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ABSTRACT

A phenomenological algebraic approach to nuclear dipole excitations is presented within the framework of the interacting boson model. The application to photon absorption and scattering by nuclei in regions of shape transitions and axially symmetric deformations is discussed. Finally, an algebraic model for the coupling of giant dipole resonance states to fermion and isospin degrees of freedom is proposed and applied.

INTRODUCTION

As is known, the coupling of the giant dipole resonance (GDR) to low-energy nuclear excitations leads to a peculiar fragmentation pattern of the GDR itself. In particular, the coupling to low-lying collective modes (surface oscillations and nuclear rotations) has been considered within the framework of the dynamic collective model /1/, the pairing-plus-quadrupole model of Kumar and Baranger /2/ and the quasiparticle-phonon model /3/.

For light-mass nuclei with nonzero ground-state isospin, there is experimental evidence for a further GDR splitting into different isospin components /4/. This fragmentation has been investigated both by a semiclassical treatment of spherical nuclei /5/ and by means of the electric dipole sum rules for different isospin channels /6,7/.

From an experimental point of view, high-resolution measurements of photon absorption and scattering cross sections in the GDR energy region allow one to investigate the fragmentation mechanisms and to extract nuclear structure information from the GDR coupling to low-energy degrees of freedom, as discussed in ref./8/ in the case of deformed nuclei with possible meson exchange effects.

In recent years, the interaction of low-lying (vibrations and rotations) and high-lying (giant resonances) collective modes has been studied /9, 10,11/ within the framework of the interacting boson model (IBM) /12/,

which has proven to be a very flexible tool for the description of nuclear structure data. The advantage of the IBM algebraic approach to GDR is twofold : first, it provides a simple, yet accurate approximation to complex shell-model calculations and an original framework to deal with topics such as shape transitions in GDR photoabsorption /13/, inelastic photon scattering /10,14/, interference between nuclear modes and quantum electrodynamics effects (Delbrück scattering) /15/, which are sometimes difficult to interpret with other nuclear models. Second, the flexible algebraic structure of IBM allows one to introduce the GDR coupling to nuclear non-bosonic degrees of freedom in a simple manner.
Essentially, four kinds of nuclear excitations have been introduced till now into an extended IBM, originally formulated /12/ in terms of s and d bosons only, for low-energy spectra of even-even (non-magic) nuclei :

1) IBM plus further particle-particle bosons, like negative-parity octupole bosons (f-boson, with $J^{\pi} = 3^{-}$) to describe negative-parity states in even-even nuclei /16/ and positive-parity bosons with intrinsic angular momentum $L \geqq 4$, to enlarge the IBM configuration space /17/.

2) IBM plus fermion degrees of freedom, to deal with odd-mass nuclei (interacting boson-fermion model) /19/, odd-odd nuclei /20/ and two-quasiparticle excitations in even-even nuclei /21/.

3) IBM plus particle-hole bosons, to describe giant resonance states and their coupling to low-energy levels /9-12,13,22/, and s' or d' bosons ($J^{\pi} = 0^{+}$ and 2^{+}, respectively) to simulate low-energy intruder states originating from two particle-two hole excitations through closed shells /18/.

4) IBM plus spin and isospin degrees of freedom, to reproduce low-energy spectra and properties of light-mass nuclei, e.g. in the 2s-1d shell (IBM-3 /23/ and IBM-4 /24/ models).

Moreover, different nuclear modes (1-4) can be combined together to yield, for instance, an interacting boson-fermion model with enlarged boson basis (s,d,g,...). In the following, we shall discuss point (3), limiting ourselves to GDR and extensions of the relevant model to include both fermion (2) and isospin (4) degrees of freedom.

FORMALISM

In IBM language, the GDR excitation can be described by means of a p-boson with $J^{\pi} = 1^{-}$; the whole nuclear system is then represented by the following basis states :

$$|\Psi\rangle = (s^{+})^{m}(d^{+})^{n}(p^{+})^{q}\,|0\rangle \quad , \qquad (1)$$

where $|0\rangle$ is the boson vacuum, m+n=N, number of effective bosons, q=0 or 1, depending on whether low-energy or GDR states are concerned, respectively. s^{+},d^{+} and p^{+} create one s,d or p boson, respectively. The relevant Hamiltonian is :

$$\hat{H} = \hat{H}_{sd} + \varepsilon_{p}\hat{n}_{p} + \hat{H}_{sdp} \; , \qquad (2)$$

where $\hat{H}_{sd}$ is the usual IBM-1 Hamiltonian /12/, $\hat{n}_p$ is the p-boson number operator and the coupling term, $\hat{H}_{sdp}$, between low- and high-energy bosons has the general form :

$$\hat{H}_{sdp} = b_0(d^+ x\tilde{d})^{(0)} \cdot (p^+ x\tilde{p})^{(0)} + b_1(d^+ x\tilde{d})^{(1)} \cdot (p^+ x\tilde{p})^{(1)} + b_2\hat{Q}_{sd}^{(2)} \cdot (p^+ x\tilde{p})^{(2)} \quad . \qquad (3)$$

