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E. Gadioli: EMISSION OF COMPLEX PARTICLES FROM HIGHLY EXCITED NUCLEI

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EMISSION OF COMPLEX PARTICLES FROM HIGHLY EXCITED NUCLEI&)

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A great deal of work has been made to investigate experimentally and predict theoretically the continuous spectra of composite particles produced in reactions induced by nucleons with energy ranging from a few to several ten MeV. Some recent results in the field are summarized. In particular the Exciton Coalescence/Pick-up model and the Exciton Knock-on model, in the case of alpha emission, are reviewed and discussed.

A quite notable amount of composite particles (d, t, ^{3}He, α) is produced in nucleon-nucleus interactions at energies ranging from a few to several ten MeV. These particles may have either a discrete energy feeding low energy levels of the residual nucleus or a continuous distribution. In this talk I will discuss some of the models that have been proposed to describe these continuous spectra.

Their general features are well known: a low energy peak, which may be greatly reduced in intensity or even absent in the case of heavy nuclei, due to evaporations from a fully equilibrated nucleus and a structureless higher energy component which extends up to the maximum available energy. The yield of these high energy particles depends very weakly on the target nucleus mass and their angular distribution is strongly forward peaked.

There is a little doubt that deuterons, tritons, ^{3}He be produced just before emission through either the coalescence of excited nucleons (1,2,3) or the pick-up of one/two nucleons of the outer shells by a nucleon which has reached the nuclear surface with a sufficiently large energy (4-7).

This interpretation is supported by the experimental observation that the cross sections for production of high energy protons, deuterons, tritons/3 He, in proton induced reactions, are scaled by a constant factor roughly equal to ten, at incident energies of either 60 (8) and 90 MeV (9) (see Fig. 1).

Fig. 1 in addition to this scaling property displays two other very interesting features: the yield of 3 He is notably smaller than the corresponding yield of tritons, and the alpha yield is by far greater than the yield of tritons and 3 He.

Any theory aiming to describe these reactions must explain these facts in a natural way. We will se that the recent calculations by Sato *et al.* (7) which are based on the coalescence-pick up mechanism fulfill this requirement. These authors show that the reduced 3 He yield reflects the fact that 3 He is a little more loosely bound system than the triton so that the phase space volume accessible to

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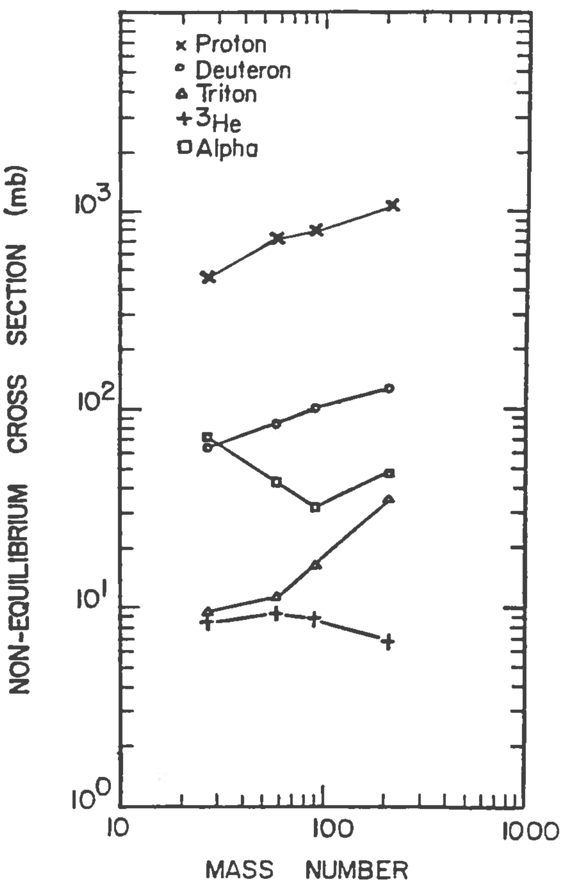

Fig. 1 - Non equilibrium component of composite charged particle spectrum, in 90 MeV proton bombardment, reported as a function of the mass number of the target nucleus (9).

the nucleons which condense in the 3 He is substantially smaller than the corresponding one for nucleons which condense in the triton.

May be the high yield of emitted alphas has the same origin; however one cannot rule out the alternative suggestion that to alpha emission an entirely different mechanism may contribute, i. e., the knock-on of preformed alphas. Detailed calculations show indeed that the spectra and the angular distribution of the emitted alphas may be quite well reproduced by such mechanism (10,11); however I think that a more convincing suggestion of the presence of a knock-on component originates from the observation that an unpaired nucleon acts as a spectator in a (p,α) or (n,α) process, thus suggesting that the protons and the neutrons constituting the alpha were already paired in the target nucleus (12,13).

In a whole series of (p, α) reactions on quasi magic odd targets from 91 Zr to 209 Bi - having a proton or a neutron outside a closed shell - it was found that the states of the residual nuclei excited in the reaction are just those of the nucleus formed by removing two pair of protons and neutrons from the outer shells of the target and replacing them by the incident proton. The lowest state excited in this way is typically about 2-3 MeV above the ground state and the strongly populated states are the ones formed by coupling the spectator nucleon (outside the closed shell) to the proton occupying one of the states vacated by the ejected particles. This would not be expected if the reaction proceeded

