ISTITUTO NAZIONALE DI FISICA NUCLEARE

Gruppo Collegato di Messina

INFN/BE-99/01
20 Maggio 1999

STUDY OF KINEMATICS OF *A* (a, b) *B* (c)*C* SEQUENTIAL REACTION IN THE FIRST AND SECOND STAGE

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Abstract

We treat the kinematics of a sequential process as A(a,b)B(c)C in its different steps, namely the two-body reaction A(a, b)B and the following B $\rightarrow$ c + C decay. By a proper choice of the reference frames used to describe the reaction, we get useful kinematical formulas whose use makes it easier to understand the physical process considered.

PACS.: 24.90 - Other topics in nuclear reactions: general.

Submitted to Nuclear Instruments and Methods

Published by ***SIS–Pubblicazioni***
Laboratori Nazionali di Frascati

1 Preliminary remarks.

To begin our analysis, let us consider the first step of the sequential reaction $A(a,b)B(c)C$, i.e.

$$a + A \rightarrow b + B$$

where the a particle impinges on target A, producing the b (projectile-like) and the B (target-like) particles in the outgoing channel (first step).

In the following, we shall label each kinematical quantity referred to the Laboratory system by **'L'** index at the top, 'C' will denote any quantity in the Center of mass system, while 'S' index will indicate the same quantity seen by the Recoil frame reference.

As Fig. 1 shows, we choose as reference frame in the laboratory a levogirous, orthonormal Cartesian coordinate system whose z-axis is orthogonal to the a particle direction; moreover, we choose the b particle direction in order to belong to the xy plane, in such a way that the b particle direction is unambiguously characterized by the $(\pi/2, \phi_b^L)$ spherical coordinates.

The Centre of mass reference system (CM) is chosen to have its origin coincident with the centre of mass of the initial system, and the axes parallel to the corresponding ones of the Laboratory system (L).

Next, we shall consider a third reference frame having its origin at B particle (which will decay into c and C particles) to which we shall refer c and C particles. This frame is the so-called 'Recoil Centre of Mass' (RCM) system.

Generally speaking, the excitation of any particle involved may be different from zero, but the kinematics of the process depends only on the Q-value of the reaction: for this reason, we shall omit to indicate excitation of the single particles (usually labelled by an asterisk).

2 Describing the *A(a, b)B(c)C* reaction following its sequential steps.

In describing the reaction, we apply the conservation laws of energy and momentum to obtain useful information.

Let us consider, in the reference system relative to an observer in the laboratory (what we called 'Lsystem'), an incident particle a having mass m_a and energy E_a^L, striking the A particle, whose mass is m_A, at rest in the L-system.

