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# PARTICLE DETECTION EFFICIENCY IN A MULTIRING TRAP

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Abstract

The multiring trap is a device consisting of a series of coaxial ring electrodes placed in a static and uniform magnetic field oriented along the system symmetry axis. When the electrodes are held at electric potentials choosed to generated into the trap volume the same electric fields that are present in a standard Penning trap, the structure can stably confine charged particles exactly as in a Penning trap. The big advantage of the multiring trap, an important feature not shared with the regular Penning trap, is that the device is particularly suitable for the in flight capture of bunches of particles. However, the monitoring of the number of trapped particles and the measure of their mean energy has been until now an open problem. The present paper offer a simple solution.

## 1. Introduction

The multiring trap was originally proposed [1] as a possible system for the in flight capture of bunches of charged particles. It consists of a coaxial series of ring shaped electrical conductors, (fig.1), isolated one from the other, and placed at electric potentials that will generate, along the trap axis ($z$ axis), and near to it, an electric force $qE_z$

$$qE_z = -kqz \tag{1}$$

of elastic nature. We indicated by $q$ the charge of the confined particle. When the trap is placed in a constant magnetic field $B$, parallel to the $z$ axis, the electric and magnetic field spatial distribution, in a region near the trap axis, is the same yielded by a regular Penning trap [2] and the two device can capture and store particles in the same way.

The main advantage of the multiring trap are of geometrical nature:

- bunches of particles can be easily brought in a cylindrical structure;

- it is possilble to adjust the lenght of the trap and its diameter to match the incoming bunches dimensions;

- the electrodes potential can be easily adjusted to any desidered capture profile before bringing them to the final trapping profile.

The main disadvantages are:

- the trap behaves as a Penning trap only in a region near the trap axis (an extension between 0.2 and 0.5 of the radius of the trap being usually acceptable);

- one needs to apply to the various electrodes many different, well stabilized, constant electric potentials;

- it is not immediately evident how to properly collect electric signals from the various electrodes to detect the particles trapped in the structure and to measure their energy.

The discussion of that last problem, and its solution, as regard the axial motion and the measurement of the axial particle energy will be the main subject of this paper.

## 2. Particle's motion

As in a Penning trap, in the paraxial region of a multiring device the trapped particles harmonically oscillate along the $z$ axis and follow a combined magnetron and cyclotron motion in a perpendicular $(x, y)$ plane. The $z$ motion may be described, in both case, by the relations

$$z(t) = A \sin(\omega t + \phi) \tag{2}$$

where

$$\omega = 2\pi/T = \sqrt{kq/m} \tag{3}$$

the constant $k$ being equal, for a multiring trap, to:

$$k = 2/L^2 \int_0^L E_z(0,0,z)dz = 2V_0/L^2 \tag{4}$$

($L$=half trap length) and being equal to:

$$k = 2/z_0^2 \int_0^L E_z(0,0,z)dz = 2V_0/z_0^2 \tag{5}$$

($z_0$= distance from the trap center to one cap) for a regular Penning trap.

## 3. Signals from a Penning trap due to the particle axial motion

In a Penning trap, any displacement of the confined particles along the $z$ axis induces image currents on the trap electrodes. To a first approximation the current $I$ induced on a trap cap is linearly related to the particle velocity, so that it is possible to write:

$$I = \frac{\alpha q}{2z_0} \frac{dz}{dt} \tag{6}$$

In the above relation, $\alpha$ is a geometrical factor [3] somewhat smaller than 1.

## 4. Signals from a multiring trap due to the particle axial motion

Let's consider a particle initially situated in the trap center and moving to the right. During an half cycle of the particle oscillation, the current flowing in the electrode named $E$ in figure 1, will be:

- zero, if the particle has a motion amplitude much lower than the distance $z_E$;

- of the kind shown in figure 2a if the particle amplitude of motion is greater than $z_E$;

- of the kind indicated in figure 2b if the particle amplitude of motion is about equal to $z_E$.

It follows from these considerations that, while the signals collected from regular Penning trap are harmonic and directly proportional to the amplitude of motion of the particle, the signals collected on one of the electrodes of a multiring trap have the same period of the oscillating particle, but are not harmonic nor simply correlated with the particle oscillation amplitude. We want to show here, however, that it is possible to get harmonic signals of height proportional to the amplitude of motion of the particle, exactly as in a regular Penning trap, if one uses all the signals coming from the various trap electrodes and properly combines these signals with the aid of an electronic network of the kind shown in fig. 3.