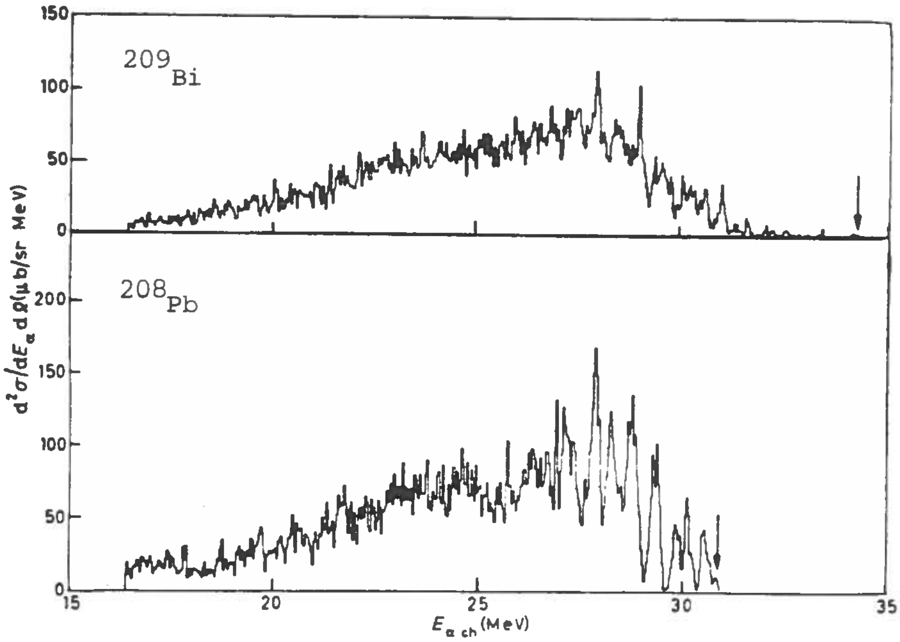

Fig. 2 - Comparison of the spectra of alpha particles emitted in the rea ctions 209 Bi(p,α) 206 Pb and 208 Pb(p,α) 205 Tl at θ∿60°30' (13). The arrows indicate the g.s. of, respectively, 206 Pb and 205 Tl.

by the pick up process, since the valence particle is available for pick up so that a much greater number of low energy states would be excited. This conjecture about the spectator role of the valence nucleons is suppor ted by the comparisons with the corresponding (p,α) reaction in the closed shell nuclei, when it is found that there is no energy gap, all states from the ground state being excited (see in Fig. 2 the comparison betwe en the alpha spectra from the 209 Bi and 208 Pb(p,α) reactions).

The comparison between the alpha spectra from (p,α) reactions on neigh bouring nuclei, one <u>magic</u> with a magic shell of protons and/or neutrons and the other <u>quasi magic</u> with a proton or a neutron outside the magic shell, also shows that <u>homologous states</u> are excited in the two residual nuclei. These states - of a very similar structure - are low lying (2 paired neutron - 1 proton) hole states in the residual nucleus from the reaction on the magic target nucleus and states of a considerable higher energy corresponding to the weak coupling of (2 paired neutron- 1 proton) holes with the spectator nucleon outside the closed shell in the reaction on the quasi magic target nucleus (Fig. 3).

In addition it was shown that in a (p,α) reaction the structure of the target nucleus affects also the transitions to high energy levels of the residual, that merge in a continuous distribution, favouring also in this case the ejection from the target nucleus of two paired neutrons and pro tons (see Fig. 4).

Let me turn now to the models that have been proposed to describe the

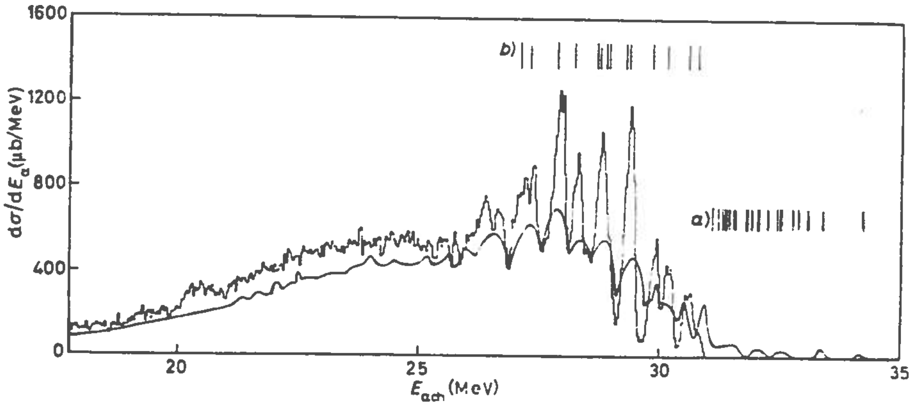

Fig. 3 - Comparison of the alpha spectra from the reactions 208 Pb(p,α) 205 Tl, histogram, and 209 Bi(p,α) 206 Pb, full line (13). In a) the bars indicate the excited levels of 206 Pb below $\sim$3.2 MeV which are weakly populated in the reaction; in b) the bars give the location of levels or group of levels of 205 Tl which are strongly populated in the 208 Pb(p,α) 205 Tl reaction and are homologous of the higher energy states preferentially populated in the 209 Bi(p,α) 206 Pb reaction.

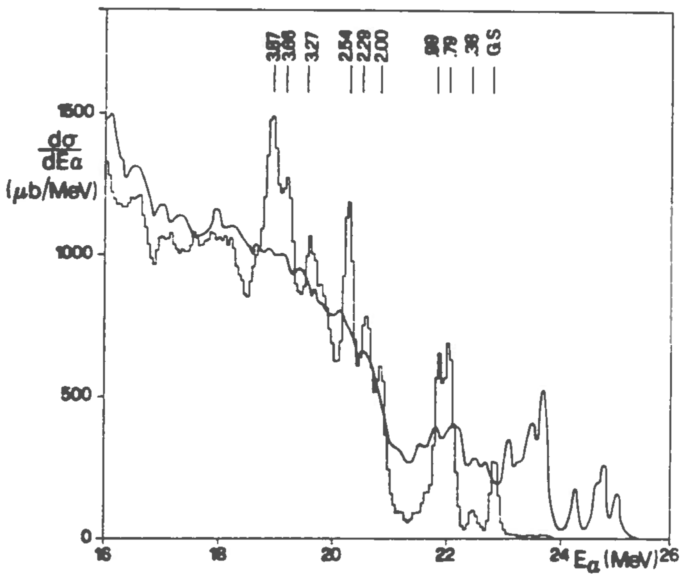

Fig. 4 - Comparison of angle integrated spectra for the reaction 90 Zr(p,α)87 Y, histogram, and 91 Zr(p,α)88 Y, continuous line (13). The absolute value and the energy dependence of the two alpha spectra show that the density of residual nucleus states excited in the reactions is the same when the excitation energy of the odd-odd 88 Y exceeds by about 2.16 MeV the one of the even-odd 87 Y. This result which is convincingly explained as originating from the spectator role of the unpaired neutron of 91 Zr, is contrary to the widespread expectation that at the same excitation energy the density of the excited states of an even-odd nucleus should be smaller than the corresponding one of the adiacent odd-odd nucleus.

emission of high energy composite particles with a continuous distribution, in nucleon induced reactions.

