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GENERALISED EXCITON MODEL THEORIES FOR THE DESCRIPTION OF PRE-EQUILIBRIUM ANGULAR DISTRIBUTIONS

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ABSTRACT

The theories proposed to reproduce, in the framework of the exciton model, the angular distributions of pre-equilibrium particles are reviewed. It is shown that, contrary to a widespread opinion, the most refined calculations based on these theories allow a satisfactory reproduction of the experimental data, that is comparable to that obtainable with quantum-mechanical multistep theories.

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1. INTRODUCTION

The exciton model, formulated by Griffin (1966), and improved by several other authors (see for instance, Williams 1970, Cline and Blann 1971, Gadioli, Gadioli Erba and Sona 1973), soon proved able to reproduce the angle integrated spectra of emitted particles and the excitation functions of a large number of reactions (Gadioli, Gadioli Erba and Hogan 1977).

However, the experimental evidence that the particles, whose emission when integrated over the angles is well described by the exciton model, have a forward angular distribution, throwed some doubts on the consistency of a formulation based on the hypothesis of the equiprobability of all the states of a given n-exciton configuration and of the decay modes of the composite nucleus. The argument is as follows. The decay rates used for evaluating the angle integrated cross sections do not include any provision to specify the direction of emitted particles. Rather, if the state densities count all the possible states corresponding to a given energy and a given exciton number, assuming them as equiprobable, also every direction of the emitted particle should be equiprobable and hence the angular distributions predicted by the model should be isotropic.

To reproduce forward peaked angular distributions one must introduce an angular dependence in the expression of the decay rates for particle emission.

It may be easily shown (Gadioli *et al* 1977) that the expression of the decay rate for particle emission $\lambda_{n,c}(E,\epsilon_c)$ may be factorised as follows (for simplicity we consider nucleon emission):

$$\lambda_{n,c}(E,\epsilon_c)d\epsilon_c = [\frac{\rho_{n-1}(U)g}{\rho_n(E)}][\frac{\sigma_{inv}(\epsilon_c)v\rho_c(\epsilon_c)}{gV}]d\epsilon_c, \quad (1)$$

where g and V are respectively the nucleon single state density and the laboratory volume. The first term of this expression (hereafter to be called for brevity $P_n(\epsilon_c)$) is simply the product of the number of excited particles times the density of probability of having a particular configuration with a particle with excitation energy equal to ϵ_c+B (B is the particle binding energy, and the energy left to the other nucleons is U). The second is the decay rate for emission of this particle in the continuum. Let, now, Ω be the direction of the emitted particle with respect to the projectile. In a generalised expression of the decay rate for particle emission, $P_n(\epsilon_c)$ should be substituted by a quantity $P_n(\epsilon_c,\Omega)$ specifying also the direction of the emitted particle. The expression of this quantity, may be easily derived, for the 3-exciton initial configuration, in the case of nucleon induced reactions. It is given by

$$P_3(\epsilon_c,\Omega) = 2P_{HKK}(\epsilon_c,\Omega) \quad (2)$$

where

$$P_{HKK}(\epsilon_c,\Omega) = \frac{1}{\int\sigma(\mathbf{k}_f)d\mathbf{k}_f}\frac{\sigma(\mathbf{k}_f)d\mathbf{k}_f}{d\epsilon_c d\Omega}. \quad (3)$$

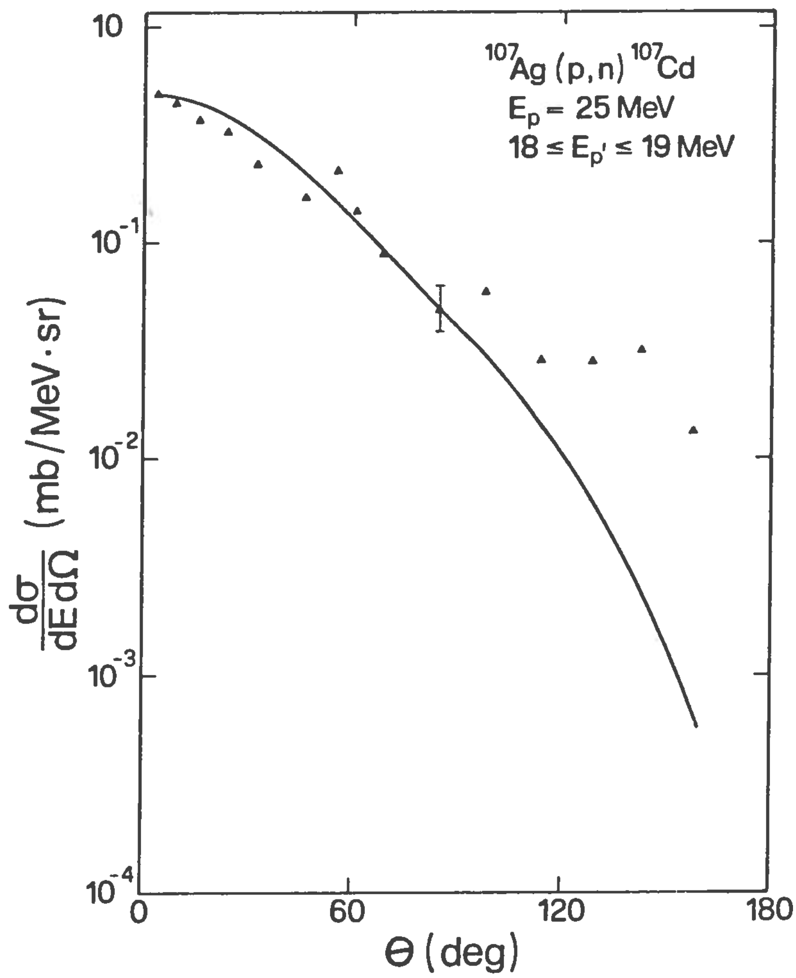

Fig. 1 - Angular distribution of ≈18.5 MeV neutrons from 25 MeV proton bombardment of ^{107}Ag. The experimental results (black triangles) are from Grimes *et al* (1976).

$\sigma(\mathbf{k}_f)\mathrm{d}\mathbf{k}_f$ is the differential cross section for a single scattering in nuclear matter, that leaves the scattered particle with momentum between $\mathbf{k}_f$ and $\mathbf{k}_f + \mathrm{d}\mathbf{k}_f$ in the laboratory system. Its expression is given by (Hayakawa *et al* 1955):

$$\sigma(\mathbf{k}_f)d\mathbf{k}_f = \frac{4d\mathbf{k}_f}{v_1(4\pi/3)k_F^3}\int \sigma(\mathbf{k},\mathbf{k}')\delta(k'^2 - k^2)d\mathbf{k}_2 \tag{4}$$

where v_1 is the velocity of the incident particle in the nucleus, k_F is the Fermi momentum, $\mathbf{k}_f$ the final momentum of the scattered particle, $\mathbf{k}_2$ is the momentum of the target nucleon and $\mathbf{k}$ and $\mathbf{k}$' are half the relative momenta of colliding nucleons before and after scattering. $\sigma(\mathbf{k}, \mathbf{k}')$ is the free nucleon-nucleon cross-section.

