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#### EMISSION OF COMPLEX PARTICLES IN PRECOMPOUND REACTIONS

#### E. Gadioli

Dipartimento di Fisica, Universita' di Milano and Istituto Nazionale di Fisica Nucleare, Sezione di Milano via Celoria 16, 20133 Milano, Italia

### ABSTRACT

Various theoretical models that have been developed to describe emission of complex particles in nucleon induced reactions are discussed and their predictions compared with the experimental data.

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Production of complex particles  $(d,t,^{s}He,\alpha)$  represents a sizeable contribution to the total reaction cross section of light projectiles.

As an example of the features that are characteristic of the spectra of all these different ejectiles, the angle integrated energy distribution of  $\alpha$ -particles emitted following the interaction of a medium energy proton with <sup>118</sup>Sn is shown in Fig. 1 [1]. In this spectrum, the contributions of different mechanisms, that, for the purpose of the present discussion, I will call one-step, multi-step and evaporative components are apparent.

One-step processes feed low energy states of the residual nucleus that, partly, are resolved. They are dominated by structure effects and the angular distributions for transitions to discrete final states greatly depend on the transferred angular and total momenta L and J. These properties make these processes an invaluable tool for nuclear spectroscopy studies.

An example of the influence of the shell structure of the target nucleus on the energy distribution of emitted particles is offered by the comparison of the  $\alpha$ -particle spectra from  $(p,\alpha)$ reactions on adjacent nuclei, one magic, with a magic neutron and/or proton shell, and one near magic with one further nucleon outside the magic shell. In Fig. 2 [2] the angle integrated spectra of  $\alpha$ -particles from  $^{208}$  Pb $(p,\alpha)^{205}$  Tl and  $^{209}$ Bi $(p,\alpha)^{206}$ Pb reactions, at 24 MeV incident proton energy, are compared as a function of the ejectile energy. This comparison shows that, while transitions to the lowest energy levels of  $^{205}$  Tl dominate the spectrum of  $\alpha$ -particles from  $^{208}$  Pb, levels of  $^{206}$  Pb below  $\approx 3.25$  MeV are populated very weakly in the  $^{209}$ Bi $(p,\alpha)^{206}$  Pb reaction. Further, transitions to the levels of  $^{206}$  Pb with energy between  $\approx 3.25$  and 7 MeV have an intensity similar to that of the transitions to the low lying levels of  $^{205}$  Tl  $(0 \le E \le 4 \text{ MeV})$ , and a striking correspondence exists between the structures appearing in the two spectra, in the two different excitation energy intervals.

This excitation of *homologous* states in the two residual nuclei (in  $^{205}$  Tl, proton hole and two neutron-one proton hole states; in  $^{206}$  Pb, the hole states of the  $^{205}$  Tl core weakly coupled to the 1h<sub>2</sub> proton outside the magic shell) is indicative of the *spectator* role of the 83th proton of  $^{209}$  Bi.

This effect, occurring also at the closure of other shells, is valuable for investigating the structure of states at an excitation energy of some MeV in near-magic nuclei, and has a notable influence also on the continuous part of the spectra.

This is demonstrated by the comparison (see Fig. 3) of the  $\alpha$  spectra in the <sup>91,90</sup>Zr(p, $\alpha$ )<sup>88,87</sup>Y reactions at about 25 MeV [2]. When, as in the previous case, the two spectra are superimposed as a function of the  $\alpha$ -particle energy one finds that, in spite of the widely different Q values, the spectrum of  $\alpha$ -particles from <sup>91</sup>Zr looks like an energy average of the more structured spectrum from <sup>90</sup>Zr, showing that the total strenght for transitions to states in a given energy interval is nearly the same when the excitation energy of the odd-odd <sup>88</sup>Y nucleus exceeds by about 2.16 MeV that of the odd-even <sup>87</sup>Y, the opposite of a pairing energy effect.

