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*Alpha scattering and excitation of isoscalar  
giant resonances in the interacting boson model*

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*Introduction*

The aim of the present work is to describe preliminary calculations of the giant monopole (GMR) and quadrupole (GQR) resonances within the framework of the interacting boson model (IBM) by means of a coupled - channel analysis of inelastic  $\alpha$  scattering by a transitional isotope chain ( $^{148-154}\text{Sm}$ ).

## Nuclear model

The starting point is the IBM-1 Hamiltonian in its standard multipole expansion<sup>(1)</sup>:

$$\hat{\mathcal{H}}(s,d) = E_0 + \varepsilon_d \hat{n}_d + a_0 \hat{P}^+ \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{U} \cdot \hat{U} + a_4 \hat{V} \cdot \hat{V}. \quad (1)$$

Here, the multipole operators are expressed in terms of creators and annihilators of s and d bosons, i.e. pairs of particles above, or pairs of holes below a major shell closure, with  $L=0$  and  $L=2$ , respectively, according to the definitions of ref.<sup>(1)</sup>. A limiting symmetry of the Hamiltonian, yielding the low-energy spectrum and transitions of a prolate rotor is obtained when, in eq.(1),  $\varepsilon_d = a_0 = a_3 = a_4 = 0$  and the quadrupole operator,  $\hat{Q}$ , is of the form:

$$\hat{Q}_\mu = (d^+ x \tilde{s} + s^+ x \tilde{d})_\mu^{(2)} - \frac{\sqrt{7}}{2} (d^+ x \tilde{d})_\mu^{(2)}, \quad (\mu = -2, \dots, +2), \quad (2)$$

with  $\tilde{d}_\mu = (-1)^\mu d_{-\mu}$ .

In that case the five components of  $\hat{Q}$  and the three of the angular momentum operator,  $\hat{L} = \sqrt{10} (d^+ x \tilde{d})^{(1)}$ , generate an  $su(3)$  algebra, whose irreducible representations (irreps) are labelled with Elliott's quantum numbers,  $(\lambda, \mu)$  corresponding to Young diagrams with  $\lambda + \mu$  boxes in the first row and  $\mu$  boxes in the second row.

In particular, the six creators  $s^+$ ,  $d^+$  ( $v = -2, \dots, +2$ ) belong to a  $(2,0)$  irrep and the ground-state band of the nucleus to a  $(2N,0)$  irrep,  $N$  being the total number of s-d bosons. According to refs.<sup>(2,3)</sup>, the GMR and GQR degrees of freedom are introduced into the model by means of S and D bosons, respectively, which simulate  $2\hbar\omega$  particle-hole excitations across major shell closures.

The total Hamiltonian can be assumed in the following form:

$$\hat{\mathcal{H}}(s,d,S,D) = \hat{\mathcal{H}}(s,d) + E_S \hat{n}_S + E_D \hat{n}_D + k \hat{Q}_r \cdot \hat{Q}_t. \quad (3)$$

Here, the total quadrupole operator,  $\hat{Q}_t$ , is a linear combination of the s-d quadrupole of eq.(2) and of the first- and second-order contributions to the S-D quadrupole,  $\hat{Q}'_\mu$ :

$$\begin{aligned} \hat{Q}_{t\mu} &= \alpha \hat{Q}_\mu + \beta^{(1)} \hat{Q}'_{\mu} + \gamma^{(2)} \hat{Q}'_{\mu} = \\ &= \alpha \hat{Q}_\mu(s,d) + \beta (D^+ + \tilde{D})_\mu + \gamma [(D^+ x \tilde{S} + S^+ x \tilde{D})_\mu^{(2)} + \chi' (D^+ x \tilde{D})_\mu^{(2)}], \\ &(\mu = -2, \dots, +2), \end{aligned} \quad (4)$$

where  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\chi'$  are hitherto unknown coefficients.

