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Measurement of the Hyperfine Splitting of Muonic Hydrogen $(\mu^- p)_{1S}$. An Experimental Method

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Abstract

We propose an experimental method to measure the hyperfine splitting of the energy level of muonic hydrogen ground state $(\mu^- p)_{1S}$ by inducing a laser-stimulated para-to-ortho transition.

The method requires an intense low energy pulsed μ^- beam and a high power tunable pulsed laser.

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1 Introduction

In the present paper an experimental method to measure the hyperfine splitting ΔE^{hfs} in the muonic atom $(\mu^- p)_{1S}$ is proposed (see Fig.1) and the results of a Monte Carlo simulation are presented. We believe that an accuracy of 10 *ppm* can be reached with a sufficiently intense pulsed tunable infrared laser [1, 2]; the results of such an experiment would be complementary to the very precise experimental result of the hyperfine splitting of the ground state of the hydrogen atom [3]:

$$\Delta E^{hfs}_{exp} = 1420.4057517864(17)\ MHz \tag{1}$$

The theoretical expression for the hyperfine splitting ΔE^{hfs}_{th} of the hydrogen-like atom $(l^- p)_{1S}$ (l^- standing for μ^- or e^-) can be written as [4]:

$$\Delta E^{hfs}_{th} = \Delta E^{F} \cdot (1 + \delta^{QED} + \delta^{FF} + \delta^{POL}) \tag{2}$$

with:

$$\Delta E^{F} = \frac{8}{3}\alpha^4 c^2 \frac{\mu_p M_p^2 M_l^2}{(M_p + M_l)^3}, \tag{3}$$

M_p – the mass of the proton;

M_l – the mass of the lepton;

α – the fine structure constant;

μ_p – the proton magnetic moment;

c – the velocity of light in vacuum.

The term δ^{QED} is the contribution from the higher-order quantum-electrodynamical effects; it is known to better than 1 *ppm* [5] and, in principle, the calculation can be improved. The terms δ^{FF} and δ^{POL} describe the contribution due to the internal structure of the proton; of course, these terms vanish for point-like "nuclei" (like in the case of muonium and positronium [6]). The part δ^{FF} depends, in first approximation, on the rigid spatial distri-

bution of the proton charge and magnetic moment, associated to the proton electromagnetic formfactors as obtained from e–p elastic scattering data. This term can be expressed [7], in the lowest approximation, as

$$\delta^{FF} = -2(m_l m_p \alpha c/\hbar(m_l + m_p))\langle R_{pr}\rangle \tag{4}$$

where $\langle R_{pr}\rangle$ is an integral of a combination of the charge and magnetic moment distributions of the proton.

For $m_l = m_e$, given the uncertainty in the proton charge and magnetic radii [8], Eq.4 gives:

$$\delta^{FF} \sim -4.5 \cdot 10^{-5}(1 \pm 0.02) \tag{5}$$

It has to be remembered that the uncertainty in Eq.5 could be reduced at present, since the proton charge formfactor can be obtained today also from the precise determination of the Lamb shift of hydrogen [9].

The term δ^{POL} incorporates the corrections to the hyperfine splitting due to the fact that the proton charge and magnetic moment distributions are not absolutely rigid but are polarized by the orbiting lepton (supposed to be point-like). There do not exist model independent expressions for δ^{POL} in terms of any experimentally observable quantities; most firmly established is the upper bound of [10]: $\left|\delta^{POL}\right| \leq 4 \cdot 10^{-6}$ for the proton polarizability correction in hydrogen $(pe)_{1S}$, though indications are that δ^{POL} might be significantly smaller [11]. It can be easily shown that in first order approximation δ^{POL} is proportional to the light particle mass m_l; we shall therefore write the above equation in the form:

$$\left|\delta^{POL}\right| \leq C^{POL} \cdot (m_l/m_{el}), \;\; C^{POL} \sim 4.10^{-6} \tag{6}$$

In Table 1 we summarize the various contributions to ΔE_{th}^{hfs} with the estimates for their uncertainties.

From Table 1 we see how the term $\delta = \delta^{FF} + \delta^{POL}$ becomes relatively important for the

case of the $(\mu^- p)$-system: one can say that sufficiently precise measurements of ΔE^{hfs} in the $(e^- p)$ and $(\mu^- p)$ systems are complementary to one another.

In what follows we are describing a possible experimental method to measure ΔE^{hfs} in $(\mu^- p)_{1S}$.

2 The experimental method and the Monte Carlo simulations

When a μ^- is stopped in a H_2 gas target, a neutral atom $(\mu^- p)$ is formed with an initial energy ≥ 1 eV [12, 13] and in a statistical mixture of the two hyperfine states $F{=}0$ and $F{=}1$ [14]. If the pressure of the H_2 gas target is $P \sim 10 Atm.$, the thermalization of the $(\mu^- p)_{1S}$ system via collisions with the H_2 molecules of the target is fast, and after a short time, compared to the μ^- lifetime, the $(\mu^- p)_{1S}$ system will be present in the $F{=}0$ state (singlet) with an average kinetic energy very near to the thermal one [12, 13].