When emission from the first stage of the cascade dominates, the angular distributions calculated using in the expression of the decay rate for particle emission $P_{HKK}(\epsilon_c,\Omega)$ instead of $P_3(\epsilon_c)$ reproduce quite reasonably the experimental angular distributions of ejectiles up to $\approx 90°$. A typical result (obtained by assuming in (4) isotropic free nucleon-nucleon cross sections varying as the inverse of the relative energy of the two nucleons and considering refraction of the incoming and outgoing particles at the crossing of the nuclear surface (Gadioli and Gadioli Erba 1980)) is shown in Fig. 1.

It may be also shown that

$$\int P_3(\epsilon_c,\Omega)d\Omega \approx P_3(\epsilon_c) \equiv \frac{\rho_{1,1}(U)g}{\rho_{2,1}(E)}, \tag{5}$$

as shown for a typical case in Fig. 2 (there P_3 is reported as a function of U=E-B-ϵ_c). This relation means that to take into account, in a two body interaction, momentum conservation results in a density of probability of finding a particle with energy between ϵ_c and ϵ_c+dϵ_c, at the beginning of the cascade, that does not differ appreciably from that calculated using state densities which count all the possible configurations characterised by a given energy and three excitons. It also follows, that in the approximation that emission to the continuum may be disregarded, a cascade of two body interactions leads to exciton distributions that are very nearly those calculated by considering all states as equiprobable. This validates the equiprobability assumption first introduced by Griffin.

Consideration of emissions only from the initial states does not allow one to reproduce the angular distributions at backward angles. In what fallows we will review the various methods that have been developed to describe emissions from subsequent stages of the intranuclear cascade.

2. GENERALISED MASTER EQUATIONS

These calculations do not take into account all possible dynamical paths in the course of the cascade and limit themselves to an approximation which consists in assuming that the classes of states reached during equilibration be characterised by the exciton number n and the direction Ω of the *fast particle*.

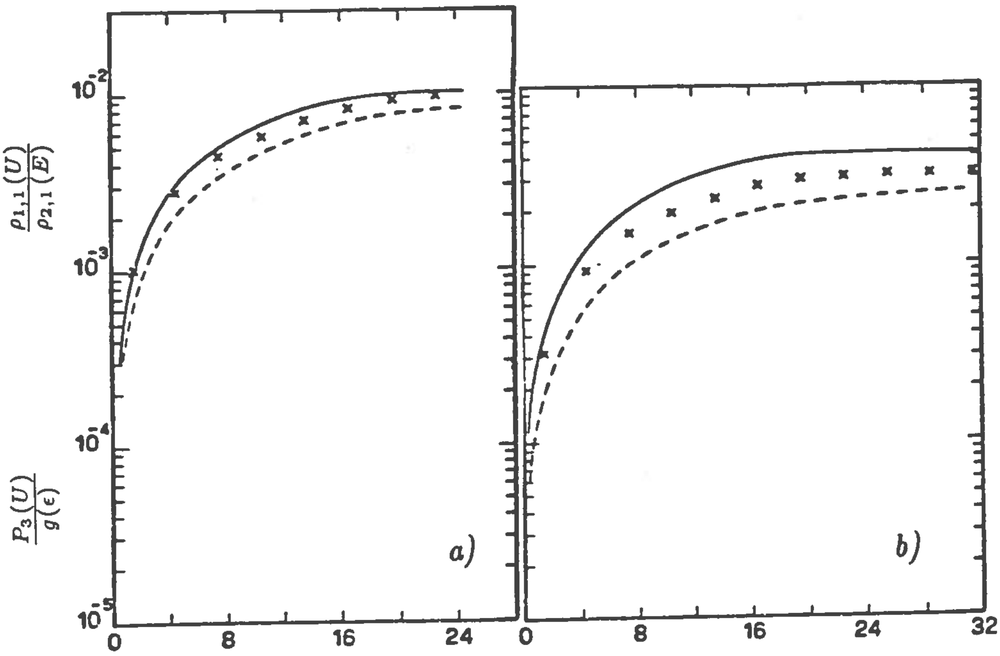

Fig. 2 - Comparison of the calculated values of $P_3(U)/g(\epsilon)$ (crosses) and $\rho_{1,1}(U)/\rho_{2,1}$ (E) (solid and dashed lines), for a proton induced reaction and (a) E=33 MeV, (b) E=68 MeV. The dashed line corresponds to level densities calculated using Fermi Gas single nucleon densities $g(\epsilon) = g_{FG}(\epsilon) \propto \epsilon^{1/2}$, the solid line to $g(\epsilon) = g_{FG}(\epsilon)$ for $\epsilon \leq \epsilon_F$ and $g(\epsilon) = g_{FG}(\epsilon_F)$ for $\epsilon \geq \epsilon_F$ (Gadioli *et al* 1977).

If $P_n(\Omega,t)$ is the probability of finding the nucleus in a state of class (n,Ω), at time t, and the decay rate for transitions to states of class (m,Ω') is $\lambda_{n,m}(\Omega \rightarrow \Omega')$, the master equation, first introduced by Cline and Blann (1971) to evaluate the time dependent occupation probability of states corresponding to a given exciton number n , may be generalised as follows (Mantzouranis, Weidenmüller and Agassi 1976):

$$\frac{dP_n(\Omega,t)}{dt} = \sum_m \int d\Omega' P_m(\Omega',t)\lambda_{m,n}(\Omega' \rightarrow \Omega)$$

$$-P_n(\Omega,t)[\sum_m \int d\Omega'\lambda_{n,m}(\Omega \rightarrow \Omega') + \lambda_n^c], \tag{6}$$

where

$$\lambda_n^c = \sum_c \int \lambda_{n,c}(E,\epsilon_c)d\epsilon_c \tag{7}$$

is the total decay rate for particle emission. The differential cross-section for transition to channel c is given by

$$\frac{d^2\sigma}{d\epsilon_c d\Omega} \propto \int_0^{t_{eq}} \sum_{n,\Delta n=2} P_n(\Omega,t)\lambda_{n,c}(E,\epsilon_c)dt. \tag{8}$$