To be more precise, we want to show that the current flowing through the tuning capacity, indicated as $C_T$ in figure 3, is given by the relation

$$I = \frac{\gamma q}{2L}\frac{dz}{dt} \tag{7}$$

where $\gamma$ is a function slowly dependent on the trapped particle energy.

The basic idea that justifies the use of the network of figure 3 in ideal situations (complete lack of parasitic capacties) was explained in reference [4]. Here we demonstrate the validity of that idea in any situation and give full details on the method that allows to completely evaluate the network performance and in particular to compute the function $\gamma$ defined by the last equation.

### 5. How to compute the induced signals

Let's consider a particle oscillating back and forth in a multiring trap whose electrodes are linked to the electronic network of figure 3 (the network being made by a series of real inductances and by the tuning capacity $C_T$). To discuss the electric signals that originate in the circuit, we introduce a set of basic charge distribution functions, $q_i(z)$, that we will define as the functions that specify the amount of charge induced on the various trap electrodes from a unitary electric charge situated along the trap axis in the position of coordinate $z$. For the particular trap having the geometry indicated in figure 1, the functions $q_i(z)$ were computed (figure 4), and we will refer to these computations [5] in our numerical examples.

The functions $q_i(z)$ will be in any case bell shaped functions satisfying the only restriction of charge conservation, namely

$$\sum_i q_i(z) = q \tag{8}$$

The exact shape of the functions $q_i(z)$ will depend on the trap geometry, essentially on the ratio between the inner radius of the ith electrode and the distance between that electrode and the nearest ones (all the functions will be identical if the electrodes are equally spaced and have the same shape).

We now write the currents $I_i(t)$, flowing trough the ith electrode, as

$$I_i(t) = \frac{dq_i}{dz}\frac{dz}{dt} \tag{9}$$

and we wrote that these currents may be considered as originating from suitable currents generators $G_i$. Accordingly, we state that the signals induced by particles oscillating in a multiring trap can be calculated studying the signals induced into the equivalent circuit of figure 5 by the current generator $G_i$. We shall suppose to tune the circuit of figure 5 (varying $C_T$) to the particle oscillation frequency, in order to get, in any situation, the maximum sensitivity. Then, the problem of calculating the electric signale generated by the particle motion is reduced to the problem of studying the fundamental harmonic transfer function of the electric network of figure 5. This study, and the related computations, can be greatly simplified approximating the functions $q_i(z)$ with a sum of a few number of function of the kind shown in figure 6.

The advantages of that approximation are:

- the Fourier component of the injected current $I_i(t)$ can be easily calculated in analytical form (approximating the functions $q_i(z)$ with a mixtilinear curve, the current is approximated by a series of step functions and ones needs only to compute the Fourier components of constants functions).

- Equation (8) is simply satisfied.

Figure 7 shows the two mixtilinear functions whose sum we used to approximate the function indicated in figure 4 and to perform many of the computations that we shall present later.

## 6. Results and discussions

Using the above outlined procedures, we computed $\gamma$ values for different particle energies and for different trap geometries and we found that the features of the computations are simple to describe, easy to understand and mainly dependent only on the following parameters:

a) the ratio $A/d$, that is the oscillation amplitude $A$ of the trapped particles measured using as unit the interelectrode distance $d$;

b) the ratios between the grounding capacities $C_{i,g}$ and the tuning capacity $C_T$;

c) the shape, the dimensions and the distances of the central electrode and of the two neighboring ones.

We shall list the main features of the behaviour of the $\gamma$ parameter, as a function of the above listed variables, referring (figure 8) to the model multiring trap of figure 1 and to its electrostatic properties (figures 4 and 7). Figure 8 gives $\gamma$ as a function of the ratio $A/d$, for different possible values of the ratio $C_g/C_T$ and of the ratio $C_{el}/C_T$ in the model trap.

