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STOCHASTIC COOLING OF THE RADIAL MOTION

IN A PENNING TRAP

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ABSTRACT

We did already present results on the stochastic cooling of the axial motion of samples of N particles stored in a Penning trap.

We complete here our previous results discussing a simple method to stocastically cool even the particle radial motion.

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1. INTRODUCTION

A possible technique to stochastically cool the axial motion of particles trapped in Penning traps was already theoretically discussed for regular [1] and multiring [2] traps and experimentally verified in a model regular trap [3].

In the present paper we want to discuss the stochastic cooling technique for the radial motion of particles confined in a regular Penning trap.

The present discussion is organized in the following way: first we recall the pertinent laws for the motion of particles both into an ideal and into a real Penning trap; then we briefly revise the main concepts and definitions about the technique of stochastic cooling when applied to trapped particles and we discuss what type of electric pulses can lead to the control of the radial motion of one trapped particle. Finally we examine the overall results of the proposed cooling technique applied to a sample of N particles.

2. IDEAL PENNING TRAP

Let's first recollect the basic results about the particle motion in a Penning trap.

A voltage V_0, applied between the ring and two cups (fig.1) of an ideal Penning trap [4] placed in a constant magnetic field B parallel to the trap z axis, generates inside the trap an electric potential given by:

$$V(x,y,z) \;=\; \frac{V_0}{r_0^2+2z_0^2}\,(2z^2-x^2-y^2) \qquad (1)$$

Under the action of that potential:
i) the z coordinate of a trapped particle changes with time as

$$z(t) = r_z\cos(\omega_z t+\phi_z) \qquad (2)$$

where

$$\omega_z \;=\; \left[\frac{4qV_0}{m(r_0^2+2z_0^2)}\right]^{\frac{1}{2}} \qquad (3)$$

ii) the particle moves radially with a motion (in ideal condition fully decoupled from the axial motion) that can be described, as superposition of two circular motions, by the equation

$$x(t)+iy(t) = r_c e^{i[\omega_c(t-t_0)+\phi_c]} + r_m e^{i[\omega_m(t-t_0)+\phi_m]} \qquad (4)$$

As it is well known the first type of motion is called the cyclotron motion and is characterized by an angular frequency:

$$\omega_c = \frac{1}{2}\left\{\frac{qB}{m} + \sqrt{\left(\frac{qB}{m}\right)^2 - 2\omega_z^2}\right\} = 2\pi f_c \tag{5}$$

while the second is called the magnetron motion and is characterized by an angular frequency:

$$\omega_m = \frac{1}{2}\left\{\frac{qB}{m} - \sqrt{\left(\frac{qB}{m}\right)^2 - 2\omega_z^2}\right\} = 2\pi f_m \tag{6}$$

Usually

$$\omega_c >> \omega_m \tag{7}$$

The parameters r_z, ϕ_z and r_c, r_m, ϕ_c, ϕ_m are constants whose values depend upon the axial and radial particle velocity and position at the time t_0.

3. REAL PENNING TRAP

In a real Penning trap geometrical imperfections, forces due to space charge and magnetic field inhomogeneities change the ideal motion of the particles and give rise to a broadening of the linewidth of the three characteristic frequencies $\omega_c, \omega_z, \omega_m$.

In practice, the standard deviation σ_c of the cyclotron angular frequency from its mean value ω_{0c} is almost enterely linked to the magnetic field inhomogeneities, while geometrical imperfections and space charge effects are mainly responsible for the broadening, characterized by standard deviation σ_z and σ_m, of the axial and magnetron frequencies from their mean values ω_{0z} and ω_{0m}.

To say that the axial, cyclotron and magnetron oscillation frequencies ω_z, ω_c and ω_m of a particle have dispersions σ_z, σ_c and σ_m, obviously means that the motion of a particle, when analyzed in term of Fourier components turns out to be a superposition of motions with differents frequencies and cannot be described by simple harmonic functions of the type given by the equations (3) and (4), thinking to the nine parameters $r_z, \omega_z, \phi_z, r_c, \omega_c, \phi_c, r_m, \omega_m, \phi_m$ as constants of the motion.

To simplify some of our next computations, whenever it will be possible without altering the final results, we shall assume that the cyclotron, magnetron and axial motions of any particle can be described by the equations (2) and (4), thinking to the nine parameters r_z, ω_z, ϕ_z, r_c, ω_c, ϕ_c, r_m, ω_m, ϕ_m as constants of the motion (each particle moves as in an ideal trap), but that these single particle parameters are a set of stochastic uncorrelated variable. In that case we will assume that the phases are uniformly distributed between 0 and 2π, that the angular frequencies float around their mean values ω_{0c}, ω_{0m} and ω_{0z} with a dispersion measured by the real standard deviations σ_z, σ_c, σ_m, and that the particle oscillation amplitudes change according to the proper physically significant distribution.