Assuming that $\lambda_{m,n}(\Omega \rightarrow \Omega')$ may be factorized as the product of an energy times an angle dependent factor one has

$$\lambda_{m,n}(\Omega \rightarrow \Omega') = \lambda_{m,n}G_{m,n}(\Omega \rightarrow \Omega'), \tag{9}$$

where $\lambda_{m,n}$ is the usual exciton-exciton decay rate. Diagonalising the intranuclear scattering kernel $G_{m,n}(\Omega \rightarrow \Omega')$, according to

$$\int G_{m,n}(\Omega \rightarrow \Omega')P_l(cos\theta')d\Omega' = \mu_l(m,n)P_l(cos\theta) \tag{10}$$

and expanding the occupation probability $P_n(\Omega,t)$ in series of Legendre polinomials

$$P_n(\Omega,t) = \sum_l \eta_l(n,t)P_l(cos\theta), \tag{11}$$

the generalised master equation (6) reduces to

$$\frac{d\eta_l(n,t)}{dt} = \mu_l(n-2,n)\lambda_{n-2,n}\eta_l(n-2,t) + \mu_l(n+2,n)\lambda_{n+2,n}\eta_l(n+2,t)$$

$$-[\lambda_n^c + \lambda_{n,n+2} + \lambda_{n,n-2} + (1-\mu_l(n,n))\lambda_{n,n}]\eta_l(n,t). \tag{12}$$

Defining

$$\xi_l(n) = \int_0^\infty \eta_l(n,t)dt, \tag{13}$$

one obtains the time integrated master equation

$$-\eta_l(n,t=0) = \mu_l(n-2,n)\lambda_{n-2,n}\xi_l(n-2) + \mu_l(n+2,n)\lambda_{n+2,n}\xi_l(n+2)$$
$$-[\lambda_n^c + \lambda_{n,n+2} + \lambda_{n,n-2} + (1-\mu_l(n,n))\lambda_{n,n}]\xi_l(n), \tag{14}$$

(Akkermans, Gruppelaar and Reffo 1980 and Costa, Gruppelaar and Akkermans 1983).

The double differential cross sections are given by

$$\frac{d^2\sigma}{d\epsilon_c d\Omega}(a,c) = \sigma_a \sum \lambda_{n,c}(\epsilon_c)\tau(n,\Omega), \tag{15}$$

where

$$\tau(n,\Omega) = \sum_l \xi_l(n)P_l(cos\theta). \tag{16}$$

Mantzouranis *et al* (1976) proposed for G($\Omega \rightarrow \Omega'$) the expression

$$G(\Omega \rightarrow \Omega') = \frac{d\sigma(\Omega \rightarrow \Omega')}{d\Omega'} / \int \frac{d\sigma(\Omega \rightarrow \Omega')}{d\Omega'}, \tag{17}$$

using for d$\sigma(\Omega \rightarrow \Omega')$/d$\Omega'$ the free differential nucleon-nucleon cross section

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega_L}\frac{d\Omega_L}{d\Omega} \propto cos\theta_L H(\frac{\pi}{2} - \theta_L)\frac{d\Omega_L}{d\Omega}, \tag{18}$$

where Ω and Ω_L refer to, respectively, the CM system of the two colliding nucleons and the Lab. System and H is the Heaviside function. With this *Ansatz* analytical expressions may be derived for the eigenvalues μ_l appearing in (9) (which also become independent on the exciton number n) and for the ξ_l (Akkermans, Gruppelaar and Reffo 1980).

This approximation allows only a qualitative agreement with the data, and neglects important effects like the Fermi motion of the struck nucleons and the Pauli principle. To account for these effects Sun *et al* (1982) used for G($\Omega \rightarrow \Omega'$) the energy averaged value of the differential cross section for nucleon nucleon scattering in nuclear matter as given by (3) and (4):

$$G(\Omega \rightarrow \Omega') = \int_{E_F}^{\epsilon_{max}} P_{HKK}(\epsilon,\Omega)d\epsilon, \tag{19}$$

where ϵ=ϵ_c+E$_F$+B is the nucleon energy, after the scattering, inside the nucleus, and E_F is the Fermi energy. For P_{HKK} they used analytical expression reported by