The introduction of S-D bosons and their type of interaction with s-d bosons could be microscopically justified in the frame of symplectic model of collective motion<sup>(4)</sup> as being produced in the  $u(3) \oplus hw(6)$  contraction of the symplectic algebra when the number,

$N_0$  ( $\approx 0.9 A^{4/3}$ ), of harmonic oscillator quanta describing the nuclear ground state goes to infinity<sup>(5)</sup>. This preliminary work is devoted to the study of the gross features of Hamiltonian (3), therefore, two drastic approximations have been made, (i): the one-boson approximation has been adopted ( $n_s, n_d = 1$  or  $0$ ); (ii): only the leading term of the  $\hat{Q}, \hat{Q}$  interaction, responsible for GMR and GQR fragmentation, has been kept in eq.(3):

$$\hat{\mathcal{H}}(s,d,S,D) = \hat{\mathcal{H}}(s,d) + E_S \hat{n}_S + E_D \hat{n}_D + c \hat{Q} \cdot {}^{(2)}\hat{Q}', \quad (5)$$

where  ${}^{(2)}\hat{Q}'$  is the S-D analogous of  $\hat{Q}$ , when  $\chi' = -\sqrt{7}/2$ . The five components of  ${}^{(2)}\hat{Q}'$  and the three of  $\hat{L}' = \sqrt{10} (D^+ x \tilde{D})^{(1)}$  generate an  $su(3)$  algebra, too, and, provided  $E_S = E_D$ , the Hamiltonian of eq.(5) is a linear combination of quadratic Casimir operators of  $SU(3)$  and  $SO(3)$  in the three representations  $(\hat{Q}, \hat{L})$ ,  $({}^{(2)}\hat{Q}', \hat{L}')$ ,  $(\hat{Q} + {}^{(2)}\hat{Q}', \hat{L} + \hat{L}')$ . Since the creators  $S^+, D_\mu^+$  ( $\mu = -2, \dots, +2$ ) form a  $(2,0)$  irrep, coupling an S-D boson to the ground-state band of an axisymmetric rotor amounts to the following reducible product of  $SU(3)$  representations:

$$(2N,0) \otimes (2,0) = (2N+2,0) \oplus (2N,1) \oplus (2N-2,2), \quad (6)$$

The GMR and GQR excitations correspond to the  $L=0$  and  $L=2$  members of the  $SO(3)$  irreps contained in the three  $SU(3)$  irreps on the right-hand-side of eq.(6), labelled with a quantum number,  $K$ , corresponding to the projection of the angular momentum on the nuclear symmetry axis; the  $(2N+2,0)$  irrep contains a  $K=0$  band with  $L=0,2,\dots,2N+2$ ;  $(2N,1)$  a  $K=1$  band with  $L=1,2,\dots,2N+1$ ;  $(2N-2,2)$  a  $K=0$  band with  $L=0,2,\dots,2N-2$  and a  $K=2$  band with  $L=2,3,\dots,2N$ . Therefore, the  $SU(3)$  limit of the model predicts two GMR components and four GQR components in an axially symmetric nucleus; the GMR states are close in energy to the GQR states with  $K=0$ , one of which has the same energy as the  $K=2$  state, since they belong to the same  $(2N-2,2)$  irrep.

Breaking the  $SU(3)$  symmetry of the S-D Hamiltonian amounts to choosing  $\chi' \neq -\sqrt{7}/2$  in eq.(4) and  $E_S \neq E_D$  in eq.(5).

