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**JACOBIAN'S INFLUENCES IN THE  $d+{}^7\text{Li} \rightarrow \alpha+\alpha+n$  REACTION**

## Jacobian's influences in the $d+{}^7\text{Li}\rightarrow\alpha+\alpha+n$ reaction

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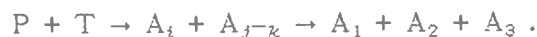
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A kinematically complete study of the  $d+{}^7\text{Li}\rightarrow\alpha+\alpha+n$  reaction has been performed at deuteron energies between 1.4 and 2.5 MeV. Aim of the work was to find the best experimental arrangement to determine the spectroscopic characteristics of the first  ${}^5\text{He}$  excited state. The obtained results show how the LS $\rightarrow$ RCS transformation Jacobians can influence the measurement of the above state excitation energy and width.

Processes having three bodies in the final state of the kind



are an excellent tool to enable the study of the resonances ( $A_{j-k}$ ) which form in the intermediate state of the same reaction



To this end, kinematically incomplete (single-particle spectra) or kinematically complete (two particles in coincidence spectra) experiments can be performed.

The former experiments are easier to perform but often have the disadvantage that the resonant peaks are placed on a strong and continuous background, whose contribution is difficult to evaluate during the data analysis. As is well known, when this is the case, kinematically complete, rather than incomplete, experiments have to be carried out in order to eliminate or reduce the background. However, when the widths to be measured are narrow, the uncertainties associated with any method projecting the data onto some appropriate curve of the  $(E_1, E_2)$  plane make accurate measurements impossible. When this occurs, one is forced to carry out experiments which are kinematically incomplete reducing the background in some way. Usually, however, kinematically complete experiments are preferred rather than the incomplete ones. If this is the case, the experimental apparatus consists of two detection chains in coincidence which allow us to obtain, in the  $(E_1, E_2)$  plane, the bidimensional spectra relative to the angle pairs  $(\theta_1, \phi_1; \theta_2, \phi_2)$  at which the detectors are placed.

The best way of treating the bidimensional spectrum data is to project them onto an axis or curve of the  $(E_1, E_2)$  plane. Various factors show that the central kinematic curve (the one corresponding to the angles defined by the beam direction and detector axes) is a good projection locus. Both the finite angular and energy resolutions of the detecting system contribute to the spreading of the events of the  $(E_1, E_2)$  plane, so if we are to extract the true distribution of events from the projected data, we

must separate the geometrical effects from the energy ones before projection. This is not necessary, however, when the angular resolution is quite good. Indeed, in this case, all the events can be considered to belong to the central kinematical curve. We found this to be true in the present experiment. By assuming a Lorentzian spreading due to the finite overall energy resolution of the detecting system the projected data can automatically be deconvoluted from the effects of the finite-energy resolving power of the experimental apparatus. Therefore, it produces a distribution of events on the central kinematical curve which ought to be a good approximation of the true one<sup>(1,2)</sup>.

The density  $N(s)$ , obtained as a function of the curvilinear abscissa along the kinematical curve, will be referred to RCS by the suitable Jacobian of transformation. Obviously<sup>(3)</sup>

$$\begin{aligned} N(E_i, \Omega_i, \Omega_j) dE_i d\Omega_i d\Omega_j &= N(s) \frac{ds}{dE_i} dE_i d\Omega_i d\Omega_j = \\ &= N(E_{i-j\kappa}, \Omega_{i-j\kappa}, \Omega_{j-\kappa}) dE_{i-j\kappa} d\Omega_{i-j\kappa} d\Omega_{j-\kappa} \end{aligned}$$

then

$$N(E_{i-j\kappa}, \Omega_{i-j\kappa}, \Omega_{j-\kappa}) = J_{i-j\kappa} N(s)$$

with

$$J_{i-j\kappa} = \frac{\partial(E_i, \Omega_i, \Omega_j)}{\partial(E_{i-j\kappa}, \Omega_{i-j\kappa}, \Omega_{j-\kappa})} \left| \frac{\partial s}{\partial E_i} \right|$$

Since it is possible

$$\frac{\partial(\vec{p}_{i-j\kappa}, \vec{p}_{j-\kappa})}{\partial(\vec{p}_i, \vec{p}_j)} = 1$$

it will be

$$p_{i-j\kappa}^2 dp_{i-j\kappa} p_{j-\kappa}^2 dp_{j-\kappa} d\Omega_{i-j\kappa} d\Omega_{j-\kappa} = p_i^2 dp_i p_j^2 d\Omega_i d\Omega_j$$

and then

$$\begin{aligned} J_{i-j\kappa} &= \frac{\mu_{i-j\kappa} p_{i-j\kappa} p_{j-\kappa}^2 dp_{j-\kappa}}{m_i p_i^2 p_j^2 dp_j} \left| \frac{\partial s(E_i, E_j)}{\partial E_i} \right| = \\ &= \left( \frac{m_\kappa}{M^3} \right)^{1/2} \frac{(E_{i-j\kappa} E_{j-\kappa})^{1/2}}{E_i E_j} (E_i \Delta_i + E_j \Delta_j)^{1/2} \end{aligned}$$

if 'i' and 'j' are the detected particles and 'i' the first emitted one.