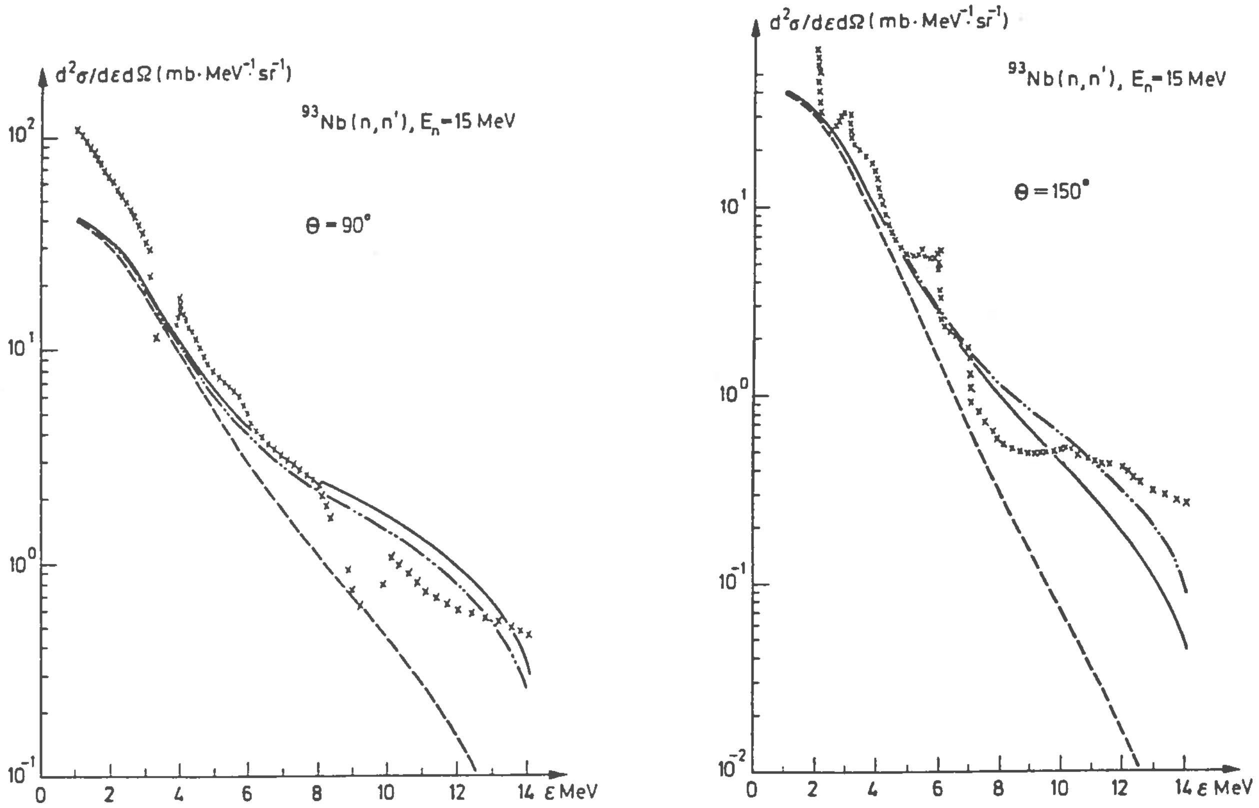

Fig. 3 - Comparison of calculated double differential cross sections (Sun *et al* 1982) with experimental results (Hermsdorf *et al* 1974) for the reaction and the emission angles indicated in the figure. The dashed line corresponds to the use of relation (17), the solid line to the use of relation (19), the double dot-dash line to use of (19) but neglecting in the calculation the effect of the Pauli principle.

Kikuchi and Kawai (1968) in the approximation of energy independent and isotropic free nucleon-nucleon cross sections. The same approximation was also made in the papers we will discuss later. Since the energy distribution of each excited particle is now depending on n, also the eigenvalues μ_l display such dependence.

The ability of the theory to reproduce the experimental data is greatly improved expecially at backward angles as shown in Fig. 3 for a typical case. However, the factorisation of $\lambda_{m,n}(\Omega \rightarrow \Omega')$ in the product of an energy times an angle dependent factor is certainly incorrect for the first few collisions of the projectile with the target nucleons when the energy and the angle of the fast particle after the collision are strongly correlated (a small loss of energy means a small deviation from the original direction). This correlation manifests itself also in forbidding that in the first scattering the fast particle be emitted at an angle exceeding a value $\theta_{kl}(\epsilon)$ depending on its energy. The use of energy averaged kernels G($\Omega \rightarrow \Omega'$), to which also very low values of the final energies contribute, violates this kinematical rule, as shown by Iwamoto and Harada (1984). To take into account the angle-energy correlation the classes of states reached during equilibration must be characterised not only by n and Ω but also by the energy of the fast particle. The expression of the generalised master equation becomes (Iwamoto and Harada 1984)

$$\frac{dP_n(\Omega,\epsilon,t)}{dt} = \sum_m \int d\Omega' \int d\epsilon' P_m(\Omega',\epsilon',t)\lambda_{m,n}(\Omega'\epsilon' \rightarrow \Omega\epsilon)$$

$$-P_n(\Omega,\epsilon,t)[\sum_m \int d\Omega' d\epsilon' \lambda_{n,m}(\Omega\epsilon \rightarrow \Omega'\epsilon') + \lambda_n^c)]. \tag{20}$$

$\lambda_{n,m}(\Omega\epsilon\rightarrow \Omega'\epsilon')$ is now given as

$$\lambda_{n,m}(\Omega\epsilon \rightarrow \Omega'\epsilon') = \lambda_{n,m} G_n(\Omega\epsilon \rightarrow \Omega'\epsilon'), \tag{21}$$

where $\lambda_{n,m}$ denotes the total transition rate from the n-exciton state to the m-exciton state irrespective of Ω and ϵ. Iwamoto and Harada suggest for $G_n(\Omega\epsilon \rightarrow \Omega'\epsilon')$ the expression

$$G_n(\Omega\epsilon \rightarrow \Omega'\epsilon') = \alpha_n G(\Omega\epsilon \rightarrow \Omega'\epsilon') + \beta_n \delta(\Omega - \Omega')\delta(\epsilon - \epsilon'), \tag{22}$$

with α_n and β_n satisfying the relation

$$\alpha_n + \beta_n = 1. \tag{23}$$

Expression (22) considers both the interactions of the fast particle and of the other excited particles. Notwhistanding such improvement, the *fast particle* approximation is still operative, and in a sense is made stronger. In fact, as shown by Iwamoto and Harada (1984), the angular distributions calculated using $\beta_n \neq 0$ are more forward peaked than those corresponding to $\beta_n=0$, since the direction of the fast particle is preserved to the next stage when in the previous one another particle interacts.

Relaxing the fast particle approximation means to allow for emission to the continuum of *slow particles* that thus increase the large angle yield being their direction much less correlated to that of the projectile.

Relation (10) applies also to $G_n(\Omega\epsilon \rightarrow \Omega'\epsilon')$ resulting in eigenvalues $\mu_l(n,\epsilon,\epsilon')$ depending both on *n* and the initial and final energy of the fast particle

$$\mu_l(n,\epsilon,\epsilon') = \alpha_n \mu_l(\epsilon,\epsilon') + \beta_n \delta(\epsilon - \epsilon'), \tag{24}$$

where $\mu_l(\epsilon,\epsilon')$ satisfies the relation

$$\int d\Omega' G(\Omega\epsilon \rightarrow \Omega'\epsilon') P_l(cos\theta') = \int d\Omega' P_{HKK}(\epsilon, \Omega \rightarrow \epsilon', \Omega') P_l(cos\theta') =$$

$$= \mu_l(\epsilon,\epsilon') P_l(cos\theta). \tag{25}$$

Use of relations (11) and (13) in this more general case and integration of (20) over *t* leads finally to the time integrated master equation

$$-\frac{2l+1}{4\pi}\mu_l(n,\epsilon_{inc},\epsilon)\delta_{n,n_o} = \sum_m \lambda_{m,n} \int d\epsilon' \xi_l(m,\epsilon')\mu_l(n,\epsilon',\epsilon)$$

$$-\xi_l(n,\epsilon)[\sum_m \lambda_{n,m} + \lambda_n^c]. \tag{26}$$

Iwamoto and Harada (1984), have shown that, in the hypothesis of

$$\lambda_{n,n+2} \gg \lambda_{n,n}, \lambda_{n,n-2}, \tag{27}$$

$$\xi_l(n,\epsilon) = \tau(n)\frac{2l+1}{4\pi}\int d\epsilon_1 \int d\epsilon_2 ..... \int d\epsilon_{(n-n_o)/2}$$

$$\times \mu_l(n_o,\epsilon_{inc},\epsilon_1)\mu_l(n_o+2,\epsilon_1,\epsilon_2)...\mu_l(n,\epsilon_{(n-n_o)/2},\epsilon), \tag{28}$$

where $\epsilon_1,\epsilon_2,...\epsilon_j$ are the energies of the fast particle after 1,2,...,j collisions, and

$$\tau(n) = (\prod_{i=n_o,\Delta i=2}^{n-2} \frac{\lambda_{i,i+2}}{\lambda_{i,i+2}+\lambda_i^c})/(\lambda_{n,n+2}+\lambda_n^c). \tag{29}$$