The  $L=0,2$  transitions between an S-D state and a member of the ground-state band (pure s-d state, in the present approximation) are given by the matrix elements of the following operators<sup>(3)</sup>:

$$\hat{T}_0 = \alpha_0 (S^+ + S), \quad (7)$$

$$\hat{T}_{2\mu} = \alpha_2 (D^+ + \tilde{D})_\mu, \quad (\mu = -2, \dots, +2), \quad (8)$$

where the coefficients,  $\alpha_0$  and  $\alpha_2$ , are fixed by assuming that the GMR and GQR excitations exhaust the corresponding energy - weighted sum rules (EWSR):

$$\sum_m E(0_m^+) |\langle 0_m^+ | \hat{T}_0 | 0_1^+ \rangle|^2 = \frac{2\hbar^2}{m_N} A \langle r^2 \rangle_m, \quad (9)$$

$$\sum_n E(2_n^+) |\langle 2_n^+ | \hat{T}_2 | 0_1^+ \rangle|^2 = \frac{25\hbar^2}{4\pi m_N} A \langle r^2 \rangle_m, \quad (10)$$

Here, index  $m$  ( $n$ ) runs over the GMR (GQR) states and  $\langle r^2 \rangle_m$  is the mean square radius of the ground-state density.

## Alpha scattering analysis

The energies of the ( $0^+ - 2^+ - 4^+$ ) members of the ground-state band and the GMR and GQR states, as well as the reduced matrix elements of the allowed ( $L=0, 2, 4$ ) transitions, evaluated by means of the nuclear model of the previous section, are utilized in the coupled channel calculations of  $\alpha$  particle scattering by even-even samarium isotopes, carried out with the ECIS88 code<sup>(6)</sup>. The transition densities between members of the ground-state band have the radial dependence and normalization discussed in ref.<sup>(7)</sup>. The  $L=0$  transition density between S-D and s-d states has the phenomenologic form given in ref.<sup>(8)</sup>:

$$\hat{\rho}^{(0)}(r, \theta, \phi) = A_0(r) (S^+ + S) Y_0^0, \quad (11)$$

where:

$$A_0(r) = -\frac{\sqrt{4\pi}}{A} \frac{\alpha_0}{4R_m \langle r \rangle_m - X_m \langle r^2 \rangle_m} \left( R_m \frac{d\rho_m}{dr} + X_m \rho_m \right). \quad (12)$$

Here,  $\alpha_0$  is given by eq.(9),  $\rho_m = \left[ 1 + \exp\left(\frac{r - R_m}{a_m}\right) \right]^{-1}$  is the radial dependence of the ground-state density,  $\langle r^k \rangle_m = \int_0^\infty \rho_m r^{k+2} dr / \int_0^\infty \rho_m r^2 dr$  and  $X_m = 2R_m \langle r^{-1} \rangle_m$ .

The  $L=2$  transition density is written in the form:

$$\hat{\rho}^{(2)}(r, \theta, \phi) = A_2(r) \sum_{\mu=-2}^{+2} (D^+ + \tilde{D})_\mu Y_2^\mu(\theta, \phi), \quad (13)$$

where:

$$A_2(r) = -\frac{\pi}{A} \frac{\alpha_2}{R_m \langle r \rangle_m} R_m \frac{d\rho_m}{dr}, \quad (14)$$

with  $\alpha_2$  given by eq.(10).

The optical potential is given by a complex Woods - Saxon well:

$$V(r) = \frac{(V_0 + iW_0)}{1 + \exp\left[\frac{(r - R_v)}{a_v}\right]}, \quad (15)$$

whose depth is adjusted so as to reproduce the experimental cross section for elastic scattering at  $E_\alpha = 120$  MeV<sup>(9)</sup>.

Between the transition potentials and the optical potential we assume the same relation as between the transition densities and the ground-state density: therefore, the radial

dependence of the transition potentials is given by eq.(12) for  $L=0$  and eq.(14) for  $L=2$ , with  $R_v$  instead of  $R_m$  and  $\langle r^k \rangle_v = \int_0^\infty V(r)r^{k+2}dr / \int_0^\infty V(r)r^2dr$  instead of  $\langle r^k \rangle_m$ . Such an assumption for the radial form factors is equivalent to an implicit folding procedure, valid for a density independent nucleon - nucleon interaction, as shown in refs.<sup>(10,11)</sup>. A standard Coulomb form factor contributes to the  $L > 0$  transitions, but plays a minor role in the excitation of the GQR.