Usually when one wants to study a process with three bodies in the final state (to give, *e.g.*, information about the spectroscopic characteristics of nuclear states formed in the intermediate step) one has to pay particular attention to the following points:

- i) choice of the incident energy;
- ii) choice of the detection geometry;
- iii) check of the accuracy of the geometrical resolving power.

With regard to i), the choice of the incident energy has to be such to excite the state of the residual nucleus object of the research. Usually this choice is made bearing in mind the trend of the excitation function.

The choice of the detection angles (linked to the incident energy choice) is made keeping into account, among all the triggered processes, these ones that can give contributions to the kinematical locus of  $A_1+A_2+A_3$  in the plane  $(E_1, E_2)$ . One tries to have the contribution of interest in a region as free as possible from other contributions which could complicate the data analysis.

Finally, the geometrical resolving power of the detecting system (iii) has to be so good as to believe that the spread around the kinematical curve is due only to energy. In such a case, as described in Refs. (1) and (2), by projecting the coincidences on the kinematical curve one obtains a distribution which is an excellent approximation of the true one.

Recently(4), in order to determine the spectroscopic characteristics of the first  $^5\text{He}$  excited state ( $J^\pi=1/2^-, T=1/2$ ), we measured the  $\alpha\alpha$  bidimensional spectra of the  $d+^7\text{Li} \rightarrow \alpha+\alpha+n$  reaction at incident energies between 1.4 and 2.5 MeV.

The experiment was carried out at the LNL (Padova). Incident deuterons were accelerated by means of the 7.0 MV Van de Graaf while the target was obtained by evaporating, onto a carbon backing, LiF enriched to 99.9% in  $^7\text{Li}$ . The detectors in coincidence ( $D_1$  in the  $\phi_1=0^\circ$  plane,  $D_2$  in the  $\phi_2=180^\circ$  one) were of 100  $\mu\text{m}$  and thick enough to stop all the  $\alpha$  particles produced in the reaction.

The choice of the geometrical arrangements of the detectors was made also keeping into account the suggestions by Fou *et al.*<sup>(5)</sup>. So we obtained the bidimensional spectra with the forward-backward detectors arrangements, besides to the ones obtained with detectors in the usual forward-forward arrangements.

Figs. 1,2,3 and 4 show the trend of the excitation energies ( $E_{2-3}$  and  $E_{1-3}$ ) for the  $\alpha$ -n system and of the LS $\rightarrow$ RCS transformation Jacobians ( $J_{1-23}$  and  $J_{2-13}$ ) versus the curvilinear abscissa  $s$  for the  $d+{}^7\text{Li}\rightarrow\alpha+\alpha+n$  reaction at the different incident energies. In the same figures the regions in which one expects the first  ${}^5\text{He}$  excited state contributions of our interest, are indicated.

$E_{2-3}$  and  $J_{1-23}$  are associated to the hypothesis that the first emitted  $\alpha$  particles go to detector  $D_1$ ;  $E_{1-3}$  and  $J_{2-13}$  to the hypothesis that  $\alpha$  particles go to detector  $D_2$ . Derivatives of the energies ( $\frac{dE_{2-3}}{ds}$  and  $\frac{dE_{1-3}}{ds}$ ) give informations about the resolving power of the experimental apparatus in the recoil coordinate system (RCS). It is obvious that the larger the slope of  $E_{j-k}(s)$ 's the more difficult the evaluation of the peak energy.

Similar considerations can be made for the LS $\rightarrow$ RCS transformation Jacobians since

$$\frac{d^3\sigma}{dE_{j-k}d\Omega_{i-jk}d\Omega_{rel}} = J_{i-jk} \frac{d^3\sigma}{dsd\Omega_1d\Omega_2}.$$

It can happen that  $J_{i-jk}(s)$ 's, in the kinematical region where the contributions of interest are expected, are functions with a strong slope. In such a case, the transformation from the laboratory system (LS) to the relative coordinate one (RCS) is very problematic, since errors can be introduced: indeed, by building the relationship existing between the slope of the Jacobians ( $\frac{dJ_{1-23}}{ds}$  and  $\frac{dJ_{2-13}}{ds}$ ) and the resolving power  $\Delta s$  on the kinematical curve and the statistical errors one realizes that the different portions of the spectrum under analysis could be treated with the incorrect values of Jacobians.