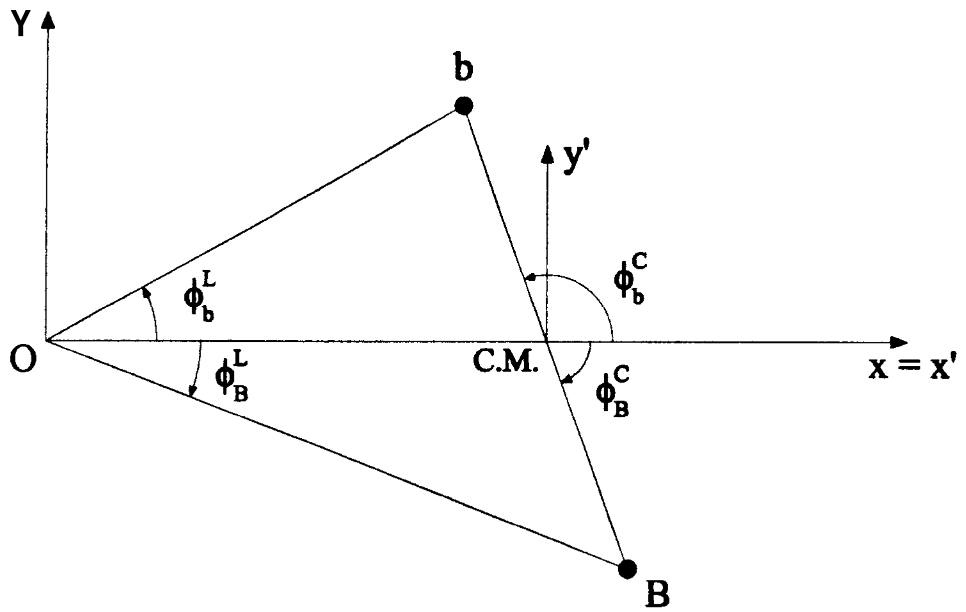

Figure 1: From Laboratory to Centre of Mass system during the first step of the reaction.

2.1 Studying the first step of the reaction, $a + A \rightarrow b + B$: from Laboratory to Centre of Mass system.

We get for the velocity of the Centre of mass in L-system the following expression:

$$v^L_{c.m.} = \frac{m_a}{m_a + m_A} v^L_a, \tag{1}$$

while the corresponding kinetic energy is

$$E^L_{c.m.} = \frac{m_a}{m_a + m_A} E^L_a. \tag{2}$$

from which the relative energy for the a – A system is given by:

$$E^L_{aA} \equiv E^C_{aA} = \frac{m_A}{m_a + m_A} E^L_a. \tag{3}$$

The Q-value of the first step of the reaction is given by

$$Q_2 = (m_a + m_A - m_b - m_B)c^2$$

where each m_i mass keeps into account the excitation energies of the particles, i.e.

$$m_i c^2 = m^0_i c^2 + \epsilon^*_i.$$

In the Centre of Mass reference system (CM) the outgoing particles b and B will have the following energies, respectively:

$$E^C_b = \frac{m_B}{m_b + m_B}\left[Q_2 + E^L_a \frac{m_A}{m_A + m_a}\right], \tag{4a}$$

$$E^C_B = \frac{m_b}{m_b + m_B}\left[Q_2 + E^L_a \frac{m_A}{m_A + m_a}\right], \tag{4b}$$

while in the L-system one gets

$$E^L_b = \frac{m_b}{m_b + m_B} E^L_{c.m.} \left[\cos\phi^L_b \pm \sqrt{\frac{E^c_b}{E^L_{c.m.}} \frac{m_b + m_B}{m_b} - \sin^2\phi^L_b}\right]^2, \tag{5a}$$

$$E^L_B = \frac{m_B}{m_b + m_B} E^L_{c.m.} \left[\cos\phi^L_B \pm \sqrt{\frac{E^c_B}{E^L_{c.m.}} \frac{m_b + m_B}{m_b} - \sin^2\phi^L_B}\right]^2. \tag{5b}$$

and the corresponding angles, in the Laboratory reference frame, for the b and B particles are:

$$\phi^L_b = \arcsin\left[\sqrt{\frac{E^C_b}{E^L_b}} \sin\phi^C_b\right], \tag{6a}$$

$$\phi^L_B = \arcsin\left[\sqrt{\frac{E^C_B}{E^L_B}} \sin\phi^C_B\right] \tag{6b}$$