As a consequence of these structure effects, an accurate reproduction of the measured spectra in the region corresponding to the excitation of low energy levels of the residual nucleus requires the use of microscopic or semi-microscopic models. As an example of such a calculation, in Fig. 4 are compared the experimental and the calculated angle integrated spectra of  $\alpha$ -particles from the <sup>91,90</sup>Zr(n, $\alpha$ )<sup>88,87</sup>Sr reactions at 18.15 MeV incident neutron energy [3]. The  $\alpha$ -particle spectrum, in the case of <sup>90</sup>Zr, displays a sharp maximum around one MeV of excitation energy; it has been reproduced, in the framework of the the semi-microscopic pick-up model [4], taking into account of all known low energy levels of <sup>87</sup>Sr that may be populated in the (p, $\alpha$ ) process. In case of <sup>91</sup>Zr, only the contribution of the transition to the to g.s. of <sup>88</sup>Sr and to the excited states between  $\approx$ 4 and 6.5 MeV of excitation energy that are homologous of the states of <sup>87</sup>Sr. Fig. 1 - Angle integrated spectrum of  $\alpha$  particles from the <sup>118</sup>Sn(p, $\alpha$ ) <sup>115</sup>In reaction at  $E_p=44.3$  MeV, as a function of the  $\alpha$ -particle channel energy [1].



Fig. 3 - Angle integrated  $\alpha$ -particle spectra from the reactions  ${}^{90}\text{Zr}(p,\alpha)^{87}$ Y, histogram, and  ${}^{91}\text{Zr}(p,\alpha)^{88}$ Y, full line, at  $E_p \approx 24$  MeV [2].





in the energy interval 0 - 2.5 MeV, has been calculated. In these calculations there is only one free parameters, normalizing the calculated to the experimental spectra, that is the same for both nuclei and different incident energies. All other input parameters are optical model parameters obtained by best fit analyses of elastic scattering data and spectroscopic factors that are calculated as suggested by Smits and Siemssen [4] using for the neutron spectroscopic amplitudes the experimental values obtained in the analysis of neutron pick-up reactions [3].

Results similar to those just discussed are obtained in the analysis of the  $\alpha$ -particle spectra in the  $(n,\alpha)$  reactions induced on <sup>142,143,144</sup>Nd by 12-20 MeV neutrons [5].

In the multi-step part of the spectra, structure effects are washed out to a large extent.

Tamura, Udagawa and Lenske [6] have developed a multistep-direct-reaction (MSDR) theory to describe these processes. Without entering into mathematical details, the main assumptions that have been introduced by these authors are the following:

(i) these processes feed states of the residual nucleus that are a complicated superposition of pure shell model states, so in principle one should expect, in measured cross sections, interference between amplitudes for transitions to different shell model states. However, at high excitations, the residual nucleus states are highly overlapped and interference terms cancel to a large extent. Then, to reproduce the data, one needs only to evaluate the cross sections for transitions to pure shell model states and sum them incoherently. Assuming that a given state can only be excited in a particular process, also interference between one-step and multi-step amplitudes need not to be considered and the contributions of the various steps to the total cross sections are added incoherently.;

(ii) the cross section for the transitions to the levels within a given excitation energy interval is proportional to a spectroscopic density that is evaluated for any given process in the hypothesis of a dominant reaction mechanism. For instance, in the case of one-step  $(p,\alpha)$  reactions, one considers the pick-up of a triton. In this case, of the many possible three nucleon-hole final states which might be excited only those corresponding to three particle wavefunctions which have a non vanishing overlap with a triton wavefunction of the correct spin-isospin nature and zero relative internal momentum must be considered. This overlap integral  $G([\bar{n}_i])$  is a function of the quantum numbers  $\bar{n}_i=2n_i+l_i$  of the transferred nucleons and may be evaluated following a procedure suggested by Ichimura *et al.* [7] in the framework of the harmonic hoscillator shell model. By summing  $G^2([\bar{n}_i])$  over the possible triads of nucleons  $[\bar{n}_i]$  that may be excited in the unit excitation energy interval one obtains the desired spectroscopic density  $\rho_l^{(3h)}(U)$ , where *l* and *U*, are the transferred orbital angular momentum and the excitation energy of the residual nucleus. Then, the cross section is simply given by

$$\sigma_R^{(p\alpha)}(E_\alpha,\theta) = \sum_l \rho_l^{(3h)}(E_p - E_\alpha + Q_{g.\theta.})\sigma_{R,l}^{(BA)}(E_\alpha,\theta), \tag{1}$$

where  $\sigma_{R,l}^{(BA)}(\mathbf{E}_{\alpha},\theta)$  is the first order DWBA cross section.;