The analysis of that figure suggests the following, generally valid, conclusions:

a) both $C_g$ and $C_{el}$ tend to decrease the values of the function $\gamma$, but, while the effects of $C_g$ are of importance (a fraction of the induced current is directly short circuited by $C_g$) and should always be considered, the effects of $C_{el}$ are small, even for physically large values of the ratio $C_{el}/C_T$ (these capacities does not short circuit the circulating current and, furthemore, they act, in a sense, in series).

b) For small values of the ratio $A/d$ ($A/d$ less than 0.15) the values of $\gamma$ is constant: the trap behaves like a Penning trap because only three electrodes are involved in the induced charge displacement. The $\gamma$ value is sensibly smaller than one, exactly as in a Penning trap, because a fraction of the induced charge is on the central electrode and never moves. (In our actual computations the exact constancy of $\gamma$ for $A/d < 0.15$, comes from the approximations involved in the use of mixtilinear functions).

c) When at least five electrodes are definitively interested by the motion of the induced charge, the value of $\gamma$ rapidly approches an almost constant values (unitary in ideal conditions) because now there are no fixed (dead) induced charge on the central electrode.

d) the transition of the $\gamma$ value from its small oscillation value to an almost constant large oscillation value, takes place in a region whose length $\lambda$, in the direction of the $z$ axis, depending on the shape of the central electrode and on the distance of that electrode from the two neighboring ones : the distance $\lambda$ is roughly equal to the sum of the central electrode width plus the minimum distance ($\delta$ in figure 3) between the central and one of the two next electrodes.

## REFERENCES

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[3] G. Gabrielse *Phys. Rev. A* 29 (1984) 462

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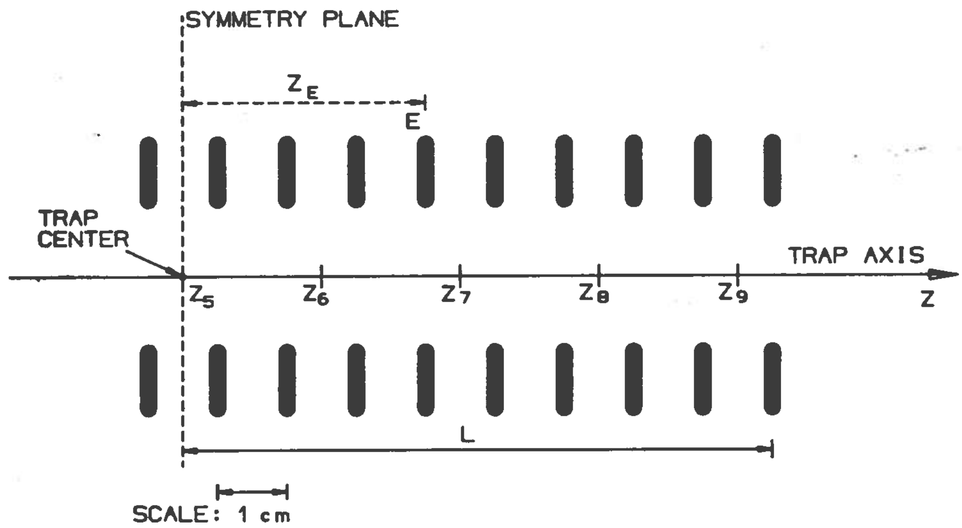


**Figure 1** - Section, in a plane containing the rotation symmetry axis of the trap, of the right part of an eighteen electrode multiring trap.

