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RECENT DEVELOPMENTS IN THE TIME ANALYSIS OF TUNNELLING PROCESSES.[*]

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ABSTRACT – In this paper we critically review and analyse the main theoretical definitions and calculations of the sub-barrier tunnelling and reflection *times*. Moreover, we propose a new, physically sensible definition of such durations, on the basis of a recent general formalism (already tested for other types of quantum collisions). At last, we discuss some surprising results regarding the temporal evolution of the tunnelling processes.

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1 – INTRODUCTION.

During the last years, *many* attempts were devoted to theoretically defining and calculating the time spent by a sub-barrier–energy particle for tunnelling through a potential barrier; so that critical reviews were already in order. Actually, a first, valuable review article, by Hauge and Stoveneng, did recently appear (at the end of 1989) as ref.[1]. We deem it useful, however, a second review paper since: (i) on one hand, it appears convenient to deepen and extend the criticism to the existing approaches, and (ii) on the other hand, we can show —as a consequence of such a criticism— how a definite proposal can be put forth for introducing suitable, self-consistent, physically meaningful definitions of the tunnelling times: a question that was regarded still as an open problem in ref.[1].

The problem of defining the tunnelling times has a long history, since it arised in the fortieth and fiftieth[2–4], simultaneously with the problem of a general definition in quantum mechanics of the collision durations.

The advent of high-speed experimental devices, based on tunnelling processes in semiconductors, did revive an interest in the whole question;[5,1] whose relevance has always been apparent in nuclear physics: For instance, tunnelling plays an essential role in the physics of nuclear fission and fusion.

As already mentioned, aims of this paper are a comparative analysis of the various, known definitions for the tunnelling durations; and the introduction of a possible, rigorous way for theoretically evaluating the time spent by a particle *inside* a potential barrier: with the final presentation of some peculiarities of the tunnelling–process time evolution. Our starting point will be the formalism in refs.[6,8].

First of all, in Sect.**2**, we critically review the main definitions of the tunnelling times appeared up to now, to the best of our knowledge. Then, Sect.**3** is devoted to the application of our own formalism[6,8] to tunnelling. At last, Sect.**4** is dedicated to the conclusions and to some proposals for further developments.

2 – THE EXISTING DEFINITIONS OF TUNNELLING TIMES.

2.1 – The simplest stationary picture of Tunnelling.

Let us confine ourselves to the ordinary case of particles moving only along the x–direction, and consider a time–independent barrier $V(x)$ in the interval $(0, a)$: see Fig. 1, in which —for later convenience— a larger interval (x_1, x_2), containing the barrier region, is also indicated. We assume the stationary scattering problem to have been solved exactly for every kinetic energy $E = \hbar^2 k^2/2m$, where k is the wave-number and m the particle mass. The wave function $\psi(x; k)$ will have the general form:

$$\begin{aligned} \psi \equiv \psi_{\rm I} = \psi_{\rm in} + \psi_{\rm R} \qquad & \text{for} \quad x < 0, \\ \psi \equiv \psi_{\rm II} \qquad & \text{for} \quad 0 < x < a, \\ \psi \equiv \psi_{\rm III} = \psi_{\rm T} \qquad & \text{for} \quad x > a; \end{aligned} \tag{1}$$

where: $\psi_{\rm in} \equiv e^{ikx}$; $\psi_{\rm R} \equiv A_{\rm R}\, e^{-ikx}$; $\psi_{\rm T} \equiv A_{\rm T}\, e^{ikx}$; the lower indices R,T staying for "reflected" and "transmitted", respectively. In the simple case of a rectangular barrier $[V(x) = V_0]$, it is $\psi_{\rm II} = \alpha e^{-\kappa x} + \beta e^{\kappa x}$ [with $\kappa \equiv \sqrt{2m(V_0 - E)}/\hbar$], where the coefficients [amplitudes] $A_{\rm R}, A_{\rm T}, \alpha$ and β can be analytically calculated, and are known to be

$$\alpha = \frac{2k}{D_+}\,\frac{k + i\kappa}{(k^2 - \kappa^2)D_-/D_+ \,+\, 2ik\kappa}; \quad \beta = \frac{2k}{D_+}\,\frac{-k + i\kappa}{(k^2 - \kappa^2)D_-/D_+ \,+\, 2ik\kappa}\,\exp(-2\kappa a);$$
$$A_{\rm R} = \alpha + \beta - 1; \quad A_{\rm T} = (\alpha e^{\kappa a} + \beta e^{\kappa a})\, e^{-ka}; \quad D_\pm \equiv 1 \pm \exp(-2\kappa a). \tag{2}$$

The amplitudes $A_{\rm R}$, $A_{\rm T}$ satisfy the probability conservation law

$$|A_{\rm R}|^2 \;+\; |A_{\rm T}|^2 \;=\; 1. \tag{3}$$

The flux density

$$j = {\rm Re}[\frac{i\hbar}{2m}\psi(x)\frac{\partial\psi^*(x)}{\partial x}] \tag{4}$$

does not depend on x. Before the barrier it equals the difference $(1 - |A_{\rm R}|^2)$ between the incoming and reflected wave flux–densities. It is less known that, inside the barrier, the fluxes for the separate components of $\psi_{\rm II}$ (exponentially decreasing and increasing, respectively: $\alpha e^{-\kappa x}$ and $\beta e^{\kappa x}$) do vanish. Only their interference does provide the conservation of j.

2.2 - Construction of the wave–packet

For later use in the non-stationary description of actually moving wave–packets, let us consider a wave–packet constructed in terms of the solutions $\psi(x;k)$ of the stationary Schroedinger equation: namely, by integrating over E from 0 to ∞ with a weight-amplitude $g(E - \overline{E})$

$$\Psi(x,t) = \int {\rm d}E\; g(E - \overline{E})\; \psi(x;k)\; \exp(-iEt/\hbar) \tag{5}$$

where we introduced the resolution of the time-evolution operator, with the normalization condition $\int {\rm d}E\; |g|^2 = 1$, quantity $\overline{E}$ being the average kinetic energy.