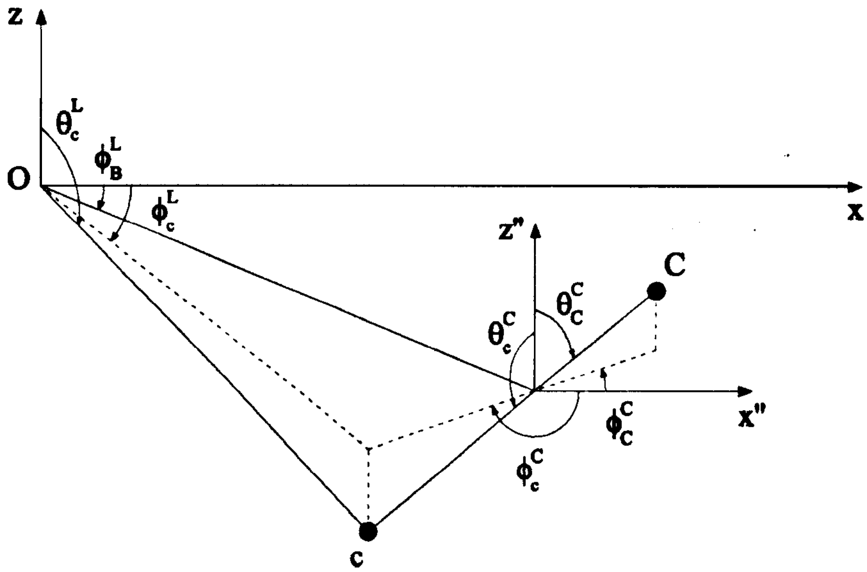

Figure 2: From Laboratory to Recoil Centre of Mass system after B particle decay, i.e. at the second step.

We must impose the following conditions to avoid ambiguities on $\phi^L_{b,B}$ polar angle of b, B particles:

$$\cos\phi_b^C > -\sqrt{\frac{m_b}{m_b+m_B}\frac{E^L_{CM}}{E^L_b}} \Rightarrow \phi_b^L < \frac{\pi}{2}, \qquad (7a)$$

$$\cos\phi_B^C > -\sqrt{\frac{m_B}{m_b+m_B}\frac{E^L_{CM}}{E^L_B}} \Rightarrow \phi_B^L < \frac{\pi}{2} \qquad (7b).$$

In the Centre of Mass system, we have the obvious formula between ϕ_b^C and ϕ_B^C angles, *i.e.*

$$\phi_b^C + \phi_B^C = \pi,$$

while in the 'L' system the two ϕ_b^L and ϕ_B^L angles are related as follows (see Fig. 1):

$$\phi_B^L = -\arcsin\left[\sqrt{\frac{m_b E_b^L}{m_B E_B^L}}\sin\phi_b^L\right] \qquad (8)$$

Moreover, the $Q_2 - value$ of the first step of the reaction will be espressed by means of measurable quantities:

$$Q_2 = E_b^L \frac{m_b + m_B}{m_B} + \frac{m_a m_b - m_A m_B}{m_B(m_a + m_A)} E_a^L - \frac{2}{m_B}\sqrt{E_a^L E_b^L m_a m_b \frac{m_b + m_B}{m_a + m_A}} \cos\phi_b^L$$

The above formulas are aimed at allowing us to transform the cross section of our process from the Centre of Mass system (as theoretically obtained) to the Laboratory reference one, where experimental data are collected, in order to compare them:

$$\frac{d\sigma}{d\omega_b^C} = J_1 \frac{d\sigma}{d\omega_b^L} \tag{9}$$

by means of the J_1 Jacobian of the transformation from L- to CM-system:

$$J_1 = \frac{d\omega_b^L}{d\omega_b^C} = \frac{E_b^C}{E_b^L}\sqrt{1 - \frac{m_a m_b}{(m_a + m_A)(m_b + m_B)} \frac{E_a^L}{E_b^C} \sin^2 \phi_b^L}. \tag{10}$$

2.2 Studying the second step of the reaction, $B \rightarrow$ c + C: from Centre of Mass to Recoil system.

2.2.1 a) general case.

The best way to treat the second step of the sequential reaction under examination is to choose a reference frame whose origin is placed at B particle (i.e., the **'recoil particle'),** while axes are parallel to the corresponding ones of the coordinate systems previously considered (see Fig. 2).