## Results and comments

The excitation of isoscalar resonances through the scattering of 129 MeV  $\alpha$  particles by  $^{154}\text{Sm}$  has been discussed in ref.<sup>(12)</sup>: both the angular distributions at excitation energy  $E^* = 11.8$  MeV and 14.9 MeV have been interpreted as a mixture of  $L=0$  and  $L=2$  modes, the quadrupole being dominant at the lower and the monopole at the higher energy.

The differential cross sections evaluated in the SU(3) limit of the model, with the IBM parameters and optical model parameters listed in Table I, are compared with the experimental data in fig.1. The high-energy cross section is the sum of an  $L=0$  contribution at  $E^* = 14.41$  MeV, exhausting 42% of the monopole EWSR and of an  $L=2$ ,  $K=0$  contribution at  $E^* = 14.48$  MeV containing 15% of the quadrupole EWSR.

TABLE I : IBM AND OM PARAMETERS

IBM :	154-Sm (SU(3))LIMIT	154-Sm (BROKEN SU(3))	148-Sm (BROKEN SU(3))
N	11	11	8
$\epsilon_d$ (KeV)	0	371.0	770.3
$a_0$ (KeV)	0	8.0	52.3
$a_1$ (KeV)	7.1	0.5	4.0
$a_2$ (KeV)	- 17.5	- 19.6	- 12.6
$x$	- $\sqrt{7/2}$	- 1.20	- 0.65
$a_3$ (KeV)	0	8.4	29.6
$a_4$ (KeV)	0	5.7	19.7
$E_S$ (MeV)	11.30	14.18	15.50
$E_D$ (MeV)	11.30	11.75	12.50
$C_{QQ}$ (KeV)	130	80	80
$x'$	- $\sqrt{7/2}$	- 0.15	- 0.15
$\alpha_0$ (fm <sup>2</sup> )	197.7	177.2	169.1
$\alpha_2$ (fm <sup>2</sup> )	88.2	86.4	81.6
OM :	154-Sm ( $E_\alpha = 129\text{MeV}$ )		148-Sm ( $E_\alpha = 115\text{MeV}$ )
$V_0$ (MeV)	50.80		58.80
$W_0$ (MeV)	36.90		32.90
$R_V$ (fm)	7.50		7.41
$a_V$ (fm)	0.73		0.73