Here, $\hat{Q}_{sd}^{(2)} = \left[(s^+ x\tilde{d} + d^+ x\tilde{s})^{(2)} + \chi (d^+ x\tilde{d})^{(2)} \right]$ and $\tilde{a}_{j,m} = (-1)^{j+m} a_{j,-m}$.
In eq.(3) the dominant term, responsible for the GDR splitting due to nuclear deformations or surface oscillations, is the quadrupole-quadrupole operator, while the monopole-monopole interaction renormalizes the free p-boson energy, ε_p. Finally, the dipole-dipole interaction shifts the GDR states to lower or higher energies, according to the sign of b_1, and modifies the relevant dipole strengths.
The transition strengths from GDR to low-lying states can be easily evaluated by means of the following dipole operator :

$$\hat{D}^{(1)} = D_0(p^+ + p) \quad . \qquad (4)$$

For a comparison with experimental data, one calculates the photon absorption, σ_{abs}, and differential scattering, $d\sigma / d\Omega$, cross sections in the GDR region. They are expressed, as usual, in terms of nuclear polarizabilities /1/, P_L :

$$P_L = \delta_{LL_f} \frac{e^2}{[3(2L_f+1)]^{1/2}} \frac{EE'}{(\hbar c)^2} \sum_{1}^{n_{max}}{}_n \langle L_f^+ || \hat{D}^{(1)} || 1_n^- \rangle \langle 1_n^- || \hat{D}^{(1)} || 0_1^+ \rangle$$
$$\times \left[\frac{1}{E_n + E' + i\Gamma_n/2} + \frac{(-1)^L}{E_n - E - i\Gamma_n/2} \right] - \delta_{L0}\delta_{LL_f} \frac{\sqrt{3}(Ze)^2}{Am_N c^2} \quad . \qquad (5)$$

$$\sigma_{abs} = \frac{4\pi}{\sqrt{3}} \frac{\hbar c}{E} \mathrm{Im} P_0 \quad . \qquad (6)$$

$$\frac{d\sigma}{d\Omega}(E, E', \theta; 0_1^+ \to L_f^+) = \frac{E'}{E} | P_L |^2 g_L(\theta) \delta_{LL_f} \, , \qquad (7)$$

with $g_0(\theta) = \frac{1}{6}(1+\cos^2\theta)$, $g_2(\theta) = \frac{1}{12}(13+\cos^2\theta)$. Here, E (E') is the incident (emitted) photon energy, ϑ the scattering angle, L the transferred angular momentum and L_f the spin of final nuclear state ; the last term on the r.h.s. of eq.(5) accounts for the Thomson amplitude in the elastic channel, Z and A being the charge and mass number, respectively, of the target nucleus and m_N the atomic mass unit.
Moreover, $| 1_n^- \rangle$ is the nth GDR state, at energy E_n. Since its intrinsic width, Γ_n, cannot be evaluated within the IBM framework, we assume the following power law /1/ :

$$\Gamma(E) = kE^{\delta} \, , \qquad (8)$$

where k and δ have to be adjusted on experimental data. We have thus six parameters left, namely b_0, b_1, b_2, D_0, k and δ , whose values are determined,