Defining

$$\tau(n,\Omega,\epsilon) = \int_0^\infty P_n(\Omega,\epsilon,t)dt = \sum_l \xi_l(n,\epsilon)P_l(cos\theta), \tag{30}$$

one finally obtains the cross section for the double differential cross section as

$$\frac{d^2\sigma}{d\Omega d\epsilon_c}(a,c) = \sigma_a \sum \tau(n)\lambda_{n,c}(E,\epsilon_c)\frac{\tau(n,\Omega,\epsilon)}{\int \tau(n,\Omega,\epsilon)d\Omega}. \tag{31}$$

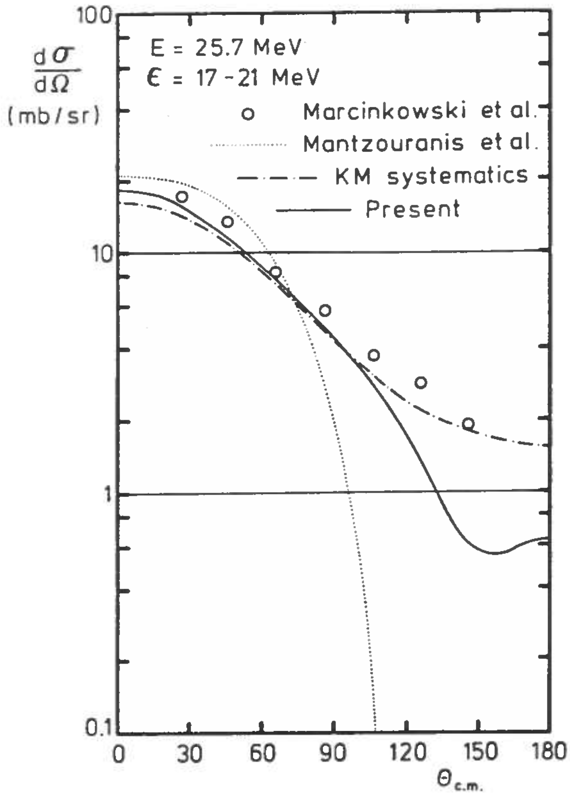

Fig. 4 - Comparison of calculated (Costa *et al* 1983), full line, and experimental angular distributions (Marcinkowski *et al* 1983) for neutron inelastic scattering at 25.7 MeV incident neutron energy. The emitted neutron energy is in the 17 - 21 MeV energy interval. The experimental data have been averaged over ten elements around ^{93}Nb.

Angle-energy correlation (by evaluating the eigenvalues of the intranuclear scattering kernel $G_n(\Omega\epsilon \rightarrow \Omega'\epsilon')$ using (25)) was first considered by Costa, Gruppelaar and Akkermans (1983) in the case of (n,n') reactions. However, these authors limited themselves to considering the effect of this correlation on decays from the initial n=3 stage, using for later stages the energy averaged kernels (19). In addition to that, they considered also the effect of refraction on the projectile and the ejectile as suggested by Gadioli and Gadioli Erba (1981). A typical results is shown in Fig. 4. The agreement with the data is quite good up to about 110°, but at more backward angles there is yet a substantial underestimation of the measured angular distributions. Calculations by Iwamoto and Harada (1984) who consider the angle-energy correlation at all stages of the cascade reproduce less accurately the data.

Sato (1985) suggests that the agreement between calculated and experimental angular distributions at backward angles could be improved including finite nuclear size effects that the semiclassical calculation of $G_n(\Omega\epsilon \rightarrow \Omega'\epsilon')$ disregards. To do that, the nucleon-nucleon differential cross-section is evaluated in plane wave Born approximation assuming a delta interaction $V_o\delta(\mathbf{r}-\mathbf{r}_t)$ between the fast nucleon and a bound target nucleon moving in an harmonic oscillator potential. After smearing the calculated cross section for a nucleon-nucleon scattering leading the struck nucleon from an n-fold to an m-fold degenerate state, with a Gaussian or Lorentzian weight function the intranuclear scattering kernel is calculated by relation (17). The angular distribution of particles emitted at the backward angles is now reproduced quite accurately as shown in Fig. 5. These results compare quite favourably with one and two step calculations made with multistep direct reaction theory by Tamura *et al* (1982).

We conclude this discussion of the results obtained in the framework of the generalised exciton model by mentioning the suggestion by De *et al* (1985) that in evaluating the angular distribution of the fast particle one should take into account that after the first interaction the nuclear temperature is no longer zero. Consideration of this effect increases emission at backward angles thus improving the agreement between calculated and experimental angular distributions.

3. LINEAR MOMENTUM DEPENDENT PARTIAL STATE DENSITIES

In these calculations, in the framework of the exciton model, the phase space is extended to total energy and linear momentum **P** of the exciton gas, and all excitons in a given class of states (n,E,**P**) are treated statistically (this assumption is, then, in some sense opposite to the *fast particle* approximation). This approach, disregarding the dynamics of the cascade of individual scatterings, leads quite naturally, to a closed form expression for evaluating the angular distribution (Mädler and Reif (1980), Iwamoto (1987)). The decay rate for nucleon emission to the continuum becomes

$$\lambda_{n,c}(E,\epsilon_c)d\epsilon_c = \frac{2s+1}{\pi^2\hbar^3}\mu\epsilon_c\sigma_{inv}(\epsilon_c)\frac{\rho_{p-1,h}(U,P^f_{\parallel},P^f_{\perp})}{\rho_{p,h}(E,P^i_{\parallel},0)}, \qquad (32)$$

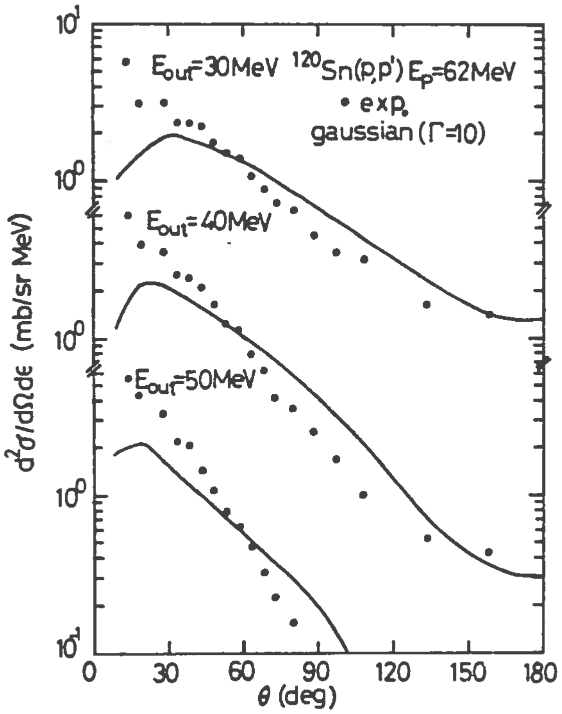

Fig. 5 - Comparison of calculated (Sato 1985) and experimental angular distributions (Bertrand and Peelle 1973) for proton inelastic scattering on ^{120}Sn at 62 MeV.