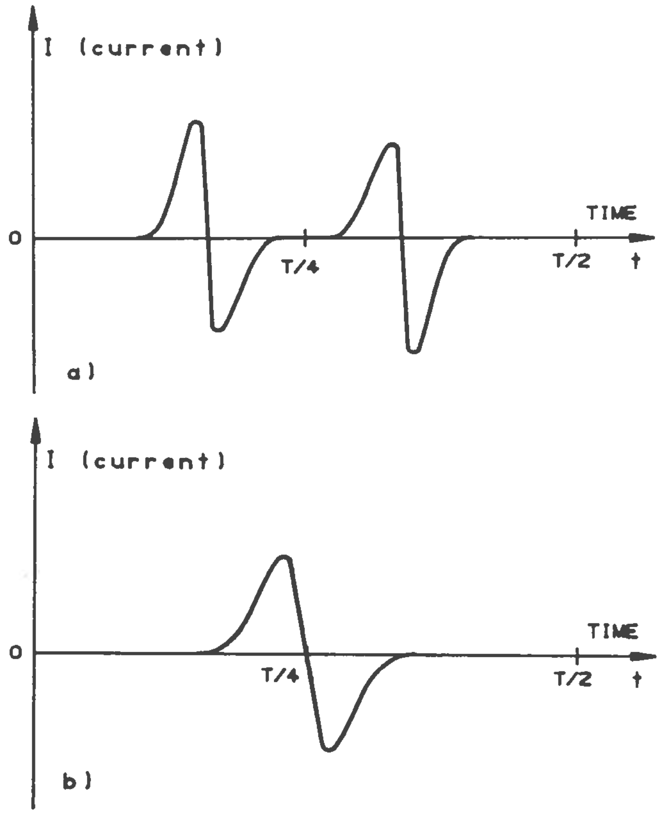


**Figure 2** - Possible currents flowing in one of the electrodes of a multiring trap for different values of the amplitude of motion of a trapped particle. (During the second half of the period, the current is in any case equal to zero).

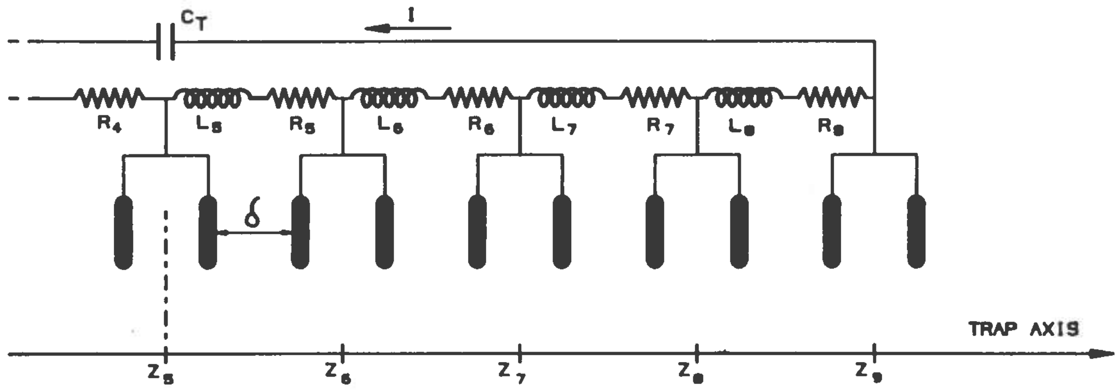


**Figure 3** - How to combine the signal from the trap electrode:

- near electrode are eventually grouped in parallel. The connection is convenient when the actual distance between real electrodes is less than the trap diameter. From now on, we shall always treat such groups of electrodes as one single electrode.

- the various electrodes are connected to a a series of real inductances and that series is closed on a tuning capacitor $C_T$.