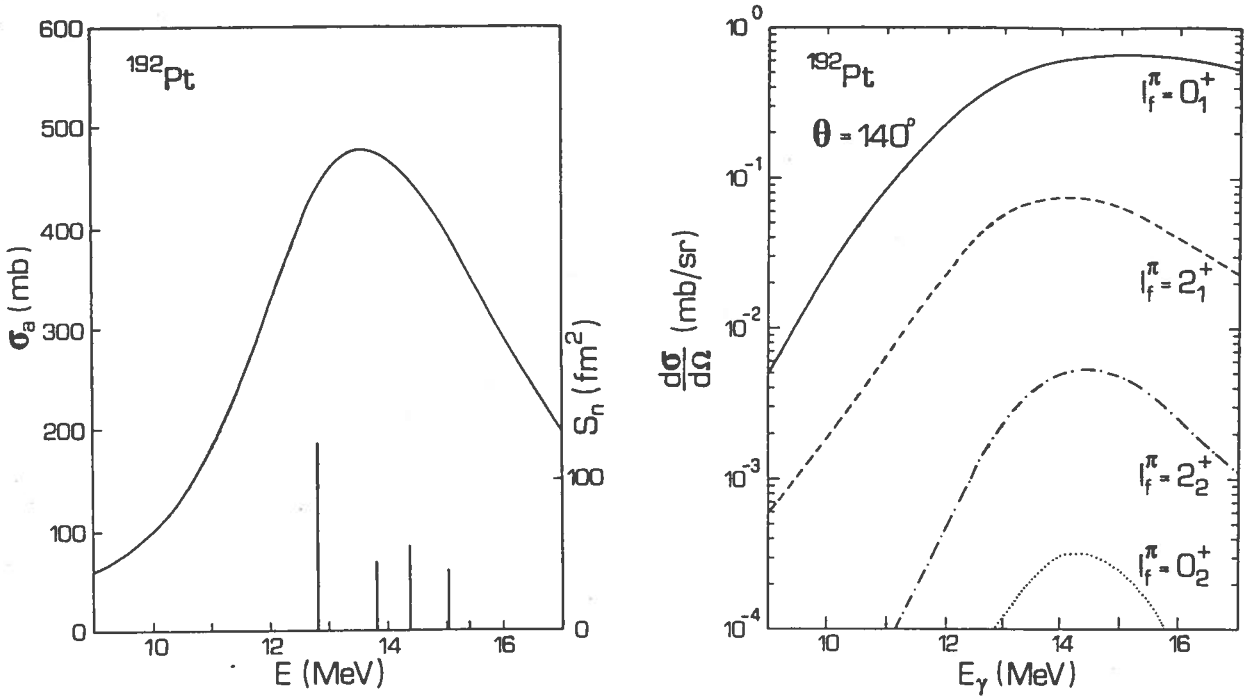

Fig.1. Photon absorption (on the left) and scattering (on the right) by ^{192}Pt. The straight-line segments represent the dipole strengths, $S_n = |\langle 1_n^- \| \hat{D}^{(1)} \| 0_1^+ \rangle|^2$. The scattering cross sections, evaluated at a scattering angle $\vartheta = 140°$, are labeled with spin and parity of the final nuclear states.

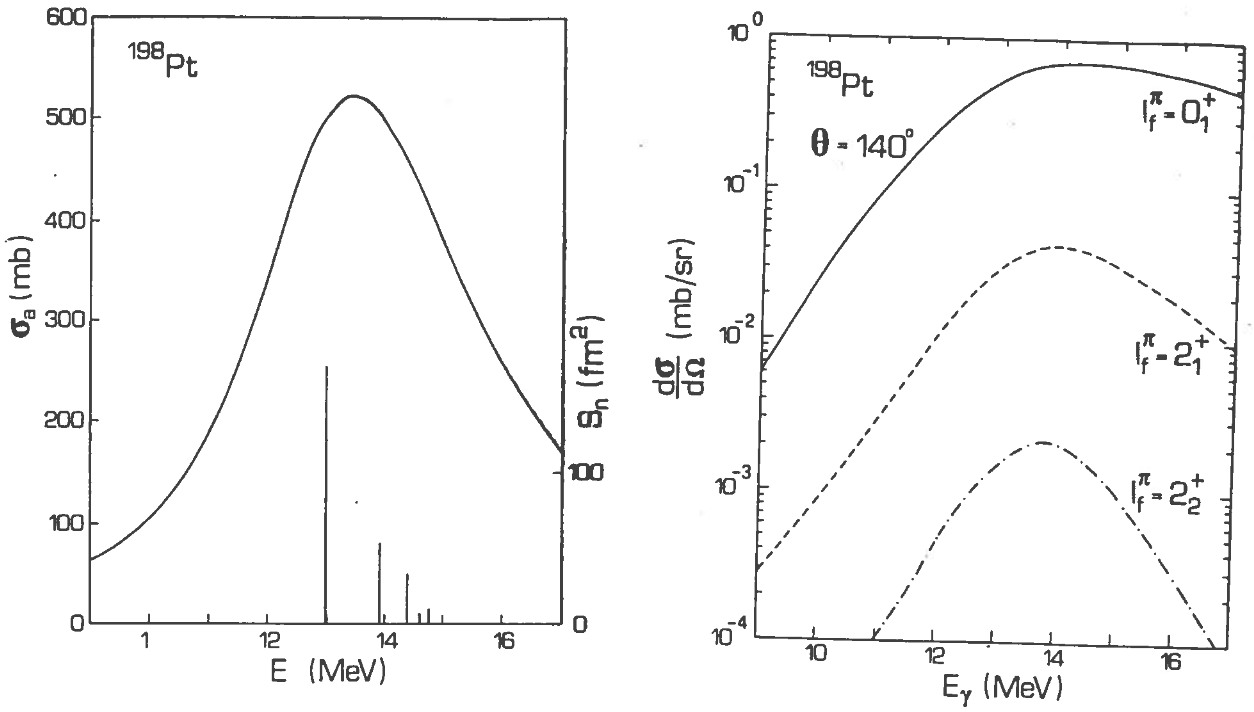

Fig.2. Photon absorption and scattering by ^{198}Pt. See captions of fig.1.

for a given isotope chain, by the best fit to the photoabsorption cross section. The p-boson energy satisfies the semiempirical law $\varepsilon_p \simeq 77.5\ A^{-1/3}$ MeV. Once the parameters are adjusted on the experimental photoabsorption data, differential scattering cross sections can be calculated without further adjustable parameters and, therefore, represent a good test for the model.