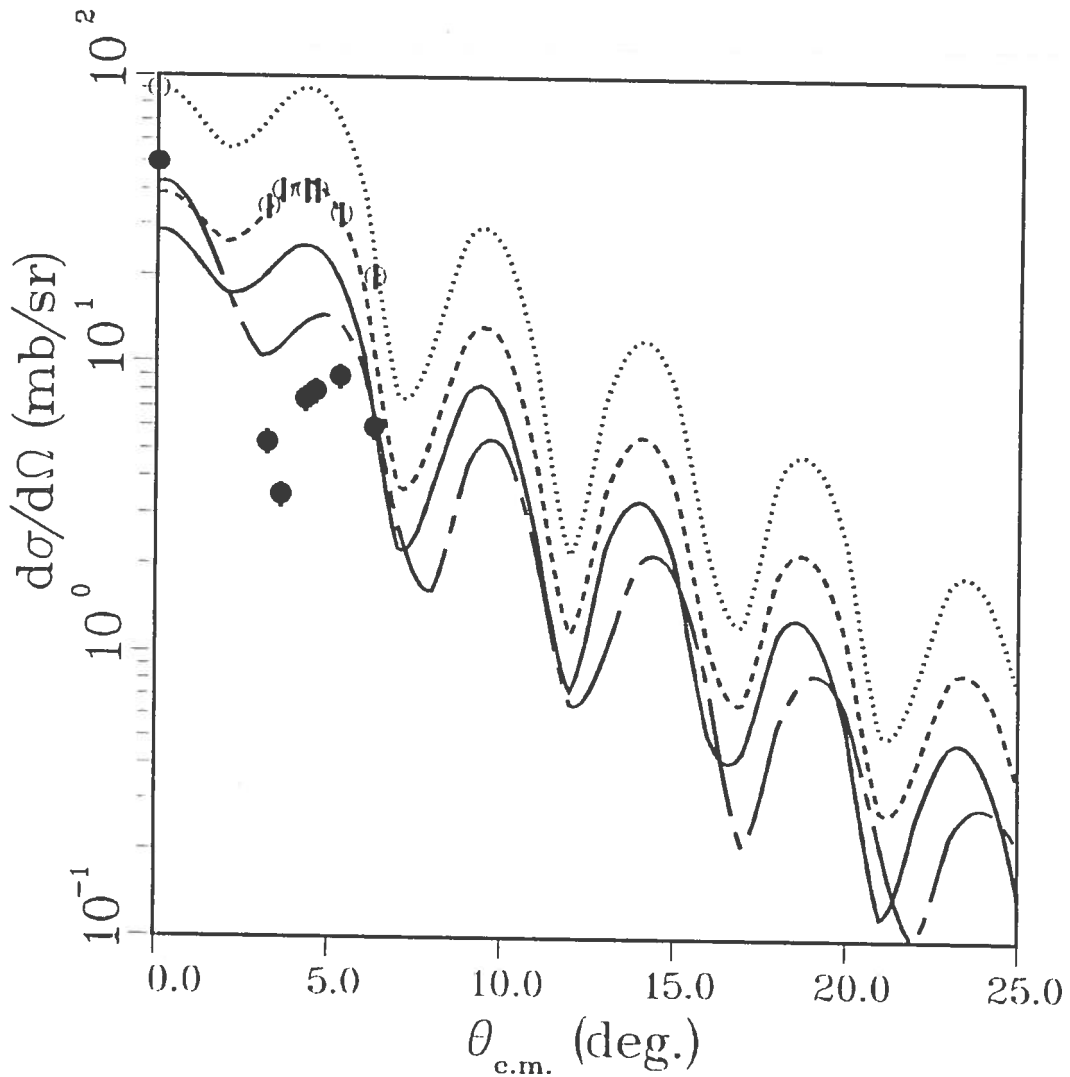


Fig. 1. GMR and GQR excitations through  $\alpha$  scattering at  $E_\alpha$  MeV by  $^{154}\text{Sm}$  in the SU(3) limit.  $\phi$  : expt. data at  $E = 11.8$  MeV from ref.<sup>(12)</sup>;  $\blacklozenge$  : expt. data at  $E = 14.9$  MeV<sup>(12)</sup>;  $\text{---}$   $E = 14.4$  MeV: L=0 (42% EWSR) + L=2 (15% EWSR);  $\text{— — —}$   $E = 12.2$  MeV:L=2 (42% EWSR);  $\text{---}$  L=2 (50% EWSR);  $\text{.....}$  L=2 (85% EWSR).

The calculation reproduces the experimental datum at  $\theta_{c.m.} = 0^\circ$  and is slightly higher than the experiment in the  $2^\circ \div 6^\circ$  angular range.

The disagreement is stronger at low energy, where the L=2, K=1 excitation at  $E^* = 12.20$  MeV, with strength  $S = 42\%$  EWSR, underestimates the experimental cross section, which could be reproduced in the  $2^\circ \div 6^\circ$  range by adding the L=2 contribution at  $E^* = 9.89$  MeV so as to exhaust 50% EWSR. Adding the fourth L=2 contribution at  $E^* = 9.83$  MeV, with total strength  $S = 85\%$  EWSR, the datum at  $\theta_{c.m.} = 0^\circ$  is reproduced, but the values at higher angles are overestimated by more than a factor of 2. The excitation of a strong L=0 component at  $E^* = 9.80$  MeV, with  $S = 58\%$  EWSR, seems to be quite incompatible with the experimental results.