Let's now discuss the motion of the center of mass of N particles. being the center of mass x coordinate given by

$$x_B(t) = \frac{\sum_{k=1}^{N} x_k(t)}{N} \tag{8}$$

we can write

$$x_B(t) = \frac{\sum_{k=1}^{N} r_{ck}\cos\theta_{ck}(t)}{N} + \frac{\sum_{k=1}^{N} r_{mk}\cos\theta_{mk}(t)}{N} \tag{9}$$

where

$$\theta_{ck}(t) = \omega_{ck}t + \phi_{ck} \tag{10}$$

$$\theta_{mk}(t) = \omega_{mk}t + \phi_{mk} \tag{11}$$

A similar formula with sin functions holds for the center of mass y coordinate $y_B(t)$.

It is often convenient to define "instantaneous" cyclotron and magnetron radii and phases, writing:

$$\frac{\sum_{k=1}^{N} r_{ck}\cos\theta_{ck}(t)}{N} = R_c(t)\cos\Theta_c(t) \tag{10}$$

$$\frac{\sum_{k=1}^{N} r_{mk}\cos\theta_{mk}(t)}{N} = R_m(t)\cos\Theta_m(t) \tag{11}$$

$$\frac{\sum_{k=1}^{N} r_{ck}\sin\theta_{ck}(t)}{N} = R_c(t)\sin\Theta_c(t) \tag{12}$$

$$\frac{\sum_{k=1}^{N} r_{mk}\sin\theta_{mk}(t)}{N} = R_m(t)\sin\Theta_m(t) \tag{13}$$

It follows immediately from the above definitions that

$$R_c^2 = \frac{\sum_{k=1}^{N} r_{ck}^2}{N^2} + \frac{\sum_{k\neq j}^{N} r_{ck}r_{cj}\cos(\theta_{ck}-\theta_{cj})}{N^2} \tag{14}$$

$$\cos\Theta_c(t) = \frac{\sum_{k=1}^{N} r_{ck}\cos\theta_{ck}(t)}{NR_c} \tag{15}$$

$$\sin\Theta_c(t) = \frac{\sum_{k=1}^{N} r_{ck}\sin\theta_{ck}(t)}{NR_c} \tag{16}$$

Similar formula hold for R_m and Θ_m.

The four quantities $R_c(t)$, $R_m(t)$, $\Theta_c(t)$, $\Theta_m(t)$, now fully defined, clearly are four time dependent stochastic variables whose time behaviour is mainly characterized by two characteristic times, that we shall call cyclotron and magnetron mixing times, defined as the inverses of the standard deviations σ_c and σ_m by the two relations

$$\tau_c = \frac{1}{\sigma_c} \tag{17}$$

$$\tau_m = \frac{1}{\sigma_m} \tag{18}$$

The mixing times practically are a measure of the coherence time of the stochastic functions $R_c(t)$, $R_m(t)$, $\Theta_c(t)$, $\Theta_m(t)$. We showed that extensively in reference [1] for the particular case of the axial position and velocity of the center of mass (means values of the z coordinate and of its derivative) of a system of N particles but the same result holds for the radial case.

Therefore we can say that during time intervals short compared with the mixing time τ_c (τ_m) the functions $R_c(t)$ and $\Theta_c(t)$ ($R_m(t)$ and $\Theta_m(t)$) are almost constant and the cyclotron (magnetron) motion of the center of mass can be considered almost harmonic. On the contrary during time intervals greater than the mixing time the functions $R_c(t)$ and $\Theta_c(t)$ ($R_m(t)$ and $\Theta_m(t)$) show significant fluctuations.

4. STOCHASTIC COOLING

The basic idea about the stochastic cooling of any type of motion of particles confined in a trap is the following:

- after a measure of the oscillation amplitude A and phase Θ of the center of mass motion component under consideration (fig. 2) a suitable electric field is applied of amplitude and duration selected to stop that motion component of the center of mass. In that way the energy of one (the center of mass) of the N degrees of freedom of that type of motion is taken away;
- after that, the system starts to evolve toward a new equilibrium situation, in which the energy will be again equally shared between all the system degrees of freedom of the considered motion.

The equilibrium will be reached after a time of the order of the proper mixing time. At this point, it is possible to repeat the process and to take away another (roughly $1/N$ of the total) amount of energy from the motion.