GDR EXCITATION IN SHAPE TRANSITIONAL REGIONS

Since the IBM parameters are fixed for a given isotope chain (see Table I), it is useful to investigate shape transitional regions where different GDR fragmentation patterns appear as a consequence of the coupling to different ground-state shapes. The original $SU_B(6)$ symmetry of IBM-1 can be reduced to three subsymmetries corresponding to different low-energy spectra : $SU_B(3)$ which describes an axially symmetric rotor ; $SU_B(5)$, a vibrational nucleus ; $O_B(6)$, a gamma-soft nucleus.
The $SU_B(5) \longrightarrow SU_B(3)$ transition from a spherical to a deformed shape is exemplified by Nd and Sm isotopes /16/; the relevant GDR fragmentation along this chain is discussed in ref./13/. The $SU_B(3) \longrightarrow O_B(6)$ case goes from axially symmetric rotors, with two main GDR components like in ^{186}Os, to γ-soft nuclei, where the GDR state at the lowest energy is strongly excited and the remaining dipole strength is uniformly distributed among several GDR components, like in ^{192}Os /13/. In figs.1 and 2, we show analogous results for Pt isotopes : ^{192}Pt has the GDR splitting typical of a γ-soft nucleus, while ^{198}Pt exhibits a fragmentation pattern close to that of an oblate rotor. Finally, the $O_B(6) \longrightarrow SU_B(5)$ transition is observed, for instance, in Ru isotopes and neighbouring nuclei /12/. Calculations for ^{100}Pd and ^{106}Pd, presented in figs.3 and 4, are representative of GDR fragmentation for nuclear shapes intermediate between these two symmetries, but closer to the $SU_B(5)$ limit. In this case, the dipole strength decreases with increasing excitation energy.
We observe that the elastic scattering in figs.1-4 is evaluated in kinematical ranges ($E \geqq 8$ MeV, $\vartheta = 140°$) where both atomic Rayleigh and Delbrück contributions are negligible. As for the inelastic Raman scattering, the present IBM approach has been proven successful in reproducing experimental data /25/, in particular excitation of rotational side bands /10,13/, which cannot be easily described within the framework of other collective models.

TABLE I

IBM parameters for the giant dipole resonance

Isotope	Pd	Sm	Os	Pt
b_0 (MeV)	0.	0.	0.30	0.30
b_1 (MeV)	0.	1.0	0.	0.
b_2 (MeV)	0.50	0.48	0.35	0.35
D_0 (e.fm)	6.4	8.0	9.4	9.5
k ($MeV^{1-\delta}$)	0.08	0.06	0.09	0.08
δ	1.5	1.6	1.4	1.5

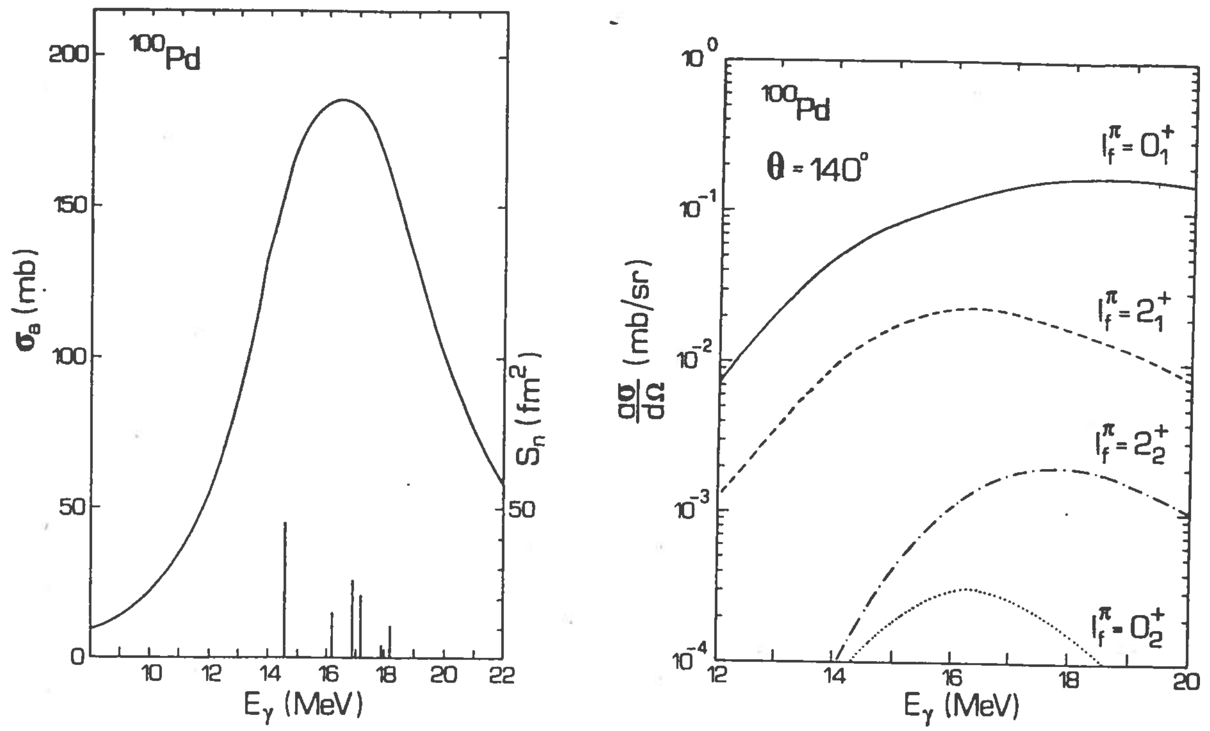

Fig.3. Photon absorption and scattering by ^{100}Pd. See captions of fig.1.