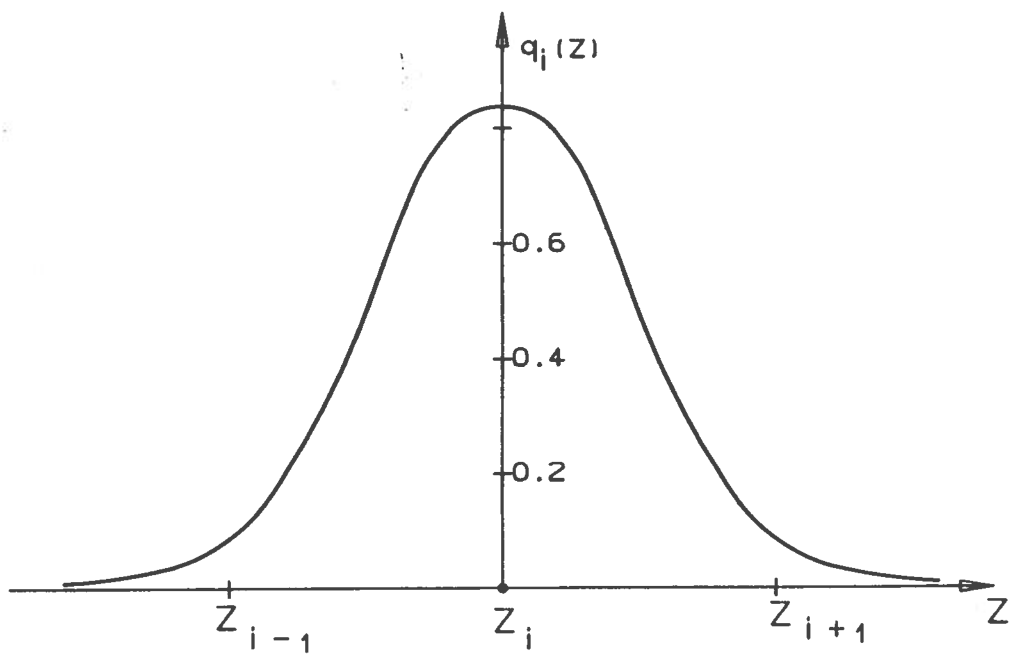


**Figure 4** - The function $q_i(z)$, as deduced from the results reported in reference [5], for every electrode (in fact a couple of real electrodes) of figure 3.

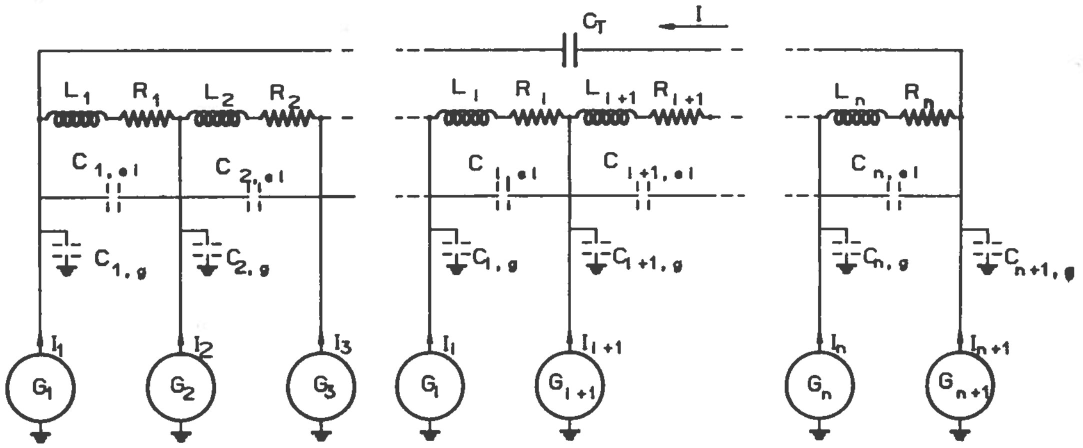


**Figure 5** - An equivalent circuit to study the signals coming from the real circuit of figure 3.

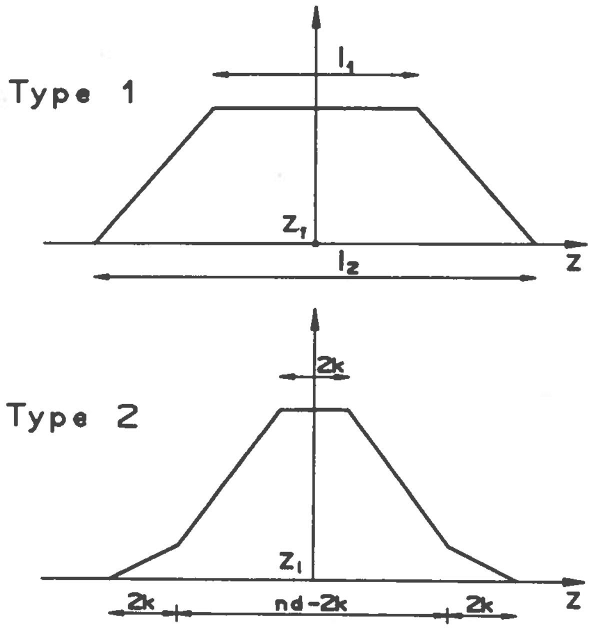


**Figure 6** - Two types of mixtilinear functions that can be used to approximate the induced charge functions $q_i(z)$. Using these kinds of approximation it is possible tp satisfy in a simple way the condition [8] and to simplify the computations. The function of type 1 is a trapezoidal function of adjustable height, centered around one of the electrodes and satisfying the two condiutions: $l_1 = nd + 2K$ and $l_2 = (n + 2m)d + 2K$ (where $n$ and $m$ are integers 0,1,2..., $d$ is the distance between two near electrode and $K$ in any distance less than $d$. Note that a particular case of the type 1 function is the very simple case of a triangular function exetending $n$ times the distance $d$. The function of type 2 is a little more elaborate. The parameter $n$ must be a posistive integer while $k$ is again a distance shorter than $d$.

