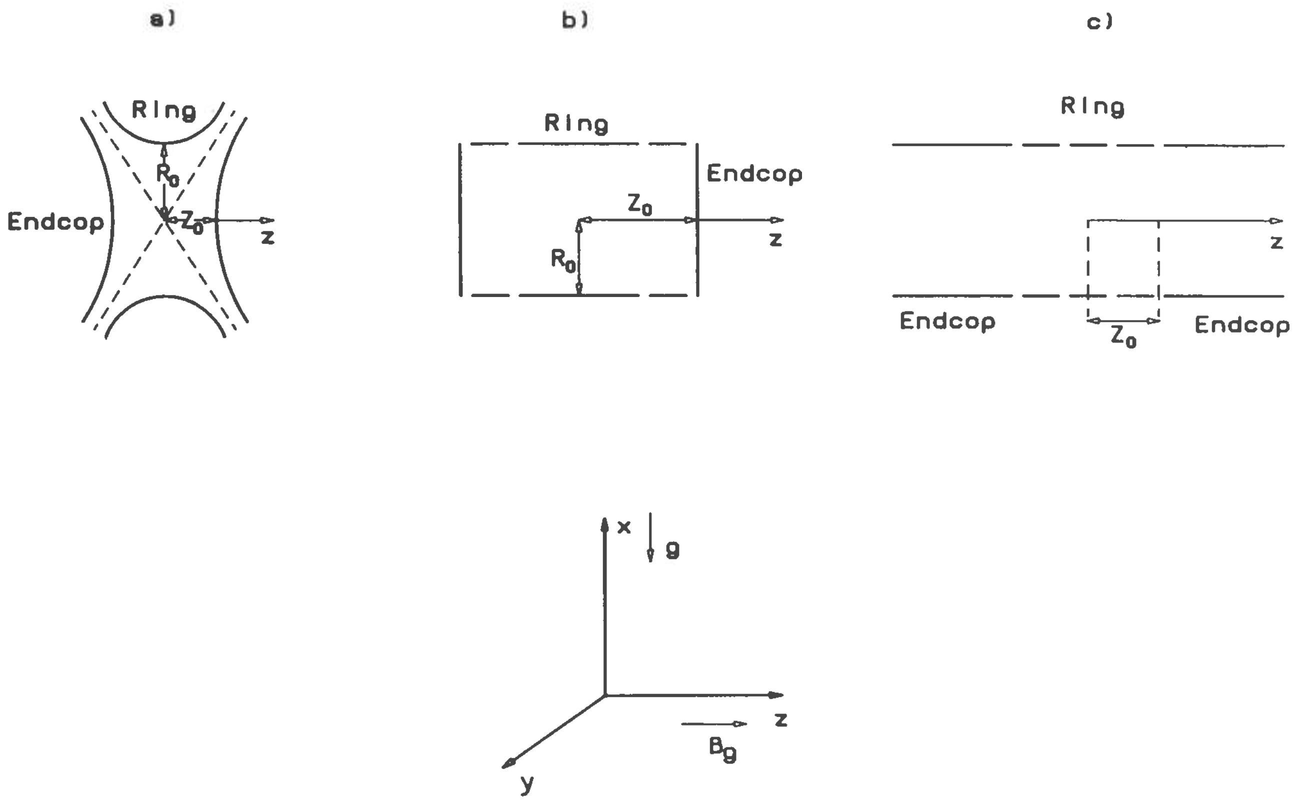

FIGURE 1.

This figure shows a cross section of Penning traps. The real electrode shapes are obtained by rotating the figure around the z axis. Fig 1a) shows a regular Penning trap having 3 electrodes (2 endcaps and 1 ring) shaped as revolution hyperboloids. A constant potential difference applied between the endcaps and the ring allows to obtain into the trap the electric field described in paragraph 2. Fig 1b) and 1c) show cylindrical Penning traps. By applying proper constant potentials to the trap electrodes, the field mentioned above can be obtained near the centre of the trap.