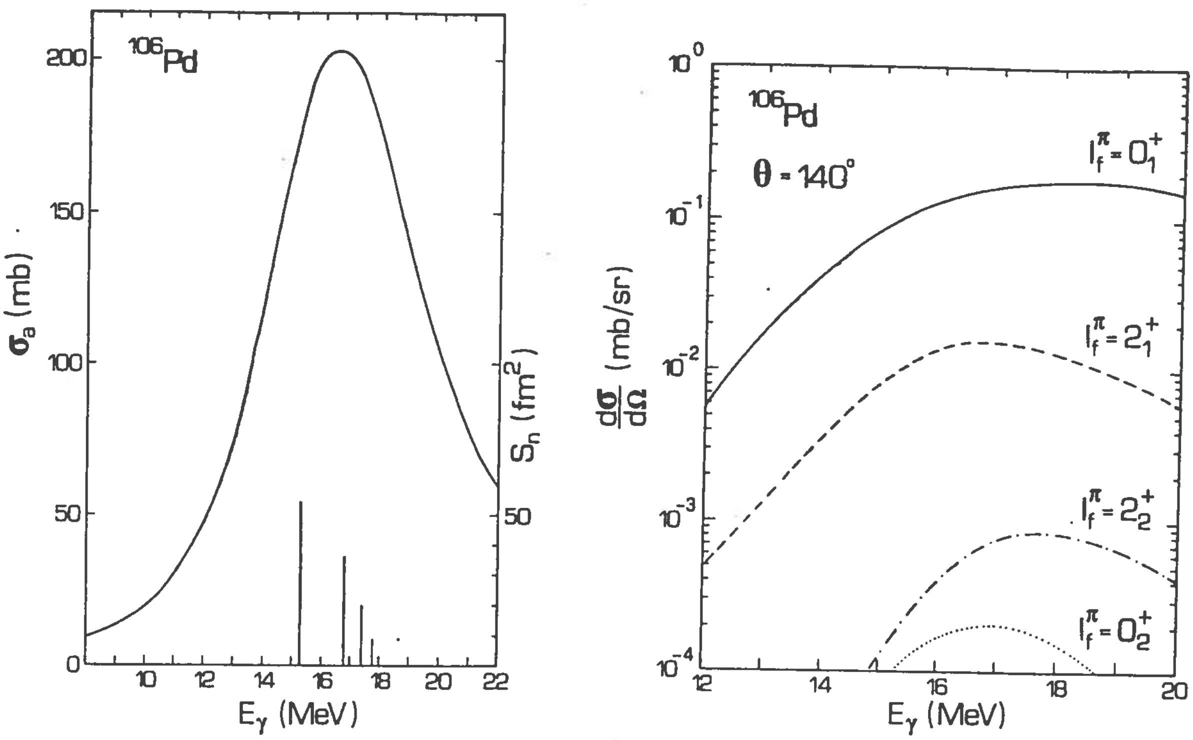

Fig.4. Photon absorption and scattering by ^{106}Pd. See captions of fig.1.

EXACT $SU_B(3)$ SYMMETRY

In the $SU_B(3)$ limit, it is possible to derive analytic formulae for the GDR splitting due to the ground-state deformation, since the p-boson operator transforms under SU(3) like a first-rank tensor, belonging to the (1,0) irreducible representation (irrep). The GDR states are then coupled only to the 0_1^+ and 2_1^+ levels in the ground-state rotational band, which belongs to (2N, 0). Thus,

$$(2N,0) \otimes (1,0) = (2N+1,0) \oplus (2N-1,1) \ . \tag{9}$$

The energies of the two GDR states contained in the (2N+1,0) and (2N-1,1) irreps are expressible in terms of expectation values of relevant SU(3) Casimir operators /11/. In fact, the Hamiltonian

$$\hat{H} = a_1 \hat{L}^{(1)}_{sd} \cdot \hat{L}^{(1)}_{sd} + \varepsilon_p \hat{n}_p + \alpha \hat{Q}^{(2)}_{sd} \cdot (p^+ x \tilde{p})^{(2)} \ , \tag{10}$$

with $\hat{L}^{(1)}_{sd} = \sqrt{10}(d^+ x \tilde{d})^{(1)}$, can be written as a combination of SU(3) Casimir operators, $\mathcal{C}_2$:

$$\mathcal{C}_2[SU(3)] = 2\hat{Q}^{(2)} \cdot \hat{Q}^{(2)} + \frac{3}{4} \hat{L}^{(1)} \cdot \hat{L}^{(1)} \ , \tag{11}$$

if one defines $\hat{Q}^{(2)}_p = -\frac{\sqrt{3}}{2}(p^+ x \tilde{p})^{(2)}$ in order to satisfy the commutation rules of su(3) algebra.
Therefore, neglecting the energy differences of levels in the ground-state band (the first term on the r.h.s. of eq.(10)) in comparison with the GDR splitting, one obtains /11/ :

$$E_n = \varepsilon_p - \frac{\alpha}{2\sqrt{3}}\Big[\mathcal{C}_2[SU_{sdp}(3)] - \mathcal{C}_2[SU_{sd}(3)] - \mathcal{C}_2[SU_p(3)]\Big] = \begin{cases} \varepsilon_p - 2N\alpha/\sqrt{3} \ ; \\ \varepsilon_p + \alpha(2N+3)/(2\sqrt{3}) \ . \end{cases} \tag{12}$$

