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DENSITY PARAMETER AT HIGH EXCITATION ENERGIES**

ENERGY DEPENDENCE AND LOCAL SYSTEMATICS OF THE LEVEL
DENSITY PARAMETER AT HIGH EXCITATION ENERGIES

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Abstract

The behaviour of the intrinsic (without collective effects) and effective (with collective effects) level density parameters as an energy function is shown for several typical nuclei (⁵⁴Cr, ⁵⁷Zn, ⁶⁰Zn, ⁶⁶Zn, ⁷¹Ge, ⁷⁸Zn, ⁷⁸Se, ¹¹⁵Cd, ¹²⁸Sb, ¹²⁹Te, ¹³¹I, ¹⁴⁸Pm, ¹⁷³Yb, ¹⁹²Ir, ¹⁹⁸Au and ²⁴⁸Cm). Local systematics of the parameter a_{eff} as a function of the neutron number N, also for nuclei extremely far from the stability line, are shown for some typical nuclei (Rb, Pd, Sn, La, Er, Hf, Os and Hg) at excitation energies of 15, 80 and 150 MeV.

Most of the decay properties of the composite nuclei formed in the heavy-ion reactions can be described in the framework of the statistical theory of nuclear reactions. In such a context the knowledge of the nuclear level densities over a wide range of the excitation energy and in particular the energy dependence of the level density parameter a is of fundamental importance.

The dependence of the level density parameter on the excitation energy U was studied by Ignatyuk *et al.* (1) who proposed the following phenomenological formula

$$a(U) = \bar{a} \left[1 - \frac{f(U) \delta W}{U} \right] \quad (1)$$

It was obtained on the basis of the thermodynamic treatment of shell effects in hot nuclei (2). Here \bar{a} is the asymptotic value of the level density parameter a at high excitation energies, δW is the shell correction in the mass formula, while the function

$$f(U) = 1 - \exp(-\gamma U) \quad (2)$$

determines the energy trend of the parameter a at low energies. This method uses two fit coefficients to determine \bar{a} . In addition it assumes the validity of the $f(U)$ function at the higher energies even though the coefficient γ is determined at the neutron binding energy.

In this paper we use a different formalism (3) to determine the level density at high excitation energy, which does not use any fitting parameters and can be applied to any nucleus, even if experimental data (like neutron resonance spacing or evaporation spectra) are not available. This method uses the shell-model hamiltonian eigenvalues and an exciton distribution to calculate the single-particle state density g near the Fermi energy. The nucleus is considered as a system made up of two non-interacting Fermi gases (proton-gas and neutron-gas) at the same thermodynamic temperature. Therefore, the single-particle state (s.p.s.) density g for the nucleus is

$$g = g_\nu + g_\pi \quad (3)$$

where g_ν and g_π are the s.p.s. densities for the neutron- and proton-gas respectively calculated according to the formula

$$g_\mu = 2 \frac{\int_{U_1}^{U_2} \sum_i \delta(w_\mu - w_{\mu,i}) \phi(w_\mu, \bar{n}_\mu) dw_\mu}{\int_{U_1}^{U_2} \phi(w_\mu, \bar{n}_\mu) dw_\mu} \quad (4)$$

where $\phi(w_\mu, \bar{n}_\mu)$ is the exciton distribution function and μ stands for ν or π .

The knowledge of g allows us to calculate the intrinsic level density parameter ($a_{i\pi\tau} = g\pi^2/6$) and subsequently the level density $\rho_{i\pi\tau}$ related to the intrinsic single particle motion. The level density that accounts for the collective effects is calculated using the relation

$$\rho_{col}(U,J) = \rho_{int}(U,J) K_{rot}(U) K_{vibr}(U) \quad (5)$$

where for the intrinsic level density we use the well-known Bethe expression

$$\rho_{int}(U,J) = \frac{\sqrt{\pi}}{12} \frac{2J+1}{2\sqrt{2\pi} \sigma^3} \frac{\exp\left(2\sqrt{a_{int} U}\right)}{a_{int}^{1/4} U^{5/4}} \exp\left[-\frac{(J+1/2)^2}{2\sigma^2}\right] . \quad (6)$$

In the adiabatic approximation (used up to about 20 MeV) $K_{rot}^{adiab}(U)$ and $K_{vibr}^{adiab}(U)$ are the enhancement coefficients (4) due to the collective motions (rotational and vibrational) and U is the effective excitation energy obtained by subtracting the pairing correction Δ by Gilbert and Cameron (5) from the excitation energy E .

