ISTITUTO NAZIONALE DI FISICA NUCLEARE

Sezione di Catania

INFN/BE–93/04
2 Agosto 1993

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PACS: 24.10

Servizio Documentazione
dei Laboratori Nazionali di Frascati

INFN – Istituto Nazionale di Fisica Nucleare
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Temporal description of interference phenomena in light-ion collisions.

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A stationary approach was used in previous papers to calculate the contribute to the cross section due to interference between different channels in light ions scattering. The aim of the present work is to show, on a formally correct base which makes use of a temporal description of the scattering processes, the validity of the former approach.

Interference phenomena arising when two particles are simoultaneously detected (in a sense that will be specified below) in processes with three particles in the final state have previously been considered in the stationary theory [1], [2], [3].
The original idea was presented by Podgoretskij and Kopylov [4] for two particles emission (evaporation) from heavy nuclei.
Here the interference between prompt direct and delayed resonance processes in reaction of the type

$$x + X \rightarrow y + z + U \tag{1}$$

is considered. In Fig.1 two possible mechanisms for reaction (1) are pictorially represented.
The symbols a and b enclosed in boxes stand for detectors located at macroscopic distances r_1 and r_2 from the scattering point C_0.
In Fig. 1a the direct (like quasi-free or so called one and two step direct) process of simoultaneous prompt emission at point C_0 of all the three final particles is described. Fig. 1b presents delayed successive decay process with emission of particle y and formation of an intermediate excited nucleus Z^* which subsequently decays into z and U at point C_1, according to the reactions

$$x + X \rightarrow y + Z^*, \quad Z^* \rightarrow z + U. \tag{2}$$

In Fig. 2 the superposition of direct and sequential emission of one of the final particles is displayed in the same picture. For macroscopic distances and under the condition specified below angles θ_2 and $\tilde{\theta}_2$ as well as impulses k_2 and $\tilde{k}_2$ can be considered practically coincident.

The asymptotic wave packet, near detectors a and b can be described by the following expression:

$$\Psi_{ab}(r_1, r_2 \rightarrow \infty) \longrightarrow C \times \int d\vec{k}_x g_i(\vec{k}_x) \int d\vec{k}_2 g_{f,2}(\vec{k}_2) \int d\vec{k}_1 g_{f,1}(\vec{k}_1) \times$$
$$\times \int d\vec{k}_3 \delta(E_i - E_f) \delta(\vec{K}_i - \vec{K}_f) \left[f_{dir}^{(\mathcal{L})}(E_1, E_2, E_3, \theta_1, \theta_2, \theta_3) e^{(\sum_{j=1}^{j=3} i k_j r_{j C_0})} + \right.$$
$$\left. + \frac{f_{Z^*}^{\mathcal{L}}(E_1, E_2, E_3, \theta_1, \theta_2, \theta_3)}{\varepsilon_Z^* - \varepsilon_{res,Z} + i\Gamma_Z/2} e^{(i k_1 r_{1 C_0} + \sum_{j=2}^{j=3} i k_j r_{j C_1})} \right] e^{-iE_f t/\hbar}. \tag{3}$$

In this equation C is a normalization constant, g_i, $g_{f,1}$, $g_{f,2}$ are amplitude weight factors describing the impulse spread of the incident particle x and that of the final particles y and z due to detectors resolution;

$$f_{dir}^{(\mathcal{L})} = \sqrt{J_{C \rightarrow \mathcal{L}}}\, f_{dir}^{(C)} \tag{4}$$

and

$$f_{Z^*}^{(\mathcal{L})} = \sqrt{J_{R\to C} J_{C\to\mathcal{L}}}\, f_{xy}^{(C)}\, \mathcal{J}_Z^{(C)} \tag{5}$$