Reference [1] gave a complete discussion of the stochastic cooling of the axial particle motion and in the present discussion we will follow the same lines.

5. THE TWO PULSES TECHNIQUE FOR THE COOLING OF THE SINGLE PARTICLE RADIAL MOTION

In the present paper we will define as "cooling" of the radial motion the reduction of both the cyclotron and magnetron radii. Therefore, to cool the radial motion, we shall have to extract energy from the cyclotron motion, while we shall have to increase the energy associated with the magnetron motion.

To add or to subtract energy from a given type of motion, exactly as for the case of the axial motion, we will think to use pulsed electric fields, that is electric fields of duration τ much shorter than the cyclotron (and magnetron) period, having constant amplitude E_x during the time τ and directed along the trap x axis (fig. 2).

Using such assumptions it is very easy to evaluate the effect of one electric field pulse: while the x and y coordinates of one particle and its y velocity component are unchanged, the x velocity component is suddenly increased of an amount δv_x given by

$$\delta v_x = \frac{qE_x\tau}{m} \tag{19}$$

The cooling of the particle radial motion looks, at the beginning, more complex than the cooling of the particle axial motion: one needs to cool two different types of motion and the controlling electric field pulse changes the amplitudes and phases of both type of motion.

That can be easily demonstrate computing the new amplitude and phases r'_{ck}, r'_{mk}, θ'_{ck}, θ'_{mk} of a given particle immediately after the electric field pulse application at the time t_1, as function of the corresponding values r_{ck}, r_{mk}, θ_{ck}, θ_{mk} immediately before the electric field pulse action.

The results are

$$r'_{mk}\cos\theta'_{mk}(t_1) = r_{mk}\cos\theta_{mk}(t_1) \tag{20}$$

$$r'_{ck}\cos\theta'_{ck}(t_1) == r_{ck}\cos\theta_{ck}(t_1) \tag{21}$$

$$r'_{mk}\sin\theta'_{mk}(t_1) = r_{mk}\sin\theta_{mk}(t_1) + \frac{\delta v_x}{\omega_{ck}-\omega_{mk}} \tag{22}$$

$$r'_{ck}\sin\theta'_{ck}(t_1) = r_{ck}\sin\theta_{ck}(t_1) - \frac{\delta v_x}{\omega_{ck}-\omega_{mk}} \tag{23}$$

and from them it follows

$$(r'_{mk})^2 = (r_{mk})^2 + \left(\frac{\delta v_x}{\omega_{ck}-\omega_{mk}}\right)^2 + 2\ \frac{\delta v_x}{\omega_{ck}-\omega_{mk}}\ r_{mk}\ \sin\theta_{mk}(t_1) \tag{24}$$

$$(r'_{ck})^2 = (r_{ck})^2 + \left(\frac{\delta v_x}{\omega_{ck}-\omega_{mk}}\right)^2 - 2\ \frac{\delta v_x}{\omega_{ck}-\omega_{mk}}\ r_{ck}\ \sin\theta_{ck}(t_1) \tag{25}$$

From these equations it easy to conclude that, even for the simple case of a single trapped particle, it is possible to freeze the particle cyclotron or magnetron motion but not the two simultaneously. If we choose

$$\theta_{ck}(t_1) = \frac{\pi}{2} \tag{26}$$

and

$$\delta v_x = (\omega_{ck} - \omega_{mk}) r_{ck} \tag{27}$$

the cyclotron motion amplitude is reduced to zero but

$$(r'_{mk})^2 = (r_{mk})^2 + (r_{ck})^2 + +2r_{ck}r_{mk} \quad \sin\theta_{mk}(t_1) \tag{28}$$

This last equation proves that the cooling of one of the two types of motion can substantially alter the other. The same conclusion holds, with additional complications, when we deal with a number of trapped particles greater than one.

Luckly it is possible to fully decouple the cooling rate of the two types of motion if every elementary cooling action is done using two consecutive pulses instead of only one pulse of electric field, the second pulse following the first one after a fixed time interval equal to one half of the cyclotron period (two pulses technique).

Let's call δv_x and $(\delta v_x)'$ the changes of the x velocity component induced by the first and second pulses and let's use the symbols r''_{mk}, r''_{ck}, θ''_{mk} θ''_{ck} to indicate the magnetron and cyclotron radii and phases immediately after the applications of the second pulse at the time t_2.