The expression for the vibrational coefficient is

$$K_{vibr}^{adiab}(U) = \exp\left\{1.69 \left[\frac{3m_0 A}{4\pi\sigma_{l.d.}} \frac{C_{l.d.}}{C}\right]^{2/3} t^{4/3}\right\} . \quad (7)$$

In general for a deformed axially symmetric nucleus, the rotational coefficient is given by the following expression

$$K_{rot}^{adiab}(U) = \frac{\sigma_{\perp}^2(U)}{2J+1} \sum_{K=-J}^J \exp\left[-\frac{K^2}{2\sigma_{e,f}^2(U)}\right] \quad (8)$$

where the effective spin cut-off parameter $\sigma_{e,f}$ may be expressed as

$$\sigma_{e,f}^2 = \left[\frac{1}{\sigma_{\parallel}^2} - \frac{1}{\sigma_{\perp}^2}\right]^{-1} . \quad (9)$$

Assuming that $\bar{K}^2 \ll 2\sigma_{e,f}^2$, we can write expression (8) as

$$K_{rot}^{adiab}(U) \simeq \sigma_{\perp}^2(U) . \quad (10)$$

Obviously, the assumption $\bar{K}^2 \ll 2\sigma_{e,f}^2$ must be relaxed for high spins and the calculation must be specialized for a compound nucleus obtained by a particular incoming channel.

At relatively high excitation energies the adiabatic approximation used to estimate the collective enhancement ceases to be valid (through a transition energy region of 20+60 MeV) thus we use a certain function $q(U, \epsilon)$ (6) to damp collective effects in the level density. By this we phenomenologically take into account the coupling of the collective to the intrinsic degrees of freedom due to the nuclear viscosity. To this end the

rotational and vibrational coefficient values are determined by expressions

$$K_{rot}(U) = [K_{rot}^{adlab}(U) - 1] q(U, \epsilon) + 1 \quad (11)$$

$$K_{vibr}(U) = [K_{vibr}^{adlab}(U) - 1] q(U, \epsilon) + 1 \quad (12)$$

where $q(U, \epsilon)$ has the following form

$$q(U, \epsilon) = \exp[-U/U_1] \quad (13)$$

and where

$$U_1 = k A^{1/3} \epsilon^2 \quad (14)$$

while k is determined in such a way that $q(U, \epsilon)$ approaches 0.5 at the excitation energy of about 30 MeV.

Clearly, the total level densities, intrinsic and collective, can be obtained performing a summation over J

$$\rho_{int}(U) = \sum_J (2J+1) \rho_{int}(U, J) = \frac{\sqrt{\pi}}{12} \frac{\exp[2\sqrt{a_{int}} U]}{a_{int}^{1/4} U^{5/4}} \quad (15)$$

$$\rho_{col}(U) = \rho_{int}(U) K_{rot}(U) K_{vibr}(U) \quad (16)$$

After having determined the $\rho_{col}(U)$ using the relation (16) and expressions (11), (12), (13) and (14) one can extract the effective level density parameter a_{eff} (where the collective effects are included) using the relation

$$\rho_{col}(U) = \frac{\sqrt{\pi}}{12} \frac{\exp[2\sqrt{a_{eff}} U]}{a_{eff}^{1/4} U^{5/4}} \quad (17)$$

Using the above-discussed formalism one can calculate the intrinsic (a_{int}) and the effective (a_{eff}) level density parameters as a function of the excitation energy U .

In our calculations we have used the shell-model spacing determined by Nix and Möller (7) and assumed the expression $K_{rot}^{adlab}(U) = \sigma_{\perp}^2(U)$. Figures 1 and 2 show the energy dependence of the ratio a_{int} / \tilde{a} in the 15-100 MeV range for nuclei ^{54}Cr , ^{57}Zn , ^{60}Zn , ^{71}Ge , ^{78}Zn , ^{78}Se , ^{128}Sb , ^{129}Te , ^{131}I , ^{173}Yb , ^{198}Au and ^{248}Cm . As one can observe the energy trend of the ratio a_{int} / \tilde{a} for the ^{71}Ge , ^{78}Zn , ^{78}Se , ^{128}Sb , ^{129}Te and ^{131}I nuclei is different from the one for the ^{54}Cr , ^{57}Zn , ^{60}Zn , ^{173}Yb , ^{198}Au and ^{248}Cm nuclei. This trend reflects the structure of the s.p.s. spectrum near the Fermi level. At the increase of the excitation energy, the numerator in eq. (4) may increase faster than the denominator - this is due to the inclusion of one or more single particle states in the integral so that the resulting g_{μ} (and