(iii) when one considers two step processes also the spectroscopic density appropriate for the other step of the reaction has to be considered. So, in the case of a  $(p,\alpha')(\alpha',\alpha)$  process the total cross section becomes

$$\sigma_{R}^{(p\alpha'\alpha)}(E_{\alpha},\theta) = \sum_{l_{1},l_{2}} \int \rho_{l_{2}}^{(ph)}(E_{\alpha'}-E_{\alpha})\rho_{l_{1}}^{(3h)}(E_{p}-E_{\alpha'}+Q_{g,s})\sigma_{R,l_{1},l_{2}}^{(BA)}(E_{\alpha},E_{\alpha'},\theta)dE_{\alpha'}$$
(2)

where  $\rho_{l_2}^{(ph)}(\mathbf{E}_{\alpha'}-\mathbf{E}_{\alpha})$  is the spectroscopic density for creating a *ph* pair appropriate for inelastic scattering, and  $\sigma_{R,l_1,l_2}^{(BA)}(\mathbf{E}_{\alpha},\mathbf{E}_{\alpha'},\theta)$  is the second order DWBA cross section.

Fig. 4 - Comparison of the experimental angle integrated  $\alpha$ -particle spectra from the reactions  ${}^{90}$ Zr(n, $\alpha$ )<sup>87</sup>Sr, and  ${}^{91}$ Zr(n, $\alpha$ )<sup>88</sup>Sr, black points with error bars, at E<sub>n</sub>  $\approx$ 18 MeV, with those calculated by pick-up theory [3].



Fig. 5 - Comparison of  $(p,\alpha)$  cross sections and analyzing powers calculated with MSDR theory [6] with the experimental data of Ref. [8]. Solid lines include one step and two-step contributions  $[(p,\alpha')(\alpha',\alpha)$  and  $(p,p')(p',\alpha)]$ .



Fig. 6 - Triton formation factor  $F_{l,m}$  as a function of triton energy  $\epsilon_t$  (the triton energy  $E_t$  in the Fermi gas is also shown) [11].

In spite of all these simplifying assumptions, at the increase of the number of steps, the calculation becomes very time consuming so, in practice, only one and two-step terms have been considered, in a few instancies, and also in these cases only a few of the possible reaction paths have been taken into account [6].

A systematic study of the possibilities offered by this approach is highly advisable; in fact, though the various authors who employed this theory to reproduce double differential spectra and analyzing powers all claim to have obtained a very satisfactory reproduction of the data (a typical result is shown in Fig. 5), the many different approximations and the different choices of basic parameters that have been made do not allow one to conclude that one can make reliable predictions of measured quantities. Also the relative importance of one and multistep contributions cannot be convincingly established (see Ref. [9] for a detailed discussion of this point).

Other, more phenomenological approaches, assume that particle emission occurs, before evaporation, at various stages of an equilibration cascade of nucleon-nucleon interactions that distributes the initial excitation energy among an ever increasing number of degrees of freedom. The basic assumption is made of a statistical competition between the various decay modes of the composite (projectile plus target) nucleus during this equilibration cascade.

Here I will discuss three different theoretical models of complex particle emission based on this general description of the multistep mechanism.

(a) In a statistical decay, it is possible that  $p_x$  excited particles condense into a cluster x which is emitted with energy  $\epsilon_x$ . The probability of coalescence of  $p_x$  particles into the cluster x is given by the square modulus  $f(p_x, \epsilon_x)$  of the overlap between the wavefunction of the cluster (factorised into the product of a center of mass motion wavefunction  $\chi(\epsilon_x, \mathbf{R})$  times an internal motion wavefunction  $\Psi_x$ ) and the  $p_x$ -nucleon wavefunction [10]

$$f(p_x, \epsilon_x) = |\langle \chi(\epsilon_x, \mathbf{R}) \Psi_x | \psi_1 \psi_2 ... \psi_{\mathbf{p}_x} \rangle|^2.$$
(3)

The sum of  $f(p_x, \epsilon_x)$  over all possible combinations  $1, 2, ..., p_x$  is equal to unity since the  $p_x$  nucleon wavefunctions form a complete orthogonal set.