The Recoil Centre of Mass system (RCM) is - with respect to the Laboratory system - an inertial reference frame moving in the direction characterized by (θ_B^L, ϕ_B^L) angles having a velocity

$$v_B^L = \sqrt{\frac{2E_B^L}{m_B}}.$$

After B particle decay, its center represents the center of mass of its products, that is to say the origin of RCM system: we shall label by 'S' any quantity we shall refer to this reference frame.

One can show that during the second step the following relationships must hold:

$$E_c^S = \frac{m_C}{m_C + m_c} m_B (Q_3 - Q_2),$$

Q_3 being the total Q-value of the reaction, given by

$$Q_3 = (m_a + m_A - m_b - m_c - m_C)c^2$$

$$E_c^L = \frac{m_c}{m_c + m_C} E_B^L \left[\sin\theta_c^L \cos(\phi_c^L - \phi_B^L) \pm \sqrt{\frac{E_c^S}{E_B^L}\frac{m_c + m_C}{m_c} - (1 - \sin^2\theta_c^L \cos^2(\phi_c^L - \phi_B^L))} \right]^2 . \quad (11)$$

We may deduce now the reality conditions by imposing the existence of the radicand and studying the sign of this expression making it explicit with respect θ_C^L *or* ϕ_C^L, as we show in the following.

Namely, if

$$\frac{E_c^S}{E_B^L}\frac{m_c + m_C}{m_c} > 1,$$

we can choose only + sign, and ϕ_C^L can have any value. If instead

$$\frac{E_C^S}{E_B^L}\frac{m_c + m_C}{m_c} < 1,$$

one can take both signs and must have

$$\cos(\phi_c^L - \phi_B^L) \geq \frac{1}{\sin\theta_c^L}\sqrt{1 - \frac{E_c^S}{E_B^L}\frac{m_c + m_C}{m_c}},$$

that, *for physical reasons,* implies

$$\phi_B^L - \delta_1 \leq \phi_c^L \leq \phi_B^L + \delta_1$$

where

$$\delta_1 = \arccos\left[\frac{1}{\sin\theta_c^L}\sqrt{1 - \frac{E_c^S}{E_B^L}\frac{m_c + m_C}{m_c}}\right] > 0.$$

On the other hand, imposing the same reality conditions with respect to θ_c^L, one gets

$$\sin^2\theta_c^L \geq \frac{1}{\cos^2(\phi_c^L - \phi_B^L)}\left(1 - \frac{E_c^S}{E_B^L}\frac{m_c + m_C}{m_c}\right),$$

that is to say

$$\delta_2 \leq \theta_c^L \leq \pi - \delta_2$$

where

$$\delta_2 = \arcsin\left[\frac{1}{\cos(\phi_c^L - \phi_B^L)}\sqrt{1 - \frac{E_c^S}{E_B^L}\frac{m_c + m_C}{m_c}}\right], 0 \leq \delta_2 \leq \frac{\pi}{2}$$

while the expression for ϕ_c^S is

$$\phi_c^S = \phi_B^L + \arccos\left[\pm\frac{1}{\sin\theta_c^S}\sqrt{1 - \frac{E_c^L}{E_c^S}\left[1 - \sin^2\theta_c^L\cos^2(\phi_c^L - \phi_B^L)\right]}\right]. \quad (12)$$

The ambiguity on arccos function can be avoided observing that

$$\phi_c^S > \phi_B^L \Rightarrow \phi_c^L > \phi_B^L; \tag{13}$$

the $\pm$ signs correspond to the signs of the expression

$$\left[\sqrt{\frac{E_c^L}{m_c}}\sin\theta_c^L\cos(\phi_c^L-\phi_B^L)-\sqrt{\frac{E_B^L}{m_c+m_C}}\right],$$