consequently also a_μ) increases - in the opposite case the level density parameter a_μ may decrease. This behaviour is difficult to predict and schematize for all nuclei (in various cases of light or heavy nuclei constituted by Z protons and N neutrons that can separately be near or far from the shell closures), but our experience suggests that a behaviour with a strong curvature of the level density parameter a_{int} is usually found for those nuclei having Z or N close to the shell closures.

We determined \bar{a} only to make a qualitative comparison of the behaviour of the ratio a_{int}/\bar{a} with Ignatyuk *et al.* (1) calculations and our results are analogous to the ones they found.

Figures 3 and 4 show the intrinsic (dashed line) and effective (full line) level density parameters (in the 15÷180 MeV excitation energy range) for ^{56}Zn , ^{78}Se , ^{115}Cd , ^{129}Te , ^{148}Pm , ^{173}Yb , ^{192}Ir and ^{248}Cm nuclei, when the non-adiabatic approximation was adopted in the a_{eff} calculation.

In table 1 we report the effective level density parameter a_{exp} and a_{cal} at the neutron binding energy for the above-mentioned nuclei whose experimental data are available (8).

Figures 5 and 6 show a series of the local systematics of the effective level density parameter a_{eff} as a function of the neutron number N , at fixed excitation energies of 15, 80 and 150 MeV, for the isotopes of Rb, Pd, Sn, La, Er, Hf, Os and Hg. For the nuclei with a deformation parameter $\epsilon \leq 0.05$, $K_{rot}^{\alpha\alpha b} = 1$ was assumed. One observes that the shell-closure effects are evident at the magic numbers $N=50, 82$ and 126 at the excitation energy of 15 MeV, while the above mentioned effects are strongly attenuated at 80 MeV and apparently disappear at 150 MeV even if for the Os and Hg nuclei a sharp change in the a_{eff} trend after the shell closure at $N=82$ is observed. In these figures (for $E=15$ MeV) one also observes the effects due to the non-uniform distribution of the eigenvalues for the different isotopes of the same element and the effects due to the different deformations of the various isotopes.

Analyzing the local systematics one is able to extract the trend of the level density parameter a_{eff} for the various isotopes even for those extremely far from the stability line.

Table 1
Comparison of the experimental and calculated values of the effective level density parameters at the neutron binding energy.

Compound nucleus	a_{exp} (MeV) ⁻¹	a_{cal} (MeV) ⁻¹
⁵⁴ Cr	7.34	7.61
⁷¹ Ge	13.54	13.47
⁷⁸ Se	13.01	14.70
¹¹⁵ Cd	20.72	19.35
¹²⁹ Te	20.14	19.50
¹⁴⁸ Pm	22.73	22.63
¹⁷³ Yb	21.06	22.03
¹⁹² Ir	24.55	24.91
¹⁹⁸ Au	19.17	18.01
²⁴⁸ Cm	27.21	26.30

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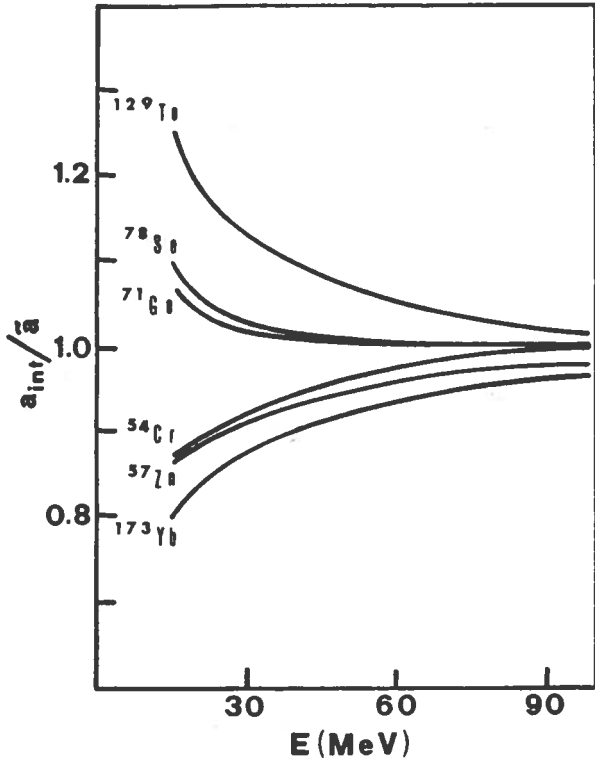


