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FLUCTUATION ANALYSIS.

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## CROSS-CORRELATION EFFECTS IN PREEQUILIBRIUM FLUCTUATION ANALYSIS

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### 1. - INTRODUCTION.

Some recent works<sup>(1)</sup> have evidenced the presence of statistical fluctuations in the excitation functions of preequilibrium reactions. The problems connected to the determination of coherence energies have been pointed out too. This paper has the aim to present in detail some of the most significant results, which may be useful for people wishing to go deep into these arguments.

### 2. - FESHBACH'S THEORY OF PREEQUILIBRIUM REACTIONS.

Feshbach's theory<sup>(2)</sup> is a quantum-mechanical theory which describes the entire spectrum of a nuclear reaction. Two types of processes are distinguished:

- a) multi-step direct emission (MSDE), with cross-sections slowly varying with energy and forward peaked angular distributions;
- b) multi-step compound emission (MSCE), with fluctuating excitation functions and angular distributions symmetric about  $90^\circ$ .

The reaction cross-section is obtained by incoherently adding the two contributions. As we intend to study statistical fluctuations in the excitation functions, we'll point our interest on the general features of MSCE.

In the MSCE the various stages of the cascade process are formed by the bound states of an intermediate system with a relatively long lifetime, so that in each stage statistical equilibrium may be reached. The transition amplitudes relative to the stages are as-

sumed not to interfere, all the stages giving so independent contributions to the cross-section; nevertheless cross correlations may still exist.

In order to distinguish the contribution of MSCE from the one of MSDE we need to separate the transition matrix into two non interfering parts: the first rapidly varying with energy to be associated with MSCE, the latter slowly varying with energies to be associated with MSDE. Generally the separation is achieved by associating to the direct processes the mean value of the transition matrix on an energy interval larger than the expected fluctuation widths, and to the compound processes the part of the transition matrix fluctuating about this mean value. Of course the mean value of the transition matrix should be independent of the averaging interval and the fluctuating part should have a 0 mean value for each amplitude of the averaging interval. In this way:

$$S = \bar{S} + S^f \quad (1)$$

with:

$$\langle \bar{S} \rangle_I = \bar{S} \quad \text{and} \quad \langle S^f \rangle_I = 0 \quad (2)$$

for each energy interval I.

If  $S^f = \sum_{n=1}^r S_n^f$ ,  $S_n^f$  being the amplitude relative to stage n, and moreover  $\langle S_n^f \rangle_I = 0$  for each energy interval I, we obtain:

$$\langle \sigma \rangle = \sigma_{\text{dir}} + \langle \sum_{n=1}^r \sigma_n^f \rangle \quad (3)$$

with no interference between the various stages.

### 3. - CORRELATION FUNCTION.

An important quantity for the evaluation of the coherence energies is the correlation function:

$$F(\epsilon) = \langle \sigma(E) \sigma(E+\epsilon) - \bar{\sigma}(E) \bar{\sigma}(E+\epsilon) \rangle \quad (4)$$

As  $\sigma(E) = SS^*$ , by using (1), we'll have:

$$\sigma(E) = |\bar{S}|^2 + |S^f|^2 + 2 \operatorname{Re}\{\bar{S}S^{f*}\}$$

$$\sigma(E+\epsilon) = |\bar{S}_\epsilon|^2 + |S_\epsilon^f|^2 + 2 \operatorname{Re}\{\bar{S}_\epsilon S_\epsilon^{f*}\}$$

and the correlation function becomes:

$$F(\epsilon) = \langle (|\bar{S}|^2 + |S^f|^2 + 2 \operatorname{Re}\{\bar{S}S^{f*}\})(|\bar{S}_\epsilon|^2 + |S_\epsilon^f|^2 + 2 \operatorname{Re}\{\bar{S}_\epsilon S_\epsilon^{f*}\}) - \langle |\bar{S}|^2 \rangle + \langle |S^f|^2 \rangle \rangle \langle |\bar{S}_\epsilon|^2 \rangle + \langle |S_\epsilon^f|^2 \rangle \rangle$$

By remembering relations (2) and that moreover  $\langle S^{\text{f*}} S_{\epsilon}^{\text{f*}} \rangle_I = \langle S^{\text{f}} S_{\epsilon}^{\text{f}} \rangle_I = 0$ , and making use of the following expression for the scattering amplitude :

$$\langle f_1 f_2 f_3 f_4 \rangle = \langle f_1 f_2 \rangle \langle f_3 f_4 \rangle + \langle f_1 f_3 \rangle \langle f_2 f_4 \rangle + \langle f_1 f_4 \rangle \langle f_2 f_3 \rangle$$

we obtain :

$$\begin{aligned} F(\epsilon) &= \langle S^{\text{f}} S_{\epsilon}^{\text{f*}} \rangle \langle S^{\text{f*}} S_{\epsilon}^{\text{f}} \rangle + \overline{S S_{\epsilon}^{\text{f*}}} \langle S^{\text{f*}} S_{\epsilon}^{\text{f}} \rangle + \overline{S^{\text{f*}} S_{\epsilon}} \langle S^{\text{f}} S_{\epsilon}^{\text{f*}} \rangle = \\ &= \left| \langle S^{\text{f}} S_{\epsilon}^{\text{f*}} \rangle \right|^2 + 2 \sigma_{\text{dir}} \text{Re} \langle S^{\text{f}} S_{\epsilon}^{\text{f*}} \rangle. \end{aligned} \quad (5)$$