Breaking the SU(3) symmetry of the Hamiltonian, when  $E_S \neq E_D$  and  $\chi' \neq -\sqrt{7}/2$ , allows us to change the L=0 and L=2 strengths in the two cross sections. In particular, the choice of a weaker GMR - GQR coupling brings the theoretical curves to better agreement with the experiment, as shown in fig.2. In that case the high energy cross section is mainly L=0, with S = 87% EWSR, while L=2 is reduced to S = 5% EWSR only; the low energy cross section almost exhausts the L=2 strength (S = 95% EWSR), but contains a sizeable L=0 contribution (S = 13% EWSR) which reproduces the experimental value at  $\theta_{c.m.} = 0^\circ$ .

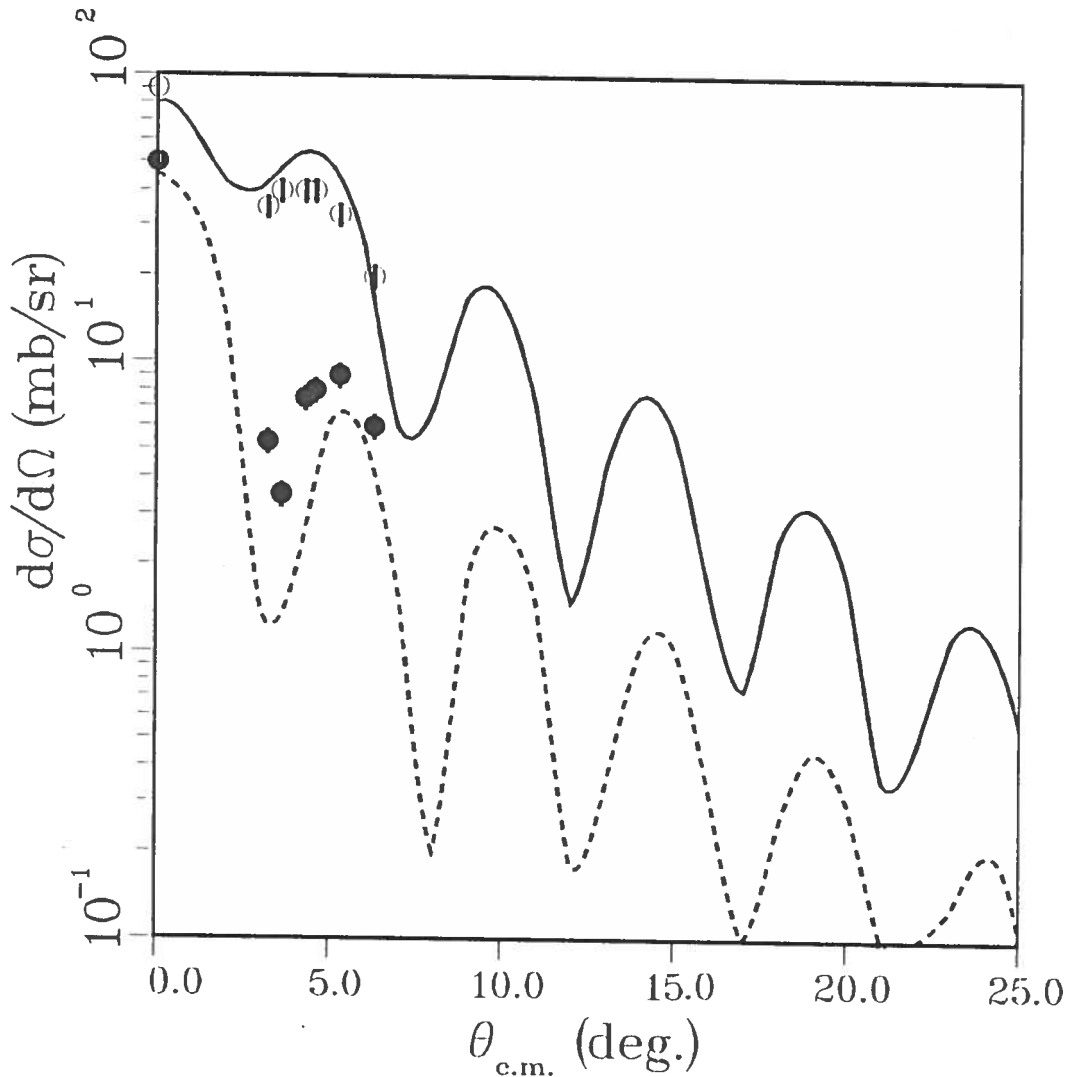


Fig. 2.  $^{154}\text{Sm}(\alpha, \alpha')$ ,  $E_\alpha = 129$  MeV, broken SU(3) symmetry; expt. data as in fig. 1; \_\_\_\_\_  $E = 12$  MeV: L=0 (13% EWSR) + L=2 (95% EWSR); - - -  $E = 15$  MeV: L=0 (87% EWSR) + L=2 (5% EWSR).

The reliability of the model in describing the GMR and GQR excitation in a transitional isotope chain can be checked by evaluating inelastic  $\alpha$  scattering by the spherical nucleus  $^{148}\text{Sm}$  with a small adjustment of the S and D boson energies so as to reproduce GMR and GQR energy centroids, but without changing the parameters, c and  $\chi'$ , of the interaction Hamiltonian.