are the amplitudes for direct and sequential processes (the subscripts $\mathcal{L}$ and C refer to laboratory and center of mass system respectively), $f_{xy}^{(C)}$ and $\mathcal{J}_Z^{(C)}$ being the amplitude of the first step direct process $x + X \to y + Z^*$ and the reduced-width amplitude of the decay process $Z^* \to z + U$ respectively; ε_Z^*, $\varepsilon_{res,Z}$ and Γ_Z are the excitation energy, the energy and total width of the resonant state of the nucleus Z^*; $J_{R\to C}$ and $J_{C\to\mathcal{L}}$ are the Jacobians of the coordinates transformation from the Recoil system to the C.M. and from C.M. to Laboratory one respectively; r_{km} are the distances from points m $(m = C_0, C_1)$ to particles k (with $k = 1,2,3$ corresponding to y, z, U); E_i, $\vec{k}_i$ and E_f, $\vec{k}_f$ are the total energies and impulses in the initial and final channels respectively; $E_j = \hbar^2 k_j^2/2m_j$ is the kinetic energy of j-th particles, θ_j and $\vec{k}_j$ being the angle of motion (relative to beam, i.e. incident particle x, direction) and the wave vector of particle j respectively. In expression (3) $\delta(E_i - E_f)$ and $\delta(\vec{K}_i - \vec{K}_f)$ take care of energy and impulse conservation.
Expression (3) is written on the base of the general formalism described in [5] with application of the asymptotic stationary functions introduced in [1],[2], [3] and taking into account particle U explicitly.
For the sake of simplicity a factor $(r_{1C_0}^{-1} r_{2C_0}^{-1} r_{3C_0}^{-1})$ in eq. (3) has been omitted as well as spin and internal coordinates.

The factor $e^{-iE_f t/\hbar}$ can be rewritten as

$$e^{-i(E_1+E_2+E_3)\frac{t}{\hbar}} e^{-iE_f' \frac{t}{\hbar}}. \tag{6}$$

The first three factors of the previous expression can be formally put in the integrals of eq. (3) as follows:

$$\int dk_1 g_{f,1} e^{ik_1 r_{1m} - iE_1 \frac{t}{\hbar}} \ldots ,$$

$$\int dk_2 g_{f,2} e^{ik_2 r_{2m} - iE_2 \frac{t}{\hbar}} \ldots ,$$

$$\int dk_3 e^{ik_3 r_{3m} - iE_3 \frac{t}{\hbar}} \ldots .$$

In order to perform the previous integrals a transformation from variables $K_{1,2,3}$ to variables

$$y_{1,2,3} = \left(\frac{i\hbar t}{m_{1,2,3}}\right)^{\frac{1}{2}} \left(k_{1,2,3}^0 - \frac{m_{1,2,3} r_{1,2,3}}{\hbar t}\right) \tag{7}$$

is useful. Here only projections of $k_{1,2,3}$ over the mean vectors $k_{1,2,3}^0 \equiv < k_{1,2,3} >$ are taken, the other components of $k_{1,2,3}$ remaining in other parts of (3).
The factor $g_{f1,2}$ can be assumed to have the form

$$g_{f1,2} \approx \frac{c_{1,2}}{E_1 - E_{1,2}^0 - i\Delta E} \tag{8}$$

and ΔE to be very small ($\Delta E \ll \Gamma_Z$), as well as the energy spread of the incident particle x. Using a known result for a similar calculation (see, e. g., [6], [7]), the wave function becomes

$$\Psi_{ab} \approx 0 \tag{9}$$

for

$$t < t_i + \frac{r_1 C_0}{v_1^0}, \quad t < t_i + \frac{r_2 C_0}{v_2^0}, \quad t < t_i + \frac{r_3 C_0}{v_3^0},$$
$$t < t_i + \tau + \frac{r_2 C_1}{v_2^0}, \quad t < t_i + \tau + \frac{r_3 C_1}{v_3^0} \tag{10}$$

and

$$\Psi \propto C \times e^{-iE_f^0 t/\hbar} \times$$
$$\times e^{i\sum_j k_j^0 r_j C_0} \times \left[f_{dir}^{\mathcal{L}} e^{-\Delta E[(t-t_i-\frac{r_1C_0}{v_1^0})+(t-t_i-\frac{r_2C_0}{v_2^0})+(t-t_i-\frac{r_3C_0}{v_3^0})]/\hbar} + \right.$$
$$\left. + \frac{f_{Z^*}^{\mathcal{L}}}{\varepsilon_Z^* - \varepsilon_{res,Z} + i\Gamma_Z/2} e^{-\Delta E[(t-t_i-\frac{r_1C_0}{v_2^0}+(t-t_i-\tau-\frac{r_2C_1}{v_2^0}+(t-t_i-\tau-\frac{r_3C_1}{v_3^0})]/\hbar} \times \right.$$
$$\times e^{ik_2^0 \Delta r_2 + ik_3^0 \Delta r_3}, \tag{11}$$