The cluster x may be made of l particles above the Fermi level and m=x-l particles below. The probability  $f(l,m,\epsilon_x)$  of forming this cluster is

$$f(l,m,\epsilon_x) = \sum_{1,\ldots,l \le \epsilon_\ell^N; l+1,\ldots,p_x > \epsilon_\ell^N} |\langle \chi(\epsilon_x,\mathbf{R})\Psi_x | \psi_1 \psi_2 \ldots \psi_{p_x} \rangle|^2,$$
(4)

where  $\sum_{l,m} f(l,m,\epsilon_x) = 1$ .

Indeed, since the cluster x should be formed at the nuclear surface, one has to impose further restrictions when evaluating the overlap integrals in (3) and the corresponding probability of forming the cluster x will be  $F(l,m,\epsilon_x) \le f(l,m,\epsilon_x)$ .

The calculation of  $F(l,m,\epsilon_x)$  may be made in the framework of Fermi gas model, assuming that the internal wavefunction of cluster x is the ground state of an harmonic oscillator intrinsic Hamiltonian whose constant  $\nu = m\omega/\hbar$  is that corresponding to the measured *rms* radius of x. When one evaluates the phase space volume that divided by  $h^{3(x-1)}$  gives the number of states contributing to  $F(l,m,\epsilon_x)$ , one requires that the centre of mass of the x particles be at the surface of the residual nucleus ( $|\mathbf{R}|=R_{res}$ ) and that, for each particle,  $|\mathbf{r}_i| < R_{res} + \Delta R$ , where  $\Delta R$  is a free parameter. The decay rate for continuum emission of the cluster x made by 1 particles above and m particles below the Fermi level, when the composite nucleus is in an n exciton configuration, is now given by

$$W_{c}^{n,x(l,m)}(E,\epsilon_{x})d\epsilon_{x} = \frac{2s_{x}+1}{\pi^{2}\hbar^{3}}\mu_{x}\sigma(\epsilon_{x})\epsilon_{x}F(l,m,\epsilon_{x})\frac{\omega_{n(l,m)}(U)}{\omega_{p,h}(E)}d\epsilon_{x}$$
(5)

where  $s_x$ ,  $\mu_x$ ,  $\sigma(\epsilon_x)$  are, respectively, the spin, the reduced mass and the inverse cross section of x and  $\omega_{n(l,m)}(U)$  and  $\omega_{p,h}(E)$  the residual and composite nucleus state densities. The total decay rate for emission of cluster x is

$$W_c^{n,x}(E,\epsilon_x) = \sum_{l,m} W_c^{n,x(l,m)}(E,\epsilon_x).$$
(6)

The emission of the cluster x leads from a p,h configuration to a p-l,h+m configuration. However in evaluating  $F(l,m,\epsilon_x)$  one has already taken into account the state density of the newly created m holes and Iwamoto and Harada [10] postulate that the residual nucleus state density be  $\omega_{n(l,m)}(U) \approx \omega_{p-l,h}(U)$ .

The calculation of  $F(l,m,\epsilon_x)$  shows that for ejectile energies smaller than  $\approx 50$  MeV the probability that a cluster be made by coalescence of particles all above the Fermi level is very small. In addition the residual nucleus state density  $\omega_{p-l,h}(U)$  further enhances the probability of emission of clusters with l as small as possible.

Because the dominant contribution to the complex particle yield, at low energies, corresponds to l=1, this approach may be considered a *statistical* description of multi-step pick-up.

Fig. 6 shows the calculated formation factors  $F(l,m,\epsilon_x)$  for tritons as a function of the triton energy both outside and inside the nucleus [11]. A comparison between the experimental data of Bertrand and Peelle [12] and the calculated angle-integrated spectra of complex particles from 62 MeV proton bombardment of <sup>197</sup>Au shows a large discrepancy at the highest energies, especially evident for d and <sup>3</sup>He (Fig. 7) [11]. This is attributed to the presence of one-step pick-up processes (the incident nucleons picks up directly the requisite number of nucleons without creating an intermediate composite system) that are not calculated by the model. It is interesting to note that, according to these calculations, this contribution does not seem to contribute noticeably to the  $\alpha$  spectrum.