In the case of a Gaussian wave–packet it is convenient to pass from the energy to the impulse representation, by recalling that ${\rm d}E = (\hbar^2 k/2m){\rm d}k$; when the spread in E of $g(E - \overline{E})$ is much smaller than $\overline{E}$, one easily gets

$$g(E - \overline{E})\; {\rm d}E \approx G(k - \overline{k})\; {\rm d}k \equiv \frac{\hbar^2\overline{k}}{2m}\;\; g(\frac{\hbar^2}{2m}\,[k^2 - \overline{k}^2])\; {\rm d}k, \tag{6}$$

with $G \equiv C \exp[-b(k-\overline{k})^2]$. Of course, the (initial) wave–packet of the incoming waves will have a Gaussian shape also in the configuration space.

By inserting in the integral of eq.(5) the reflected (ψ_R) or transmitted (ψ_T) wave, instead of the total wave ψ, we obtain the *final* reflected or transmitted wave–packets, respectively, carrying a time-delay due to the interaction. Notice that one could expect a distortion in the wave–packet form due to the energy dependence of A_R and A_T; but it has been already shown, for a wide class of weight amplitudes, such a distortion to be negligible.[7] Furthermore, we shall get rid also of the wave components with above-barrier energies by introducing the additional transformation

$$g(E-\overline{E}) \longrightarrow g(E-\overline{E})\,\Theta(E-V_0), \tag{7}$$

in order to avoid distortions of the sub-barrier penetration (tunnelling).

For simplicity's sake, we shall in general address ourselves to quasi-monochromatic packets, for which the energy spread ΔE is so much smaller than $\overline{E}$ that it is possible to adopt the approximation

$$|g(E-\overline{E})|^2 \simeq \delta(E-\overline{E}) \tag{8}$$

in all the final expressions for our quantities, when averaged over $\rho\,\mathrm{d}x$ or $J\,\mathrm{d}t$; where

$$\rho \equiv |\Psi(x,t)|^2; \quad J \equiv \mathrm{Re}[\frac{i\hbar}{2m}\Psi(x,t)\frac{\partial\Psi^*(x,t)}{\partial x}] \tag{9}$$

are the probability density [for a particle to be located in the unitary space–interval centered at x] and the probability–density flux [for a particle to pass through position x during the unitary time–interval centered at t], respectively.

2.3 – The ordinary phase–times

Following the usual procedure, introduced in[2–4], it is easy to get the ordinary phase–times for quasi-monochromatic packets, in the stationary–phase approximation:

$$\tau_T^{Ph}(x_i, x_f;\ E) = \frac{1}{v}\,(x_f - x_i) + \hbar\,\frac{\mathrm{d}(\arg A_T + ka)}{\mathrm{d}E} \tag{10}$$

and

$$\tau_R^{Ph}(x_i, x_i; E) = \frac{1}{v}\,(-2x_i) + \hbar\,\frac{\mathrm{d}(\arg A_R)}{\mathrm{d}E} \tag{11}$$

where $v \equiv \hbar k/m$ is the group–velocity. Eqs.(10) and (11) refer to a transmitted [from the initial position x_i to the final position x_f] particle and to a reflected [from the initial position x_i to the same position] particle, respectively; cf., *e.g.*, [1].

For a rectangular barrier with height V_0, the phase–times (10) and (11), when linearly extrapolated[1] to the barrier region ($x_i = 0$; $x_f = a$) would become

$$\tau_{\rm T}^{\rm Ph}(0,a;E) = \tau_{\rm R}^{\rm Ph}(0,a;E) = \frac{m}{\hbar k \kappa D} \, [2\kappa a k^2(\kappa^2 - k^2) + k_0{}^4 {\rm Sinh}(2\kappa a)] \ , \qquad (12)$$

which, for $\kappa a >> 1$, would simply yield $2/v\kappa$. In eqs.(12), it is $D \equiv 4\kappa^2 k^2 + k_0{}^4 {\rm Sinh}^2(\kappa a)$; and $k_0 \equiv 2mV_0/\hbar$. In other words,[7] for sufficiently wide —*i.e.*, opaque— (or high) barriers, eqs.(12) do not depend on the barrier width a, and the effective tunnelling–velocity $a/\tau^{\rm Ph}$ may become arbitrarily large [Hartmann and Fletcher effect[9,10]].

One of the main objections against extrapolations (12) is that they do not describe the actual asymptotic behaviour of the phase–times; since they disregard the fact that both the [magnitude of the] initial packet mean–position, $|x_{\rm i}|$, and quantity $x_{\rm f} - a$ (where $x_{\rm f}$ is the transmitted packet mean–position) must be large with respect to the packet spatial extension [of the order of $\hbar v/\Delta E$], in order to avoid "interference" effects between physically quite different processes (*i.e.*, between incident and reflected waves).

Therefore, it is not quite correct to attribute to the extrapolated phase–times the physical meaning of "times spent in the barrier region (= inside the barrier)". Moreover, one cannot separate in $\tau_{\rm T}^{\rm Ph}$ and $\tau_{\rm R}^{\rm Ph}$ the self-interference delays from the time spent inside the barrier.

Before going on, let us clarify the behaviour of the phase–times at the very *top* of the barrier, and check whether there is any continuity —there— between the values of the sub-barrier tunnelling time and those for the above-barrier case. Let us compare eqs.(12) with the following expression for the above-barrier transmission time:

$$\tau_{\rm T}^{\rm Ph}(0,a;E>V_0) = \frac{2m}{\hbar k q} \; \frac{-(k^2-q^2)^2 \tan(qa) + 4qak^2(k^2+q^2)/\cos^2(qa)}{4k^2q^2 + [(k^2+q^2)\tan(qa)]^2} \qquad (13)$$

which was obtained(*), by the stationary–phase method, for the case of a rectangular barrier. In such a case, it is $\psi_{\rm II} = \gamma e^{iqx} + \delta e^{-iqx}$ with $q \equiv \sqrt{2m(E-V_0)}/\hbar$, and the coefficients γ and δ are analytically evaluable. By comparing eqs.(12) and (13) one gets

$$\lim_{\kappa\to 0} \tau_{\rm T}^{\rm Ph}(0,a;E>V_0) = \frac{mka^3}{6\hbar(1+k^2a^2/4)} \;\; \overrightarrow{_{a\to\infty}} \;\; \frac{2ma}{3\hbar k} \ , \qquad (12')$$

$$\lim_{\kappa\to 0} \tau_{\rm T}^{\rm Ph}(0,a;E<V_0) = \frac{mka^3}{6\hbar(1+k^2a^2/4)} \;\; \overrightarrow{_{a\to\infty}} \;\; \frac{2ma}{3\hbar k} \ . \qquad (13')$$

In other words, we find that that the two limits (12'), (13') do coincide with each other, and linearly depend on a for "opaque" barriers (provided that the condition $\kappa a \to 0$ holds). Notice that such a result does not contradict the Hartmann and Fletcher effect, since the latter takes place only when $\kappa a \to \infty$, while it is absent for finite values of κa.