During this evolution, if in the target are present (thin) foils of a given material (e.g. gold) some of the $(\mu^- p)$ atoms will reach the gold's surface and transfer the μ^- to form an excited muonic-gold ion:

$$\mu^- p + Au \rightarrow (\mu^- Au)^* + p.$$

Therefore prompt characteristic deexcitation X rays (of energy $\sim$ 5.8 MeV) will be emitted from the $(\mu^- Au)^*$ when the $(\mu^- p)$ atoms reach the gold foils.

The time distribution dN_X/dt of these high energy X rays can be easily observed [12, 13]: clearly such a distribution will be mainly function of the scattering cross section $\sigma_{0\rightarrow 0}$ of the muonic system against the H_2 molecules, the distance d between the gold plates, the pressure and temperature of the target and of the velocities of the colliding systems. A Monte Carlo calculation has been performed, taking for the cross sections the values from [15]. The $(\mu^- p)$

atoms in these calculations disappear either because they meet the gold surfaces or because the μ^- decays. The lower curve on Fig.2 (labeled "no laser shot") has been calculated for the typical values $P = 10Atm$, $T = 300^oK$ and $d = 1mm$; it is in a qualitative agreement with what has been experimentally observed earlier [12, 13].

Suppose now that at a time t_l (chosen so that the $(\mu^- p)_{1S}$ is at this time "nearly thermalized") a laser pulse (say 20 *ns* wide) of a proper wavelength λ is sent into the target in order to induce singlet (*F=0*) to triplet (*F=1*) transitions. If such transitions occur, it is known that due to the "jump mechanism" [16] in a very short time the *F=1* state will be converted (via collisions with the H_2 target molecules) back to *F=0* singlet state ($\sigma_{1\to 0} \sim 10^{-18} cm^2$ [15]). This singlet state will now retain about 0.12 eV of kinetic energy[2], that is much higher than the thermal kinetic energy possessed before t_l. Therefore, if the transition has occured, after time t_l the diffusion process will be altered and a change in the dN_X/dt distribution will occur.

The resonance value $\bar{\lambda}$ of the triplet to singlet difference ΔE^{hfs} can be measured by searching, in a proper time gate Δt (following the laser shot time t_l) for the increase in the number of X rays $N_X(\Delta t)$ while tuning the laser wavelength around the $\bar{\lambda}$ value.

In our Monte Carlo simulation we have been considering a laser pulse starting at time t_l with the resonance wavelength $\bar{\lambda} = 44 \cdot 10^3\ GHz$, supposing its intensity to be high enough to give 1% transition probability. The change in the dN_X/dt yield (the upper curve labeled "laser pulse at 300 ns" on Fig.2) compared to the lower curve ("no laser shot") is clearly visible. We ran our Monte Carlo program for a large set of values of the parameters P, T and d, and looked for the optimal gate Δt and laser shot time t_l. It seems that a pressure $P \sim 10Atm$, a room temperature $T \sim 300^oK$, a laser shot time $t_l = 300ns$ and a gate Δt of about 200 *ns* are appropriate.

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[2] $m_{H_2}/(m_{H_2} + m_{\mu^- p}) \cdot \Delta E^{hfs} \sim 0.12eV$

3 Concluding remarks

i) We have taken optimistically 1% for the transition probability $P(\bar{\lambda})$: the Monte Carlo simulation shows that even $P(\bar{\lambda})$ ~0.01% gives a seizable effect if 300000 $(\mu^- p)$ atoms are formed in the target.

ii) The broadening of the singlet-to-triplet line is mainly due to the Doppler effect from the $\mu^- p$ thermal motion; the triplet (and singlet) state lifetimes contribute very little and similarly does the laser pulse width.

iii) We have used for the scattering cross sections the values from [15]. It was shown in [17] that at low energy (near the thermal one) the cross sections of $(\mu^- p)$ atoms on H_2 molecules are much higher; however, this increase is only for very small scattering angles (less than a degree) while for larger angles the scattering is essentially dominated by the S-wave.

iiii) In order to have enough energy per laser pulse, most probably the repetition rate of the laser will be quite small (1-2 per second). Therefore, to realize this experiment, a low energy pulsed μ^- beam of sufficient intensity must be used. This requires an accelerator machine where pulsed ($< 100\ ns$ wide) muon beam without electron contamination is available. Consequently a highly segmented fast X ray detector together with a small H_2 target (with gold foils spaced at $d \sim 1mm$) where fast laser pulses are injected must be used.

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Table 1: Hyperfine splitting of the ground state of hydrogen and muonic hydrogen atoms: order of magnitude of the correction terms to the Fermi formula and uncertainties

	hydrogen		rescaling	muonic hydrogen	
	order of magnitude	relative error	factors	order of magnitude	relative error
ΔE^F	1.42 GHz	10^{-7}	$(m_l/m_e)^2$	56.8 THz	10^{-7}
δ^{QED}	$O(\alpha)$	10^{-6}	1	$O(\alpha)$	10^{-6}
δ^{FF}	10^{-4}	2%	(m_l/m_e)	10^{-2}	$\sim 2\%$
δ^{POL}	$< 4.10^{-6}$	100%	(m_l/m_e)	$< 10^{-3}$	100%
δ	10^{-4}	10%	(m_l/m_e)	10^{-2}	10%

Figure captions

Fig.1. Hyperfine structure of the ground state of muonic hydrogen $(\mu^- p)_{1S}$.