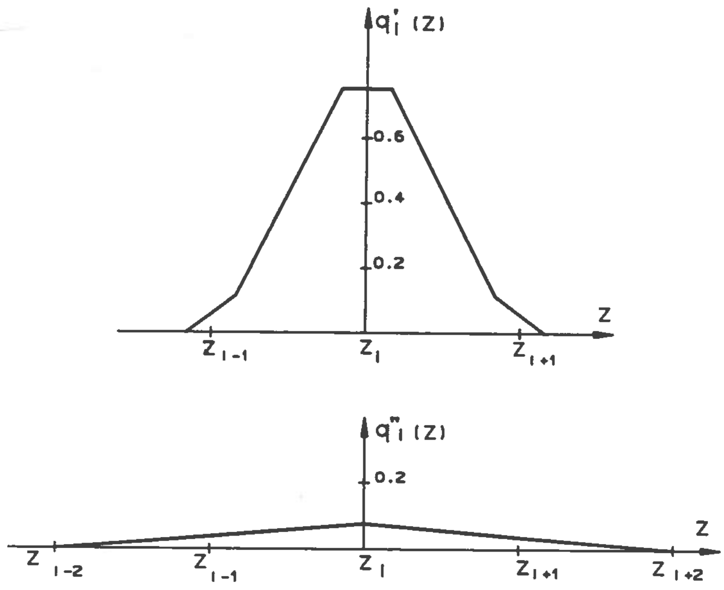


**Figure 7** - The two mixtilinear functions whose sum we used to approximate the function of figure 4.

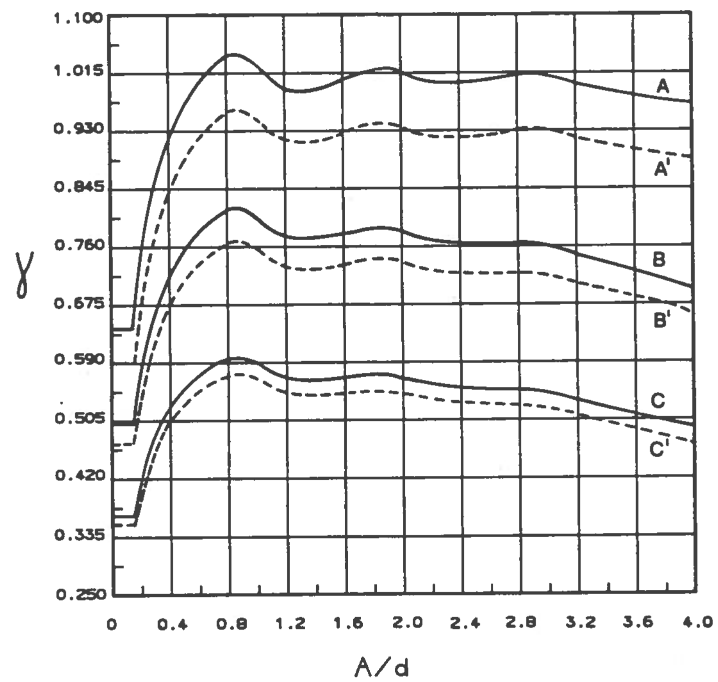


**Figure 8** - Values of the function $\gamma$ for different values of the amplitude $A/d$. The three continuos curves $A$, $B$, $C$ refer to $\gamma$ values computed assuming the ratio $C_{el}/C_T$ always equal to zero and the ratio $C_g/C_T$ respectively equal to 0, 0.4, 1. The three curves $A'$, $B'$, $C'$ are calculated assuming the ratio $C_{el}/C_T$ always equal to 0.7 and the ratio $C_g/C_T$ respectively equal to 0, 0.4, 1.