where $P_{\|}^{i}=\sqrt{2m\epsilon_{inc}}$,

$$P_{\|}^{f} = P_{\|}^{i} - \sqrt{2m(\epsilon_c + \epsilon_F + B)}cos\theta,$$

and

$$P_{\perp}^{f} = \sqrt{2m(\epsilon_c + \epsilon_F + B)}sin\theta.$$

To evaluate the momentum dependent state densities one starts from the partition function (Mädler and Reif 1980)

$$Z_{p,h}(\beta,\overline{\mathbf{v}}) = \sum_k \exp[-\beta E_k + \overline{\mathbf{v}}\cdot\mathbf{P}_k). \tag{33}$$

Introducing a single particle level density $g(\epsilon,\Omega)=g/4\pi$, replacing the summation by an integration over single-particle states, if one neglects the Pauli principle, one obtains

$$Z_{p,h}(\beta,\overline{\mathbf{v}}) = \frac{1}{p!}[\int\int_{(p)} d\epsilon d\Omega g(\epsilon,\Omega)\exp(-\beta\epsilon + \overline{\mathbf{v}}\cdot\mathbf{p})]^{\mathrm{p}}$$

$$\times\frac{1}{h!}[\int\int_{(h)} d\epsilon d\Omega g(\epsilon,\Omega)\exp(\beta\epsilon - \overline{\mathbf{v}}\cdot\mathbf{p})]^{\mathrm{h}}. \tag{34}$$

The level density is the inverse Laplace transform of the partition function

$$\rho_{p,h}(E,\mathbf{P}) = \frac{1}{(2\pi i)^4}\int\int_{i\infty}^{+i\infty} d\beta d\overline{\mathbf{v}} Z_{p,h}(\beta,\overline{\mathbf{v}})\exp(\beta E - \overline{\mathbf{v}}\cdot\mathbf{P}). \tag{35}$$

The application of saddle point approximation to evaluate the integrals yields

$$\rho_{p,h}(E,\mathbf{P}) = \frac{\exp(S_{p,h})}{4\pi^2\sqrt{D_{p,h}}}, \tag{36}$$

where $D_{p,h}$ is a symmetrical determinant whose elements are the second derivatives of the natural logaritm of the partition function with respect to β and the components of $\overline{\mathbf{v}}$. $S_{p,h}$ is the entropy given by

$$S_{p,h} = \ln Z_{p,h}(\beta,\overline{\mathbf{v}}) + \beta E - \overline{\mathbf{v}}\cdot\mathbf{P}, \tag{37}$$

and β and $\overline{\mathbf{v}}$ are obtained by solving the saddle-point equations

$$\frac{\partial Z_{p,h}}{\partial\beta} = -E, \qquad \frac{\partial Z_{p,h}}{\partial\overline{\mathbf{v}}} = \mathbf{P}. \tag{38}$$

Fig. 6 reports the calculated density of states $\rho_{p-1,h}(U,P_{\|}, P_{\perp})$ as a function of the angle of emission of the neutron in the case of the reaction ^{93}Nb(n,n') for ϵ_{inc}=14.38 MeV and ϵ_c=5.5 MeV.

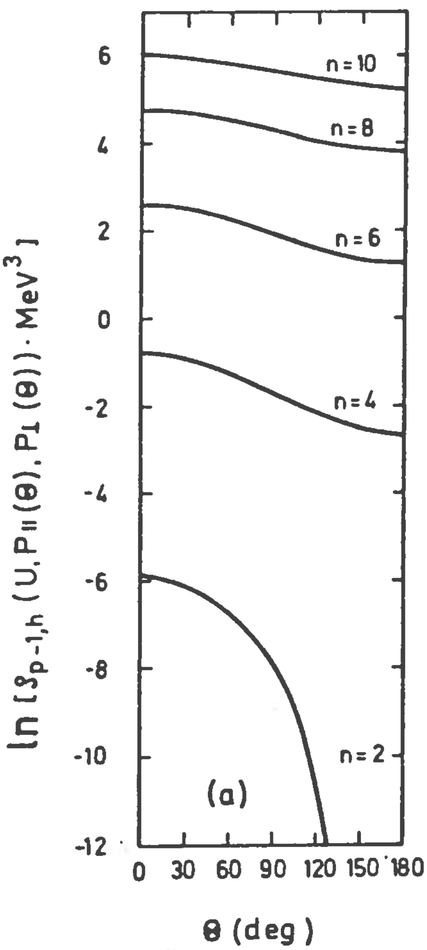

Fig. 6 - Momentum dependent densities of states of the residual nucleus as a function of the emitted nucleon angle and the exciton number n. The case considered is neutron inelastic scattering on ^{93}Nb at 61.1 MeV incident energy, the emitted nucleon energy is 5.5 MeV (Mädler and Reif 1980).

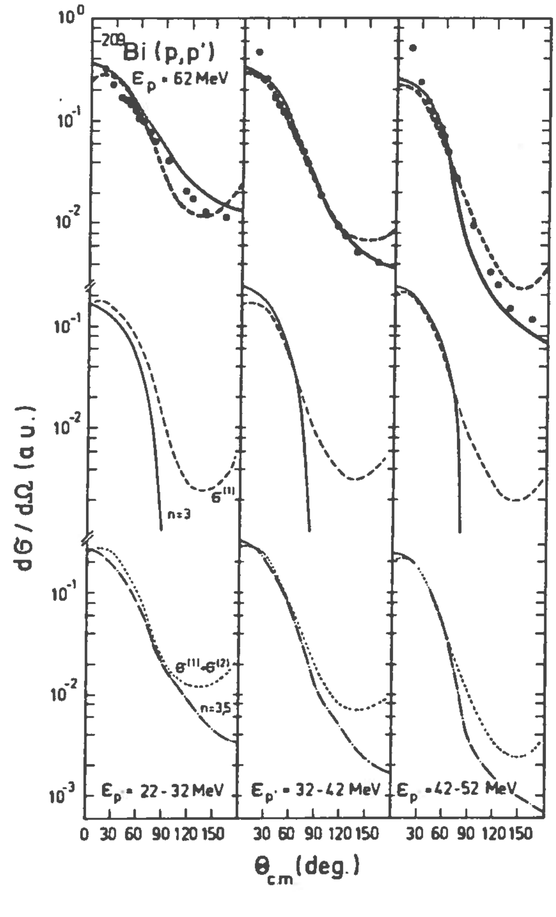

Fig. 7 - Comparison between experimental (Bertrand and Peelle 1973) and calculated angular distributions of protons inelastically scattered by ^{209}Bi at 62 MeV. The solid curve gives the result of Mädler and Reif (1980) calculations, the dashed curve of MSDR calculations by Tamura *et al* (1977).