Alternatively, we can recover the same result by an exact solution of the corresponding two mixed-level problem /9/. The 2x2 matrix Hamiltonian (10) which couples GDR components to 0_1^+ and 2_1^+ levels, is :

$$\hat{H} = \varepsilon_p \mathbb{1} + \begin{pmatrix} 0 & \frac{\alpha}{\sqrt{15}}\langle 0_1^+ \| \hat{Q}^{(2)}_{sd} \| 2_1^+ \rangle \Phi \\ \frac{\alpha}{\sqrt{15}}\langle 2_1^+ \| \hat{Q}^{(2)}_{sd} \| 0_1^+ \rangle \Phi & E(2_1^+) + \frac{\alpha}{10}\sqrt{\frac{7}{3}}\langle 2_1^+ \| \hat{Q}^{(2)}_{sd} \| 2_1^+ \rangle \Phi \end{pmatrix}, \tag{13}$$

with $\Phi = \langle (1,0),0,1 \,\|\, (p^+ x p)^{(2)} \,\|\, (1,0),0,1 \rangle = \sqrt{5}$ and $\langle 0_1^+ \| \hat{Q}^{(2)}_{sd} \| 2_1^+ \rangle = \langle 2_1^+ \| \hat{Q}^{(2)}_{sd} \| 0_1^+ \rangle = \sqrt{N(2N+3)}$, $\langle 2_1^+ \| \hat{Q}^{(2)}_{sd} \| 2_1^+ \rangle = -\sqrt{5}(4N+3)/(2\sqrt{7})$.
By diagonalizing Hamiltonian (13), where $E(2_1^+)$ is assumed negligible, we obtain the eigenvalues $E_n = \varepsilon_p - \alpha(4N+3 \pm 3\sqrt{16N^2+24N+1})/(8\sqrt{3})$, corresponding to the energies of the two GDR components ; in the limit of large N, they reduce to :

$$E_n = \begin{cases} \varepsilon_p - 2\alpha N/\sqrt{3} \ ; \\ \varepsilon_p + \frac{\alpha}{2\sqrt{3}}(2N+3/2) \ , \end{cases} \tag{14}$$

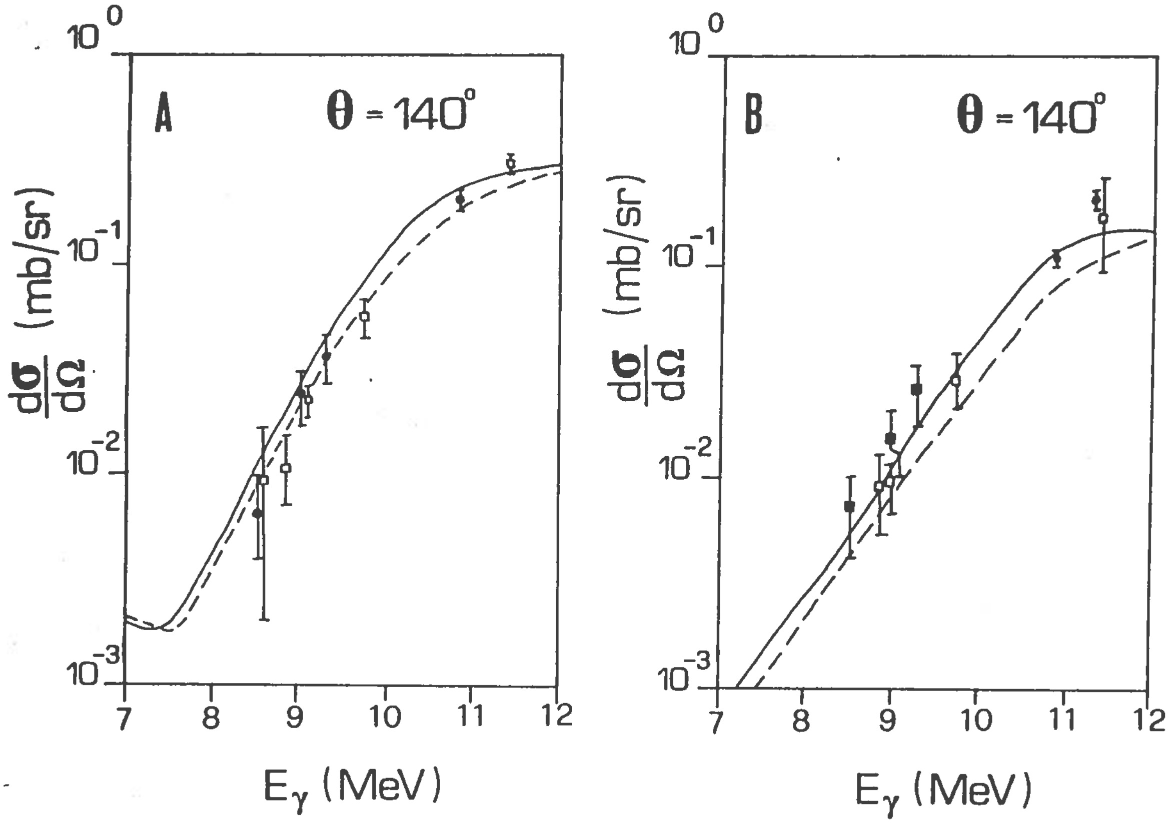

Fig.5. $SU_B(3)$ calculations of (a) photon elastic scattering and (b) inelastic scattering to the 2_1^+ state of ^{232}Th, at an angle $\vartheta = 140°$. Solid line : broken $SU_B(3)$ symmetry /14/ ; dashed line : exact $SU_B(3)$ symmetry (see text). Experimental data are quoted in ref./14/.

to be compared with formula (12). Moreover, the two dipole transition strengths are proportional /11/ to the reduced Wigner coefficients of the $SU(3) \supset SO(3)$ decomposition /26/.
In fig.5, results of IBM calculations for ^{232}Th in both exact and broken $SU_B(3)$ symmetry are compared with experimental data. Breaking the $SU_B(3)$ symmetry allows the low-energy spectrum to be accurately reproduced /14/.