The angular and energy distribution of these particles may be evaluated in a Multistep Direct Reaction approach, as it has been done by Tamura and Udagawa (14). In such a theory one introduces a statistical distribution for some quantity, e.g., the spectroscopic density, but the transition matrix element is calculated by a procedure that being quantum-mechanical is in principle much preferable to the semiclassical one which is utilized in all the phenomenological pre-equilibrium models. However, in my opinion, apart from the different physical assumptions, a M S D R approach cannot really do the same as a semiclassical model does. First of all, part of the accuracy is lost when one is obliged to introduce quite notable approximations to simplify the calculations. These, strongly depend on parameters like the Optical Model ones and the bound particle geometrical factors (15) and just at the second step one has to choice a particular reaction path, thus loosing some of the generality. However, in my opinion, the most important fact is that the phenomenological models allow one to do a comprehensive calculation of the cross sections of all processes taking place, while these approaches do not. So, I will limit myself to discuss the approaches that have been proposed in the framework of the phenomenological models and in particular of the Exciton model (&).

The first unified description of the emission of composite particles was proposed by Kalbach Cline (17). She reported the following expression for the decay rate for emission of a composite particle with energy between ε and $\varepsilon+d\varepsilon$, which constitutes a quite natural generalization of the decay rates for nucleon emission:

$$W_c^{n,x}(E,\varepsilon)d\varepsilon=\frac{2s_x+1}{\pi^2\hbar^3}\,\mu_x\sigma_x(\varepsilon)\,\varepsilon\,\frac{\omega_{p-px,h}(U)}{\omega_{p,h}(E)}\,R_x d\varepsilon \qquad (1)$$

s_x and μ_x are the spin and reduced mass of particle x and px the number of its constituent nucleons.
σ_x is the inverse cross section, U and E, respectively, the residual nucleus and the composite nucleus energies. R_x is the probability of getting the right proton-neutron configuration in the outgoing particle.

The physical meaning of (1) is that - in a statistical process - it is possible that px excited particles condense into a composite x which is experimentally observed with energy ε.

Formula (1) predicts too low cross sections and to improve the reproduction of data it was further multiplied by a factor px!. An explanation of this factor was not given and thereafter it was abandoned. However, also with a factor px! expression (1) does not allow a reasonable reproduction of spectra on heavy nuclei and high incident energies.

To improve the situation, Ribansky and Oblozinsky (1) proposed the following expression for the decay rate:

&) It is well known that more formal theories of pre-equilibrium emission have been recently proposed (16); however, until now I am not aware of extensive applications of such theories to the analysis of data concerning composite particle emission.

$$W_c^{n,x}(E,\varepsilon)d\varepsilon = \frac{(2s_x+1)}{\pi^2\hbar^3}\,\mu_x\sigma_x(\varepsilon)\,\varepsilon\left\{\frac{\omega_{p-px,h}(U)}{\omega_{p,h}(E)}\;\omega_{px,0}(\varepsilon+B_x)\gamma_x\right\}\frac{R_x}{g_x}\,d\varepsilon \tag{2}$$

$\omega_{px,0}(\varepsilon+B_x)$ is the density of px particle states at the excitation energy, inside the nucleus, $\varepsilon+B_x$; B_x is the binding energy of x, g_x the single x state density, a quantity usually taken independent on the energy. γ_x is defined as the condensation probability of px particles in x.
Calculations by Wu and Chang (2) showed that γ_x may be assumed independent on the energy, and Machner (3), to give a physical interpretation of this quantity suggested that the excited nucleons may coalesce only if their relative momenta are smaller than a momentum p_{ox}, whose value must be obtained from the analysis of the experimental data. Then γ_x may be expressed as:

$$\gamma_x = \left[\frac{4}{3}\pi\left(\frac{p_{ox}}{mc}\right)^3\right]^{px-1} \tag{3}$$

m is the nucleon mass and c the velocity of light.

In this approach, then, γ_x is simply related to the momentum space accessible to the nucleons which coalesce.

This exciton-coalescence model has been used to reproduce the angle integrated cross sections and also the angular distributions of reactions induced by several different projectiles with a fair degree of success; however the use of expression (2) with γ_x energy independent and not related to g_x might be not justified.

To discuss this point, let me re-write (2) in the following way:

$$W_c^{n,x}(E,\varepsilon)d\varepsilon = \left\{\frac{\omega_{p-px,h}(U)}{\omega_{p,h}(E)}\;\omega_{px,0}(\varepsilon+B_x)\;\gamma'_x\,\frac{(p-px)!\,px!}{p!}\,d\varepsilon\right\}\cdot R_x\cdot\left\{\frac{\sigma_x(\varepsilon)\,v_x\,\omega_{c,x}(\varepsilon)}{g_x\,V}\right\} \tag{4}$$

where

$$\gamma'_x\,\frac{(p-px)!\,px!}{p!} \equiv \gamma_x \tag{5}$$

v_x is the velocity of particle x and $\omega_{c,x}(\varepsilon)$ its continuum state density given by

$$\omega_{c,x}(\varepsilon) = \frac{(2s_x+1)8\pi\mu_x\varepsilon V}{h^3 v_x} = \frac{1}{\pi^2\hbar^3}\,\mu_x\varepsilon\,V(2s_x+1)/v_x \tag{6}$$

V is the Lab. volume.