for

$$t > t_i + \frac{r_1 C_0}{v_1^0}, \quad t > t_i + \frac{r_2 C_0}{v_2^0}, \quad t > t_i + \frac{r_3 C_0}{v_3^0},$$
$$t > t_i + \tau + \frac{r_2 C_1}{v_2^0}, \quad t > t_i + \tau + \frac{r_3 C_1}{v_3^0} \tag{12}$$

Here $v_{1,2,3}^0 = \hbar k_{1,2,3}^0 / m_{1,2,3}$, the initial time t_i is defined by the phase of the amplitude weight factor g_i and the mean time τ of the nucleus Z^* motion before its decay is given by the well known expression:

$$\tau = \frac{\hbar \Gamma_Z / 2}{(\varepsilon_Z^* - \varepsilon_{res,Z})^2 + \Gamma_Z^2/4}, \tag{13}$$

and

$$\Delta r_{2,3} = V_{\perp(2,3)} \tau,$$

$V_{\perp(2,3)}$ being the projection of the velocity of the nucleus Z^* onto the direction of $\vec{k}_{2,3}$.

The energy spread for particle U is of the order ΔE, according to energy-impulse conservation.

Interference phenomena can occur only in case of simultaneous arrival (within the time resolution of the detectors) of particles y and z on a and b. The coincidence-rate intensity is defined by a temporal integration of

$$\Psi_{ab}^* \hat{j}_1 \hat{j}_2 \Psi_{ab}$$

($\hat{j}_{1,2}$ being the flux probability density operator for particles y and z) over a time interval ΔT, which is great with respect to the temporal extension of the wave packets, and a spatial intergration over particle U coordinates, i.e.:

$$P \approx \int_{t_{min}}^{\infty} dt \int_{r_{3_{min}}}^{r_{3_{max}}} dr_3 \Psi^*_{ab} \hat{j}_1 \hat{j}_2 \Psi_{ab} \approx$$

$$\approx \int_{t_{min}}^{\infty} dt \int_{0}^{v_3^0(t-t_i-\frac{r_{3C_0}}{v_3^0})} dr_3 |\Psi_{ab}|^2 \ , \qquad (14)$$

where t_{min} is the smallest value among

$$t_i + \frac{r_{1C_0}}{v_1^0},\ t_i + \frac{r_{2C_0}}{v_2^0},\ t_i + \tau + \frac{r_{2C_1}}{v_2^0},$$

$$t_i + \frac{r_{3C_0}}{v_3^0},\ t_i + \tau \frac{r_{3C_1}}{v_3^0},$$

$r_{3_{max}}$ is the maximum between $v_3^0(t - t_i - (r_{3C_0}/v_3^0))$ and $v_3^0(t - t_i - \tau - (r_{3C_1}/v_3^0))$, $r_{3_{min}} \to 0$ for ordinary small wave packets.

Under standard experimental conditions, i.e. when

$$\Delta E \tau / \hbar \ll 1 \qquad (15)$$

and

$$\delta t = \frac{r_l}{v_l^0} - \frac{r_m}{v_m^0} \ll \Delta T, \quad (l, m = 1, 2, 3,\ l \neq m) \qquad (16)$$

(ΔT is the time resolution of the coincidence scheme)
it is possible to write

$$P = P_0 + P_1 \,, \qquad (17)$$

$$P_0 = |f_{dir}^{\mathcal{L}}|^2 + \frac{|f_{Z^*}^{\mathcal{L}}|^2}{(\varepsilon_Z^* - \varepsilon_{res,Z})^2 + \Gamma_Z^2/4} \qquad (18)$$

and

$$P_1 = 2\,|f_{dir}^{\mathcal{L}} \frac{f_{Z^*}^{\mathcal{L}}}{\varepsilon_Z^* - \varepsilon_{res,Z} + i\Gamma_z/2}| \cos\phi \qquad (19)$$