One remarkable aspect of these calculations is the fact that they reproduce quite naturally the ratio of *triton* to <sup>3</sup>*He* yield showing that in this model the reduced helion emission is due to the <sup>3</sup>*He* being a little more loosely bound than the triton, so that the phase space volume accessible to the condensing particles is substantially reduced.;

(b) A second approach is based on the hypothesis (found to be valid for explaining the production of complex particles in relativistic light and heavy ion reactions [13,14] and in relatively low energy heavy ion reactions [15]) that excited nucleons have an high probability of coalescing into a cluster if their relative momentum is low, *i. e.* if they share the same volume of momentum space.

The probability P of finding a nucleon within the coalescence volume centered around a momentum **p** is given by [15]

$$P = \frac{4\pi}{3} P_o^3 \frac{1}{\overline{m}} \frac{dN(\mathbf{p})}{d\mathbf{p}} \tag{7}$$

where  $\overline{m}$  is the average nucleon multiplicity and  $\frac{1}{\overline{m}} \frac{dN(p)}{dp}$  represents the differential nucleon multiplicity normalised to unity. P<sub>o</sub> is the coalescence radius usually treated as a free parameter.



Fig. 7 - Comparison between experimental, bar graph [12], and calculated, full line [11], angle integrated energy spectra for charged particles emitted in reactions induced by 62 MeV protons on <sup>197</sup>Au.



Fig. 8 - Comparison of experimental differential cross sections of <sup>3</sup>He from (p, <sup>3</sup>HeX) reactions at  $E_p=72$  MeV with those predicted by relation (12) using proton differential cross sections from Ref. [8] (from Ref. [16]).

If the nucleons that are excited are m, the probability for n of them to be within the coalescence volume is

$$P_{m,n} = {m \choose n} P^n (1 - P)^{m-n}.$$
 (8)

If f(m) is the probability distribution of the excited nucleon multiplicity, the probability of coalescence of n nucleons,  $P_n$ , is given by  $P_n = \sum_{m \ge n} f(m) P_{m,n}$ . Assuming for f(m) a Poisson distribution one easily finds

$$\mathcal{P}_{n} = \frac{(\overline{m}P)^{n} exp(-\overline{m}P)}{n!} \approx \frac{(\overline{m}P)^{n}}{n!},\tag{9}$$

since the product of the average nucleon multiplicity times P is usually small. The probability of coalescence of N neutrons and Z protons is then

$$P_{N,Z} = \frac{(\overline{m}_N P_N)^N (\overline{m}_Z P_Z)^Z}{N!Z!}$$
(10)

where  $P_N$  and  $P_Z$  are the values of P for, respectively, the neutrons and the protons. If one assumes that the protons and the neutrons have the same momentum distribution  $\frac{dN(\mathbf{p})}{d\mathbf{p}}$  except for a numerical coefficient which simply affects its absolute value that is assumed to be unity for the protons and equal to  $\frac{N_T + N_P}{Z_T + Z_P}$  for neutrons (T=target, P=projectile) one obtains

$$\frac{\mathcal{P}_{N,Z}}{\frac{4\pi}{3}P_o^3} \approx \frac{dN_{N,Z}(\mathbf{p}_A)}{d\mathbf{p}_A} = \left(\frac{N_T + N_P}{Z_T + Z_P}\right)^N \frac{1}{N!Z!} \left(\frac{4\pi}{3}P_o^3\right)^{A-1} \left(\frac{dN(\mathbf{p})}{d\mathbf{p}}\right)^A. \tag{11}$$

A=N+Z is the cluster mass and  $\overline{p}_A = A\overline{p}$ . Further refinements taking into account the Coulomb energy per unit charge  $E_c$  of the cluster created near the nuclear surface lead to the following expression for the double differential spectrum of the cluster at large distancies from the nucleus, as a function of the proton distribution

$$\frac{dN_{N,Z}^2(E_A)}{dE_A d\Omega} = \left(\frac{N_T + N_P}{Z_T + Z_P}\right)^N \frac{A^{-1}}{N!Z!} \left[\frac{\frac{4\pi}{3}P_o^3}{2^{\frac{1}{2}}m^{\frac{3}{2}}(E - E_c)^{\frac{1}{2}}}\right]^{A-1} \left(\frac{d^2N(E)}{dEd\Omega}\right)^A,\tag{12}$$

where m is the nucleon mass and  $E_A = AE - NE_c$ .