The quantity within the first bracket in expression (4) has the meaning of the probability that the energy E of the composite nucleus in a (p,h) configuration be partitioned between px particles, which coalesce into a cluster x, with energy in the interval $(\varepsilon+B_x) - (\varepsilon+B_x+d\varepsilon)$, and p-px particles, h holes with the rest of the energy.
The term (p-px)!px!/p! is inserted because the integral over the energy of all the quantities appearing in the first bracket, except γ'_x, must give unity.
The quantity within the second bracket is the decay rate for emission of this cluster in the continuum.
The quantity within the first bracket could also be written:

$$\frac{\omega_{p-px,h}(U)}{\omega_{p,h}(E)} \; g_x^{*} d\varepsilon \qquad (7)$$

where g_x^{*} is a single state density for cluster x, which in general will be energy dependent. Indeed, since the clusters with energy $(\varepsilon+B_x)$--$(\varepsilon+B_x+d\varepsilon)$ are not necessarily made only by condensation of px excited particles, g_x is a partial single cluster state density and

$$g_x^{*} \le g_x \; . \qquad (8)$$

This considerations show that the decay rate (1) is an upper estimate of the true decay rate corresponding to emission of clusters made only by excited particles.

It may be useful to define a quantity γ_x^{*} through the relation

$$g_x \gamma_x^{*} = \gamma_x \qquad (9)$$

γ_x^{*} has the meaning of an average condensation probability of px excited particles into a cluster x.

Then, we have

$$g_x \gamma_x^{*} \omega_{px,0}(\varepsilon+B_x) = g_x^{*} \qquad (10)$$

so that

$$\gamma_x^{*} \omega_{px,0}(\varepsilon+B_x) \le 1 \qquad (11)$$

a relation which shows that γ_x^{*} cannot be energy independent as assumed in the so called Exciton-Coalescence model previously quoted, where both g_x and γ_x are energy independent. In addition, previous considerations indicate that g_x and γ_x must be related and not independent quantities as usually assumed.

The same conclusions are reached by Iwamoto and Harada[6] following a different procedure.

According to these authors, the overlap of the cluster x wave function and px particles's wave function may be written as

$$<\phi_x \chi^{(\varepsilon)}(\vec{R}) \,|\, \phi_1 \phi_2 \ldots \phi_{px}> \qquad (12)$$

where $\chi^{(\varepsilon)}(\vec{R})$ is the center of mass wave function and ϕ_x the internal wave function of cluster x, characterized by px-1 intrinsic freedoms; ϕ_i is the wave function of the i-th constituent nucleon.

We have obviously to expect that

$$\sum_{1,2,..,px} \left| < \phi_x \chi^{(\varepsilon)}(\vec{R}) \,|\, \phi_1 \phi_2 \ldots \phi_{px} > \right|^2 = 1 \qquad (13)$$

and, if we restrict the summation only to terms corresponding to nucleons with energy greater that the Fermi energy, ε_f,

$$\sum_{\substack{1,2,\ldots,px \\ \varepsilon_i > \varepsilon_f}} \left| < \phi_x \chi^{(\varepsilon)}(\vec{R}) \,|\, \phi_1 \phi_2 \ldots \phi_{px} > \right|^2 \equiv \gamma_x^{*} \omega_{px,0}(\varepsilon+B_x) \le 1 \qquad (14)$$

Iwamoto and Harada, further suggest that the cluster x might be made of l particles above the Fermi level and m=px-l particles below. The probability of forming such cluster is

$$f(l,m,\varepsilon)=\sum_{\substack{\varepsilon_i>\varepsilon_f (i=1,\dots,l)\\ \varepsilon_j>\varepsilon_f (j=l+1,\dots,px)}} \left| \langle \phi_x \chi^{(\varepsilon)}(\vec{R}) \mid \phi_1\ \phi_2 \dots \phi_{px} \rangle \right|^2 \qquad (15)$$

Obviously, for the relation (13),

$$\sum_{l+m=px} f(l,m,\varepsilon)=1 \ . \qquad (16)$$

Indeed, since the cluster x should be formed in the surface region, we have to impose some restrictions when evaluating the overlap integrals in (15) and the corresponding probability of forming the cluster x will be indicated by $F(l,m,\varepsilon) < f(l,m,\varepsilon)$.

Now the expression of the decay rate for emission of a cluster x made by l particles above and m particles below Fermi level, when the composite nucleus is in a n exciton configuration is given by

$$W_c^{n,x(l,m)}(E,\varepsilon)d\varepsilon=\frac{(2s_x+1)}{\pi^2\hbar^3}\mu_x\varepsilon_x\sigma_x(\varepsilon)\ F(l,m,\varepsilon)\ \frac{\omega_{n(l,m)}(U)}{\omega_{p,h}(E)} \qquad (17)$$

and the decay rate for emission of x will be:

$$W_c^{n,x}(E,\varepsilon) = \sum_{l,m} W_c^{n,x(l,m)}(E,\varepsilon) \qquad (18)$$

The emission of x leads from a (p,h) configuration to a (p-l,h+m) configuration, so that the density $\omega_{n(l,m)}(U)$ of the residual nucleus states should be given by $\omega_{p-l,h+m}(U)$.

However in evaluating $F(l,m,\varepsilon)$ one has already taken into account the state density of the newly created m holes and Iwamoto and Harada, assuming further that the excitation energy of the m holes be small when compared to the total excitation energy U, postulate that

$$\omega_{n(l,m)}(U) \sim \omega_{p-l,h}(U) \qquad (19)$$

This is a quite significant approximation, that also partly conflicts with the way $F(l,m,\varepsilon)$ is calculated in this approach, since one does not impose that the energy of each of the m particles of x which are below the Fermi level, be almost equal to Fermi energy.

The calculation of $F(l,m,\varepsilon)$ is quite cumbersome. It is made in the framework of Fermi Gas model, assuming that the internal wave function of x is the ground state of an intrinsic Hamiltonian evaluated with the harmonic oscillator model, whose frequency is determined from the condition which relates ω to the rms radius, r_x^2, of x ($r_x^2 \div 1/\omega$).