GDR IN ODD-MASS NUCLEI

It is possible to generalize to odd systems the above treatment of GDR in even-even nuclei by adding suitable fermion degrees of freedom /27/. In particular cases we can exploit the limit symmetries to derive closed-form expressions for the GDR energy splitting, in analogy with the exact $SU_B(3)$ symmetry. For instance, we shall consider a deformed nucleus whose even-even core is described by $SU_B(3)$, where the unpaired nucleon occupies a j=1/2 or 3/2 orbital. The fermion angular momenta, j=1/2 and 3/2, can be decomposed into a pseudo-orbital part, k=1, which transforms like a first-rank tensor under $SU_F(3)$, and a pseudo-spin part, s=1/2 /28/.
The relevant group decomposition chain for the description of low-energy spectrum is :

$$U(6/6) \supset U_B(6) \otimes U_F^{(j)}(6) \supset SU_B(3) \otimes SU_F^{(k)}(3) \otimes SU_F^{(s)}(2) \supset$$
$$SU_{BF}(3) \otimes SU_F^{(s)}(2) \supset SO_{BF}(3) \otimes SU_F^{(s)}(2) \supset Spin(3) \ . \qquad (15)$$

The rotational ground-state band belongs to the (2N+1,0) irrep of $SU_{BF}(3)$ and includes states with J=1/2,3/2,5/2,...,2N+3/2. Only the three lower-spin levels, namely J=1/2,3/2 and 5/2, are coupled to the GDR, according to the following decomposition :

$$(2N+1,0) \otimes (1,0) = (2N+2,0) \oplus (2N,1) \ , \qquad (16)$$

where five GDR components arise ; two of them,with angular momentum J=1/2 and 3/2 and parity opposite to the ground-state, belong to the (2N+2,0) symmetric irrep. The remaining three states, with J=1/2,3/2 and 3/2, are contained in the (2N,1) irrep.
Their energies are obtained by diagonalizing two Hamiltonian matrices, as in the $SU_B(3)$ limit. The 2x2 Hamiltonian matrix gives the energies of the J=1/2 GDR components, coupled to the low-energy J=1/2 and 3/2 levels, while the other 3x3 Hamiltonian matrix yields the energies of the J=3/2 GDR states, interacting with the J=1/2,3/2 and 5/2 levels of the ground-state band.
Instead of solving the exact Hamiltonian, the energy splitting of the two J=1/2 GDR components can be directly related to the expectation values of SU(3) Casimir operators, in analogy with eqs.(11) and (12) :

$$\Delta E = E\left[(2N,1),1/2\right] - E\left[(2N+2,0),1/2\right] \cong \bar{\alpha}\sqrt{3}(N+1) \ . \qquad (17)$$

GDR WITH ISOSPIN

Another interesting problem is the isospin splitting of the GDR in light-mass nuclei, where the isospin, T, is a good quantum number and the low-energy spectrum has to be described within the framework of IBM-3 version /23/ of the model, which contains neutron-proton bosons (T=1, T_z=0), in addition to the usual neutron-neutron and proton-proton pairs.
Moreover, the electric dipole excitation carries one isospin unit with third component equal to zero. The relevant group decomposition chains are /29/ :

$$U_B^{(sd)}(6) \otimes SU_T^{(sd)}(3) \otimes SU_B^{(p)}(3) \otimes SU_T^{(p)}(3) \supset U_B^{(sd)}(6) \otimes SU_B^{(p)}(3) \otimes SU_T^{(sdp)}(3)$$

or : (18a)

$$\supset U_B^{(sd)}(6) \otimes SU_B^{(p)}(3) \otimes SU_T^{(sd)}(2) \otimes SU_T^{(p)}(2)$$
$$\supset U_B^{(sd)}(6) \otimes SU_B^{(p)}(3) \otimes SU_T^{(sdp)}(2) \ , \qquad (18b)$$

depending on whether strong (18a) or weak (18b) isospin coupling is considered (see ref./29/ for a detailed discussion). If eq.(18a) holds, the isospin irreps for the ground-state band /23/, (N,0), and the GDR excitation /29/, (1,0), are decomposed as follows :