(*) These calculations have been explicitly performed by V.S. Sergeyev.

2.4 – The dwell time

The total scattering time duration has been defined in[11] as the probability for the particle to be localized in the interval between the initial (maybe, source) position and the final (maybe, detector) position, divided by the incident particle flux density; that is to say, as the time spent by a particle while travelling inside such space–interval: the so–called *dwell time*. In the chosen case of particles moving only along x, the dwell time is therefore defined as[12]:

$$\tau^{\rm Dw}(x_{\rm i}, x_{\rm f}; k) = \frac{1}{v} \int_{x_{\rm i}}^{x_{\rm f}} {\rm d}x \; |\psi(x;k)|^2. \tag{14}$$

For a rectangular barrier, the (dwell) time spent inside the barrier becomes:[12]

$$\tau^{\rm Dw}(0, a; k) = \frac{mk}{\hbar \kappa D} \; [2\kappa a(\kappa^2 - k^2) + \frac{2mV_0}{\hbar^2} {\rm Sinh}(2\kappa a)] \tag{15}$$

which, for $\kappa a >> 1$, would give $\hbar k / \kappa V_0$. The results (12) and (15) are in sharp contrast with each other with regard to the k-dependence. Let us comment on this point.

The stationary definition (14) for the dwell time, according to us, is not self-consistent from its very beginning, and appears to be in contradiction with its physical meaning. In fact, the time variable is firstly discarded (in passing from the time–dependent to the stationary Schroedinger equation), and later on it is re-introduced in an artificial, *ad hoc* way: namely, through the introduction of a localization–probablility expressed in terms of time–independent wave functions, instead of actually moving wave–packets.

Moreover, even the modified "dwell time approaches" with time–dependent wave functions[13–15], in which

$$\bar{\tau}^{\rm Dw}(x_{\rm i}, x_{\rm f}; k) = \frac{\int_{-\infty}^{\infty} {\rm d}t \int_{x_{\rm i}}^{x_{\rm f}} {\rm d}x \; \rho(x,t)}{\int_{-\infty}^{\infty} {\rm d}t \; J_{\rm in}(x_{\rm i}, t)} \quad , \tag{16}$$

do still contain *formal* time–averages that are not actual averages over the physical time (*i.e.*, the time $t(x)$ at which the considered particle passes through the position x). In fact —at least when the direction of flux J is time–independent— by integrating $J{\rm d}t$, and not $\rho {\rm d}t$.[6,16] In eq.(16) quantity $J_{\rm in}$ is defined as in eq.(9), just replacing $\psi(x;k)$ of eq.(5) by $\psi_{\rm in}(x;k) \equiv e^{ikx}$. At last, the "dwell–time approaches" are unable[6] to define and study the time *distributions* for any kind of collision process.

2.5 – The local Larmor times

In[17] a *gedanken experimente* was proposed for measuring the scattering duration as the ratio θ/ω, where θ is the angle by which the magnetic moment μ of the considered particle due to a small homogeneous magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ (directed along z) supposedly present in the scattering region, and $\omega \equiv 2\mu B_0/\hbar$ is the Larmor precession frequency. For a magnetic field existing in the interval (x_i, x_f), and for an incident particle (moving along x and) with spin $\frac{1}{2}$ polarized along the x-direction (see Fig. 2), θ results to be proportional to the average spin component $< s_y >$: namely, $\theta = -2 < s_y >_T /\hbar$, or $\theta = -2 < s_y >_R /\hbar$, for the transmitted or reflected waves, respectively. In this case, the Larmor times $\tau^{La}_{yT}(x_i, x_f; k)$ and $\tau^{La}_{yR}(x_i, x_f; k)$ become equal to the Phase Times, *plus* —however— terms which oscillate as kx_i and kx_f vary.[1] In the particular case of a rectangular barrier

$$\tau^{La}_{yT}(0, a; k) = \tau^{La}_{yR}(0, a; k) = \tau^{Dw}(0, a; k). \tag{17}$$

In Baz's approach, as it was shown in refs.[6], the expressions for the collision duration [*e.g.*, eqs.(17)] are artificially distorted by the sharp boundaries attributed to the magnetic–field region; in other words, are influenced by the mathematical, rather than physical, assumptions. Actually, the oscillating terms do depend on the kind of [for instance, smoothed] boundary that one adopts. Moreover, both those oscillating terms vanish when $x_i \to -\infty$ and $x_f \to \infty$, once one does average over the incident particle energy–spread; so that the final expressions do coincide with the Phase Times.

It it also known[1] that the mathematical behaviour assumed for the magnetic–fiels boundary does not influence only the spin components along y. In fact (see Fig. 2), the incident particle has finite probabilities of being spin-up or spin-down along the field–direction z. As pointed out in ref.[12], the spin-up components will be preferentially *transmitted* (except when $\mathrm{d}|A_T|^2/\mathrm{d}E < 0$): so that one gets the noticeable result that $< s_z >_T \;\; >> \;\; < s_y >_T$. On the basis of what precedes, Büttiker in ref.[12] introduced the new Larmor times: $\tau^{La}_{zT}(x_i, x_f; k)$ and $\tau^{La}_{zR}(x_i, x_f; k)$, both defined analogously to quantities τ^{La}_{yT} and τ^{La}_{yR}, respectively; as well as the *hybrid* Larmor times, defined as follows:

$$(\tau^{La}_{T,R})^2 = (\tau^{La}_{yT,R})^2 + (\tau^{La}_{zT,R})^2. \tag{18}$$

However, the introduction of so many time durations for a single collision (*e.g.*, trasmission and reflection processes) seems to us physically unjustified.