Fig. 1. Energy dependence of the ratio $a_{int}/\tilde{\alpha}$ for ^{54}Cr , ^{57}Zn , ^{71}Ge , ^{78}Se , ^{129}Te and ^{173}Yb nuclei, where a_{int} and $\tilde{\alpha}$ are the internal and asymptotic level density parameter, respectively.

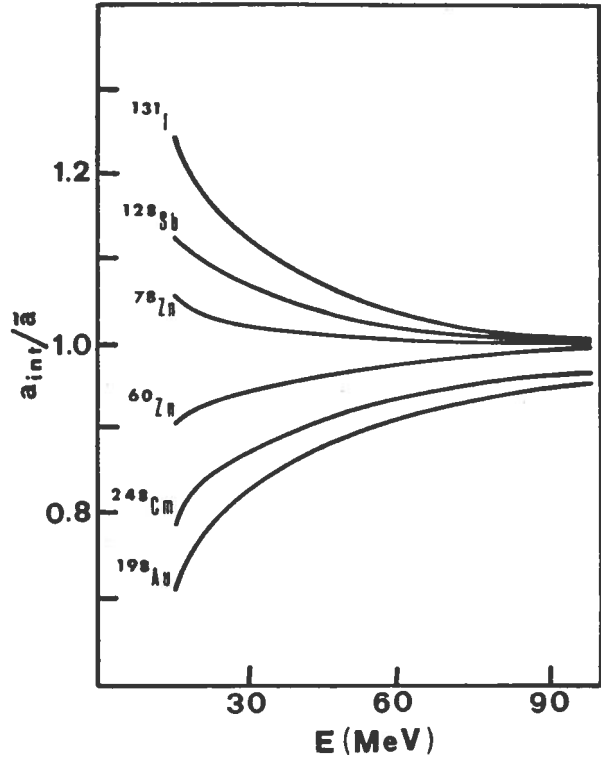


Fig. 2. Same as figure 1 but for ^{60}Zn , ^{78}Zn , ^{128}Sb , ^{131}I , ^{198}Au and ^{248}Cm nuclei.

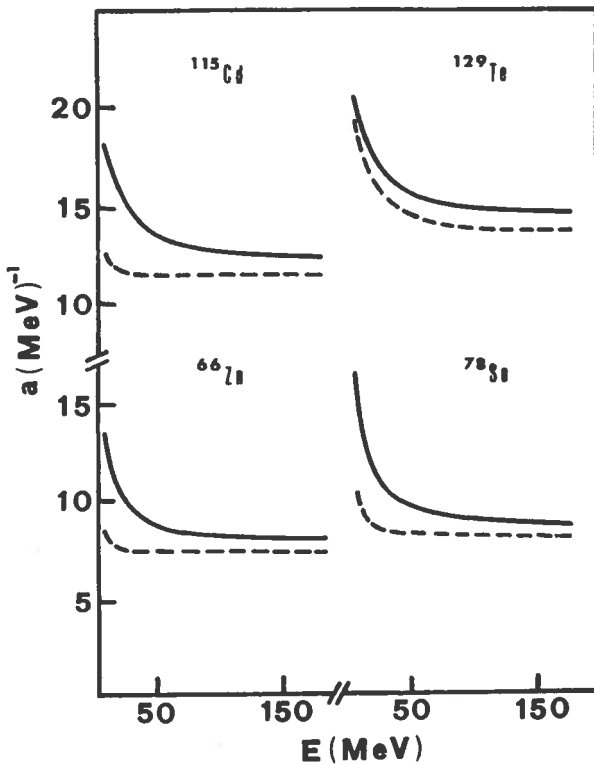


Fig. 3. Energy dependence of the intrinsic and effective level density parameter calculations a_{int} (dashed lines) and a_{eff} (full lines) for ^{66}Zn , ^{78}Se , ^{116}Cd and ^{129}Te nuclei.