Now, with regard to the measurement of the spectroscopic

characteristics of the first  ${}^5\text{He}$  excited state by means of the  $\alpha\alpha$  spectra analysis of the  $d+{}^7\text{Li}\rightarrow\alpha+\alpha+n$  reaction studied at incident energies between 1.4 and 2.5 MeV, we are in the same situation described above: indeed, in the spectra with detectors  $D_1$  and  $D_2$  placed in a forward-backward arrangement in the kinematical regions where the  $J^\pi=1/2^-$ ,  $T=1/2$  state contribution appears, the Jacobians vary from  $\sim 5\times 10^{-2}$  to  $\sim 26\times 10^{-2}$  differently from what happens in the spectra where the detector arrangement is the classical forward-forward one. Here the Jacobian limit values are  $\sim (3.2\div 5.5)\times 10^{-2}$  and  $\sim (1.8\div 4.2)\times 10^{-2}$ .

It is obvious that, in such conditions, also treating all the obtained spectra analytically, we believe the results coming from those spectra with forward-forward geometrical arrangement are reliable.

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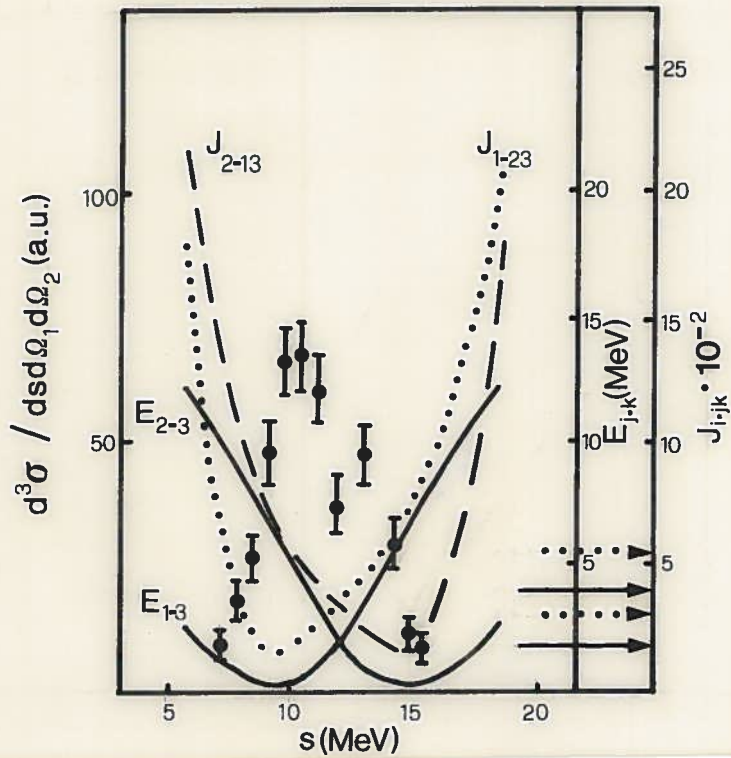


Fig. 1.  $E_{j-k}$  (solid lines) and  $J_{i-jk}$  (dashed and dotted lines) versus  $s$  for the  $d+{}^7\text{Li}\rightarrow\alpha+\alpha+n$  reaction at  $E_d=1.4$  MeV with  $\theta_1=70^\circ$  and  $\theta_2=90^\circ$ . The two couples of arrows indicate the Jacobian's intervals corresponding to the kinematical regions where the first  ${}^5\text{He}$  excited state decay contributions are expected. The experimental values of the measurement are also reported.