The cluster double differential spectrum may thus be evaluated if the proton spectrum  $\frac{d^2 N(E)}{dE d\Omega}$  is known.

In Fig. 8, as an example, the experimental double differential cross sections for  $(p, {}^{3}\text{HeX})$  reactions are compared with those calculated with relation (12) [16]. Results of the same quality are obtained also in the case of  $(p, \alpha)$  reactions [16].

Assuming that (12) holds also when one considers the polarization of particles after the interaction process, one easily relates the analysing power of the cluster with energy  $E_C$ , emitted in a ( $\vec{p}$ ,CX) reaction induced by a proton beam with polarisation  $p_y^C$ , to that of protons of energy  $E_p = \frac{E_Q}{A_C} + \frac{N_Q}{A_C} E_c$  ( $N_C$  and  $A_C$  are, respectively, the number of neutrons and the mass number of the cluster C), emitted in a ( $\vec{p}$ ,p'X) reaction induced by protons of the same energy and polarisation  $p_y^P$  [17]. For instance, in the case of a ( $\vec{p}$ , <sup>3</sup>HeX) reaction the relation is

$$A_{\nu}(\vec{p},^{3} HeX) = \frac{p_{\nu}^{p}}{p_{\nu}^{^{3}He}} \frac{A_{\nu}(\vec{p},p'X)[3+(p_{\nu}^{p}A_{\nu}(\vec{p},p'X))^{2}]}{1+3(p_{\nu}^{p}A_{\nu}(\vec{p},p'X))^{2}}.$$
(13)

If the proton analysing power  $A_{y}(\vec{p},p'X)$  is small  $A_{y}(\vec{p},^{3}HeX) \approx \frac{p_{y}^{2}}{p_{y}^{3}He} 3A_{y}(\vec{p},p'X)$  [17]. To assume that relation (13) holds means to neglect the effect of the spin dependent final state interaction of the nucleons which coalesce in the  $\alpha$ -particle. Kozlowski *et al* [16,17] assume that this final state interaction may alter only the absolute value of the measured analysing power of the cluster without modifying its angular dependence. In Fig. (9) from Ref. [16] the analysing power of <sup>3</sup>He from ( $\vec{p}$ ,<sup>3</sup>He) reactions measured at 72 MeV is compared to that predicted with previous approach using data on proton analyzing power at 65 MeV reported by Sakai *et al* [8] for the same or neighbouring nuclei.;

(c) Many calculations of  $\alpha$  spectra and angular distributions and of excitation functions of  $(n,\alpha)$  and  $(p,z\alpha xnyp)$  reactions have been based on the hypothesis of a knock-out mechanism [18,19]. Here, I will discuss briefly the calculations described in Ref. [19] made in the framework of the Exciton model.

It is assumed that the  $\alpha$ -particle is already present in the nucleus and is knocked out by the projectile or by an excited nucleon during the equilibration cascade. Thus the picture is that of the usual exciton model with the further assumption that one of the excitons may be an  $\alpha$ particle. The possible configurations at each stage are divided in two groups: that which contains only nucleon-excitons and that which also contains  $\alpha$ -excitons. Since the number of  $\alpha$ -exciton states is small compared with the total number of possible excited states, the decay rates for exciton-exciton interactions,  $W_{eq}^n$ , are assumed to be unaffected by the presence of  $\alpha$ -excitons. On the other hand, the decay rates for  $\alpha$ -particle emission into the continuum strictly depend on the number of  $\alpha$ -exciton configurations and the energy distribution of excited  $\alpha$ -particles resulting from nucleon- $\alpha$  scattering inside the nucleus. To do the calculation one introduces the basic hypothesis that if it is not immediately emitted an excited  $\alpha$  particle dissolves into its constituents, so the  $\alpha$ -particles excited in the k-th stage of the de-excitation cascade are produced in the decay of states of the (k-1)-th stage through the interaction of an excited nucleon with a preformed  $\alpha$  cluster. The nucleon energy distribution in the (k-1)-th stage is assumed to be that resulting from a statistical partition of the excitation energy so it may be evaluated using state density functions  $\omega_{p,h}(E)$ . If p and h are the number of particles and holes in the k-th stage (p+h=n), E is the composite nucleus excitation energy,  $\Phi_{\alpha}=N_{\alpha}/A$  the density of preformed  $\alpha$ -clusters in terms of nuclear density, the  $\alpha$ -particle energy distribution  $P_n(E,\epsilon_\alpha)$ , is given by