Let us notice that, for an *opaque* rectangular barrier, we obtain

$$\tau^{La}_{zT}(0, a; k) \simeq \frac{ma}{\hbar\kappa} \; , \tag{19}$$

which results to be different from both the extrapolated Phase Times (12') and the Dwell Times (15).

2.6 – A complex time approach

It is known, a formal generalization of the classical time spent by a particle inside the barrier can lead, for $E < V_0$ (when the actual presence of the particle, there, is forbidden by classical mechanics), to the introduction of a complex time. More generally, an analogous extension of classical time *to the quantum domain* has been recently proposed in[18] (see also, *e.g.*, refs.[19–21], and refs. therein). For one–dimensional motion, following ref.[18], one of the natural "quantum generalizations" of the classical expression

$$\tau[P(t)] = \int_{t_i}^{t_f} \mathrm{d}t \int_V \mathrm{d}x \;\; \delta[x - P(t)] \tag{20}$$

for the time spent inside a region V —where $P(t)$ is the classical path going from $x_i(t_i)$ to $x_f(t_f)$— is the path–integral average

$$\tau^{\mathrm{Qu}}(x_i, t_i; x_f, t_f) \; = \; < \tau[P(\;\;)] >_{\mathrm{paths}} \;\; , \tag{21}$$

in which $P(\;\;)$ is any arbitrary path between the given end–points.

For the process relative to Fig. 1 and to eq.(1), one has:[5,19]

$$\tau_{\mathrm{T}}^{\mathrm{Qu}} = i\hbar \int_V \mathrm{d}x \; \frac{\delta \log A_{\mathrm{T}}}{\delta \Omega(x)} \;\; ; \qquad \tau_{\mathrm{R}}^{\mathrm{Qu}} = i\hbar \int_V \mathrm{d}x \; \frac{\delta \log A_{\mathrm{R}}}{\delta P(x)} \;\; , \tag{22}$$

where V is nothing but the interval (x_i, x_f) [or, in particular, $(0, a)$]; and $\delta / \delta P(x)$ is a functional derivative. In general, quantities $\tau_{\mathrm{T,R}}^{\mathrm{Qu}}$ are complex; and are connected with the Larmor Times by the relations

$$\mathrm{Re}\, \tau_{\mathrm{T,R}}^{\mathrm{Qu}} = \tau_{y\mathrm{T,R}}^{\mathrm{La}} \; ; \qquad \mathrm{Im}\, \tau_{\mathrm{T,R}}^{\mathrm{Qu}} = -\tau_{z\mathrm{T,R}}^{\mathrm{La}} \;\; . \tag{23}$$

Of course, complex time is a useful theoretical tool; even if the ordinary tunnelling–times should be real. The physical meaning of the imaginary part is still controversial.[22]

2.7 – The Büttiker–Landauer time

In refs.[23] the tunnelling times were studied via a new kind of "gedanken experimente", namely by supposing the barrier to possess, besides the ordinary (time–independent) part, an additional part oscillating in time:

$$V(t) = V_0 + V_1 \cos \omega t \;\; . \tag{24}$$

Since the potential V varies with time, the incident particles —if endowed with electric charge (or magnetic moment)— can absorb or emit "modulation quanta" $\hbar\omega$ during the tunnelling, which leads to the appearance of *sidebands* with energies $E + n\hbar\omega$; $[n = \pm 1, \pm 2, ...]$. In the first–order approximation in V_1, it is enough to consider only the neighbouring sidebands with energies $E \pm \hbar\nu$. Büttiker and Landauer did obtain the following expressions for the relative sideband intensities

$$I_{\rm T}^{(\pm1)}(\omega) \;=\; |\,\frac{A_{\rm T}^{(\pm1)}(\omega)}{A_{\rm T}^{(0)}}\,|^2 \simeq\; [\frac{V_1}{2\hbar\omega}\;\exp(\pm\omega\tau_{\rm T}^{\rm BL}) - 1]^2\;,$$

$$I_{\rm R}^{(\pm1)}(\omega) \;=\; |\,\frac{A_{\rm R}^{(\pm1)}(\omega)}{A_{\rm R}^{(0)}}\,|^2 \simeq\; (\frac{V_1\tau_{\rm R}^{\rm BL}}{2\hbar})^2(1\pm\omega\tau_\kappa)\;, \tag{25}$$

where $A_{\rm T}^{(\pm1)}$ and $A_{\rm T}^{(0)}$ are the perturbed (sideband) and unperturbed transmission–amplitudes, respectively; and similarly for the reflection–amplitudes $A_{\rm R}^{(\pm1)}$ and $A_{\rm R}^{(0)}$. In eqs.(25) the last equalities ($\simeq$) hold only for the case of *opaque* [rectangular] barriers and not too high frequencies: *i.e.*, for $\hbar\omega$ small with respect to both E and $V_0 - E$. Moreover, $\tau_{\rm T}^{\rm BL} \equiv ma/\hbar\kappa$; $\tau_{\rm R}^{\rm BL} \equiv 2mk/[\hbar\kappa(\kappa^2+k^2)]$; and $\tau_\kappa \equiv m/\hbar\kappa^2$. One can see that $\tau_{\rm T}^{\rm BL}$ is identical to the Larmor time $\tau_{z{\rm T}}^{\rm La}$ as given in eq.(19).

In our opinion, it is not worthwhile to report about the discussions originated by Büttiker–Landauer's approach, since they seem to us to be too technical and unsufficiently justified; let us only quote, here, the refs.[24-27] However, two results should be mentioned.