respectively. The $Q_3 - value$ expression is given by

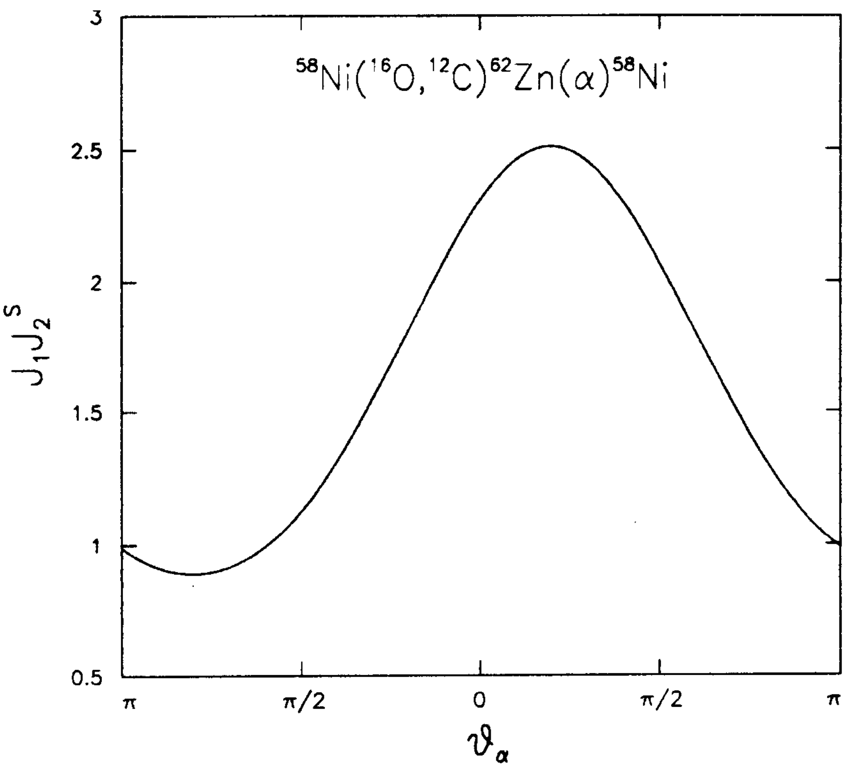

Figure 3: Plot of $J_1 J_2^S$ Jacobian versus ϑ_α-angle for $\vartheta_b^L = -35°$.

$$Q_3 = Q_2 + E_B^L\frac{m_c}{m_C} + E_c^L\frac{m_c+m_C}{m_C} - \frac{2}{m_C}\sqrt{E_c^L E_B^L m_c(m_c+m_C)}\sin\theta_c^L\cos(\phi_c^L-\phi_B^L).$$

Our goal is to connect the theoretical cross-section to the experimental one. This is accomplished by a term given by the product of two Jacobians, namely

$$\frac{d\sigma}{d\omega_b^C d\omega_c^S} = J_1 J_2^S \frac{d\sigma}{d\omega_b^L d\omega_c^L}, \tag{14}$$

where J_1 is the Jacobian from Laboratory to Centre of Mass and J_2^S is the corresponding from the Laboratory to Recoil Center of Mass.

The expression for J_S^2 in the general case is

$$J_2^S = \frac{d\omega_c^L}{d\omega_c^S} = \frac{E_c^S}{E_c^L}\sqrt{1 - \frac{m_c}{(m_c + m_C)}\frac{E_B^L}{E_c^S}(1 - \sin^2\theta_c^L\cos^2(\phi_c^L - \phi_B^L))}, \qquad (15)$$

while the trend of the final transformation Jacobian $J_1 J_2^S$ versus angle is reported in Fig. 3.

2.2.2 *b)* in-plane case.