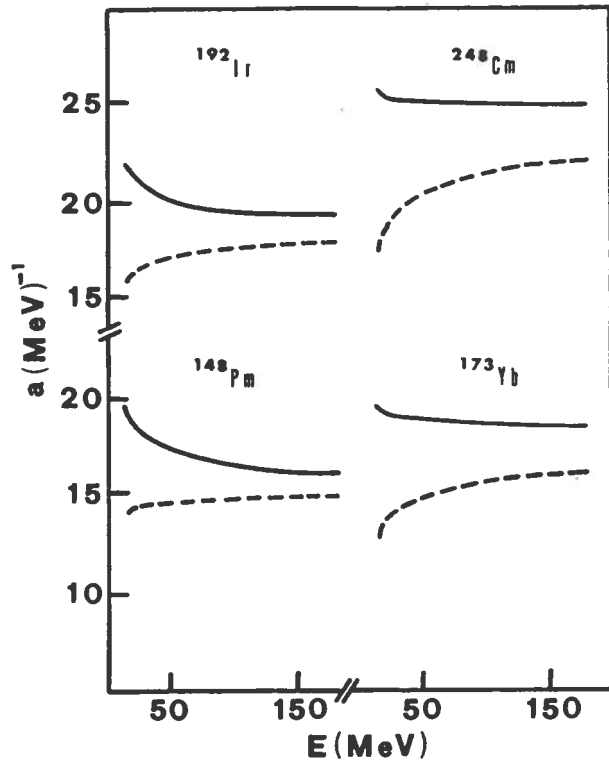


Fig. 4. Same as figure 3 but for ^{148}Pm , ^{173}Yb , ^{192}Ir and ^{248}Cm nuclei.