The above formalism may be applied when the number of states in an element $\Delta U \Delta P_{\|} \Delta P_{\perp}$ is much greater than unity

$$\mathcal{N}_{p-1,h} = \Delta U \Delta P_{\|} \Delta P_{\perp} \rho_{p-1,h}(U, P_{\|}, P_{\perp}) \gg 1$$

to justify a statistical description of the reaction. Previous inequality is hardly verified for states of the initial 3-exciton configuration at low incident energies, so the model should be utilised at rather high incident energies and in cases where emissions from $n>3$ states give a substantial contribution.

Fig. 7 show the comparison between data (Bertrand and Peelle 1973) and theory (Mădler and Reif 1980) in the case of the reaction ^{209}Bi(p,p') at 62 MeV. The experimental angular distributions are reproduced with accuracy comparable to that obtained by Sato (1985) using the generalised master equation and Tamura *et al* (1982) with MSDR theory.

4. GENERALISED HAUSER AND FESHBACH EXPRESSIONS.

Plyuiko (1978) and Fu (1988) propose the following expression for the double differential cross section of a pre-equilibrium process a+X→ CS → b+Y (CS=composite nucleus)

$$\frac{d^2\sigma(a,b)}{d\epsilon_b d\Omega_b} = k \sum_L (D_L + B_L) P_L(cos\theta), \tag{39}$$

where

$$k = \frac{\lambda_a^2}{4(2J_a+1)(2J_X+1)},$$

$$D_L = \sum (-1)^{j_a - j_b} Z_a Z_b \sum_{n=n_o, \Delta n=2}^{\overline{n}} T_a^n T_b^n \rho(n - n_b, U_Y, J_Y, \Pi_Y) \tag{39a}$$

and

$$B_L = C \sum{}' (-1)^{j_a - j_b} Z'_a Z'_b (T_a^{n_o} T_a^{n'_o} T_b^{n_o} T_b^{n'_o})^{\frac{1}{2}} \rho(n_o - n_b, U_Y, J_Y, \Pi_Y), \tag{39b}$$

J are total angular momenta (J without index refer to the CS), Π is the parity, $Z_a = Z(l_a J l_a J; j_a L)$, $Z_a' = i^{\,l_a - l'_a - L} Z(l_a J l'_a J'; j_a L)$ (analogous expressions hold for Z_b and Z_b'), l and j are the orbital momenta and channel spins.

Plyuiko (1978) compared the angle integral of (39) with the expression provided by a generalised master equation including spin

$$\frac{d\sigma}{d\epsilon_b} = \sum_J \sigma_a(J, \epsilon_a, J_X) \sum_{n=n_o, \Delta n=2}^{\overline{n}} [\int_0^\infty dt P(n, J, t)] \lambda_{n,b}(E, J, \epsilon_b, J_Y). \tag{40}$$

and, after having expressed the absorption cross section σ_a(J,ϵ_a,J_X) and the inverse cross section appearing in the decay rate for emissions to the continuum $\lambda_{n,b}$(E,J,ϵ_b,J_Y)

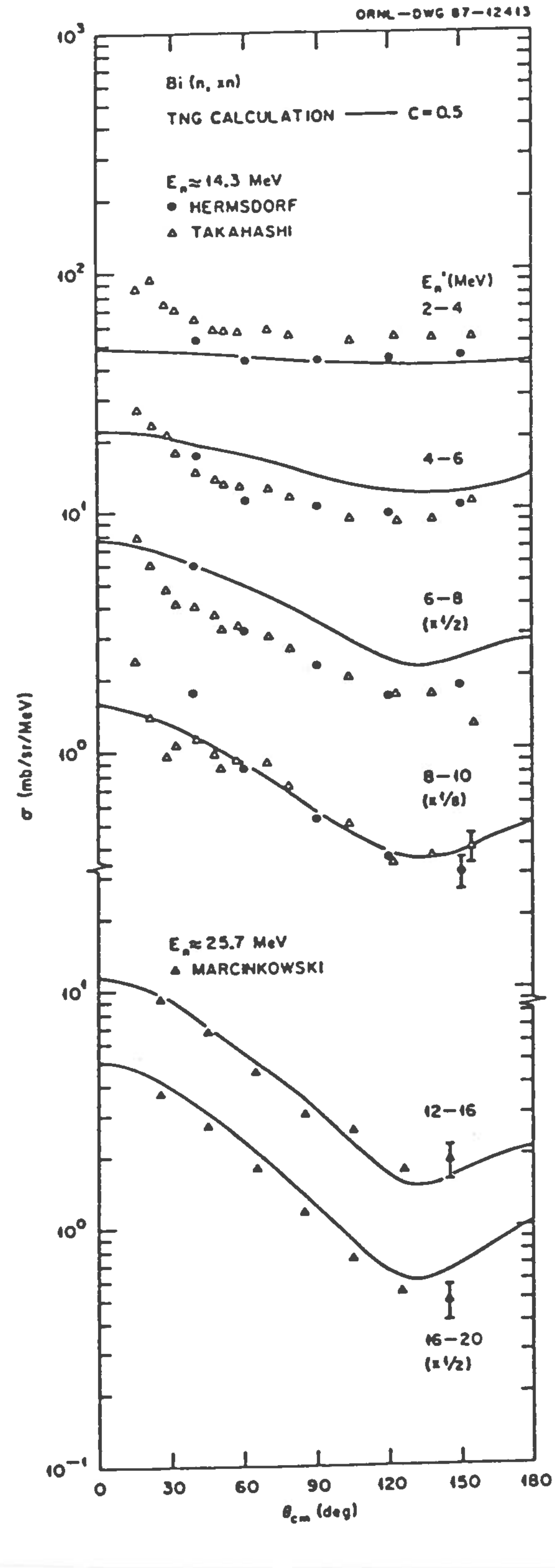

Fig. 8 - Comparison between experimental angular distributions of neutrons from ^{209}Bi(n,xn) reaction at 14.3 MeV (Hermsdorf *et al* 1974 and Takahashi *et al* 1983) and 25.7 MeV (Marcinkowski *et al* 1983) incident neutron energy, and theoretical calculations (Fu 1988).