First, Hauge and Stovneng[1] did find a simple connection between, on one side, the $\omega \to 0$ limits of $A_{\rm T}^{(\pm1)}(\omega)/A_{\rm T}^{(0)}$ and $A_{\rm R}^{(\pm1)}(\omega)/A_{\rm R}^{(0)}$, and, on the other side, the complex times of eqs.(23):

$$\frac{A_{\rm R}^{(\pm1)}(\omega)}{A_{\rm R}^{(0)}} = -i\,\frac{V_1}{2\hbar}\,\tau_{\rm T}^{\rm Qu}\;; \qquad \frac{A_{\rm T}^{(\pm1)}(\omega)}{A_{\rm T}^{(0)}} = -i\,\frac{V_1}{2\hbar}\,\tau_{\rm R}^{\rm Qu}\;, \tag{26}$$

even if the physical meaning of such a connection is not yet very clear.

Second, it is interesting to recall that Bruinsma and Bak[28] (see also ref.[1]) proposed the characteristic frequency $(\tau_{\rm T}^{\rm BL})^{-1} \equiv \hbar\kappa/ma$ to give information about the *coupling* between tunnelling and other accompaning channels, rather than about the intrinsic tunnelling times.

3 – ABOUT THE POSSIBILITY OF INTRODUCING CLEAR, UNIVOCAL DEFINITIONS OF $\tau_{\rm T}$ AND $\tau_{\rm R}$.

3.1 – A comment on Hauge and Stovneng's conclusions.

After having reviewed all the main definitions and evaluations of the tunnelling times (which we also have presented, and criticized, in Sect.2), the authors of ref.[1] concluded that no definite, acceptable approach still exists to calculating such tunnelling durations. As a necessary but not sufficient condition, to be obeyed by any physically acceptable expression of the tunnelling and reflection times $\tau_{\rm T}$ and $\tau_{\rm R}$, those authors did propose the following relation

$$\tau^{\rm Dw} = |A_{\rm T}|^2\tau_{\rm T} \;+\; |A_{\rm R}|^2\tau_{\rm R}\;, \tag{27}$$

which they required to be satisfied by the durations calculated via *any* method (besides the dwell one, of course, which *ab initio* does not separate transmission from reflection time). Let us observe that the negative conclusion of ref.[1], which is actually the main conclusion of that review, is based not only on a criticism of all the previosly existing approaches (a criticism that we made more complete and even stronger), but also on the fact that none of them satisfies condition (27).

However, relation (27) is unacceptable as a general criterion, since it attributes a special role (and meaning) to the Dwell Time $\tau^{\rm Dw}$, which on the contrary does *not* possess —in general— the physical meaning of global collision–duration, as we showed in Sect.2.4.

In the following Section, we are going to show that it is possible to define (and calculate) —in a physically meaningful and self-consistent way— those durations $\tau_{\rm T}$ and $\tau_{\rm R}$.

3.2 – A direct, general definition of the collision durations; and Applications to Tunnelling

A direct, general definition of the collision durations was put forth first by Ohmura[29], and then improved —and generalized for finite distances— by us.[6,8] Following refs.[6], the transmission and reflection durations $< \tau_{\rm T} >$, $< \tau_{\rm R} >$ (averaged over the corresponding flux densities) can be defined, in the considered case of one–dimensional motion in presence of a barrier, as follows:

$$
\begin{aligned}
< \tau_{\rm T} > \; &\equiv \; < t(x_{\rm f}) >_{\rm T}^{\rm III} - < t(x_{\rm i}) >_{\rm in} = \\
&= \frac{\int_{-\infty}^{\infty} {\rm d}t \; t \; J_{\rm T}^{\rm III}(x_{\rm f}, t)}{\int_{-\infty}^{\infty} {\rm d}t \; J_{\rm T}^{\rm III}(x_{\rm f}, t)} - \frac{\int_{-\infty}^{\infty} {\rm d}t \; t \; J_{\rm in}(x_{\rm i}, t)}{\int_{-\infty}^{\infty} {\rm d}t \; J_{\rm in}(x_{\rm i}, t)} = \\
&= \frac{\int_0^{\infty} {\rm d}E \, v \, |g A_{\rm T}|^2 \; \tau_{\rm T}^{\rm Ph}(x_{\rm i}, x_{\rm f}, \; E)}{\int_0^{\infty} {\rm d}E \, v \, |g A_{\rm T}|^2} \equiv (x_{\rm f} - x_{\rm i}) < v^{-1} > + < \Delta\tau_{\rm T} > ; \quad (28)
\end{aligned}
$$

$$
\begin{aligned}
< \tau_{\rm R} > \; &\equiv \; < t(x_{\rm i}) >_{\rm R}^{\rm II} - < t(x_{\rm i}) >_{\rm in} = \\
&= \frac{\int_{-\infty}^{\infty} {\rm d}t \; t \; J_{\rm R}^{\rm II}(x_{\rm i}, t)}{\int_{-\infty}^{\infty} {\rm d}t \; J_{\rm R}^{\rm II}(x_{\rm i}, t)} - \frac{\int_{-\infty}^{\infty} {\rm d}t \; t \; J_{\rm in}(x_{\rm i}, t)}{\int_{-\infty}^{\infty} {\rm d}t \; J_{\rm in}(x_{\rm i}, t)} = \\
&= \frac{\int_0^{\infty} {\rm d}E \, v \, |g A_{\rm R}|^2 \; \tau_{\rm R}^{\rm Ph}(x_{\rm i}, x_{\rm i}, \; E)}{\int_0^{\infty} {\rm d}E \, v \, |g A_{\rm R}|^2} \equiv 2|x_{\rm i}| \; < v^{-1} > + < \Delta\tau_{\rm R} > \; , \quad (29)
\end{aligned}
$$

which hold when the incoming, reflected and transmitted wave–packets do not interfere: *i.e.*, are totally separated in space–time. Quantity $g \equiv g(E - \overline{E})$ was defined in eqs.(5), (6); while the ordinary Phase Times $\tau_{\rm T}^{\rm Ph}$, $\tau_{\rm R}^{\rm Ph}$ have been defined in eqs.(10) and (11). Moreover, quantities $J_{\rm R}^{\rm II}$ and $J_{\rm T}^{\rm III}$ are defined as in eq.(9), just replacing $\psi(x, k)$ of eq.(5) by $\psi_{\rm R} \equiv A_{\rm R} \; e^{-ikx}$ and $\psi_{\rm III} \equiv \psi_{\rm T} \equiv A_{\rm T} \; e^{ikx}$, respectively. Let us stress that our equations (28), (29) do implicitly *define* also the time delays $< \Delta\tau_{\rm T} >$, $< \Delta\tau_{\rm R} >$

due to transmission and reflection, respectively; as well as the "average" instants $< t(x_f) >_T^{III}$, $< t(x_i) >_R^{II}$, $< t(x_i) >_{in}$ at which the corresponding wave–packets [transmitted, reflected and initial, respectively] pass through point x_f or x_i.