Applying two times equations of the type (20),(21), (22), (23) taking into account the proposed timing and taking into account (7) one easily gets

$$r''_{mk}\cos\theta''_{mk}(t_2) = r_{mk}\cos\theta_{mk}(t_1) \tag{29}$$

$$r''_{ck}\cos\theta''_{ck}(t_2) = -r_{ck}\cos\theta_{ck}(t_1) \tag{30}$$

$$r''_{mk}\sin\theta''_{mk}(t_2) = r_{mk}\sin\theta_{mk}(t_1) + \frac{\delta v_x}{\omega_{ck}-\omega_{mk}} + \frac{\delta v_x{}'}{\omega_{ck}-\omega_{mk}} \tag{31}$$

$$r''_{ck}\sin\theta''_{ck}(t_2) = r_{ck}\sin\theta_{ck}(t_1) + \frac{\delta v_x}{\omega_c-\omega_m} - \frac{\delta v_x{}'}{\omega_c-\omega_m} \tag{32}$$

From them:

$$(r''_{ck})^2 = (r_{ck})^2 + \left[\frac{\delta v_x - (\delta v_x)'}{\omega_{ck}-\omega_{mk}}\right]^2 - 2\left[\frac{\delta v_x - (\delta v_x)'}{\omega_{ck}-\omega_{mk}}\right] r_{ck}\sin\theta_{ck}(t_1) \tag{33}$$

$$(r''_{mk})^2 = (r_{mk})^2 + \left[\frac{\delta v_x + (\delta v_x)'}{\omega_{ck}-\omega_{mk}}\right]^2 + 2\left[\frac{\delta v_x + (\delta v_x)'}{\omega_{ck}-\omega_{mk}}\right] r_{mk}\sin\theta_{mk}(t_1) \tag{34}$$

It should be now very clear that, using the two pulses technique, it is sufficient to choose $\sin\theta_{ck}(t_1) = 1$ ($\sin\theta_{mk}(t_1) = -1$) and $\delta v_x = (\delta v_x)\prime$ ($\delta v_x = -(\delta v_x)\prime$) to change the cyclotron (magnetron) motion of a particle without affecting in any way its magnetron (cyclotron) motion.

6. COOLING OF THE RADIAL MOTION OF MANY PARTICLES

As previously said in the fourth paragraph to cool the cyclotron (magnetron) motion of N particles we have to repetly cool the cyclotron (magnetron) motion of their center of mass. Let's think to use the two pulses technique, with pulses of amplitude

$$\delta v_x = \frac{g}{2}(\omega_{c0} - \omega_{m0})R_c \tag{35}$$

($\delta v_x = \frac{g}{2}(\omega_{c0} - \omega_{m0})R_m$) proportional to R_c (R_m) choosing

$$\sin\Theta_c(t_1) = 1 \tag{36}$$

($\sin\Theta_m(t_1) = -1$) Every couple of pulses changes the cyclotron (magnetron) single particle radius acccording to the equations (33) and (34). The expectation values of both members of these equations are suitable quantities to describ the mean effect of a sequence of pulses applied at regular time interval of lenght τ_0. Proceeding exactly as we did in [1], supposing $\tau_0 \geq \tau_c$ ($\tau_0 \geq \tau_m$), taking $\omega_{ck} \simeq \omega_{c0}$ and $\omega_{mk} \simeq \omega_{m0}$ and assuming (with the hypothesis of complete uncorrelation between the particle amplitudes and phases) that

$$< R_c^2 >_s = \frac{< r_c^2 >_s}{N} \tag{37}$$

$$< R_c^2 >_s = \frac{< r_c^2 >_s}{N} \tag{38}$$

(the symbol $< \ >_s$ meaning the statistical expectation value of a random variable) one gets for the cyclotron motion the equation

$$\frac{d< r_c^2 >_s}{dt} = \frac{g^2 - 2g}{N\tau_0} < r_c^2 >_s \tag{39}$$

if one neglects noise effects and the equation

$$\frac{d< r_c^2 >_s}{dt} = \frac{g^2 - 2g}{N\tau_0}(< r_c^2 >_s - < r_{c,n}^2 >_s) \tag{40}$$

if one introduces the effects of noise errors that bring, at the output of the circuits of fig. 2, to uncorrect reading both of the amplitude and the phase of the center of mass cyclotron motion.

The quantity $< r_{c,n}^2 >_s$ turns out to depend only on the trap and on the detection resonant circuit of the cyclotron motion and is given by

$$\frac{g^2}{2g - g^2} \frac{4r_0^2}{(\alpha_r q)^2 RN} \frac{8KT_cB_c}{{\omega_{0c}}^2} \tag{41}$$

where K is the Boltzmann constant, α_r is a geometrical factor, T_c is the equivalent noise temperature of the detection circuit while B_c and R are the bandwith and the dynamical resistance of that circuit.