The nonrelativistic level $1S$ is splitted into a triplet 1^3S_1 and a singlet 1^1S_0 hyperfine states.

Fig.2. Simulated time distribution dN_X/dt of the events per 10 ns gate for pressure $P = 10Atm$, temperature $T = 300^oK$ and distance between the gold foils $d = 1mm$.

300000 $(\mu^- p)_{1S}$ atoms are present in the target at the initial moment. The upper curve represents the time distribution of the events when the a laser pulse of length 20 ns is injected at time $t_l = 300ns$; the lower curve represents the time distribution obtained when no laser pulse is injected.

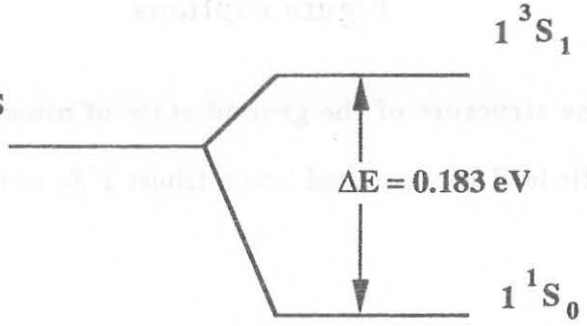
1^3S_1
1 S
ΔE = 0.183 eV
1^1S_0

FIG 1

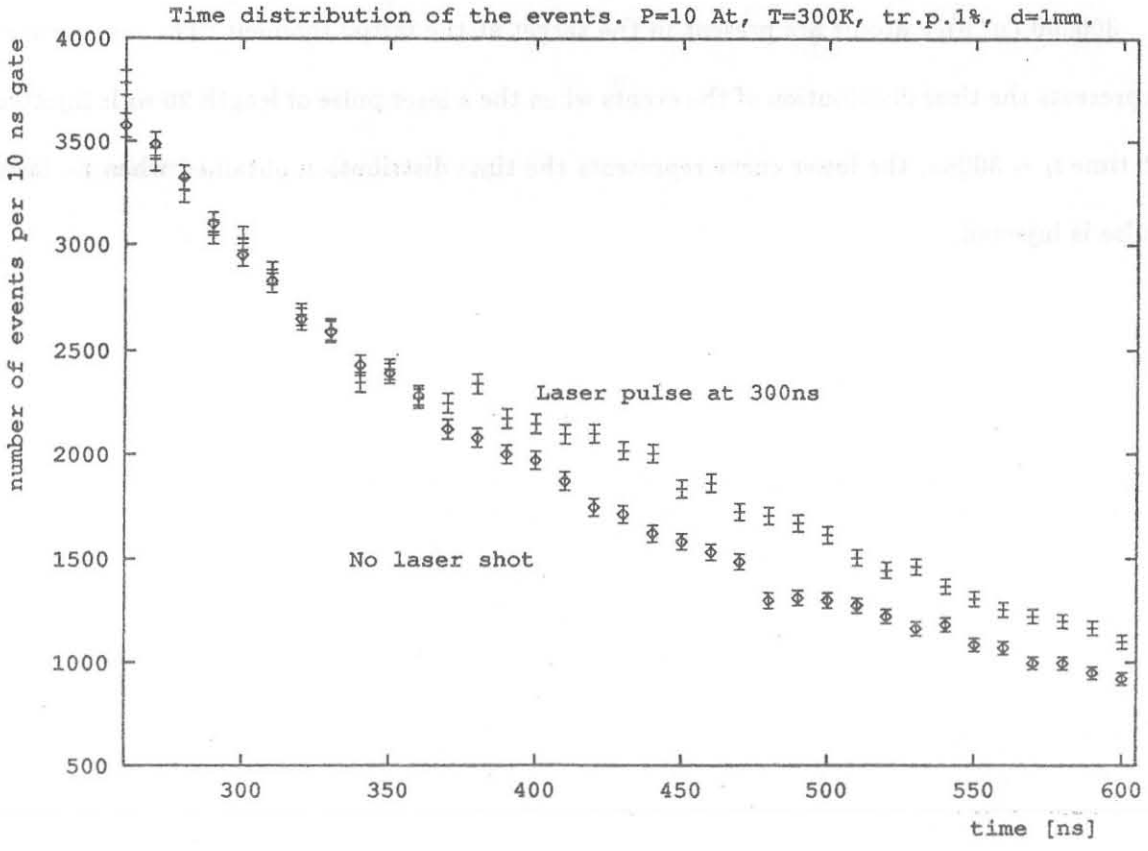
Time distribution of the events. P=10 At, T=300K, tr.p.1%, d=1mm.
number of events per 10 ns gate
time [ns]
Laser pulse at 300ns
No laser shot
4000
3500
3000
2500
2000
1500
1000
500
300
350
400
450
500
550
600

FIG. 2