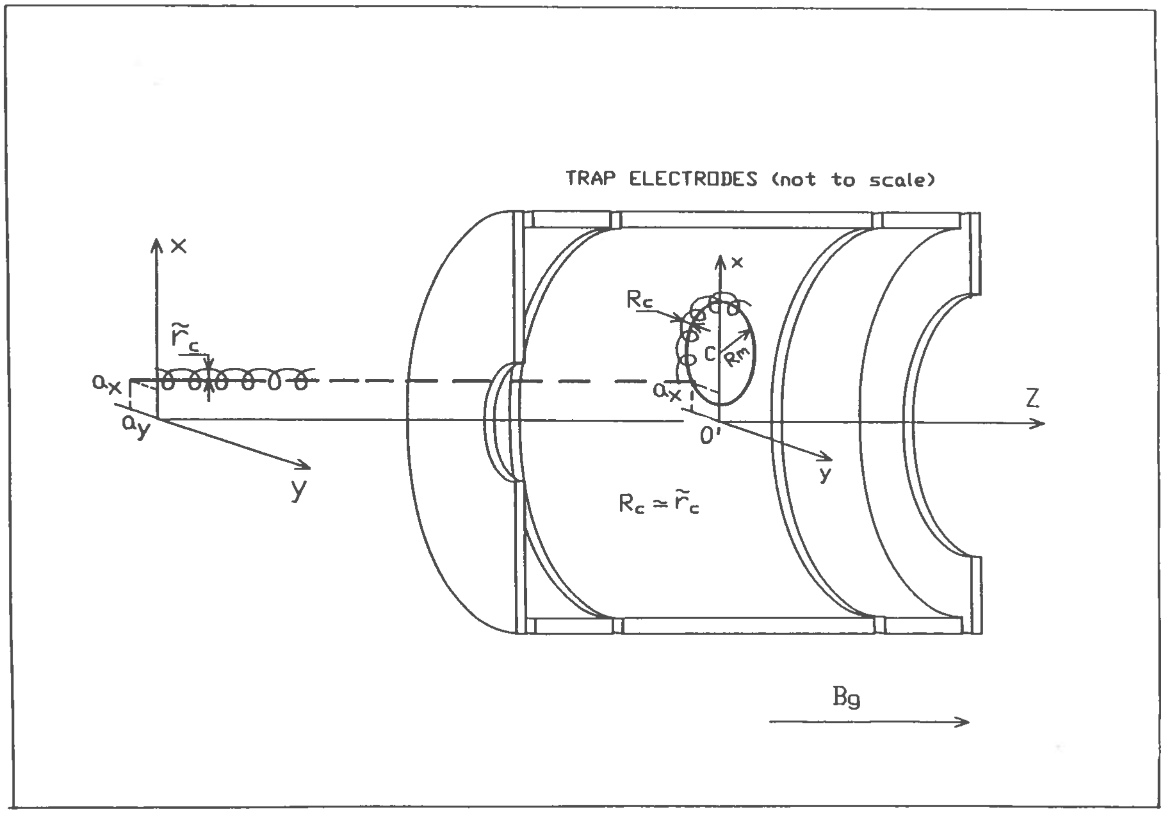

FIGURE 2. In this figure is displayed the link between the radial orbit parameters of an antiproton (or a charged particle) feeling the gravity force inside a Penning trap and the radial initial condition of that particle before its capture. For simplicity reasons the axial motion inside the trap is not shown.
With the gravity field pointing in the x direction, the center of the cyclotron motion inside the trap describes a circle centered around the point $C = (C_g, 0, 0)$. Supposing an ideal capture process, the cyclotron radius R_c inside the trap is practically equal to the cyclotron radius $\tilde{r}_c$ before the capture and the radius R_m is related to the radial particle position (a_x, a_y) before the capture. R_m gets close to C_g as long as a_x and a_y are negligeable.