Notice that for quasi–monochromatic wave packets, *i.e.*, when approximation (8) holds, eqs.(28), (29) do directly yield the ordinary Phase–Times τ_T^{Ph}, τ_R^{Ph}, given in eqs.(10), (11).

However, when x_i, x_f are not far from the barrier, then it happens that the incoming, reflected and transmitted wave–packets *can* interfere. Moreover, the flux density $J(x,t)$ does in general change its sign with time; for example, the sign of $J(0,t)$ does change from + into − approximately a time $\hbar\ \mathrm{d}(\arg A_R)/\mathrm{d}E$ after the arrival at $x = 0$ of the initial wave–packet. Therefore, the integrals $\int_{-\infty}^{\infty} \mathrm{d}t\ t\ J(x,t)$ do represent in general the algebraic sum of positive and negative quantities, so that the probability densities

$$\frac{J(x,t)\ \mathrm{d}t}{\int_{-\infty}^{\infty} \mathrm{d}t\ J(x,t)}$$

are not positive definite and do not possess a direct physical sense.

Each probability density acquires a physical meaning only during those (partial) time–intervals in which the corresponding flux–density $J(x,t)$ does *not* change its direction. As a consequence, the previous integrals are to be split into varios integrals, each one carried over a partial time–interval such that during it the sign of $J(x,t)$ is only positive, or only negative. Afterwards, one will sum over all such contributions.

In other words, we have to deal only with the *positive definite* probability densities

$$\frac{\mathrm{d}t\ J_+(x,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\ J_+(x,t)} \quad \text{and} \quad \frac{\mathrm{d}t\ J_-(x,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\ J_-(x,t)}\ ,$$

where J_+ and J_- represent the positive and negative values of $J(x,t)$, respectively.

Therefore, we do *propose* as physically adeguate definitions for the average *transmission time* and the average *reflection time* the following expressions:

$$< \tau_T > \ \equiv\ < t(x_f) >_+ - < t(x_i) >_+ =$$
$$= \frac{\int_{-\infty}^{\infty} \mathrm{d}t\ t\ J_+(x_f,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\ J_+(x_f,t)} - \frac{\int_{-\infty}^{\infty} \mathrm{d}t\ t\ J_+(x_i,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\ J_+(x_i,t)}\ ; \tag{30}$$

$$< \tau_R > \ \equiv\ < t(x_i) >_- - < t(x_i) >_+ =$$
$$= \frac{\int_{-\infty}^{\infty} \mathrm{d}t\ t\ J_-(x_i,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\ J_-(x_i,t)} - \frac{\int_{-\infty}^{\infty} \mathrm{d}t\ t\ J_+(x_i,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\ J_+(x_i,t)}\ . \tag{31}$$

Let us notice that, when $x_f \geq a$ and $x_i \to -\infty$, equation (30) goes into equation (28) since in that case $J_+(x_f,t) = J_T(x_f,t) \equiv J(x_f,t)$ and $J_+(x_i,t) = J_{in}(x_i,t)$. Analogously, when $x_i \to -\infty$, equation (31) goes into equation (29) since in such a case

$J_+(x_i,t) = J_{in}(x_i,t)$ and $J_-(x_i,t) = J_R(x_i,t)$.

What precedes, and in particular eqs.(30), (31), lead us to adopt as suitable, strict definitions for the very *Tunnelling Time* and the Reflection Time at the barrier–front (or *To-and-Fro Time*) the following ones:

$$< \tau_{\rm tun} > \;\equiv\; < t(a) >_+ - < t(0) >_+ \;, \tag{32}$$

$$< \tau_{\rm to-fro} > \;\equiv\; < \tau_{\rm R}(x_{\rm i}=0) > \;\equiv\; < t(0) >_- - < t(0) >_+ \;, \tag{33}$$

where one should recall that the barrier starts at the point $x = 0$.

According to us, eqs.(32) and (33) are the correct definitions for the "Tunnelling time" $< \tau_{\rm tun} >$ and the "Reflection-due-to-the-whole-barrier time" (*i.e.*, the Reflection time at the barrier front wall) $< \tau_{\rm to-fro} >$. In conclusion, at variance with the authors of review[1], we think that a positive answer *can* be given to their question about the possibility of a precise, meaningful, univocal definition of the Tunnelling and Reflection times; such an answer being given by our equations (30)–(33).

Unfortunately, simple analytical expressions for those time–durations *in the energy representation* exist only in particular, limiting cases. In general, even for Gaussian or quasi–monochromatic wave packets, calculations can be performed only numerically. Anyway, eqs.(30)–(33) can be qualitatively tested in an easy way.

We are left with the question of the time evolution of wave–packets *inside the barrier*: a problem which till now was paid attention to only in ref.[7]. We shall examine it in the coming Section.

3.3 – Time evolution of the tunnelling wave–packets *inside* the barrier

In ref.[7] calculations were performed of $\rho(x,t)$ and $J(x,t)$, at different points x inside the barrier, for a Gaussian wave–packet with an energy spread $\Delta E = 0.025\ E$. The results of those calculations are presented in Fig. 3, for $E = \frac{1}{2}V_0$ and $\kappa a = 5/\sqrt{2}$. From it, one can see that the times $\tau_\rho(x)$, $\tau_{J+}(x)$ and $\tau_{J-}(x)$, taken by the maximum of $\rho(x,t)$, $J_+(x,t)$ and $|J_-(x,t)|$, respectively, to penetrate the barrier till the depth $\Delta x = x$, do *not* depend *linearly* on x; and that $J(x,t)$ —inside the barrier— *does change* its sign with time, not very far from the barrier forward wall ($0 \leq x < 0.6\ a$).