$$P_n(E,\epsilon_{\alpha}) = \frac{(p-1)\Phi_{\alpha}\overline{\lambda}_{n-2}^+(E,\epsilon_{\alpha})}{(p-1)\Phi_{\alpha}\overline{\lambda}_{(n-2)T}^+(E) + W_{eq}^{n-2}(E)}$$
(14)

where the average nucleon- $\alpha$  collision probability per unit time  $\overline{\lambda}_{n-2}^+(E,\epsilon_{\alpha})$  is given by

$$\overline{\lambda}_{n-2}^{+}(E,\epsilon_{\alpha}) = \frac{1}{(p-1)\omega_{p-1,h-1}(E)} \int_{\epsilon_{\alpha}}^{E} \omega_{p-2,h-1}(E-\epsilon_{p})g(\epsilon_{p})\lambda^{+}(\epsilon_{p},\epsilon_{\alpha})d\epsilon_{p}$$
(15)

and the average total nucleon- $\alpha$  collision probability per unit time  $\overline{\lambda}^+_{(n-2)T}(E)$  by

$$\overline{\lambda}_{(n-2)T}^{+}(E) = \int_{0}^{\epsilon_{\alpha}^{max}} \overline{\lambda}_{n-2}^{+}(E,\epsilon_{\alpha}) d\epsilon_{\alpha}, \qquad (16)$$

where  $\epsilon_{\alpha}^{max}$  is the maximum  $\alpha$  energy. The nucleon and  $\alpha$  energies ( $\epsilon_p$  and  $\epsilon_{\alpha}$ ) are those in excess of respective Fermi energies. The nucleon- $\alpha$  collision probability per unit time  $\lambda^+(\epsilon_p, \epsilon_{\alpha})$  is given by

$$\lambda^{+}(\epsilon_{p},\epsilon_{\alpha}) = \sigma(\epsilon_{p} + \epsilon_{f}^{N},\epsilon_{\alpha} + \epsilon_{f}^{\alpha})v\rho$$
(17)





Fig. 9 - Comparison of  $(\vec{p}, {}^{3}\text{HeX})$  and  $(\vec{p}, p'X)$  analysing powers based on relation (13). <sup>3</sup>He data are from Ref. [16], proton data from Ref. [8] (from Ref. [16]).

Fig. 10 - Comparison of the measured angle integrated  $\alpha$  spectra in (p, $\alpha$ X) reactions on <sup>118</sup>Sn (black points and triangles) with those calculated in the hypothesis of  $\alpha$  knockout (histograms) [20].



Fig. 11 - Comparison of the measured excitation function for the  ${}^{50}\text{Ti}(p,4p5n){}^{42}\text{K}$  reaction with that calculated in the hypothesis of  $\alpha$  knockout (full line). The dashed line is the result of a calculation made by assuming that the  $\alpha$ -particles may be only emitted in the evaporation of an equilibrated compound nucleus [22]. where  $\epsilon_f^N$  and  $\epsilon_f^{\alpha}$  are the nucleon and  $\alpha$  Fermi energies,  $\sigma(\epsilon_p + \epsilon_f^N, \epsilon_{\alpha} + \epsilon_f^{\alpha})$  is the angle-integrated cross section (for the free interaction of a nucleon, with energy  $\epsilon_p + \epsilon_f^N$  inside the nucleus, with a preformed  $\alpha$ -cluster leading to an excited  $\alpha$ -particle with energy  $\epsilon_{\alpha} + \epsilon_f^{\alpha}$ ) averaged over the  $\alpha$ momentum distribution, v is the nucleon velocity and  $\rho$  the nuclear density. The Pauli principle is approximately taken into account by requiring that the nucleon and the  $\alpha$ -particle after a nucleon- $\alpha$  interaction have an energy greater than, respectively,  $\epsilon_f^N$  and  $\epsilon_f^{\alpha}$ . In the initial 2p-1hstage of the de-excitation cascade, the  $\alpha$  energy distribution is given by