When one evaluates the phase space volume that divided by $h^{3(px-1)}$ gives the number of nucleon states contributing to $F(l,m,\varepsilon)$ one imposes that the center of mass of the px particles be at the surface of the residual nucleus:

$$|\vec{R}| = |(\vec{r}_1+\vec{r}_2+\dots+\vec{r}_{px})/px| = R_{Res} \qquad (20)$$

and the values of the nucleon position vectors are further restricted by the relation

$$|\vec{r}_i| < R_{Res} + \Delta R \qquad (21)$$

Where ΔR may be suitably chosen.

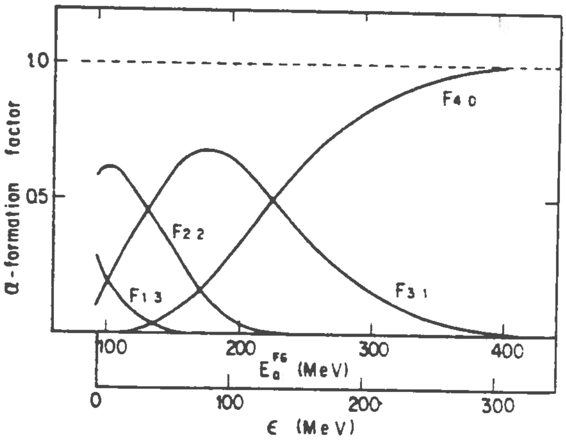

Fig. 5 - The alpha particle formation factors $F(l,m,\varepsilon)$ reported as a function of the alpha energy ε. In the abscissa the energy of the alpha in the Fermi Gas is also shown. The $F(l,m,\varepsilon)$'s are normalized to unity.

In Fig. 5 calculated alpha particle formation factors $F(l,m,\varepsilon)$ are reported as a function of the alpha energy ε (similar results are obtained for other composite particles).

One clearly notes that, below $\sim$100 MeV, the probability that the alpha be made of 4 particles all above the Fermi level is quite small. In addition one should remind that the factor $\omega_{p-1,h}(U)$ further enhances

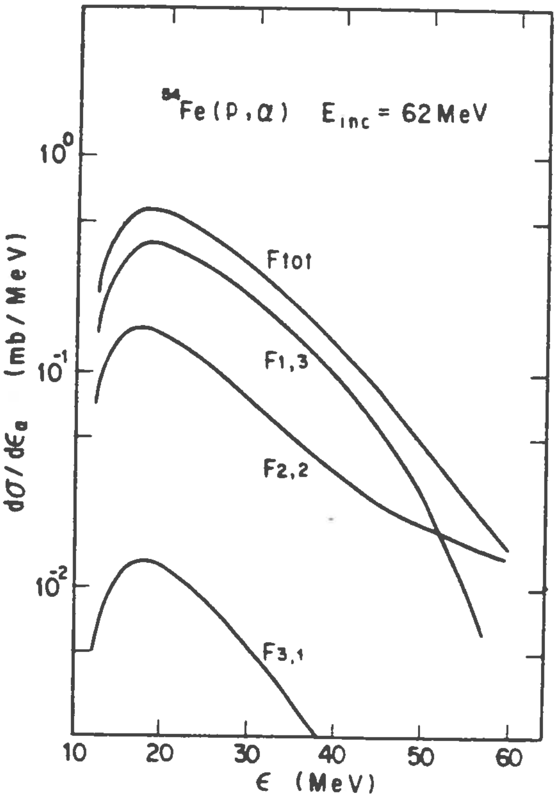

Fig. 6 - Decomposition of the calculated energy spectrum for the 54 Fe(p,α) reaction at Ep $\sim$62 MeV into the components corresponding to the different $F(l,m,\varepsilon)$.

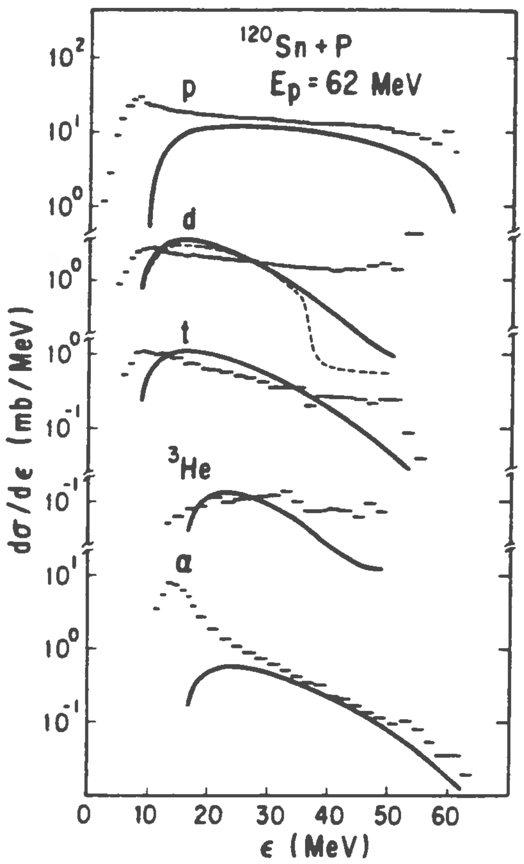

Fig. 7 - Angle integrated energy spectra of composite particles emitted when 62 MeV protons interact with 120 Sn. The bar histograms are the data and the continuous lines the calculated spectra.

the probability of emission of alpha particles with <u>l</u> as low as possible.

In Fig. 6 the decomposition of a calculated alpha energy spectrum, at an incident proton energy of ~60 MeV, into components corresponding to the different F(l,m,ε) is shown. This quite general result shows that, at these energies, the main contribution to the alpha spectrum, predicted by this model, is due to pick up of three bound nucleons by an excited nucleon. The pick up of one or two nucleons is dominant also in the case of d, t, 3 He emission at incident proton energies of ~60 MeV (7).

In Fig. (7) experimental and theoretical angle integrated spectra of composite particles are compared in the case of reactions induced by 62 MeV protons on 120 Sn.