In the particular case when the first and the second step sequential processes occur in the same plane (i.e. when $\theta_C^S = \theta_C^L = \frac{\pi}{2}$), the above formulas become simpler as follows:

$$E_c^L = \frac{m_c}{m_c + m_C}E_B^L\left[\cos(\phi_c^L - \phi_B^L) \pm \sqrt{\frac{E_c^S}{E_b^L}\frac{m_c + m_C}{m_c} - \sin^2(\phi_c^L - \phi_B^L)}\right]^2 \qquad (16)$$

where for:

$$\frac{E_c^S}{E_B^L}\frac{m_c + m_C}{m_c} \geq 1$$

only + sign gives meaningful solutions and $\forall\phi_c^L$, while for

$$\frac{E_c^S}{E_B^L}\frac{m_c + m_C}{m_c} \leq 1$$

one must have

$$|\,(\phi_c^L - \phi_B^L)\,| \leq \arcsin\sqrt{\frac{E_c^S}{E_B^L}\frac{m_c + m_C}{m_c}} \leq \frac{\pi}{2}$$

and both signs hold. Moreover, as one can deduce from equation (12) one gets

$$\phi_c^S = \phi_B^L + \alpha,$$

with

$$\alpha = \arcsin\left[\sqrt{\frac{E_c^L}{E_c^S}}\sin(\phi_c^L - \phi_B^L)\right].$$