It is worthwhile to notice that: (i) although the continuity equation $\partial\rho/\partial t + \partial J/\partial x = 0$ goes on holding inside the barrier, nevertheless the equality $J = v\rho$ (which is valid for quasi-monochromatic wave packets outside the barrier) is *not* valid —not even approximately— inside the barrier; and that: (ii) the effective velocities $v_\rho \equiv ({\rm d}\tau_\rho/{\rm d}x)^{-1}$, $v_{J+} \equiv ({\rm d}\tau_{J+}/{\rm d}x)^{-1}$ and $v_{J-} \equiv ({\rm d}\tau_{J-}/{\rm d}x)^{-1}$ of the maximum of ρ, J_+ and $|J_-|$, respectively, not only are *non-constant* as x varies, but also *do not coincide* with each other.

Passing to the *mean velocity* of the wave–packet while tunnelling through *the whole* barrier, it can be defined in a natural way as follows:

$$\overline{v_{\rm tun}} \;\equiv\; \frac{a}{< \tau_{\rm tun} >} \,. \tag{34}$$

Let us explicitly notice that, if $<\tau_{\rm tun}>$ does not increase with a, the "effective" speed $\overline{v_{\rm tun}}$ may become arbitrarily large. This would actually happen when the tunnelling time can be expressed by the ordinary Phase Time: $<\tau_{\rm tun}>=\tau_{\rm T}^{\rm Ph}(0,a;E)$; cf. Sect.**2**.3.

Moreover, we can show that even *in general*, for rectangular barriers and large values of a, quantity $<\tau_{\rm tun}>$ does not depend practically on a. In fact, it is obviously $<t(a)>_{\rm T}^{\rm III}=<t(a)>_{+}$; so that the corresponding terms, for $x_{\rm f}=a$, become equal in eq.(32) and in eqs.(28). Therefore, the same will happen —for quasi-monochromatic wave packets— for the corresponding term included in eqs.(12). As a consequence, the term $<t(a)>_{+}$ does not depend on a for *opaque* barriers. As to the second term, $<t(0)>_{+}$, of eq.(32), it differs from $<t(0)>_{\rm in}$ of eqs.(28) [owing to the effect of interference between incoming and reflected waves in the flux $J_{+}(x_{\rm i}=0,t)$] by a quantity that depends on the reflection time $\tau_{\rm R}^{\rm Ph}(0,a;E)$, on the wave packet time–extension [of the order of $\hbar/\Delta E$], and on the form of the wave–packet. Consequently, also such a "second" term does not depend on a for opaque barriers. We can conclude that the Hartmann–Fletcher's effect is valid even for our definition (32).

It is perhaps worthwhile to add the following observations. The arriving, initial wave–packet does interfere with the reflected waves, that start to be generated as soon as the packet forward tail reaches the barrier edge; in such a way that (when considering the profiles of fluxes $J(x,t)$ before the barrier) the backward tail of $J_{\rm in}$ decreases —for destructive interference with $J_{\rm R}$— in a larger degree than the forward one. This simulates an increase of the average speed of the entering–flux profile, $J_{+}(x,t)$. Hence, the term $<t(x)>_{+}^{\rm I}$ decreases for negative $x\approx 0$. In other words, the effective (average) flight–time of the approaching packet from the source to the barrier does decrease. Let us now consider what happens inside the barrier, for positive $x\approx 0$. An analogous interference effect leads to expect an *increase* of the (effective) tunnelling time $<\tau_{\rm tun}>$; which consequently will not coincide with the Phase Time $\tau_{\rm T}^{\rm Ph}(0,a;E)$: not even for quasi-monochromatic packets. Finally, it is interesting to note, and easy to recognize, that the time–flight decrease before the barrier, and the tunnelling–time increase inside the barrier, do exactly cancel each other out, so that the total effect vanishes. In any case, let us stress that the fact that the entrance time–instant $<t(0)>_{+}$ is decreased by the mentioned "distortion" does *not* obscure the physical sense of our definition of the tunnelling time $<\tau_{\rm tun}>$, eq.(32).

Coming back to the time–evolution of the wave packet inside the barrier, we cannot describe it as the quasi–classical motion of a particle. We can visually describe it, on the contrary, in terms of the "motions" of the three densities $\rho^{\rm II}(x,t)$, $J_{+}^{\rm II}(x,t)$ and $J_{-}^{\rm II}(x,t)$.

In particular, we can pictorially say that both profiles [bumps] of the "incoming" and the "reflected" flux–densities $J_{+}(0,t)$, $J_{-}(0,t)$ (depicted for $x=0$ as a function of time: see Fig. 3) do repeat themselves —with distortion— for increasing values of the penetration depth x. Such a "transmission" takes place rather rapidly (when considering the velocities v_{J+}, v_{J-} of the bump maxima) in comparison with the initial speed v, non-uniformly, and with a gradual decreasing of the bumps. In particular, such decreasing is so strong for the reflected waves, that they *disappear* at the end of the barrier. At last, we can qualitatively say that the probability density $\rho(x,t)$ does conserve the form of its temporal shape at every position x inside the barrier, however exponentially decreasing as the depths increases, and quickly, non-uniformly moving towards the final barrier wall.

4 – CONCLUSIONS AND PERSPECTIVES

In this work, besides a critical review of the previous definitions of the tunnelling times and of their consequences, we proposed for them *new* definitions, eqs.(30)–(33), that we regard as physically acceptable. We do share with the authors of review[1], however, the opinion that none of the previously known definitions were generally acceptable.

Moreover, in this work we began analysing —for the first time— also the time–evolution of the tunnelling process *inside* the barrier.

As to the future, let us before all recall that analytical expressions in the energy representation for $<\tau_{\rm tun}>$ and $<\tau_{\rm to-fro}>$ do not exist, even in the case of the simplest barriers. However, numerical calculations can be straightforwardly performed about the time evolution of the considered wave–packet inside the barrier. On the basis of such one-particle (and one-dimensional) calculations, il will be possible to start developing a *kinematical theory* for the tunnelling of bound or metastable many–particle systems, and of unbound aggregates, through various barriers.