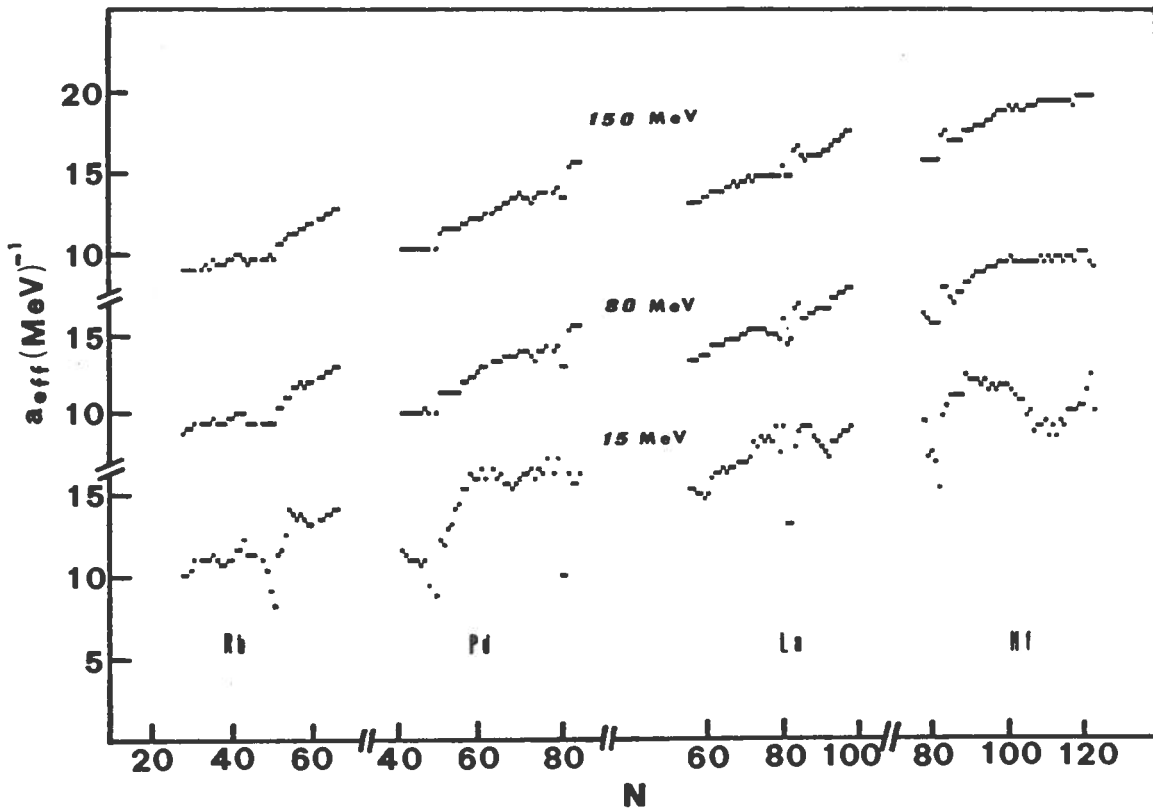


Fig. 5. Local systematics of the effective level density parameter α_{eff} as a function of the neutron number N , at 15, 80 and 150 MeV of the excitation energy, for the isotopes of Rb, Pd, La and Hf nuclei.

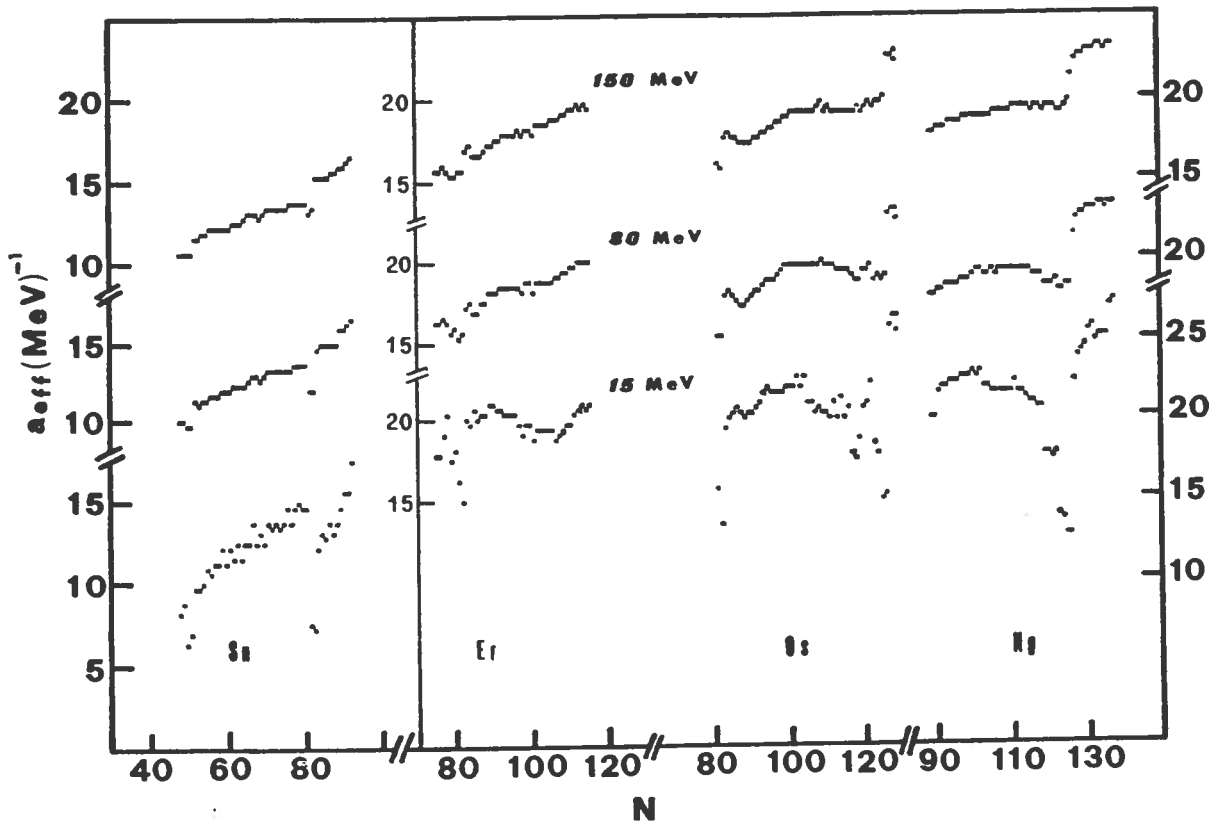


Fig. 6. Same as figure 5 but for the isotopes of Sn, Er, Os and Hg nuclei.