One remarkable aspect of these calculations is the fact that they reproduce quite naturally the ratio of triton to 3 He yield showing that, in this model, the reduced helion emission probability is explained as due to the fact that 3 He is a little more loosely bound system than the triton, so the phase space volume accessible to condensing particles is substantially reduced.

The comparison between experiment and theory shows that something is lacking in the theoretical spectra of deuterons, tritons and 3 He. This is quite reasonable since in all calculations it is assumed that the initial interaction of the proton with the nucleus excites 2 particles - 1 hole states and only <u>after</u>, excited and bound particles may condensate into a composite x. It is well known that the direct pick-up by the incident proton of, respectively, a neutron, two neutrons and a neutron-proton pair, which is not evaluated in present model, gives a large contribution in the highest energy region of the spectra (4,5). The analogous contribution does not seem to be important in the case of the (p,α) reaction and this is in contrast with what one expects from a MSDR calculation (14,18).

The Iwamoto - Harada model may be generalised to predict the angular distribution of the composite particles and calculations have been made in the case of alpha particle emission (19).

The double differential cross section is given as:

$$\frac{d^2\sigma}{d\Omega_\alpha d\varepsilon_\alpha}=\sigma_{abs} \sum_n \tau(n)\left\{\sum_{l,m} W_c^{n,\alpha(l,m)}(\varepsilon)\; X_{l,m}(n\Omega\varepsilon)\right\} \tag{22}$$

σ_{abs} is the absorption cross section and $\tau(n)$ the composite nucleus lifetime in a configuration of n excitons. $X(n\Omega\varepsilon)$ which satisfies to relation

$$\int X_{l,m}(n\Omega\varepsilon)d\Omega=1 \tag{23}$$

is given by

$$X_{l,m}(n\Omega\varepsilon) \div \int d\vec{p}_1 f(l,m,\vec{p}_1,\vec{P}_\alpha)\tau(n\Omega_1\varepsilon_1) \tag{24}$$

where

$$\tau(n\Omega_1\varepsilon_1)=\int_0^\infty dt\; q(n\Omega_1\varepsilon_1\; t) \tag{25}$$

gives the probability of finding at any time the composite nucleus in a configuration of n excitons one of which, the fast one, has energy ε_1 and direction Ω_1. This quantity may be evaluated by solving the Master Equation satisfied by the occupation probability $q(n\Omega_1\varepsilon_1 t)$ with a well established Legendre polinomials expansion technique (20,21).

$f(l,m,\vec{p}_1,\vec{P}_\alpha)$ is the probability that the alpha with Center of Mass momentum $\vec{P}_\alpha$ be made of l particles, one of which with momentum $\vec{p}_1$, above Fermi level and $m=4-l$ particles of the Fermi sea.

To evaluate $f(l,m,\vec{p}_1,\vec{P}_\alpha)$ one defines an average momentum $\vec{\bar{p}}$, whose absolute value may be evaluated when one knows the alpha internal wave function, which satisfies the relation:

$$\vec{p}_1 + \vec{\bar{p}} = \vec{P}_\alpha/4 \tag{26}$$

and one evaluates within the framework of Fermi Gas model for any fixed $\vec{p}_1$ and $\vec{P}_\alpha$ the probability that the momentum of the three other nucleons be equal to:

$$\vec{p}_3= \vec{P}_\alpha-\vec{p}_1. \tag{27}$$

The calculated angular distributions reproduce with a fair accuracy the experimental angular distributions.

At the beginning of this talk I mentioned that some experimental findings suggest that alpha particles might also be produced in a knock on process.

The first attempt to reproduce the data in this way was made by Milazzo Colli and Braga Marcazzan (22).

These authors hypothesized that the incident nucleon or one of the nucleons excited in the course of the equilibration cascade could inte ract with one alpha particle preformed in the target nucleus, supplying it with a sizable amount of energy. It was found that satisfactory results - at incident nucleon energies smaller than approximately 40 MeV - could be obtained by introducing the simplifying hypothesis that the energy distribution of the alpha , at each stage of the de-excitation cascade, is the one resulting from the statistical partition of the excitation energy among the alpha, the alpha hole and the other excited particles and holes, assuming as equiprobable all the possible states of the excited system. This approach, as shown by Ferrero et al. (10), leads to less satisfactory results at higher incident energies and requires, also at energies smaller than 40 MeV, energy dependent alpha preformation factors.

In more detailed calculations (10, 23) the alpha energy distributions have been evaluated on the basis of nucleon-alpha scattering dynamics assuming the nucleon-alpha interaction inside the nucleus to be the same, except for limitations due to Pauli principle, as the free one.

Following the theory described in ref. (10), the decay rate for alpha emission from the composite nucleus in an n-exciton configuration is given by:

$$W_c^{n,\alpha}(E,\varepsilon)d\varepsilon = P_n(E,\varepsilon')\left\{\frac{\sigma_\alpha(\varepsilon) v_\alpha \omega_{c,\alpha}(\varepsilon)}{g_\alpha V}\right\} d\varepsilon = \frac{1}{\pi^2 \hbar^3} \mu_\alpha \varepsilon_\alpha \sigma_\alpha(\varepsilon) \frac{P_n(E,\varepsilon')}{g_\alpha} d\varepsilon \qquad (28)$$

where ε' is the alpha energy in excess of Fermi energy inside the nucleus and $P_n(E,\varepsilon')$ is the alpha energy distribution.

In the initial configuration (n=3)

$$P_3(E,\varepsilon') = \frac{\phi_\alpha \lambda^+(E,\varepsilon')}{\phi_\alpha \lambda^+_T(E) + W_{1p}(E)} \qquad (29)$$

where $\lambda^+(E,\varepsilon')$ is the A independent probability per unit time and unit energy that the incident nucleon, with energy outside the nucleus $E_i = E-B$ (B is its binding energy), excites an alpha particle in the energy interval $\varepsilon' - \varepsilon'+d\varepsilon'$.