By dropping the direct contribution and showing explicitly the contributions of the various stages of MSCE (i. e.  $S^{\text{f}} = \sum_{n=1}^r S_n^{\text{f}}$ ,  $S_{\epsilon}^{\text{f*}} = \sum_{n=1}^r S_{\epsilon n}^{\text{f*}}$ ) we have :

$$F(\epsilon) = \left| \langle (\sum_n S_n^{\text{f}}) (\sum_m S_{\epsilon m}^{\text{f*}}) \rangle \right|^2 = \left| \langle \sum_n S_n^{\text{f}} S_n^{\text{f*}} \rangle \right|^2$$

as  $\langle S_n^{\text{f}} S_{m\epsilon}^{\text{f*}} \rangle_I = 0$ .

Making use of the relation  $\langle f f_{\epsilon}^{\text{f*}} \rangle = \langle f f^{\text{f*}} \rangle \frac{\Gamma}{\Gamma - i\epsilon}$  one finally has :

$$F(\epsilon) = \left| \sum_{n=1}^r \overline{\sigma}_n^{\text{f}} \frac{\Gamma_n}{\Gamma_n - i} \right|^2 \quad (6)$$

$\Gamma_n$  being the coherence energy relative to stage n.

Expression (6) has been obtained for the first time by McVoy<sup>(3)</sup>.

In order to apply the spectral density method<sup>(4)</sup> for the extraction of the coherence energies, it is convenient to write the correlation function in a more suitable form. To this aim we note that (we drop the f) :

$$\begin{aligned} F(\epsilon) &= \left| \sum_1^r \frac{\overline{\sigma}_n \Gamma_n^2}{\Gamma_n^2 + \epsilon^2} + i \sum_1^r \frac{\overline{\sigma}_n \Gamma_n}{\Gamma_n^2 + \epsilon^2} \right|^2 = \left( \sum_1^r \frac{\overline{\sigma}_n \Gamma_n^2}{\Gamma_n^2 + \epsilon^2} \right)^2 + \left( \sum_1^r \frac{\overline{\sigma}_n \Gamma_n \epsilon}{\Gamma_n^2 + \epsilon^2} \right)^2 = \\ &= \sum_{1, m}^r \frac{\overline{\sigma}_n \overline{\sigma}_m \Gamma_n \Gamma_m}{(\Gamma_n^2 + \epsilon^2)(\Gamma_m^2 + \epsilon^2)} (\Gamma_n \Gamma_m + \epsilon^2) = \\ &= \sum_{1, m}^r \left[ \left( \frac{\overline{\sigma}_n \Gamma_n^2}{\Gamma_n^2 + \epsilon^2} \right) \left( \frac{\overline{\sigma}_m \Gamma_m}{\Gamma_m^2 + \epsilon^2} \right) + \left( \frac{\overline{\sigma}_n \Gamma_n}{\Gamma_n^2 + \epsilon^2} \right) \left( \frac{\overline{\sigma}_m \Gamma_m \epsilon}{\Gamma_m^2 + \epsilon^2} \right) \right]. \end{aligned}$$

By applying the Laplace antitransform and making use of its property :

$$Y_1(p) Y_2(p) = \int_0^t y_1(\tau) y_2(t - \tau) d\tau$$

we obtain:

$$\begin{aligned}
 \mathcal{L}^{-1}\{F(\varepsilon)\} &= \sum_{n,m}^r \left| \int_0^t \bar{\sigma}_n \bar{\sigma}_m \Gamma_n \Gamma_m \sin \Gamma_n \tau \sin \Gamma_m (t-\tau) d\tau + \right. \\
 &+ \left. \int_0^t \bar{\sigma}_n \bar{\sigma}_m \Gamma_n \Gamma_m \cos \Gamma_n \tau \cos \Gamma_m (t-\tau) d\tau \right| = \\
 &= \sum_{n,m}^r \bar{\sigma}_n \bar{\sigma}_m \Gamma_n \Gamma_m \int_0^t \cos \left[ (\Gamma_n + \Gamma_m) \tau - \Gamma_n t \right] d\tau = \\
 &= \sum_{n,m} \bar{\sigma}_n \bar{\sigma}_m \frac{\Gamma_n \Gamma_m}{\Gamma_n + \Gamma_m} (\sin \Gamma_n t + \sin \Gamma_m t).
 \end{aligned}$$

Applying now the Laplace transform we have:

$$F(\varepsilon) = \sum_{n,m} \bar{\sigma}_n \bar{\sigma}_m \frac{\Gamma_n \Gamma_m}{\Gamma_n + \Gamma_m} \left( \frac{\Gamma_n}{\Gamma_n^2 + \varepsilon^2} + \frac{\Gamma_m}{\Gamma_m^2 + \varepsilon^2} \right). \quad (7)$$