In the above formula, one shall have $\alpha < \frac{\pi}{2}$ for

$$\cos(\phi_C^L - \phi_B^L) > \sqrt{\frac{E_c^L}{m_c}\frac{E_B^L}{m_c + m_C}}$$

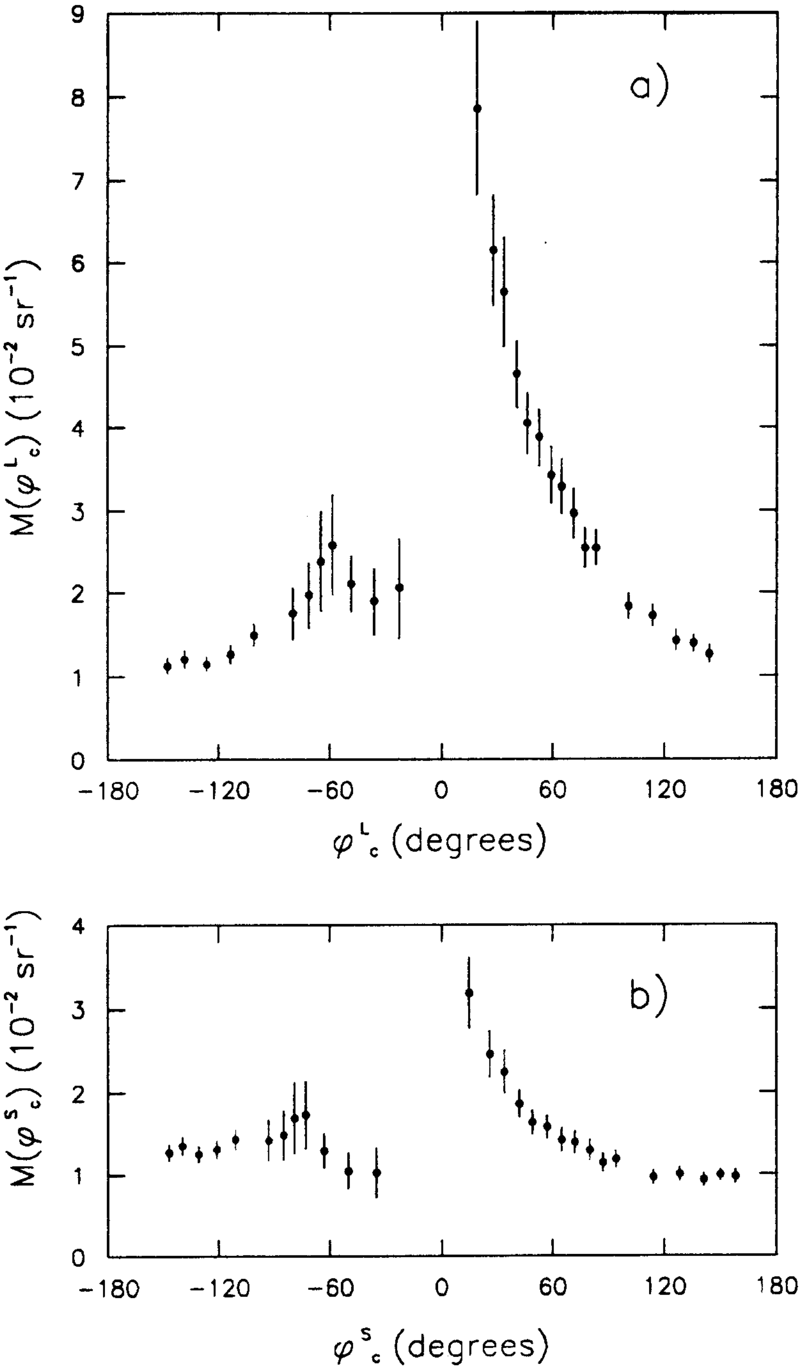

Figure 4: Experimental data for $(96 MeV)^{16}O + {}^{58}Ni$ reaction: *a)* in laboratory system (L) and *b)* in Recoil Centre of Mass system (S) versus c−particle φ_c^L and φ_c^S angles, respectively.

and $\alpha > \frac{\pi}{2}$ for

$$\cos(\phi_C^L - \phi_B^L) < \sqrt{\frac{E_c^L}{m_c}\frac{E_B^L}{m_c + m_C}},$$

respectively.

The Jacobian for this last transformation of reference frame will be given by

$$J_2^S = \frac{d\omega_c^L}{d\omega_c^S} = \frac{E_c^S}{E_c^L}\sqrt{1 - \frac{m_c}{(m_c + m_C)}\frac{E_B^L}{E_c^S}\sin^2(\phi_c^L - \phi_B^L)}. \tag{17}$$

as particular case of Eq. (15).

3 Application.

As an application of the formulas deduced above, let us consider the $^{16}O + {}^{58}Ni$ process at $E_{^{16}O} = 96$ MeV [1,2] and study the kinematics of this reaction. For a better understanding of the physical processes involved, we adopt a proper coordinate system with the same properties described in section 1, and apply the deduced formulas to this case. As one can see, the trend of the Jacobian is symmetric respect to the maximum at $\vartheta_\alpha \simeq 35°$ angle, and assumes the minimum value at $\vartheta_\alpha \simeq -145°$. As shown in Ref. [1], a quantity often considered in the $b - c$ coincidence measurements is the $b - c$ differential multiplicity defined by

$$M(\omega) = \frac{d^2\sigma}{d\omega_b d\omega_c} \Big/ \frac{d\sigma}{d\omega_b},$$

that is the number of c particles per nucleus b and per unit solid angle in the moving frame of the decaying nucleus B.

Fig. 4, showing the multiplicity data for the same reaction a) in the Laboratory System (L) and b) after transformation to the Recoil Centre of Mass system (S), represents a clear evidence of the effect of the $J_1 J_2^S$ transformation Jacobian, which induces a deformation of the experimental data distribution for the above mentioned reaction.

4 Acknowledgements

Authors are grateful to Prof. A. Strazzeri of University of Catania for his encouragement in facing this work and for helpful suggestions and stimulating discussions on the subject.

References

[1] A. Italiano, A. Trifirò, G. Pisent, A. Strazzeri, 'Study of peripheral heavy-ion collisions by residue-particle angular correlations', Nuovo Cimento **A 110** (1997) 781.

[2] J.C. Adloff, D. Disdier, A. Kamili, I. Rauch, G. Rudolf, F. Scheibling, A.Strazzeri, H. Ho, W. Khun, J. P. Wurm, P. L. Gonthier, G. Y. Fan, Sources d'emissions de particules tu dans la reaction ^{16}O+^{58}Ni, Rapport d'Activite' C.R.N., Strasbourg, 1982, p. 27-28.