The calculations are compared in fig.3 with the experimental data of inelastic scattering of 115 MeV  $\alpha$  particles taken from ref.<sup>(13)</sup>. The cross section at  $E^* = 12.0$  MeV is satisfactorily reproduced by an  $L=2$  component with  $S = 99\%$  EWSR and an  $L=0$  component with  $S = 9\%$  EWSR, but the cross section at  $E^* = 14.9$  MeV is strongly underestimated by the calculation, predicting an almost pure  $L=0$  excitation, with  $S = 91\%$  EWSR. The data of ref.<sup>(13)</sup> at  $E_\alpha = 115$  MeV, as well as the relative cross sections for the scattering of 129 MeV  $\alpha$  particles in the  $2^\circ \div 6^\circ$  angular range given in ref.<sup>(14)</sup>, could only be explained by a strong mixture of  $L=0$  and  $L=2$  modes at both excitation energies; on the other hand, recent measurements<sup>(15)</sup> of inelastic scattering of 120 MeV  $\alpha$  particles at  $\theta_{c.m.} \simeq 0^\circ$  favour an almost pure  $L=0$  excitation at  $E^* = 14.95$  MeV ( $S_{\text{expt.}} = (117 \pm 27)\%$  EWSR) and an  $L=2$  excitation at  $E^* = 12.66$  MeV ( $S_{\text{expt.}} = (87 \pm 27)\%$  EWSR) in better qualitative agreement with the present work.