So:

$$F(\varepsilon) = \sum_{n,m} F_{nm}(\varepsilon) \quad (8)$$

where:

$$F_{nm}(\varepsilon) = \bar{\sigma}_n \bar{\sigma}_m \frac{\Gamma_n \Gamma_m}{\Gamma_n + \Gamma_m} \left( \frac{\Gamma_n}{\Gamma_n^2 + \varepsilon^2} + \frac{\Gamma_m}{\Gamma_m^2 + \varepsilon^2} \right)$$

and

$$F_{nm}(0) = \bar{\sigma}_n \bar{\sigma}_m. \quad (9)$$

Note that, by definition,  $F_{nm}(0) = \bar{\sigma}_n \bar{\sigma}_m - \bar{\sigma}_n \bar{\sigma}_m$ , and that expression (9) is an obvious result, due to the Poisson distribution followed by the  $\sigma_n$ 's.

If we now consider the normalized correlation function:

$$\varrho(\varepsilon) = \frac{F(\varepsilon)}{F(0)} = \sum_{n,m} \frac{F_{nm}(0)}{F(0)} \frac{F_{nm}(\varepsilon)}{F_{nm}(0)} = \sum_{n,m} A_{nm} \frac{F_{nm}(\varepsilon)}{F_{nm}(0)} \quad (10)$$

where:

$$A_{nm} = \frac{F_{nm}(0)}{F(0)} = \frac{\bar{\sigma}_n \bar{\sigma}_m - \bar{\sigma}_n \bar{\sigma}_m}{\bar{\sigma}^2 - \bar{\sigma}^2} = A_{mn}.$$

The  $A_{nm}$  coefficients do not depend on energy and moreover they are normalized as, for  $\varepsilon = 0$ , expression (10) gives:

$$\sum_{n,m} A_{nm} = 1. \quad (11)$$

The  $A_{nm}$  coefficients represent, for  $m = n$ , the variance of the cross-section relative to a single stage and, for  $m \neq n$ , the cross correlation between two different stages in the chain.

The correlation function now becomes:

$$\varrho(\varepsilon) = \sum_{n,m} A_{nm} \frac{\Gamma_n \Gamma_m}{\Gamma_n + \Gamma_m} \left( \frac{\Gamma_n}{\Gamma_n^2 + \varepsilon^2} + \frac{\Gamma_m}{\Gamma_m^2 + \varepsilon^2} \right)$$

which, due to its symmetry with respect to  $n$  and  $m$ , may be written as:

$$\varrho(\varepsilon) = \sum_n \frac{\Gamma_n^2}{\Gamma_n^2 + \varepsilon^2} \left( \sum_m 2A_{nm} \frac{\Gamma_m}{\Gamma_n + \Gamma_m} \right) = \sum_n A_n \varrho_n(\varepsilon) \quad (12)$$

with

$$\varrho_n(\varepsilon) = \frac{\Gamma_n^2}{\Gamma_n^2 + \varepsilon^2}, \quad A_n = \sum_{m=1}^r 2A_{nm} \frac{\Gamma_m}{\Gamma_n + \Gamma_m}$$

If the hypothesis

$$\Gamma_1 \gg \Gamma_2 \gg \dots \gg \Gamma_r \quad (13)$$

holds<sup>(3)</sup>, then:

$$2A_{nm} \frac{\Gamma_n}{\Gamma_n + \Gamma_m} \approx 0 \quad n < m,$$

$$2A_{nm} \frac{\Gamma_n}{\Gamma_n + \Gamma_m} = A_{nn} \quad n = m,$$

$$2A_{nm} \frac{\Gamma_n}{\Gamma_n + \Gamma_m} \approx 2A_{nm} \quad n > m.$$

Hence:

$$A_n = A_{nn} + \sum_{m < n} 2A_{nm}$$

which now are independent from the coherence widths. It is evident, from (11), that the  $A_n$  coefficients are normalized too.

Writing down the spectral density from expression (12), by means of the Fourier transform, we obtain:

$$W(\alpha) = \sum_{n=1}^r A_n \Gamma_n e^{-\Gamma_n \alpha} \quad (14)$$

which allows the evaluation of the coherence energies in the way as described in ref. (4).

#### 4. - CONCLUSIONS.

The basic assumption of Feshbach's theory, eq. (3), states that incoherent contributions come to the cross-section, on the average, from the various stages of the cascade. However it doesn't exclude the presence of cross correlation effects, and these are taken into account in expressions (4) and (14), used in the evaluation of the coherence energies from the excitation functions.

On the other hand, expression (14) is formally identical to the one we could obtain by completely neglecting cross correlation terms, except for a different meaning of the weights  $A_n$ 's. In this way the spectral density method may be used, without any changement, for the contemporary extraction of the  $A_n$ 's and  $\Gamma_n$ 's.

#### REFERENCES.

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