The quantity

$$\lambda^+_T(E) = \int \lambda(E,\varepsilon')d\varepsilon' \qquad (30)$$

is the total nucleon-alpha collision probability per unit time. $W_{1p}(E)$ is the total nucleon-nucleon collision probability per unit time. $\phi_\alpha = N_\alpha/A$ is the density of α's within the nucleus in terms of nuclear density. Since the absolute value of $W_{1p}(E)$ that was used in the calculation is deduced from the analysis of experimental data and is noticeably smaller than the one evaluated on the basis of free nucleon-nucleon scattering, it has been assumed that the second addendum of the denominator of expression (29) is an effective quantity which already takes into account the possible presence of preformed alphas and one did not introduce the factor $(1-4\phi_\alpha)$ that otherwise would be necessary.

$$\lambda^+(E,\varepsilon') = \sigma(E,\varepsilon')\ v\ \rho \qquad (31a)$$

where v is the incident nucleon velocity in nuclear matter, ρ the nuclear density.

$$\sigma(E,\varepsilon') = (m_1/P_1) \iint v_{Rel} \frac{dn(P_2)}{d\vec{P}_2} \frac{d\sigma_{CM}(\vec{k}_1,\vec{k}_f^{\alpha})}{d\hat{k}_f^{\alpha}} d\hat{k}_f^{\alpha} d\vec{P}_2 \qquad (31b)$$

Here P_1 is the modulus of the incident proton momentum within the nucleus in the Lab. system, which coincides, for sufficiently heavy nuclei with the C M system for proton-nucleus scattering, v_{Rel} is the relative nucleon-alpha velocity.

$d\sigma_{CM}(\vec{k}_1,\vec{k}_f^{\alpha})/d\hat{k}_f^{\alpha}$ is the free nucleon-alpha cross section in C M system of the two colliding particles and the $\vec{k}$'s are their momenta in this system. The quantity $dn(P_2)/d\vec{P}_2$ is the momentum distribution of the preformed alpha within the nucleus (normalized to one alpha). A detailed description of the integration technique of (31) may be found in ref. (10).

In order to evaluate the alpha energy distribution at the subsequent stages of the equilibration cascade, it was hypothesized that if it not emitted an alpha does not survive (recent results (24) throw some doubt on the validity of this hypothesis). Then, at a given stage of the equilibration cascade, the alpha energy distribution is the one which results from the interaction of one already excited nucleon with a preformed alpha. If one retains the usual assumption that, at the previous (n-2) exciton stage of the equilibration cascade, the density of states was the one corresponding to the partition of the excitation energy in all the possible and equiprobable ways among (p-1) excited nucleons and (h-1) nucleon holes and one neglects those few states in which an alpha was excited and divided subsequently into its constituents, the alpha energy distribution in a n(=p+h) exciton configuration is given by

$$P_n(E,\varepsilon') = \frac{(p-1)\phi_\alpha \bar{\lambda}^{+}_{(n-2)}(E,\varepsilon')}{(p-1)\phi_\alpha \bar{\lambda}^{+}_{(n-2)T}(E) + W^{n-2}_{eq}(E)} \qquad (32)$$

where

$$\bar{\lambda}^{+}_{(n-2)}(E,\varepsilon') = \frac{1}{(p-1)\omega_{p-1,h-1}(E)} \int_{\varepsilon'}^{E} \omega_{p-2,h-1}(E-\varepsilon'_p)\, g(\varepsilon'_p)\, \lambda^{+}(\varepsilon'_p,\varepsilon')\, d\varepsilon'_p \qquad (33)$$

$$\bar{\lambda}^{+}_{(n-2)T}(E) = \int \bar{\lambda}^{+}_{(n-2)}(E,\varepsilon')\, d\varepsilon' \qquad (34)$$

and $W^{n-2}_{eq}(E)$ is the decay rate for exciton-exciton interaction in the (n-2) exciton stage. ε'_p is the proton energy in excess of Fermi energy.

Since in expression (29) and (32) the denominator is approximately equal to, respectively, $W_{1p}(E)$ and $W^{n-2}_{eq}(E)$, the yield and the shape of the spectra essentially depend on the ratio R between the density of preformed alphas ϕ_α and the single alpha state density g_α and the momentum or the energy distribution of preformed alphas.

R is ususlly assumed to be a free parameter since there is no recipe to predict a priori its value accurately.

A satisfactory reproduction of the angle integrated alpha spectra in (p,α) reactions induced by protons with energy varying from about 20 to about 70 MeV on nuclei spanning the entire mass table (from Al to Th) and of the angular distributions of high energy alpha particles was obtained by utilising either a Fermi Gas model momentum dis tribution or a distribution which favours even more alpha momenta near to the maximum momentum allowed; e.g., in several calculations a distribution

$$dn(P) \div P^6 \, d\vec{P} \qquad (35)$$

was used. Values of 20 and 80 MeV were assumed for the nucleon and alpha Fermi energies. Values of R equal to (1.51/A) MeV and (0.62/A) MeV were found to correspond, respectively, to the Fermi Gas Model distribution and distribution (35). A typical result is shown in Fig. 8.

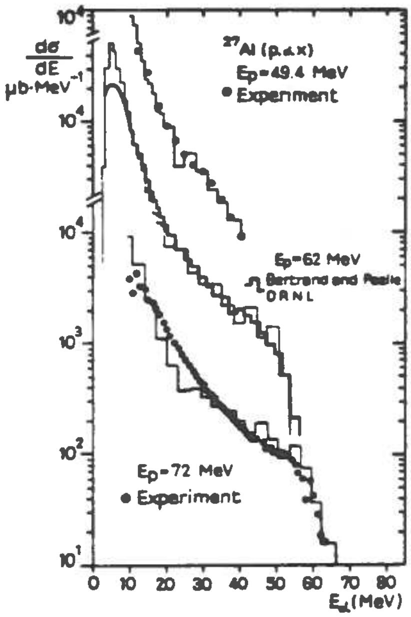

Fig. 8 - Comparison of experimental and calculated angle integrated cross section for the reaction 27 Al(p,α ...) at different proton energies. The data are from refs. (25) and (27). The calculations are represented by the thin line histograms.