by means of the optical model transmission coefficients T_a^{opt} and T_b^{opt}, obtained the expression of the generalised transmission coefficients $T^n_{a,b}$

$$T_a^n T_b^n = T_a^{opt}(\epsilon_a, l_a)[\frac{\int_0^\infty dt P(n,J,t)}{h\rho(n,E,J,\Pi)}]T_b^{opt}(\epsilon_b, l_b). \quad (41)$$

Finally, in (39), $\sum$=sum over J,Π, $J_Y, \Pi_Y, j_a, j_b, l_a, l_b$ and $\sum$'=sum over J,Π, J', Π', J_Y, Π_Y, j_a, j_b, l_a, l'_a, l_b, l_b' with the restrictions $J \neq J'$, and/or $l_a \neq l_a$', and/or $l_b \neq l_b$'.

Expression (39) is based on the hypothesis that one can apply the same expression that holds in case of statistical processes proceeding through strongly overlapped compound nucleus states to pre-equilibrium processes proceeding through strongly overlapped n-exciton states, retaining for $n > n_o$ the random phase approximation (which eliminates any cross product of reaction matrix elements with different indexes) and allowing a correlation between matrix elements with different indexes in the initial n_o state. Plyuiko suggested that this correlation should be operating only for near diagonal terms and assumed a complete correlation for matrix elements with l_a-l_a'=$\pm$1. Introduction of this correlation, by allowing odd order Legendre polynomials in expression (39) leads to asymmetric angular distributions. However, later, Fu (1988) remarked that, with this assumption, pre-equilibrium processes initiated by high energy α-particles on even-even nuclei would always lead to symmetric angular distributions (being not possible, in this case, interference of different l_a's for a fixed J), a result contrary to the experimental findings, and tentatively suggested a partial correlation ($C \approx 0.5$ in (39b)) between matrix elements with

$$\Delta J = 0 \qquad \Delta l_a = \Delta l_b = 1, \quad (42a)$$

$$\Delta J = 1 \qquad \Delta l_a = \Delta l_b = 1, \quad (42b)$$

$$\Delta J = 1 \qquad \Delta l_a = \Delta l_b = 0. \quad (42c)$$

This assumption leads to satisfactory results as shown in Fig. 8.

5. CONCLUSION

Previous theories are the ones most usually employed, often with satisfactory results, to reproduce the angular distributions in the framework of the exciton model. If we exclude the generalised Hauser and Feshbach theory (that, however, is based on assumptions that are reasonable, but not yet proved) these theories are still semiclassical, neglecting, or taking only approximately into account important quantum mechanical effects due to the nuclear finite size (Mantzouranis, Weidenmüller and Agassi 1976), the partial wave expansion of incoming and outgoing particle wavefunctions and distortion effects due to the coulomb and nuclear field. Nevertheless, they demonstrate the great flexibility of the exciton model and its internal consistency. Even if they are more involved than the theory one utilises for predicting angle integrated spectra and cross sections the simplicity and physical transparency of the model are not lost. Since the basic parameters entering the theories are weakly dependent on the mass of the nuclei involved and the energy, the most valuable feature of the exciton model is also retained: the ability of reproducing large set of data and predicting unmeasured cross sections with the use of a fixed set of input parameters.

REFERENCES

Akkermans J M, Gruppelaar H and Reffo G 1980 Phys. Rev. C**22** 73

Bisplinghoff J, Blann M and Keuser H 1978 *Proceedings of the International Workshop on Reaction Models for Continuous Spectra of Light Particles*, Bad Honnef, Report of the Institut für Strahlen und Kernphysik der Universität Bonn

Bertrand F E and Peelle R W 1973 C**8** 1045

Cline C K and Blann M 1971 Nucl. Phys. **A172** 225

Costa C, Gruppelaar A and Akkermans J M 1983 Phys. Rev. C**28** 587

De A, Ray S and Ghosh S K 1985 J Phys. G **11** L79

Fu C Y 1988 *Proceedings of a specialist's meeting on preequilibrium nuclear reactions*, Semmering, February 10-12, 1988. Edt. B. Strohmaier, NEANDC-245 'U', 1988

Gadioli E, Gadioli Erba E and Sona P G 1973 Nucl. Phys. **A217** 589

Gadioli E, Gadioli Erba E and Hogan J J 1977 Phys. Rev. C**16** 1404

Gadioli E, Gadioli Erba E and Tagliaferri G 1977 *Proceedings of the International Conference on Nuclear Reaction Mechanisms*, Varenna, June 1977, Clued Publ. Co., Milano

Gadioli E and Gadioli Erba E 1980 *Proceedings: Nuclear Theory for Applications 1980*, ICTP Report IAEA-SMR-68/1, 1981

Griffin J J 1966 Phys. Rev. Lett. **17** 478

Grimes S M, Anderson J D and Wong C 1976 Phys. Rev. C**13** 2224

Hayakawa S, Kawai M and Kikuchi K 1955 Progr. Theoret. Phys. (Kyoto) **13** 415

Hermsdorf D *et al* 1974 Zentralinstitut für Kernforschung Rossendorf bei Dresden Report ZfK-277 (1974)

Iwamoto A and Harada K 1984 Nucl. Phys. **A419** 472

Iwamoto A 1987 Phys. Rev. C**35** 984

Kikuchi K and Kawai M 1968 *Nuclear Matter and Nuclear Reactions* North Holland Publ. Co., Amsterdam

Mädler P and Reif R 1980 Nucl. Phys. **A337** 445

Mantzouranis G, Weidenmüller H A and Agassi D 1976 Z. Phys. **A276** 145

Marcinkowski A *et al* 1983 Nucl. Sci. Eng. **83** 13.

Plyuiko V A 1978 Yad. Fiz. **27** 1175 (Sov. J. Nucl. Phys. **27** 623)

Sato K 1985 Phys. Rev. C**32** 647

Sun Ziyang, Wang Shunuan, Zhang Jingshang and Zhuo Yizhong 1982 Z. Phys. **A305** 61

Takahashi A *et al* 1983 Report A-83-01 of the Intense Neutron Facility, Osaka University

Tamura T, Udagawa T, Feng D H and Kan K K 1977 Phys. Lett. **66B** 109

Tamura T, Udagawa T and Lenske H 1982 Phys. Rev. C**26** 379

Williams F C jr. 1970 Phys. Lett. **31B** 184