$$P_{3}(E,\epsilon_{\alpha}) = \frac{\Phi_{\alpha}\lambda^{+}(E,\epsilon_{\alpha})}{\Phi_{\alpha}\lambda^{+}_{T}(E) + W_{1p}(E)}$$
(18)

where  $W_{1p}(E)$  is the collision probability of the incident nucleon with energy E (in excess of  $\epsilon_f^N$ ) with a nucleon of the target nucleus.

As a function of  $P_n(E,\epsilon_{\alpha})$ , the continuum decay rate for  $\alpha$  emission is given by

$$W_{c}^{n,\alpha}(E,\epsilon_{\alpha}') = \frac{1}{\pi^{2}\hbar^{3}}\mu_{\alpha}\epsilon_{\alpha}'\sigma(\epsilon_{\alpha}')\frac{P_{n}(E,\epsilon_{\alpha})}{g_{\alpha}}d\epsilon_{\alpha}'.$$
(19)

where  $\epsilon'_{\alpha} = \epsilon_{\alpha} - B_{\alpha}$  is the  $\alpha$ -particle energy outside the nucleus,  $B_{\alpha}$  and  $g_{\alpha}$  are the binding energy and single state density of the  $\alpha$  cluster,  $\mu$  and  $\sigma$  the reduced mass and the inverse cross section.

 $W_{eq}^{n-2}$  and  $W_{1p}$  are the leading terms in (14) and (18), respectively, so (19) shows that the continuum  $\alpha$  decay rate, and, as a consequence, the absolute yield and the spectra of emitted  $\alpha$  particles depend essentially on the ratio  $R=\Phi_{\alpha}/g_{\alpha}$  (usually a free calculation parameter) and on the preformed  $\alpha$  momentum distribution. This dependence has been studied in Ref. [20].

A measure of  $g_{\alpha}$  independent of  $\Phi_{\alpha}$  may be obtained by the analysis of the spectrum of inelastically scattered  $\alpha$ -particles in  $\alpha$ -particle induced reactions as shown in Ref. [21]. The value thus found is  $g_{\alpha} \approx A/10.36 \text{ MeV}^{-1}$ . The values of  $\Phi_{\alpha}$  that corresponds to this single alpha particle state density depend, as previously discussed, by the preformed  $\alpha$ -particle momentum distribution, but in all cases are unexpectedly high (to give a quantitative information, values smaller than  $\approx 0.06$  have never been reported). So, even if the previous theory provides a reasonable reproduction of a large body of data using a unique set of parameters, as shown, for instance, in Figs. 10 and 11, a well founded theoretical justification of the values found for the  $\alpha$  density seems to be a necessary requisite for assuming that the mechanism described above is the dominant physical process contributing to the hard component of the continuous  $\alpha$ -particle spectra.

These phenomenological models are only a few of those that have been proposed to describe multi-step processes. Though, with an appropriate parametrization, they give a reasonable description of a large set of data, each of them may at most, reflect one aspect of the real process occurring. They remain only a first order approximation of more fundamental microscopic theories whose application in the multi-step region presents formidable computational difficulties.

The MSDR approach is an attempt to apply semi-microscopic theories to transitions to highly excited states. The many approximations and the lack of a comprehensive analysis of the existing data do not allow one to judge its ability to provide accurate quantitative predictions.

Further progress in this field might be also obtained through exclusive experiments that allow a more accurate test of theoretical predictions.

Previous arguments are discussed in greater detail in a review article on Nucleon-Alpha Reactions in Nuclei submitted to Rep. Prog. Phys. by Peter E. Hodgson and the author. I wish to thank Dr. Hodgson for the extensive quotations from this review article.

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