Formally exactly equal equations can be obtained for the case of the magnetron motion. The solution is again an exponential law but, from a physical point of view, it may be useful to note that the time constant for the cyclotron motion is muche shorter than the corresponding time constant for the magnetron motion bein usually $\tau_m \gg \tau_c$ (mainly owing to (7)).

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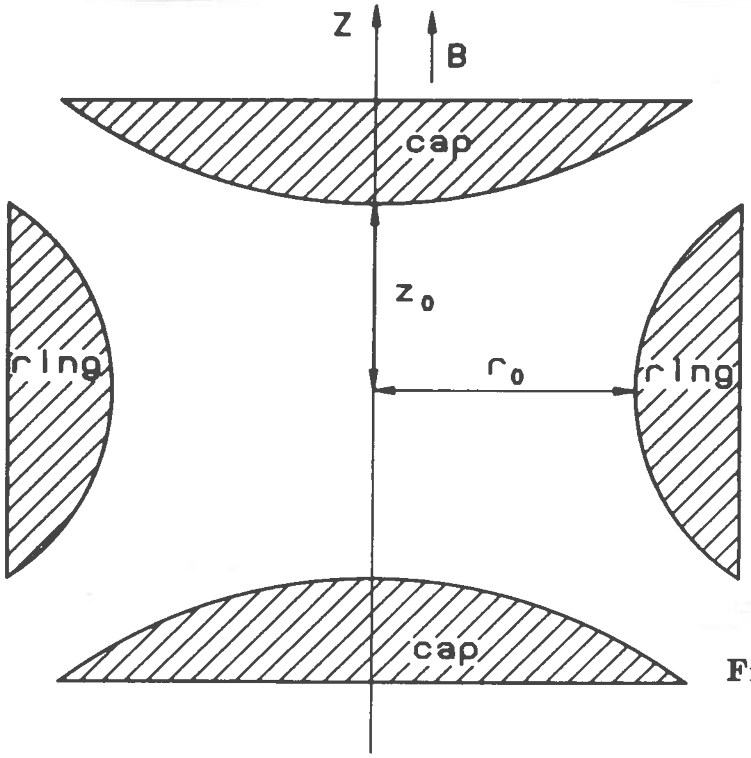

Figure 1 Axial section of a Penning trap

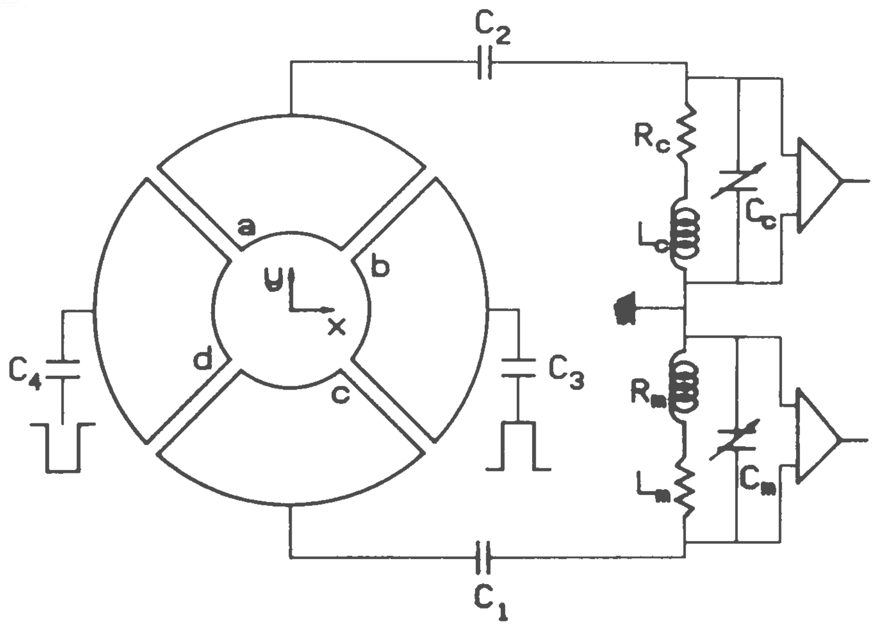

Figure 2 Scheme of the circuits detecting the cyclotron and magnetron motion.

While the electric charge associated to the center of mass moves inside a Penning trap, there is an associated motion of the image charges on the trap electrodes.To detect the particle radial motion it is customary [4] to divide the trap ring electrode in four sectors (a,b,c,d) and to look at the movement of electric charges between facing electrodes. The figure shows the four median plane (x,y) section of a 4 sectors ring electrode together with the schematic diagram of the electronic circuits that detect the cyclotron and magnetron motion components along the y axis.