Also in the calculations of Scobel <u>et al.</u> (23), in spite of the many differences between their calculation and the one quoted above, the average energy of pre-formed α's, $\bar{\varepsilon}_\alpha$, was assumed to be only 10 MeV lower than ε_f^α, the alpha Fermi energy. This estimate is approximately equal to the one corresponding to the use of a momentum distribution like (35).

The need for a high value of the energy of pre-formed alphas is due to the dynamics of nucleon-alpha scattering, which hinders the transfer of a large fraction of the energy at the disposal of the alpha particle.

A satisfactory reproduction of the data was also achieved utilising a momentum distribution like

$$dn(P) \div P^{2L} \exp(-P^2/2P_L^2)\, d\vec{P} \qquad (36)$$

calculated from a simple harmonic oscillator model for L=2 with ε_f^α and ε_f^N equal, respectively, to 40 and 10 MeV and P_2=141 MeV/c, or ε_f^α and ε_f^N equal, respectively to 60 and 15 MeV and P_2=173 MeV/c, but a considerably greater value of R was required (11).
The reduction of the Fermi energy values would simulate a process occurring at the nuclear surface.

While calculations with harmonic oscillator distributions for L=0 and 1 do not afford satisfactory results, the use of distributions with L=3 or greater would also allow satisfactory reproduction of the data.
In this case the R values would be midway between the ones corresponding to distribution (35) and an L=2 harmonic oscillator distribution.

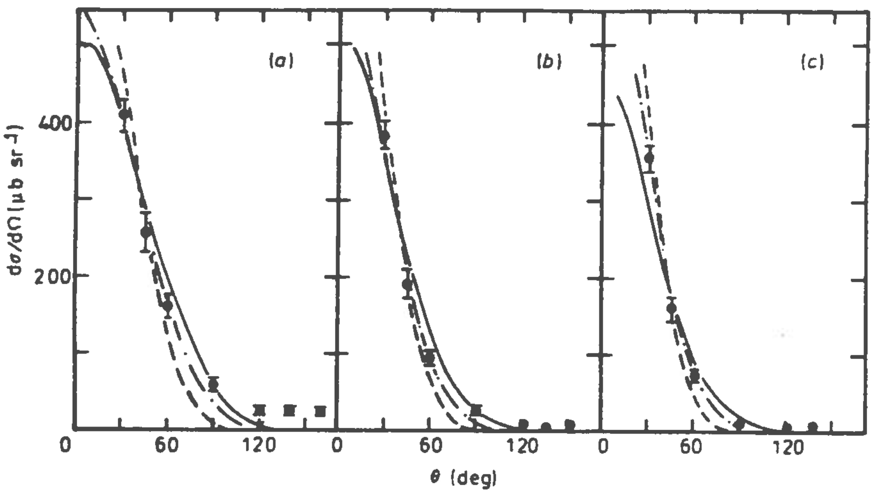

Fig. 9 - Comparison between experimental (closed circles) and calculated angular distributions of alpha particles emitted in the reaction 165 Ho(p, α) 162 Dy at different proton energies. The data are from ref. (10). Each angular distribution corresponds to alpha particles with energy in the interval ΔEα: (a) Ep=25.8 MeV, ΔEα=25-30 MeV, (b) Ep=36.5 MeV, ΔEα=35-40 MeV, (c) Ep= 44.3 MeV, ΔEα= 40-45 MeV. The theoretical curves correspond to the use of diffrent momentum distributions for the preformed alpha particle: full curves, distribution (35) and alpha and proton Fermi energies of 80 and 20 MeV; broken curves, harmonic oscillator (H O) distribution, L=2, Fermi energies of 40 and 10 MeV, P_2=141 MeV/c; chain curves, H O distribution, L=2, Fermi energies of 60 and 15 MeV, P_2=173 MeV/c.

These results confirm that the main contribution to the alpha spectrum is due to the interactions which involve preformed alphas of energy comparable with the Fermi one. The use of momentum distributions peaking to an energy considerably smaller than the Fermi energy, reduces drastically the alpha yield.

In addition to angle-integrated spectra, the angular distributions of high energy alpha particles emitted in reactions induced by neutrons and protons may be evaluated.

In the case of rather low incident projectile energies, the emission from states of the initial configuration of the excited nucleus gives by far the most predominant contribution to the process observed. Then, the calculation is made by substituting in (29) $\lambda^*(E,\varepsilon')$ by $\lambda^*(E,\varepsilon',\Omega)$ which is obtained by avoiding the integration over the final directions of the alpha particle in expression (31), and evaluating classically the refraction of the incident nucleon and outgoing alpha particle wave functions assuming that the target nucleus, considered to be infinitely heavy, acts on the incoming and outgoing particles as a charged sphere for $r > R$ and as a potential well of depth V (equal to ($\varepsilon_f^N + B_N$) and to ($\varepsilon_f^\alpha + B_\alpha$) for, respectively,nucleons and alphas) for $r < R$.

A typical results is shown in Fig. 9.

The forward peaking of the angular distributions of the emitted alpha particles derives essentially from the impossibility for the alpha to lose energy in the collision, because of the Pauli principle, and from the increase in the nucleon-alpha scattering cross section as the relative nucleon - alpha energy is lowered; both these effects favour the interaction of nucleons and alphas with the same direction of motion, i. e., the one of the incident nucleon.

We have seen that a great deal of work has been made to reproduce the continuous composite particle spectra in nucleon induced reactions.

I think that we now have quite well defined ideas concerning the mechanism of production of deuterons, tritons, 3 He, and the Iwamoto-Harada model implemented by direct pick-up processes may be a satisfactory solution of the problem.

In the case of alpha emission, two quite successfull approaches have been developed, the coalescence-pick up model and the knock on model. I think that both processes most probably contribute to the measured spectra and further work is necessary to quantify their relative importance.

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