(in arbitrary units), where

$$\phi = \delta + \beta + \varphi,$$

$$\delta = arg(f_{Z^*}^{\mathcal{L}}) - arg(f_{dir}^{\mathcal{L}}),$$

$$\beta = arg(\varepsilon_Z^* - \varepsilon_{res,Z} + i\Gamma_z/2)^{-1}$$

$$\varphi = k_2^0 \Delta r_2 + k_3^0 \Delta r_3.$$

Comparing these results with that obtained in a stationary model [1],[2],[3], the latter are confirmed by the present self-consistent temporal approach in the limit $\Delta E \ll \Gamma_Z$. The same conclusion is valid for cases in which two intermediate excited nuclei are formed, i.e.

$$x + X \rightarrow \begin{cases} y + Z^* & \rightarrow y + z + U \\ z + Y^* & \rightarrow z + y + U \end{cases} ,$$

with the conditions $\Delta E \ll \Gamma_y$, and $\Delta E \ll \Gamma_Z$

ACKNOWLEDGEMENTS: We want to thank Prof. G.Giardina, M. Lattuada, C. Spitaleri and H. Wolter for many stimulating discussions. One of us (V.S.O.) is also thankful to INFN-Sez. di Catania and INFN-Laboratorio Nazionale del Sud for the kind hospitality.

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Figure captions

Fig. 1*a* : Direct process reaction channel.

Fig. 1*b* : Sequential process reaction channel.

Fig. 2 : Simultaneous representation of direct and sequential processes.

See text for detailed description.

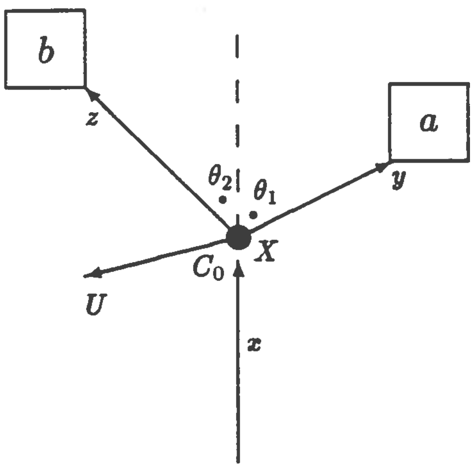

Fig.1*a*

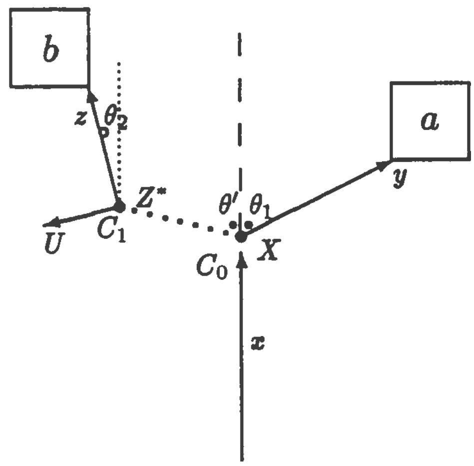

Fig. 1*b*

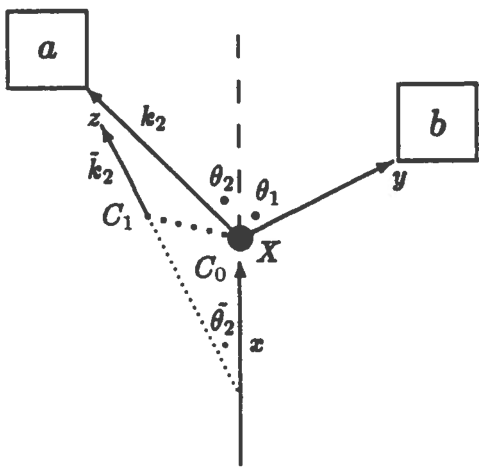

Fig.2