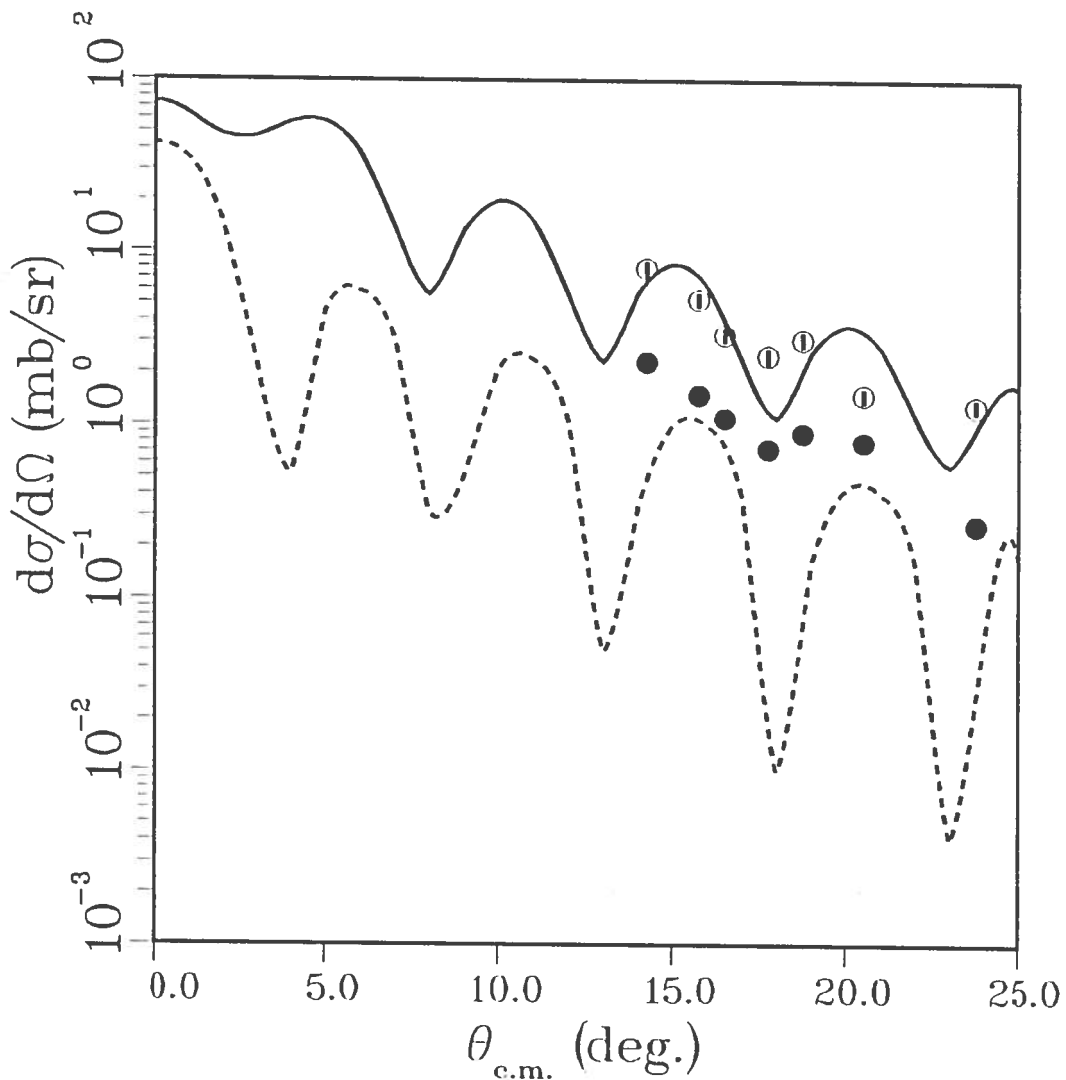


Fig. 3.  $^{148}\text{Sm}(\alpha, \alpha'), E_\alpha = 115$  MeV, broken  $\text{SU}(3)$  symmetry;  $\phi$  : expt. data at  $E^* = 12$  MeV from ref.<sup>(13)</sup>;  $\clubsuit$  : expt. data at  $E^* = 15$  MeV<sup>(13)</sup>; \_\_\_\_\_  $L=0$  (9% EWSR) +  $L=2$  (99% EWSR); .....  $L=0$  (91% EWSR).

While the experimental situation is not completely clear, mainly for  $^{148}\text{Sm}$ , the nuclear model needs to be improved by going beyond the one-boson approximation and replac-

ing the adopted quadrupole - quadrupole interaction of eq.(5) with the more general form of eq.(3), susceptible to solid microscopic interpretation. We expect, in that case, to get a deeper insight into the structure of isoscalar resonances not only for deformed nuclei, where the preliminary calculations look encouraging, but also for transitional nuclei, which can hardly be treated by any theoretical model.

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