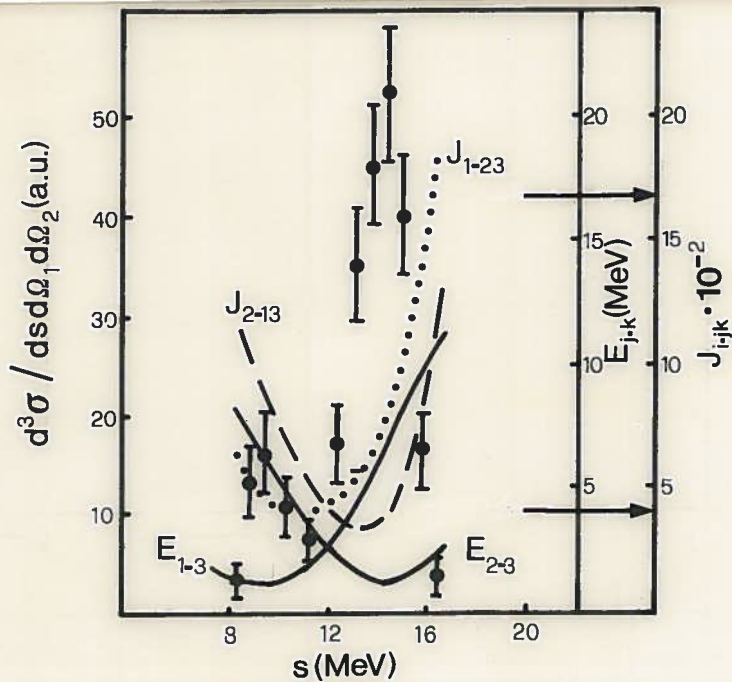


Fig. 2. Same as Fig. 1, but with  $E_d=1.4$  MeV,  $\theta_1=135^\circ$  and  $\theta_2=45^\circ$ .



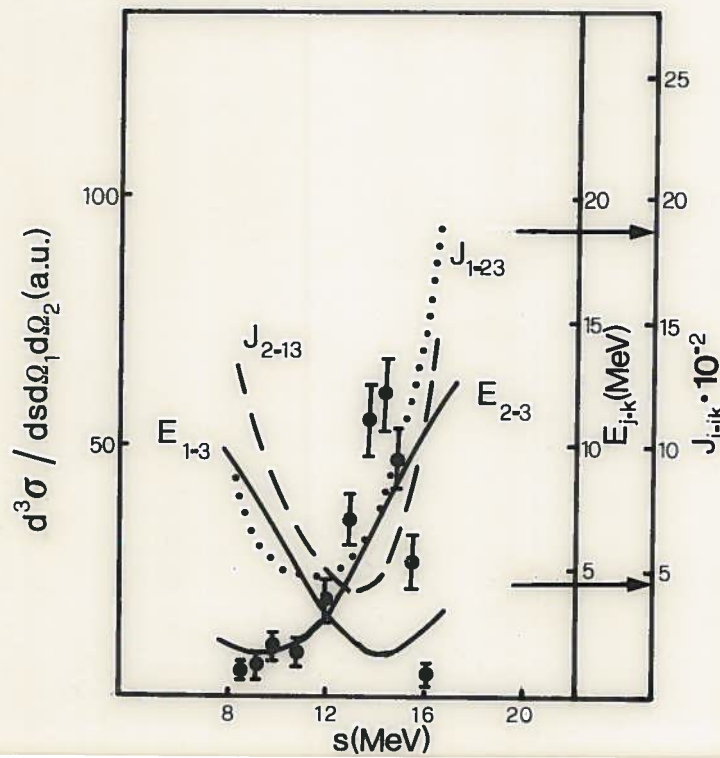


Fig. 3. Same as Fig. 1, but with  $E_d=2.1$  MeV,  $\theta_1=135^\circ$  and  $\theta_2=45^\circ$ .

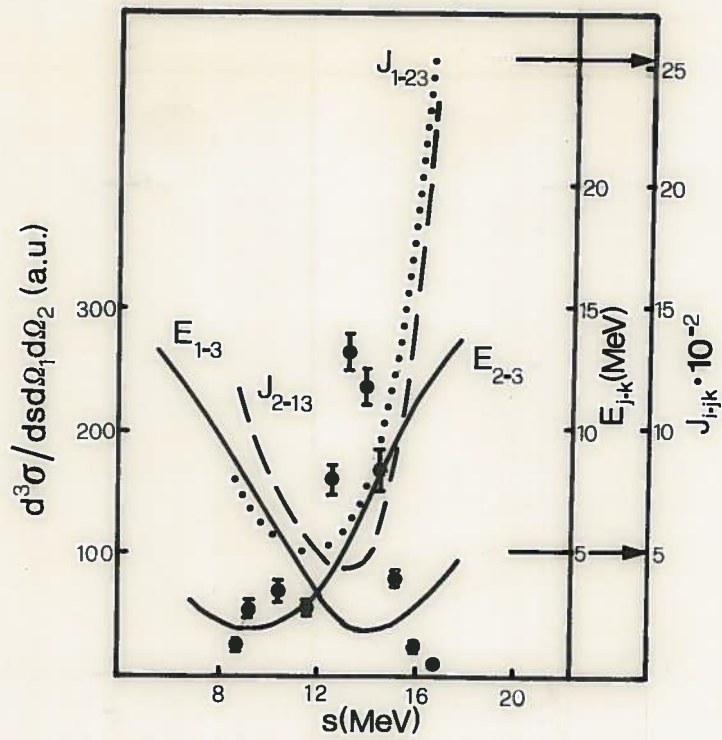


Fig. 4. Same as Fig. 1, but with  $E_d=2.5$  MeV,  $\theta_1=135^\circ$  and  $\theta_2=45^\circ$ .