$$(N,0) \otimes (1,0) = (N+1,0) \oplus (N-1,1) \ . \qquad (19)$$

The (N+1,0) irrep contains T=N+1,N-1,...,T_0+1, where T_0 is the ground-state

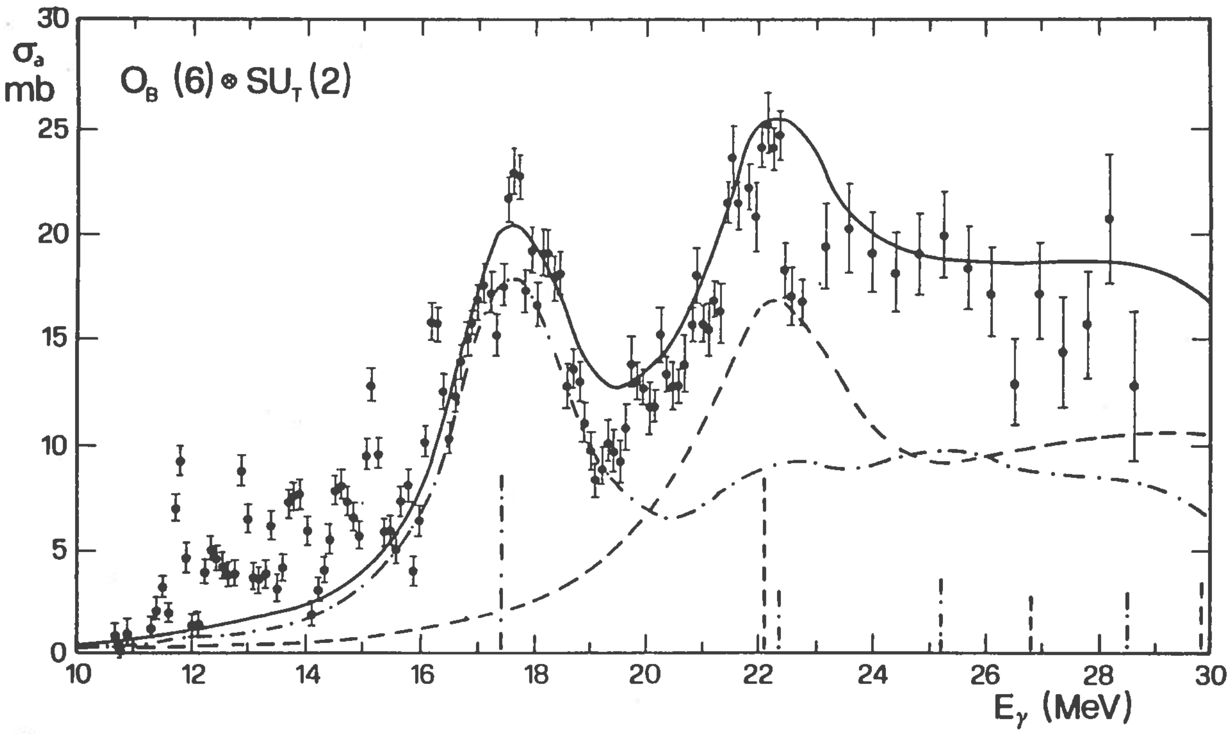

Fig.6. Experimental /30/ and calculated (solid line : broken $O_B(6) \times SU_T(2)$ symmetry) photoabsorption cross section of ^{26}Mg. Dot-dashed line : T=1 component ; dashed line : T=2 component ; the segments at the bottom represent the calculated dipole strengths, S_n, in arbitrary units.

isospin, while the non-symmetric (N-1,1) irrep contains $T=N,N-1,\ldots,T_0+1$, T_0. Only GDR components with $T=T_0$ or T_0+1 are allowed by the selection rules. In scheme (18b), only two isospin components arise, with $T=T_0+1$ and $T=T_0$, respectively, and energy separation $\Delta E(T) = 4\beta\,(T_0+1)$, where $\beta \cong 15/A$ MeV /29/.
In both cases, however, the ratio of dipole strengths in the two isospin channels (T_0 and T_0+1) is given by :

$$S(T=T_0+1)/S(T=T_0) = 1/T_0 \quad . \tag{20}$$

This isospin splitting is superimposed to the (previously discussed) deformation splitting. In fig.6, we show the photoabsorption cross section of ^{26}Mg, where both isospin and deformation (in a broken $O_B(6)$ symmetry which reproduces the low-lying spectrum /29/) effects are taken into account in the IBM calculation. The GDR parameters are : $\varepsilon_p = 21.9$ MeV, $b_2 = 2.60$ MeV and $D_0 = 1.7$ e fm.
Finally, it is worth mentioning the possible model extension which includes both spin and isospin degrees of freedom within the IBM-4 framework /24/. The p-boson excitation then carries additional intrinsic spin (s=0,1) and isospin quantum numbers ; the relevant group structure is $SU_B^{(p)}(3) \otimes SU_{ST}^{(p)}(6)$, which is coupled to the low-lying IBM-4 states /24/.
IBM-4 shows two interesting features : first, the low-energy spectra of

odd-odd nuclei can be successfully described within the model /31/ and, therefore, it is possible to consider GDR fragmentation in odd-odd nuclei, too. Second, since $SU_{ST}(6)$ contains the Wigner supermultiplet group, SU(4), the following meaningful decomposition holds :

$$U_B^{(sd)}(6) \otimes SU_{ST}^{(sd)}(6) \otimes SU_B^{(p)}(3) \otimes SU_{ST}^{(p)}(6) \supset U_B^{(sd)}(6) \otimes SU_{ST}^{(sd)}(6) \otimes SU_B^{(p)}(3) \otimes SU_{ST}^{(p)}(4) \quad (21)$$

This symmetry group incorporates the Goldhaber-Teller Oscillator (GTO) model, introduced by Donnelly and Loludice /32/ in order to couple surface vibrations to GDR excitations so as to deal with inelastic electron scattering reactions in light-mass nuclei. In fact, the GTO model corresponds to the limit symmetry, $SU_B^{(sd)}(5) \otimes SU_B^{(p)}(3) \otimes SU^{(p)}(4)$, arising as subgroup of formula (21).

Acknowledgments

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