Within the field of nuclear physics, it will be interesting —on the basis of the peculiarities of the tunnelling–process temporal evolution (Sect.**3.3** above; and ref.[7])— to investigate the possibility of *observing* effects due to the change in form, in volume, in orientation and in life–time of many–particle systems (nuclei, fragments,...) during their tunnelling. As well as effects due to "collisions", inside the barrier (*e.g.*, the ion–ion barrier), between different, successive, penetrating particles: for example, the role of those possible collisions in enhancing two–proton sub-barrier transfer in heavy ion interactions. Another future task will be developing a multi-dimensional description, in terms of tunnelling processes, of the sub-barrier fusion of two nuclei: taking account of nuclear deformation, formation of noses or a neck, and of dissipation phenomena.[30]

At last, we should comment on the fact that —when $<\tau_{\rm tun}>$ does not practically depend on the barrier width a, as we have seen to happen for opaque or high barriers (Sects. **3.3** and **2.3**)— then one apparently meets, in connection with eq.(34), speeds that (*inside the barrier*) can assume arbitrarily large values. This does not violate any postulate, *as far as* we deal with non-relativistic (quantum) physics; and in fact such a phenomenon has already been frequently met within "quantum systems".[31]

It is easy to check that the same would happen, in our case, *even* when replacing the Schroedinger equation by the Klein–Gordon or Dirac equation, at least for barriers that do not depend explicitly on time. More interesting is the occurrence of this fact in such "quantum field theories" (which deal, however, with semi–classical potentials). But those infinitely large speeds could disappear in a *self-consistent* relativistic quantum theoretical treatment.

If, on the contrary, that effect would not disappear, then it should be added to the already known results that point in the same direction: cf., *e.g.*, refs.[32]. Results that have been predicted, inside relativistic theories like QFT, by many authors, like Sudarshan[33], Van Dam and Wigner[34], Recami[35], Ne'eman[36] and others.

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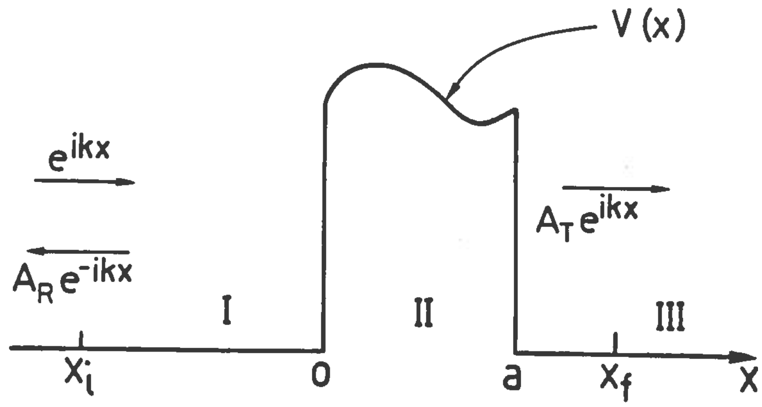

Fig. 1 – The case of *stationary* scattering and tunnelling. In this figure we depict a generic potential–barrier $V(x)$ (which does not depend explicitly on time), with the three regions generated by its presence; and the incoming, reflected and transmitted plane waves.

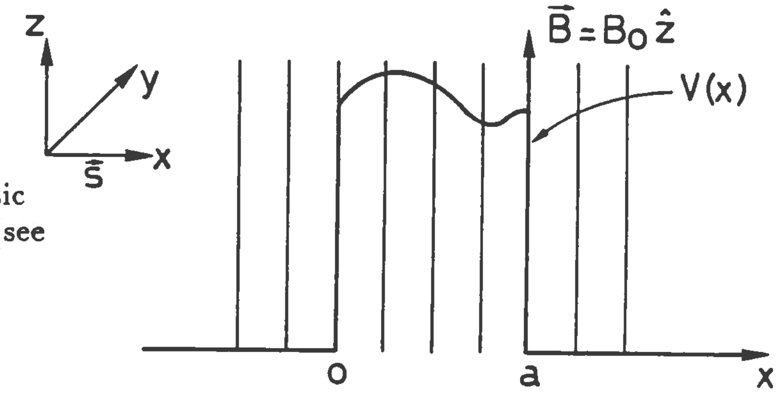

Fig. 2 – Orientations of spin and magnetic field for the case of the "Larmor clock" (see the text).

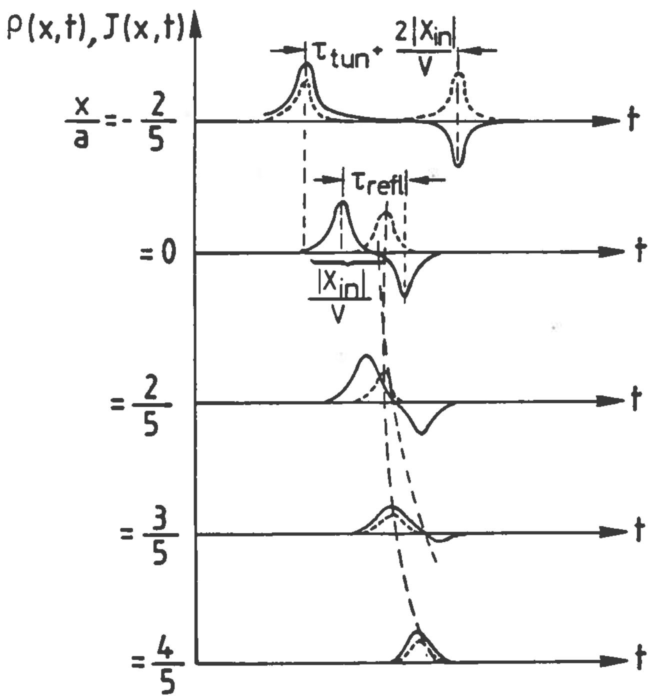

Fig. 3 – A pictorial view of the time dependence of $J(x,t)$ and $\rho(x,t)$, for various values of the penetration depth x inside a rectangular barrier (for Gaussian wave–packets). Actually, as x increases, all the "bumps" suffer an exponential damping; but, for convenience, we neglected in this figure the exponential factor $\exp(-\kappa x)$.